Demonstrations:
•scissors and elastic bands to illustrate quark confinement
Text: Mod. Phys. 4.A, 4.B, 4.C
Problems: 8, 9, 12, 18, 19 from Ch. 4

What's important:
•definition of binding energy
•systematics for particles, nuclei, planets *etc*.
•energetics of reactions: **Q**-values

Binding energy

Under some circumstances, interacting particles can form stable bound states: for example, protons and neutrons can form a nucleus ^{2}H (often referred to as the deuteron D):

$$p + n \rightarrow {}^2\text{H} \text{ (or D).}$$

We know that energy is released when a bound state is formed, and we know that it takes energy to break up a bound state. From the general energy-momentum equation, the release in energy when a bound system is formed must come from a decrease in the mass energy $\mathbf{mc^2}$ of the system. In other words, **a bound system has less mass energy than its unbound components have in isolation**.

The binding energy **B.E.** is alternatively:
•the amount of energy released when the system forms from its unbound components
•the amount of energy that must be added to the system to break it into unbound components at rest.
Hence, **B.E.** is the difference in mass energies between the bound state and the isolated components of the bound state. In symbols:

$$\mathbf{B.E.} = \sum_i \mathbf{m_i c^2} - \mathbf{m_{total} c^2}$$

where the sum over the individual components **i** refers to the masses of the particles in isolation.

For atoms and nuclei, it is more convenient to use electron-volts (eV) as an energy unit, rather than Joules.

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J} \qquad 1 \text{ MeV} = 10^6 \text{ eV}$$

Example

The deuteron (or ^{2}H) consists of a proton and a neutron and has a binding energy of 2.23 MeV (or 3.6 x 10^{-13} J). What is the decrease in mass when this nucleus forms from a proton and neutron?

Solution:

From **B.E.** = Δ**mc**^{2} ,

$$\Delta \mathbf{m} = \mathbf{B.E.} / \mathbf{c}^2 = 3.6 \times 10^{-13} / (3.0 \times 10^8)^2 = 4 \times 10^{-30} \text{ kg.}$$

The fractional change in mass is then

$$\frac{\Delta \mathbf{m}}{\mathbf{m}_p + \mathbf{m}_n} = \frac{4 \times 10^{-30}}{2 \times 1.67 \times 10^{-27}} = 0.1\%$$

The example shows that the fractional change in mass in the formation of a nucleus is small, but measurable. But the deuteron is a loosely bound system, and for most nuclei, the fractional change is closer to 1%.

Binding energy systematics

1. Quarks → hadrons

Hadrons are thought to be composed of quarks and gluons. However, since isolated quarks and gluons have not been produced in experiments, then one cannot obtain a proton's binding energy by comparing the proton's mass with those of its isolated quarks and gluons (suggesting that the interquark force grows with distance, similar, but not identical, to the way the force exerted by an elastic band grows with extension). Using high energy scattering experiments to knock a quark out of a proton only produces more hadrons, like pions; it does not produce free quarks:

$$p + p \rightarrow p + n + \pi^+.$$

We return to the nature of quark binding later.

2. Nucleons → nuclei

Protons and neutrons have almost the same mass, and are collectively referred to as **nucleons**. The strong interaction between nucleons allows them to form stable states, which we call **nuclei**. Except for the lightest hydrogen nucleus, all nuclei contain both protons and neutrons. A nucleus is then characterized by

Z = number of protons
N = number of neutrons
A = **Z** + **N** = number of nucleons.

The binding energy for a given **A** is greatest when **Z** = **N**, if the Coulomb interaction is ignored. The binding energy for nuclei is defined by

$$\mathbf{B.E.} = \mathbf{Z}\mathbf{m}_p\mathbf{c}^2 + \mathbf{N}\mathbf{m}_n\mathbf{c}^2 - \mathbf{m}(\mathbf{Z},\mathbf{N})\mathbf{c}^2$$

where **m**(**Z**,**N**) is the mass of a nucleus with **Z** protons and **N** neutrons. It is found, to about 90% accuracy, that the largest **B.E.** for a given **A** is

$$\mathbf{B.E.} \sim 8\mathbf{A} \text{ in MeV.}$$

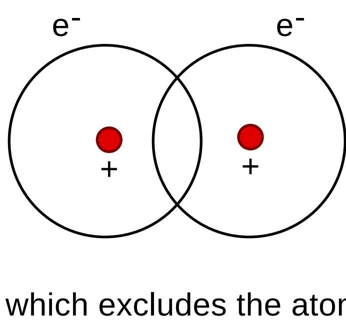
That is, each nucleon in the most stable nuclei has about 8 MeV of binding energy.

3. Nuclei + electrons → atoms

Nuclei can bind together with electrons to form atoms. The binding between nuclei and electrons is electromagnetic in origin, in contrast to the strong interactions between nucleons, and hence the binding energies of atoms are **much** lower than those of nuclei. The binding energy of a typical electron in a nucleus is tens of electron volts (*e.g.*, **B.E.** of the hydrogen atom is 13.6 eV).

4. Atoms → molecules

Atoms themselves can bind together electromagnetically to form molecules. For example, H + H → H_2 (where H_2 is the hydrogen molecule):



The molecular binding energy, which excludes the atomic binding, is given by

$$2\mathbf{m}(H)\mathbf{c}^2 - \mathbf{m}(H_2)\mathbf{c}^2 = \mathbf{B.E.}(H_2) - 2\bullet\mathbf{B.E.}(H) = 4.75 \text{ eV},$$

or about 2 eV per atom. That is, the binding energy per atom in the formation of the H_2 molecule is about 1/5 the binding energy of the H-atom itself (13.6 eV).

4. Gravitational binding

Even though the gravitational force associated with massive objects like the Sun is large, we know that on an elementary particle scale, the gravitational interaction is weak. One can show from integral calculus that the gravitational binding energy of a spherical object of mass **M**, radius **R**, and uniform density is

$$\mathbf{B.E.} = \frac{3}{5}\,\mathbf{G}\,\frac{\mathbf{M}^2}{\mathbf{R}}$$

Let's evaluate the binding energy of the Sun due to gravity, using

$\mathbf{M} = 1.99 \times 10^{30}$ kg $\qquad \mathbf{R} = 6.96 \times 10^{8}$ m.

Then

$$\mathbf{B.E.} = \frac{3}{5}\; 6.67 \times 10^{-11}\; \frac{(1.99 \times 10^{30})^2}{6.96 \times 10^{8}} = 2.3 \times 10^{41}\ \text{J}$$

Even when converted to an energy per particle, this energy is impressively large. The Sun has about 10^{57} protons, so **B.E.** per proton is about 10^3 eV.

5. Summary

Bound state	Energy released per particle (eV)	Interaction
quarks → hadrons	$> 10^8$	strong
hadrons → nuclei	10^7	strong
nuclei + electrons → atoms	10	electromagnetic
atoms → molecules	1	electromagnetic
molecules → liquids, solids	10^{-1}	electromagnetic
planets, stars	variable	gravity

Energy in reactions and decays

Reactions and decays are subject to **conservation laws**: there exist **conserved** quantities that **do not change** during a reaction. Total energy **E** and total momentum **p** are examples. Consider the reaction A + B → C + D

cons. of energy $\mathbf{E}_A + \mathbf{E}_B = \mathbf{E}_C + \mathbf{E}_D$

cons. of momentum $\mathbf{p}_A + \mathbf{p}_B = \mathbf{p}_C + \mathbf{p}_D$

The conservation of momentum is actually three equations in one: it applies separately to each component of the momentum ($\mathbf{p}_x$, $\mathbf{p}_y$, $\mathbf{p}_z$) separately. A conservation law does *not* mean that the conserved characteristic (e.g., the energy) of each individual particle remains the same throughout the interaction process.

Example: work through example of electron-positron annihilation in the text.

Q-values

If the mass energies change in a reaction or decay, so too the kinetic energy must change in the opposite manner in order that **E** be conserved. Consider the decay A → B + C in which particle A is at rest. Then

$\mathbf{E}_{\text{initial}} = \mathbf{m}_A\mathbf{c}^2$ $\mathbf{E}_{\text{final}} = \mathbf{m}_B\mathbf{c}^2 + \mathbf{m}_C\mathbf{c}^2 +$ plus kinetic energies of B and C.

If $\mathbf{m}_B\mathbf{c}^2 + \mathbf{m}_C\mathbf{c}^2$ is greater than $\mathbf{m}_A\mathbf{c}^2$, then the reaction is forbidden by conservation of energy. The initial mass energy less the final mass energy is called the **Q**-value of the reaction, and is the kinetic energy released in the decay:

$$\text{Kinetic energy released} = \mathbf{Q}\text{-value} = \mathbf{m}_A\mathbf{c}^2 - (\mathbf{m}_B\mathbf{c}^2 + \mathbf{m}_C\mathbf{c}^2).$$

The **Q**-value can be generalized to include reactions as well as decays:

$$\mathbf{Q}\text{-value} = \Sigma_{\text{initial}}\mathbf{mc}^2 - \Sigma_{\text{final}}\mathbf{mc}^2,$$

where the sums are over the initial reactants and over the final products of the reaction. The **Q**-value has similarities to binding energy, except that it deals with bound systems.