Demonstrations:
•radioactive sources, Geiger counter *etc*
Text: Mod. Phys. 6.A, 6.B, 6.C, 6.D
Problems: 3, 11, 12, 17, 18 from Ch. 6

What's important:
•nuclear binding energies
•fission and fusion
•decay rates and lifetimes
•radioactive dating techniques

Fission and fusion

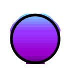
A nucleus is characterized by:

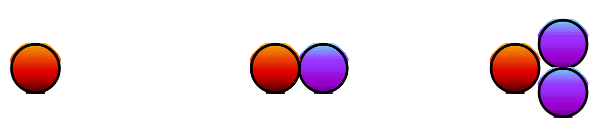
Z = number of protons
N = number of neutrons
A = mass number = **Z** + **N**.

Only two of **A**, **Z** or **N** are needed to describe a nucleus. It is conventional to label a nucleus, in part, by the elemental symbol corresponding to the **Z** of the nucleus. For example, all **Z** = 1 nuclei are called hydrogen nuclei, because they are found in hydrogen atoms or ions. The representation is:

^{A}[*elemental symbol*].

Consider the lightest hydrogen isotopes as an example:

 = proton  = neutron



^{1}H ^{2}H ^{3}H

(only one proton in ANY hydrogen isotope)

From the nuclear lexicon:
•nuclei with the same **Z** and differing **N** are called **isotopes** (*e.g.* ^{6}Li and ^{7}Li)
•nuclei with the same **N** and differing **Z** are called **isotones** (*e.g.* ^{7}Li and ^{8}Be)
•nuclei with the same **A** and differing **Z** are called **isobars** (*e.g.* ^{7}Li and ^{7}Be)
•α-particles are ^{4}He nuclei
•β-particles are electrons
•γ-rays are very short wavelength electromagnetic waves.

As applied to a nuclear reaction of the form A + B → C + D, conservation of baryon number and conservation of charge have the form

$$\mathbf{A}_A + \mathbf{A}_B = \mathbf{A}_C + \mathbf{A}_D \qquad\qquad \mathbf{Z}_A + \mathbf{Z}_B = \mathbf{Z}_C + \mathbf{Z}_D.$$

Example

Find the nucleus X in ^{14}N + ^{4}He → ^{1}H + ^{A}X.

Solution:

Cons. of A: 14 + 4 = 1 + **A** **A** = 17

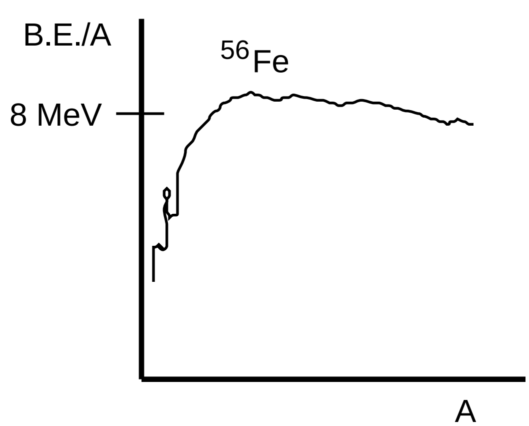
Cons. of Z: 7 + 2 = 1 + **Z** **Z** = 8

So the unknown nucleus is ^{17}O.

The energetics of nuclear reactions are governed by the binding energy:

$$\mathbf{B.E.} = \mathbf{Z}\mathbf{m}_p\mathbf{c}^2 + \mathbf{N}\mathbf{m}_n\mathbf{c}^2 - \mathbf{m}(\mathbf{Z}, \mathbf{A})\mathbf{c}^2, \qquad (1)$$

where **m**(**Z**, **A**) is the nuclear mass. The binding energy per nucleon **B.E./A** for the most deeply bound nuclei is roughly constant, typically within 10% of 8 MeV (see also Table C.7)



What's the origin of this phenomenon?

(i) The strong interaction binds the nucleons together, but extends over just a short range, so nucleons sense the presence only of their immediate neighbouring nucleons. Hence:
•nucleons in the interior are deeply bound
•nucleons on the surface are less well bound.
As **A** increases, there are more nucleons in the interior of the nucleus, and **B.E./A** should increase with **A** for light nuclei, before becoming relatively constant.

(ii) Generally speaking, the number of protons in a nucleus rises with the mass number, so that Coulomb repulsion among the protons ultimately starts to drive down **B.E.** / **A**. Ultimately, very large nuclei become unbound because of the Coulomb repulsion.

Summary: both light and heavy nuclei have lower values of **B.E./A** than do intermediate mass nuclei; **B.E./A** peaks at ^{56}Fe.

Fission: Massive nuclei (**A** > 240) may still be bound (**B.E.** > 0), but their binding energy is so low that they are unstable against breakup into smaller nuclei with a larger **B.E./A**. In general, light nuclei cannot break up into even lighter, because of the low binding energy. Fission occurs in the decay of heavy elements in the Earth.

Fusion: Small-**A** nuclei can join together to produce a heavier nucleus and liberate energy at the same time. In general, heavy nuclei cannot fuse into very heavy nuclei because of the low binding energy. Fusion powers the stars.

<u>Example</u>
How much energy is released if two protons and two neutrons fuse to form helium?

Solution:
For the reaction $2p + 2n \rightarrow {}^4\mathrm{He} + \gamma$, the **Q**-value is

$$\mathbf{Q} = 2\mathbf{m}_p\mathbf{c}^2 + 2\mathbf{m}_n\mathbf{c}^2 - \mathbf{m}_{He}\mathbf{c}^2$$
$$= 2\mathbf{m}_p\mathbf{c}^2 + 2\mathbf{m}_n\mathbf{c}^2 - [\, 2\mathbf{m}_p\mathbf{c}^2 + 2\mathbf{m}_n\mathbf{c}^2 - \mathbf{B.E.}({}^4\mathrm{He})\,]$$
$$= \mathbf{B.E.}({}^4\mathrm{He}).$$

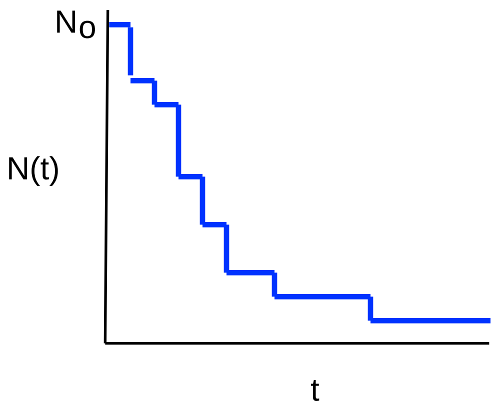
Substituting for the binding energy of ^{4}He, we find

$$\mathbf{Q} = 28.2959 \text{ MeV} = 4.55 \times 10^{-12} \text{ J}.$$

Note: For reactions in which total **Z** and **N** don't change, the **Q**-value is equal to the difference in binding energies (final - initial). If **Z** and **N**, do change, the electrons must be taken into account.

Decay Lifetimes

There is an element of randomness in the decay of a particle or fission of a nucleus. Suppose we had a system of $\mathbf{N_o}$ identically prepared neutrons that can spontaneously decay and liberate kinetic energy. The neutrons decay randomly: one cannot *a priori* determine which neutron will decay at what time. The population of neutrons $\mathbf{N(t)}$ present at any given time might look like the curve:



What form does $\mathbf{N(t)}$ have?

Consider the **rate** of neutron decay $\mathbf{R(t)}$, where the rate is defined as the number of neutrons decaying per unit time. At $\mathbf{t}$ = 0, there are $\mathbf{N_o}$ neutrons present and they decay with a rate $\mathbf{R_o}$. The ratio $\mathbf{R/N}$ is called the decay constant λ, which has units of [*time*]$^{-1}$:

$$\mathbf{R_o} = \lambda\, \mathbf{N_o}.$$

But since the decay rate is always proportional to the number of particles available to decay, then it is true at all times that

$$\mathbf{R(t)} = \lambda\, \mathbf{N(t)} \qquad (2)$$

The shape of the $\mathbf{R(t)}$ *vs.* time curve is the same as the $\mathbf{N(t)}$ *vs*. time curve, except for the overall multiplicative constant, λ.

The decay rate $\mathbf{R(t)}$ as we have defined it is a positive quantity. Defining the change in $\mathbf{N(t)}$ in a time $\Delta\mathbf{t}$ to be

$$\Delta\mathbf{N} \equiv \mathbf{N}_{\text{final}} - \mathbf{N}_{\text{initial}},$$

then

$$\mathbf{R}(\mathbf{t}) = - \Delta\mathbf{N}/\Delta\mathbf{t}$$

where the minus sign is required since **N** is decreasing with time (*i.e.*, Δ**N** is negative). In the limit of small Δ**t**:

$$\mathbf{R}(\mathbf{t}) = - d\mathbf{N}/d\mathbf{t} = \lambda\mathbf{N}(\mathbf{t}). \qquad (3)$$

Eq. (3) is a **differential equation** that relates **N**(**t**) to its derivative d**N**/d**t** at any time **t.** We claim that the solution to Eq. (3) is a function of the form:

$$\mathbf{N}(\mathbf{t}) = \mathbf{N}_o \exp(-\lambda\mathbf{t}). \qquad (4)$$

The proof is by direct substitution:

$$\begin{aligned} d\mathbf{N}/d\mathbf{t} &= d[\mathbf{N}_o \exp(-\lambda\mathbf{t})]/d\mathbf{t} \\ &= \mathbf{N}_o\, d[\exp(-\lambda\mathbf{t})]/d\mathbf{t} \\ &= \mathbf{N}_o\, (-\lambda)\exp(-\lambda\mathbf{t}) \\ &= -\lambda\, [\mathbf{N}_o \exp(-\lambda\mathbf{t})\,] \\ &= -\lambda\, \mathbf{N}(\mathbf{t}) \quad \text{(as required)} \end{aligned}$$

From this form, it follows that

$$\mathbf{R}(\mathbf{t}) = \mathbf{R}_o \exp(-\lambda\mathbf{t}) = \lambda\mathbf{N}_o \exp(-\lambda\mathbf{t}) \qquad (5)$$

Clearly, Eqs. (4) and (5) start at a value of $\mathbf{N}_o$ or $\mathbf{R}_o$ at **t** = 0, and then decrease to zero as **t** becomes infinitely large.

There are two other constants which are commonly quoted as alternatives to λ

•the lifetime τ $\quad \tau = 1/\lambda$.

•the half-life $\mathbf{t}_{1/2}$ $\quad \mathbf{t}_{1/2} = \ln 2 / \lambda$

In terms of the half-life:

$$\mathbf{N}(\mathbf{t}) = \mathbf{N}_o \exp(-\ln 2\ \mathbf{t} / \mathbf{t}_{1/2}) = \mathbf{N}_o (e^{\ln 2})^{-t / t_{1/2}} = \mathbf{N}_o\, 2^{-t / t_{1/2}}$$

where ln2 is the logarithm in base *e* of 2, or 0.693... Thus, at $\mathbf{t} = \mathbf{t}_{1/2}$, we find

$$\mathbf{N}(\mathbf{t}_{1/2}) = \mathbf{N}_o/2.$$

In other words, the half-life is the time that is takes for the sample to decline to half of its original size, or to half of its original decay rate.

Lastly, nuclear decay rates are quoted in units of *Curies* (Ci) or *Becquerels* (Bq):

$$1 \text{ Ci} = 3.7 \times 10^{10} \text{ decays per second}$$

$$1 \text{ Bq} = 1 \text{ decay per second.}$$

Further, some authors prefer to use the word *activity* instead of decay rate.

<u>Example</u>

A sample of 10^{20} particles has an initial activity (decay rate) of 2 Ci. What is the decay constant and lifetime of the particles in the sample?

Solution:

$$\lambda = \mathbf{R}/\mathbf{N} = 2 \times (3.7 \times 10^{10}) / 10^{20} = 7.4 \times 10^{-10} \text{ s}^{-1}.$$

The lifetime is then

$$\tau = 1 / \lambda = 1 / (7.4 \times 10^{-10}) = 1.4 \times 10^{9} \text{ s.}$$

Radioactive Dating Techniques

The decay of radioactive or unstable nuclei can be used as a technique for dating the age of an object. In fact, Rutherford was the first to use the technique and show that a terrestrial piece of pitchblende had to be at least 700,000,000 years old. Of the many variants of the technique, the one that we discuss here is ^{14}C dating.

While the most plentiful isotope of carbon on Earth is ^{12}C, there is another isotope of carbon, ^{14}C:

•produced continuously in the atmosphere through $^{1}n + ^{14}N \rightarrow ^{1}H + ^{14}C$

•unstable with $\mathbf{t}_{1/2} = 5.73 \times 10^3$ yrs [or $\lambda = \ln 2 / (5.73 \times 10^3 \text{ yr}) = 1.21 \times 10^{-4} \text{ yr}^{-1}$]

•observed ratio of ^{14}C:^{12}C in the atmosphere today is 1.3×10^{-12}.

If a tree or plant gets all of its carbon from the atmosphere (say, through carbon dioxide), then the ratio ^{14}C:^{12}C in the tree is the same as the atmospheric ratio as long as the tree is alive. Once the tree dies, then the ^{14}C nuclei will not be replaced as they decay in the tree. Hence, the ratio at any given time is

$$(^{14}C:^{12}C)_t = (^{14}C:^{12}C)_o \exp(-\lambda_{14}\mathbf{t}) \qquad (\lambda_{14} \text{ is the decay constant})$$

Conventionally, one makes the assumption that:

$$(^{14}C:^{12}C)_o = (^{14}C:^{12}C)_{\text{today's atmosphere}}$$

<u>Example</u>

What is the $^{14}C:^{12}C$ ratio in a sample that is 10,000 years old? If the sample is pure carbon and has a mass of 10 g, what mass of ^{14}C is present today?

Solution:

Substitution gives

$$(^{14}C:^{12}C)_t = 1.3 \times 10^{-12} \exp(-1.21 \times 10^{-4} \times 10^4)$$
$$= 3.9 \times 10^{-13}$$

With so little ^{14}C present, we can approximate $^{14}C:^{12}C = {}^{14}C:C_{total}$ so that

$$^{14}C = 3.9 \times 10^{-13} \times 10 = 3.9 \times 10^{-12} \text{ g.}$$

Limitations to carbon dating:

•$^{14}C/^{12}C$ ratio does appear to change with time (count the rings in living trees that are several thousand years old, then date the rings using ^{14}C and find just how much the $^{14}C:^{12}C$ ratio in the atmosphere has changed).

•Because of its 5000-year half-life, ^{14}C is typically used to study objects that are less than 25,000 years old.

•Much longer-lived nuclei must be used to date very old objects, *e.g.* uranium is used for dating rocks and minerals as old as the Earth.