APPENDIX B

SCATTERING EXPERIMENTS

Scattering experiments are used extensively to probe the properties of atoms, nuclei and elementary particles. As described in Chapter 1, these experiments involve the measurement of the probability for a beam particle to scatter from a target particle. There are two aspects to the experimental technique that we gloss over in Chapter 1:

- how is the beam produced?
- how do we know the number of particles in the target region?

We address these two issues in this appendix.

B.I Accelerators

Nature provides somewhat crude ways of producing beams of energetic particles. The decay of radioactive material was used in early experimental work as a source of particles. Although the decay particles may not be emitted in any preferred direction by a radioactive substance, a simple beam can be constructed by placing an amount of strongly absorbing material around the sample, and making a hole in the material to let the particles through in one direction. This technique is used today for the formation of neutron beams from nuclear reactors, where the shielding material may be a meter of concrete or other material, as in Fig. B.1. Low energy neutron beams obtained from reactors by this method are important for studying the atomic arrangements in crystalline materials, including proteins.

Another natural source of particles are high energy cosmic rays. Because cosmic rays can be scattered and degraded in the atmosphere, some cosmic ray experiments have been performed in balloons or in high altitude experimental stations (at Banff, Alberta, for example). While not terribly numerous, cosmic rays have been observed with very high

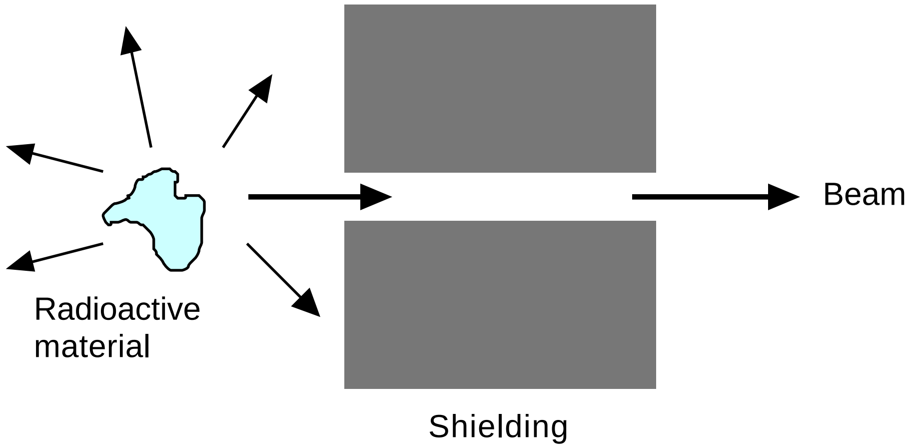

Fig. B.1 A crude beam produced by surrounding radioactive material with shielding.

energies, in excess of 1 TeV or 10^{12} eV. Although protons constitute a significant fraction of cosmic rays at high altitudes, muons are by far the largest component remaining at sea level.

The experimenter does not have a lot of control over the beam particles in these techniques. A substantial advance in the formation of charged particle beams was made through the invention of the cyclotron by Ernest O. Lawrence, working at Berkeley in the 1930s. The basic idea of the cyclotron, and most accelerators that have followed it, is to use electric fields to accelerate charged particles, and magnetic fields to manipulate their direction of motion. The principles are illustrated schematically in Fig. B.2. Charged particles are introduced in the central region of the

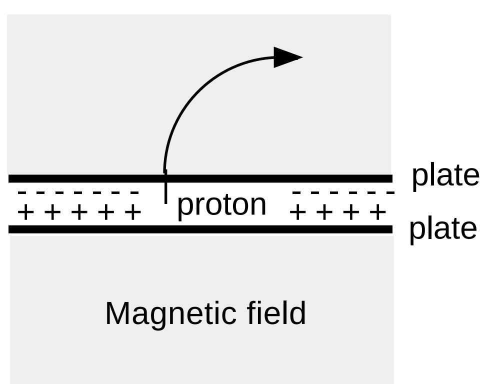

Fig. B.2 Charged particles, protons here, are introduced between two charged plates in a cyclotron.

cyclotron, and accelerated by the charged plates. In Fig. B.2, for example, positively charged protons are attracted to the negatively charged plate and repelled by the positively charged plate.

After the particles pass though a gap in the plates, they move through a magnetic field which causes them to execute an arc perpendicular to the direction of the field. While the particles circulate through the magnetic field, the charges on the plates are reversed so that the particles are accelerated again when they enter the region between the plates. As illustrated in Fig. B.3, if the magnetic field is held constant, then the particles execute semicircular orbits of ever-increasing radius while their speed steadily increases. The largest cyclotron in physical size is the TRIUMF cyclotron in Vancouver, which accelerates protons up to energies of 500 MeV. Most recently, cyclotrons have been made with superconducting magnets, thus reducing their size considerably. Examples of superconducting cyclotrons that can accelerate heavy nuclei as well as protons can be found at the National Superconducting Cyclotron Laboratory at Michigan State University, and at Chalk River Laboratories in Canada.

Since the orbital radius increases with beam energy if the magnetic field is fixed, then the physical size of the machine will impose an obvious limit on the ultimate beam energy achievable with a cyclotron. The

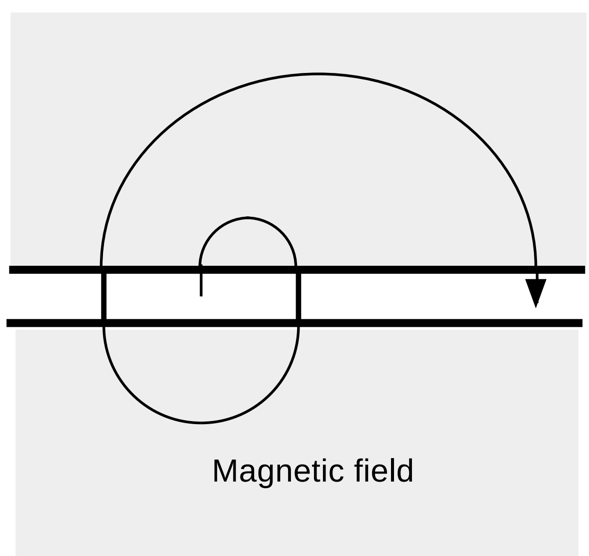

Fig. B.3 Exaggerated view of a particle’s motion as it executes a trajectory through a cyclotron.

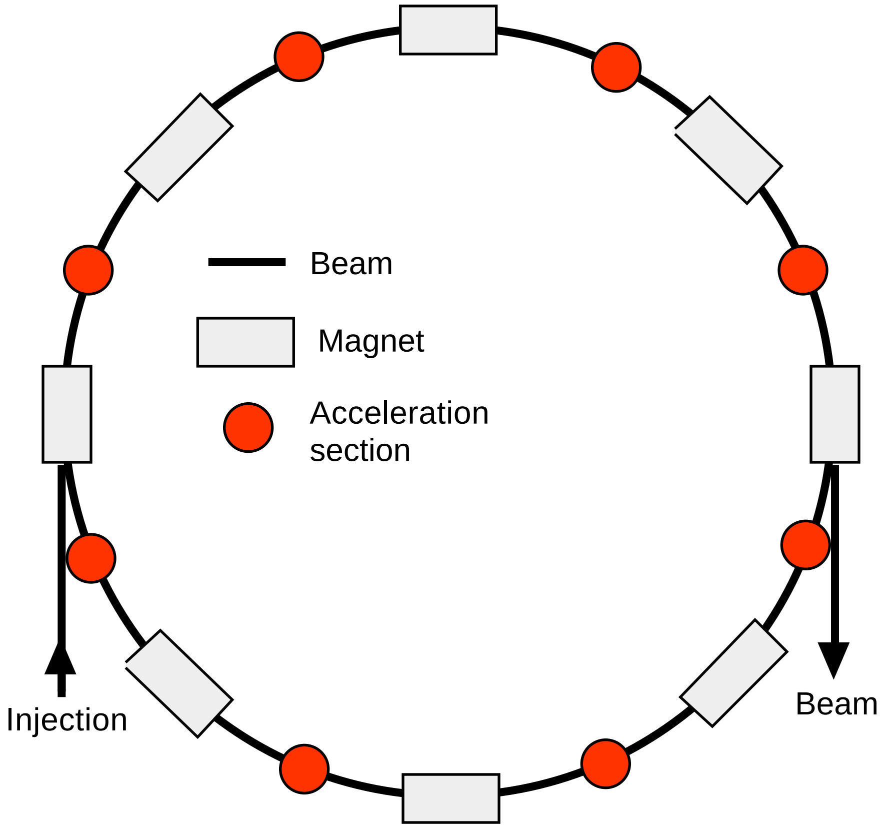

Fig. B.4 Theorist's view of a large accelerator showing bending magnets designed to steer the beam, interspersed between accelerating components.

solution to the size problem is found by making the magnetic fields variable, and breaking up the accelerating plates into segregated components. An initial low energy beam, sometimes produced by an old cyclotron, is injected into a ring of magnets and accelerating sections, as in Fig. B.4. When the beam has reached its full energy, it can be extracted from the accelerator ring. At the largest accelerators in the world, the ring may be many kilometers in circumference, with a construction cost to match.

The design advantage to the layout in Fig. B.4 is that the beam passes through the same magnets and accelerator sections repeatedly as its energy is raised, thus saving construction costs. However, not all accelerators are built this way, since some particles, such as electrons, lose energy as they are bent into a circular path. Linear accelerators, up to several kilometers long, have been built without bending magnets to produce beams of such particles. In many cases, the acceleration ring feeds a beam into a *storage ring*, in which the particles circulate with high energy through several experimental halls constructed along the ring. Although expensive to build, storage rings have the advantage of reusing the beam once it has passed through a target region, and this reduces some of the operating costs of the accelerator.

The highest energies now produced for single beams are impressive: 10^6 MeV (10^{12} eV) for protons and antiprotons, 55 x 10^3 MeV for electrons and anti-electrons. Beams of unstable particles can be produced by slamming the main beam into a target, and catching the debris of the collisions as a *secondary* beam. Such secondary beams typically have lower energy and much lower intensity than the primary beam accelerated in the ring.

B.II Target Densities

The expressions relating the scattering probability to the scattering cross section involve the quantity n_T, which is the number of particles per unit area, or an *area* number density. It is equal to the product of the target thickness t times the *volume number density* n_V,

$$n_T = tn_V. \tag{B.1}$$

The target thickness is measured in the beam direction, as illustrated in Fig. B.5. Eq. (B.1) says that the thicker the target, the more particles there are per unit area facing the beam. The units of the right hand side of Eq. (B.1) are [*length*] for t and [*length*]$^{-3}$ for n_V. This gives units of [*length*]$^{-2}$, or a number per unit area, for n_T.

Now, the volume number density is the number of particles (atoms, nuclei, apples... whatever is relevant to the scattering problem) per unit volume. This is not the same as the density ρ, which is the *mass per unit volume*. To go from ρ to n_V, we have to find how many atoms there are per unit mass. Chemistry tells us how to do this using a quantity called a *mole*.

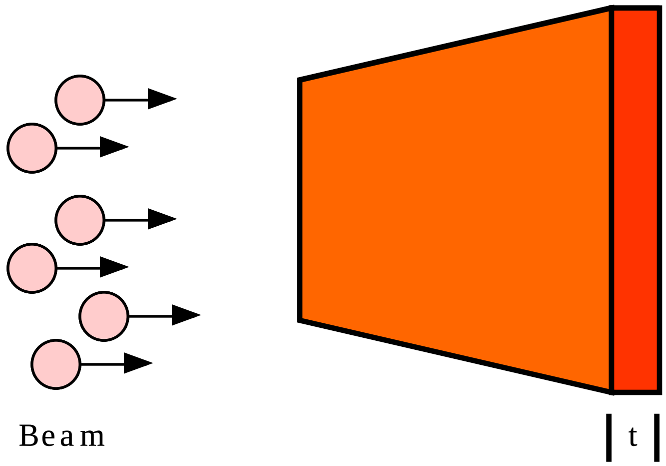

Fig. B.5. The target thickness *t* is measured in the beam direction.

Every atom has what is called an *atomic mass*, which is approximately equal to the number of protons and neutrons in the atom's nucleus. For example, the atomic mass of a hydrogen atom with one proton is 1.008, and the atomic mass of an oxygen atom with a total of 16 protons and neutrons is very close to 16. The atomic mass is standardized by a carbon atom with 6 protons and 6 neutrons (symbolically written as ^{12}C), which is defined to have an atomic mass of exactly 12. Where the *mole* comes in is the relationship between the atomic mass, which is just a number, and the mass in grams (g) of a certain amount of material: the mass of a *mole* of an element in grams is numerically equal to the atomic mass. In other words, a mole of hydrogen atoms has a mass of 1.008 g, a mole of ^{12}C atoms has a mass of exactly 12 g. Since the mass of a ^{12}C atom can be determined experimentally, then the number of particles in a mole can be calculated. A mole of particles contains 6.022×10^{23} particles, a number which is known as *Avogadro's number* N_o. Although we initially apply the idea of moles to atoms, moles can be used for molecules as well by replacing atomic mass with molecular mass.

So, to find the number of particles in a given mass *m* of material, we first find the number of moles using

$$[\textit{number of moles}] = m \; / \; [\textit{atomic mass}] \qquad \text{(B.2)}$$

and then multiply the number of moles by Avogadro's number

$$[\textit{number of particles}] = [\textit{number of moles}] \times N_o. \qquad \text{(B.3)}$$

Hence

$$[\textit{number of particles}] = mN_o \,/\, [\textit{atomic mass}]. \qquad \text{(B.4)}$$

Dividing both sides of Eq. (B.4) by the volume of the sample, then the relationship between the number density and the mass density is:

$$n_V = \rho N_o \,/\, [\textit{atomic mass}]. \qquad \text{(B.5)}$$

Finally, the number of particles per unit area can be found by substituting Eq. (B.5) into Eq. (B.1)

$$n_T = [\textit{density of target material}] \times [\textit{target thickness}] \times [\textit{Avogadro's number}] \div [\textit{atomic mass of target material}]$$

or

$$n_T = \rho t N_o \,/\, [\textit{atomic mass}]. \qquad \text{(B.6)}$$

Example B.1: *Find the number of particles per unit area* n_T *for a gold target 0.01 mm thick. Gold has an atomic mass of 197.0 and a density of 19.3 g/cm*^{3}.

The atomic mass of 197.0 means that a mole of gold (6.022 x 10^{23} particles) has a mass of 197.0 g. Hence,

$$n_T = [19.3]\,[0.01 \times 10^{-1}]\,[6.022 \times 10^{23}] \,/\, 197.0$$
$$= 5.9 \times 10^{19} \text{ atoms/cm}^2$$

Just for interest, this corresponds to 5900 atoms/Å^2 and 0.00000059 atoms/fm^2. The first of these numbers tells us that the target has a thickness in the range of tens of thousands of atoms. The second number tells us that the likelihood of a helium nucleus (with a radius of a fm or two) finding a gold nucleus in its way as it traverses the target is very small.