Demonstration: photoelectric effect
Text: Mod. Phys. 3.A, 3.B, 3.C, 3.D
Problems: 3, 4, 6, 17, 19 from Ch. 3

What's important:
•Einstein's energy equation
•photoelectric effect
•momentum of massless particles
•wavelengths of massive particles

Energy-momentum equation

We have stated that some particles, such as the photon (the elementary particle of light) have zero mass. Yet the Newtonian mechanics taught in high school deals with massive particles, through equations such as $\mathbf{F} = \mathbf{ma}$. In this lecture, we examine the kinematics of massless particles, and the wavelike properties of massive particles.

According to Einstein, a particle's energy $\mathbf{E}$ is related to its mass $\mathbf{m}$ and momentum $\mathbf{p}$ through

$$\mathbf{E}^2 = \mathbf{p}^2\mathbf{c}^2 + \mathbf{m}^2\mathbf{c}^2, \qquad (1)$$

where $\mathbf{c}$ is the speed of light, and has a value of 3.00 x 10^8 m/s. It is easy to convince oneself that $\mathbf{mc}^2$ has the units of energy. Note:

i) this equation is quadratic in $\mathbf{E}$, $\mathbf{p}$ and $\mathbf{m}$
ii) it depends on momentum $\mathbf{p}$, not velocity
iii) it has a contribution from a mass energy $\mathbf{mc}^2$.

Particles with small velocities

Now, for at low velocities, we can use the Newtonian approximation $\mathbf{p} = \mathbf{mv}$ to compare the magnitudes of each term in the energy expression:

$$\mathbf{pc} / \mathbf{mc}^2 = \mathbf{mv} / \mathbf{mc} = \mathbf{v/c} << 1!$$

So, at low velocities, the first term is much less than the second term. We can develop an approximation for Eq. (1) in this limit by

$$\mathbf{E}^2 = \mathbf{p}^2\mathbf{c}^2 + \mathbf{m}^2\mathbf{c}^4 = \mathbf{m}^2\mathbf{c}^2\,(1 + \mathbf{p}^2\mathbf{c}^2 / \mathbf{m}^2\mathbf{c}^4)$$

$$\rightarrow \quad \mathbf{E} = \mathbf{mc}^2\,(1 + \underbrace{\mathbf{p}^2\mathbf{c}^2 / \mathbf{m}^2\mathbf{c}^4}_{\uparrow})^{1/2} \qquad \text{[exact, so far]}$$

This is tiny in everyday life.

We can use the following approximation for $(1 + \mathbf{x})^{1/2}$ when $\mathbf{x}$ is small:

$$(1 + \mathbf{x})^n \simeq 1 + n\mathbf{x} \qquad \text{(for small } \mathbf{x}\text{)}$$

eg.
$$(1 + x)^2 \sim 1 + 2x$$
$$(1 + x)^1 = 1 + x$$
$$(1 + x)^0 = 1 + 0$$
$$(1 + x)^{-1} = 1 / (1 + x) \sim 1 - x$$

So $(1 + x)^{1/2} \sim 1 + x/2$.

$$\rightarrow \qquad \mathbf{E} \sim \mathbf{mc}^2 \left(1 + 1/2 \; \mathbf{p}^2\mathbf{c}^2 / \mathbf{m}^2\mathbf{c}^4\right) = \mathbf{mc}^2 + 1/2 \bullet \mathbf{p}^2/\mathbf{m}.$$

Now, <u>if</u> $\mathbf{p} = \mathbf{mv}$, then $\mathbf{E} \simeq \mathbf{mc}^2 + 1/2 \; \mathbf{m}^2\mathbf{v}^2/\mathbf{m}$

↑

$1/2 \; \mathbf{mv}^2$ (looks like kinetic energy $\mathbf{K}$).

Hence, at low velocities, Einstein's expression reduces to a mass energy term $\mathbf{mc}^2$ and a kinetic energy $\mathbf{K}$. In fact, the decomposition $\mathbf{E} = \mathbf{mc}^2 + \mathbf{K}$ of the Einstein expression is valid at all velocities, BUT, the kinetic energy is not $1/2 \; \mathbf{mv}^2$ at all velocities:

At all velocities:
$$\mathbf{E}^2 = \mathbf{p}^2\mathbf{c}^2 + \mathbf{m}^2\mathbf{c}^4$$
$$\mathbf{E} = \mathbf{mc}^2 + \mathbf{K} \quad \text{but } \mathbf{K} = \mathbf{E} - \mathbf{mc}^2 \neq 1/2 \; \mathbf{mv}^2$$

At low velocities only:
$$\mathbf{p} = \mathbf{mv}$$
$$\mathbf{K} = 1/2 \; \mathbf{mv}^2$$

Aside

The general definition of velocity is

$$\mathbf{v} = \mathbf{pc}^2/\mathbf{E}$$

where E is the total energy. At lower velocities where $E \sim mc^2$, we recover

$$\mathbf{v} \rightarrow \mathbf{pc}^2/\mathbf{mc}^2 = \mathbf{p}/\mathbf{m}.$$

At high momenta

$$\mathbf{E} \rightarrow \mathbf{pc} \qquad \text{and } \mathbf{v} \rightarrow \mathbf{c} \quad \text{(but cannot exceed } \mathbf{c}\text{)}$$

Example: Calculate $\mathbf{E}$ and $\mathbf{K}$ for a particle with momentum $\mathbf{p} = 2\mathbf{mc}$.

Start with $\mathbf{E}^2 = \mathbf{p}^2\mathbf{c}^2 + \mathbf{m}^2\mathbf{c}^4$ and substitute:

$$\mathbf{E}^2 = (2\mathbf{mc})^2\mathbf{c}^2 + \mathbf{m}^2\mathbf{c}^4 = 5 \; \mathbf{m}^2\mathbf{c}^4$$

$\rightarrow$ $\mathbf{E} = \sqrt{5}\,\mathbf{mc}^2$

Then

$\mathbf{K} = \mathbf{E} - \mathbf{mc}^2 = (\sqrt{5} - 1)\,\mathbf{mc}^2 = 1.24\ \mathrm{mc}^2$ (note, $\mathbf{K} > \mathbf{mc}^2$)

Finally, just for completeness,

$\mathbf{v} = \mathbf{pc}^2/\mathbf{E} = 2\mathbf{mc} \bullet \mathbf{c}^2 \,/\, \sqrt{5}\,\mathbf{mc}^2 = 2/\sqrt{5}\ \ \mathbf{c}$

Note that: 1. $\mathbf{v} < \mathbf{c}$

2. $\mathbf{v} \neq \mathbf{p}/\mathbf{m} = 2\mathbf{mc}/\mathbf{m} = 2\mathbf{c}$.

Momenta of Massless Particles

Some particles, such as the photon or neutrino, appear to be massless ($\mathbf{m}_\gamma < 5 \times 10^{-63}$ kg; $\mathbf{m}_\nu < 3 \times 10^{-35}$ kg). According to Newtonian mechanics, $\mathbf{p} = \mathbf{mv}$ and a massless particle should have zero momentum. But according to Einstein,

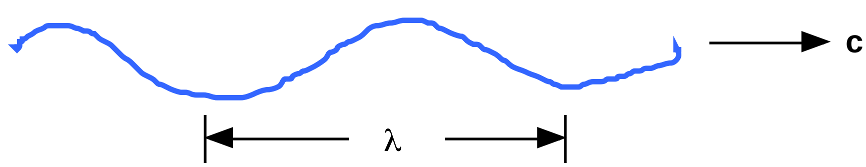
$\mathbf{E}^2 = \mathbf{p}^2\mathbf{c}^2 + \mathbf{m}^2\mathbf{c}^4$

$\rightarrow$ $\mathbf{E} = \mathbf{pc}$ if $\mathbf{m} = 0$.

Therefore, a massless particle should have momentum. This relationship can be tested by doing a scattering experiment in which the momenta of the ejected electrons are measured.

Photoelectric Effect

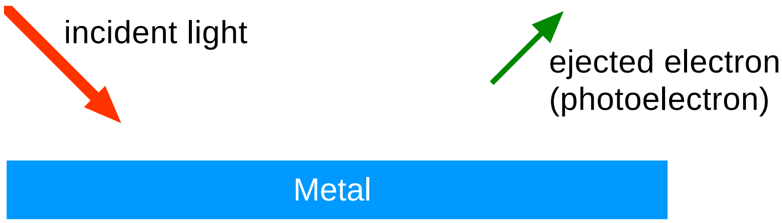
A wave is characterized by a wavelength λ and a speed $\mathbf{c}$.



The number of waves passing by a point per unit time is the frequency $\mathbf{f}$, and

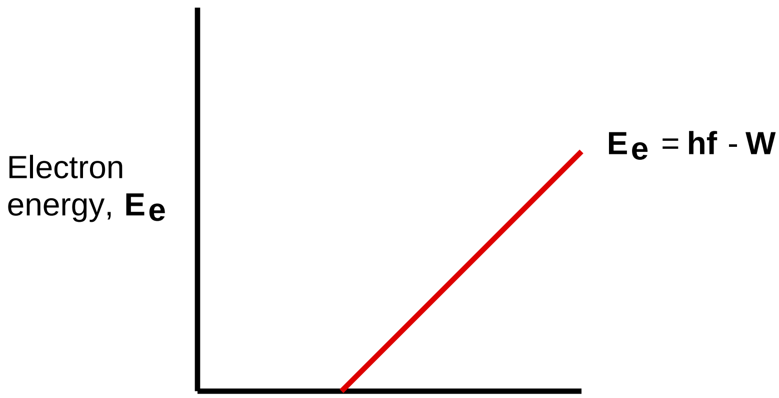
$\mathbf{c} = \lambda\mathbf{f}$.

It was known by the 1800's that light had wavelike properties, including a wavelength λ. In 1887, Hertz discovered that light could knock electrons out of certain metals. In the late 1800's to about 1900, many studies were done on the dependence of the ejected electrons on the various characteristics of light.



The observations of most importance are:
i) as the **intensity** of light increases (but no change in frequency) the **number** of photoelectrons increases, but their energy is unchanged.
ii) as the **frequency** of the incident light increases (blue light instead of red) the **energy** of the photoelectrons increases.
iii) even at very low intensity, there are still a few photoelectrons and they are emitted as soon as light is shone on the metal.
iv) there is a **cutoff frequency** below which no photoelectrons are emitted.

Observations (ii) and (iv) are summarized by the diagram



Now, these observations are <u>all</u> in <u>disagreement</u> with classical electromagnetism. In 1905, Einstein explained the observations by proposing that:

a) light comes in the form of particles called photons. The **intensity** of light is just the **number** of photons per unit area. Hence, observations (i) and (iii) just say the number of photoelectrons is determined by the number of photons.

b) each photon has energy **hf**. (**h** = Planck's constant = 6.63 x 10^{-34} J-s.) Observations (ii) and (iv) just say that a photon brings in energy **hf** to the metal, but loses energy **W** to overcome the binding energy of the electron in the metal.

Thus, light is a massless particle that has <u>both</u> energy and momentum.

Example Find the energy and momentum of light with a frequency of $6 \times 10^{14}\ s^{-1}$ (green light).

From $\mathbf{E} = \mathbf{hf}$,

$$\mathbf{E} = 6.63 \times 10^{-34} \bullet 6 \times 10^{14} = 4 \times 10^{-19} \text{ J.}$$

This is not a lot of energy. The corresponding momentum is

$$\mathbf{E} = \mathbf{pc} \rightarrow \mathbf{p} = \mathbf{E}/\mathbf{c}.$$

$$\rightarrow \mathbf{p} = 4 \times 10^{-19} \ / \ 3 \times 10^{8} = 1.3 \times 10^{-27} \text{ kg m/s.}$$

The momentum is tiny, but measurable.

Wavelength and Momentum

Let's use the expression we have for light to find wavelength as a function of momentum:

$$\mathbf{E} = \mathbf{hf} \quad + \quad \mathbf{f}\lambda = \mathbf{c}$$

$$\downarrow$$

$$\mathbf{E} = \mathbf{pc} \quad + \quad \mathbf{E} = \mathbf{hc} / \lambda$$

$$\downarrow$$

$$\mathbf{pc} = \mathbf{hc} / \lambda$$

$$\downarrow$$

$$\boxed{\lambda = \mathbf{h} / \mathbf{p}}$$

This equation is derived for light. Count Louis de Broglie proposed that <u>it is valid for all</u> particles, whether they have mass or not. The wavelength for massive particles is called the <u>de Broglie</u> wavelength. The de Broglie wavelength has been tested experimentally by scattering monoenergetic neutrons on a crystal lattice and observing a wave-like scattering pattern.