CHAPTER 1

ATOMS, NUCLEI AND PARTICLES

You've probably heard it said that the microscopic atomic and nuclear worlds have length scales and mass scales that are much smaller than our familiar macroscopic world. How do we know this? Although scientists had no means of directly measuring sizes in their early exploration of the atomic world, they were able to deduce order of magnitude estimates based on a number of observations. For example, in the nineteenth century Lord Rayleigh (1842-1919) postulated that oil spread on water could form a layer only one molecule thick. He calculated approximately the thickness of this layer by spreading a known volume of oil on a calm lake and estimating, through the observation of reflected light, the area of the lake covered by oil. Assuming that the total volume of oil did not change during spreading, he found the thickness of the layer had to be in the range of 10^{-9} m or 1 nanometer (nm). We know today his estimate of molecular sizes is approximately correct.

Most studies of atomic and subatomic dimensions in the twentieth century have used the scattering of particles from targets to probe very small distances. Sir Ernest Rutherford (1871-1937) pioneered the use of this technique and used it to deduce the nuclear model of the atom in 1911 while working at Manchester University. Rutherford, a New Zealander by birth, worked at McGill University from 1898 to 1907 where he performed work on the transmutation of the elements.

In the last two decades, several new microscopy techniques have been developed that allow materials to be observed on atomic distance scales. The new techniques are known by their three-letter acronyms (TLAs) of scanning tunneling microscopy (STM) and atomic force microscopy (AFM), to name but two examples. An STM image of a liquid crystal layer is shown in Fig. 1.1, in which the ordered arrangement of the liquid crystal molecules is apparent. At the molecular level, most liquid



Fig. 1.1. Scanning tunneling microscope image of a layer of liquid crystal molecules called 8CB. An outline of the atomic arrangement in the molecule, two rings with a long tail, is indicated in the upper left section. The molecules appear as double rows, with a kink every fourth molecule. the br in the lower left corner indicates the scale of the image: 5 nm (courtesy of Jeff Hutter and John Bechhoefer, Simon Fraser University).

crystals have highly non-spherical geometries that encourage the axes of neighbouring molecules to align.

1.A How We Measure What We Can't See

Suppose we want to find the size of an apple. One way we can proceed is to wrap a flexible tape measure around the apple and measure its circumference, or use calipers to measure its diameter. A less direct measurement is to shine light on the apple and measure its shadow with a ruler. The light source can't be too close to the apple, or the shadow will not be a true representation of the apple's size. The ruler provides us with a standard measure. In each of these measurement techniques, the geometry of the apple is compared directly to a standard length scale inscribed on a ruler.

At the atomic level, we are forced to use indirect means to determine sizes since we can't see with our eye exactly where the atoms or their shadows are. Consider how we could modify the measurement of the apple's shadow if the apple were placed in a box at a location that was unknown to us. We'll let the box have an open top and bottom, and not worry about how the apple is suspended inside. An indirect measurement of the apple's size might involve dropping sticky marbles through the top of the box, and seeing how many of the marbles pass out the bottom of the box. The marbles that are stuck to the apple and do *not* pass through the box represent the apple's shadow.

Since we don't know precisely where the apple is, then we'll have to drop marbles at many random positions through the top of the box. For example, if we dropped all of our marbles through the centre of the box while the apple was off in one corner, then not only would we conclude from the large number of marbles coming out the bottom that the apple was very small, we might conclude that it wasn't there at all. This marbledropping experiment is an indirect or probabilistic measurement. We use the probability of a marble getting past the apple to deduce something about the apple's size.

Example 1.1: We place an ideal "spherical" apple of radius 3 cm in a box of length 1 m to the side. We drop 10,000 very small and very sticky marbles into the box at random positions. Estimate how many marbles stick to the apple.

We assume that if any one of these very small marbles hits the apple, it sticks. That is, if any marble has a trajectory which passes within 3 cm of the apple's centre, it sticks. Obviously, if the marbles were large, then we would need to take into account more than just the 3 cm radius of the apple. This 3 cm radius defines an area of $3^2 = 28.3 \text{ cm}^2$ which the apple presents to the marbles raining down on it. Now, the area of the top face of the box is 1 m^2 , or $100^2 (=10^4) \text{ cm}^2$. Since 10^4 marbles have been dropped into the box, then there is one marble for every cm² of area on average. Hence, about 28 marbles should stick to the apple, given that there is an average of 1 marble per cm² of area and that the apple presents an area of 28 cm² to the falling marbles.

In Example 1.1, the area of the apple is used to deduce the number of marbles sticking to it. The situation is different in an experiment, where the area of the target is what we are trying to measure. To use the apple/marble example, in an experiment we would know the number of marbles per unit area entering the box and we would count the number of marbles passing through the box. From these two numbers (the number of marbles in and the number of marbles out), the apparent cross sectional area of the apple can be determined. This cross sectional area, which is the area of the apple measured perpendicular to the direction of the incoming marbles, is simply called the *cross section* and denoted by the Greek letter sigma: σ . In Section 1.B, we repeat the analysis of Example 1.1 and derive some algebraic expressions for the cross section.

The idea that we have described is the essence of a scattering experiment. There are some obvious problems of which we should be aware in interpreting scattering experiments, and there are some important improvements that can be made to the technique.

Firstly, the experiment is a *statistical* measurement. The fewer marbles that are dropped into the box, the less accurately the size of the apple is determined. In our example, if 1,000 marbles were dropped into the box instead of 10,000, then we would expect only 3 marbles to stick to the apple on average. If the experiment were repeated again and again with 1,000 incoming marbles (removing the attached marbles between experiments!), then sometimes we would find 1 or 2 marbles, sometimes 4 or 5, would be stuck to the apple. This variation arises because the marbles are dropped into the box randomly, not on a regular grid of points. Hence, good accuracy is obtained only with a large number of marbles.

Secondly, the measurement process as we've described it is cumbersome. Counting all of the marbles is a nuisance, particularly since we are trying to determine how many marbles did *not* pass through the target region. It takes a lot of labour to count up 9972 marbles when all we want to know is that 28 marbles are missing. In the experimental implementation of a scattering measurement, we count the "stuck" marbles directly rather than determine them as a difference between two large numbers.

There are several ways in which scattering experiments are performed in subatomic physics to determine sizes. Usually, small nuclei

or particles are accelerated to form a particle *beam* and used to bombard larger nuclei. Some beam particles stick to the target nuclei, thereby changing the target nucleus to one which is physically different. Experimentally, one can measure the radioactive decay of the transformed nuclei or one can manipulate the target chemically (e.g., dissolve it in acid) and separate out the transformed nuclei. In either case, it is like measuring the number of apples that have marbles stuck to them and deducing the likelihood that an incoming marble finds an apple to stick to.

A different approach is to use a sticky box rather than sticky marbles. Then, marbles bounce off the apple but stick to the inside of the box. In a subatomic physics experiment, the sticky box is usually an electronic device called a *detector*. Detectors can directly count the number of times they are struck by a scattered particle. Experiments using helium nuclei as the marbles and heavy metals such as gold and platinum as the apples were performed by Hans Geiger (1882-1945; later to invent the Geiger counter with his student W. Muller in 1928) and Ernest Marsden (1889-1970) at Manchester in 1909. In 1911, Rutherford developed a theory for analysing data from the Geiger/Marsden scattering experiments and was able to deduce the size of a nucleus.

1.B Cross Sections

Section 1.A gives a general description of the scattering process and how it can be used to estimate sizes. We now cast these ideas into a more mathematical form. The statistical nature of a scattering experiment requires us to introduce the idea of probability, which we denote by P. The probability is the measure of the likelihood for a given result to be found. The maximum value of *P* is one, and this corresponds to a given result always occuring. For example, if it always rains on the weekend then we would say that the probability of it raining on the weekend is 1 (P = 1). If a particular result is not always obtained in a set of measurements, then the probability of the result is less than one. If it only rains on half the weekends during the year, then the probability of rain on the weekend is one-half (P = 0.5). If it never rains on the weekend (not likely in Vancouver) then P = 0. So the range of P is 0 Ρ 1.

In a probabilistic experiment to probe subatomic systems, a beam of particles is sent towards a target. The beam is set up so that it is roughly uniform over its own width transverse to its direction of motion. The target is generally larger in area than the beam is, so not all of the target is hit by the beam. The target is made sufficiently thin that a beam particle is not likely to scatter a second time once it scatters off a particle in the target. We measure the probability of scattering, P, as

$$P = [number of beam particles scattered] \div$$

[total number of beam particles shot at the target]. (1.1)

If all of the beam particles are scattered by the target, then the numerator and denominator on the right hand side of Eq. (1.1) are the same and P = 1. If none of the beam particles are scattered, then P = 0.

To use the marble/apple example, the probability P that a given marble is scattered is equal to the number of scattered marbles divided by the total number of marbles dropped into the box. The words "shot at the target" in Eq. (1.1), as applied to the apple/box system, means all of the marbles dropped into the box, not just those that are heading towards the apple.



Fig. 1.2. The scattering probability is equal to the number of particles scattered by the target divided by the number of particles shot into the target region.

Example 1.2: Suppose that 28 marbles actually stuck to the apple during a trial experiment of Example 1.1. What is the measured scattering probability?

From the definitions in Eq. (1.1) we have

[number of beam particles scattered] = 28

[number of beam particles shot at the target] = 10^4

 $P = 28 / 10^4 = 0.0028 = 2.8 \times 10^{-3}$.

Therefore, the scattering probability is 2.8×10^{-3} .

Example 1.3: In a scattering experiment, a beam of particles strikes a target at the rate of 10^{12} per second. What is the scattering probability if 10^8 particles are scattered by the target per second?

Using the definitions in Eq. (1.1), in one second we have

[number of beam particles scattered] = 10^8

[number of beam particles shot at the target] = 10^{12}

so that

 $P = 10^8 / 10^{12} = 10^{-4}$.

Therefore, the scattering probability is 10⁻⁴ or 0.01%.

Now, we argue in Example 1.1 that the scattering probability should be equal to the effective area which the target presents to the beam, divided by the real area of the target actually exposed to the beam. That is,

P = [total effective area of target particles exposed to beam] ÷ [total real area of target exposed to beam]. (1.2)

In the marble/apple example, the effective area of the target particles is the cross sectional area of the apple (28 cm^2), and the total target area exposed to the beam is the area of the box (10^4 cm^2). If there were two apples in the box then the effective area of the target particles would be 56 cm², assuming that one apple did not hide the other.

Example 1.4: Predict the scattering probability for the situation in Example 1.1 by using Eq. (1.2).

Since there is only one apple in the box, then the total effective target area is just that of the single apple. Using the definitions in Eq. (1.2):

[total effective target area] = $3^2 = 28.3 \text{ cm}^2$

[total area of target exposed to beam] = 10^4 cm².

Hence, we predict that

 $P = 28.3 / 10^4 = 2.8 \times 10^{-3}$.

Therefore, the scattering probability is 2.8×10^{-3} .

Let's cast Eq. (1.2) into symbols. We define $A_{\rm T}$ as the area of the target exposed to the beam and $n_{\rm T}$ as the number of target particles per unit area (of the target exposed to the beam). The number of target particles exposed to the beam is then $n_{\rm T}A_{\rm T}$. In Section 1.A, the effective area of a single target particle is defined as σ . Hence, the summed effective area of all of the target particles equals the number of target particles times σ , or $n_{\rm T}A_{\rm T}\sigma$, as illustrated in Fig. 1.3. Eq. (1.2) can be written symbolically as:

$$P = n_{\rm T} A_{\rm T} \sigma \,/\, A_{\rm T} = n_{\rm T} \sigma. \tag{1.3}$$

The $A_{\rm T}$ terms cancel out in Eq. (1.3), showing that the scattering probability has no explicit dependence on the target area exposed to the beam.



Fig. 1.3. The total area of the target exposed to the beam is A_{T} , while the effective area of a single target particle is σ .

Example 1.5: Predict the scattering probability for the situation in Example 1.1 by using Eq. (1.3).

The number of target particles per unit area is one apple in 1 $\ensuremath{\text{m}}^2,$ or

 $n_{\rm T} = 1 \text{ m}^{-2} \text{ (or } 10^{-4} \text{ cm}^{-2}\text{)}.$

The effective area of a single apple is

 $\sigma = x \ 0.03^2 = 0.0028 \ m^2.$

Hence

 $P = n_{\rm T}\sigma = 1 \ge 0.0028 = 2.8 \ge 10^{-3}$.

Therefore, we find that the scattering probability is 2.8×10^{-3} , just as we did in Example 1.4.

The quantity $n_{\rm T}$ is sometimes called an area number density and it depends on the material of which the target is made and also on the target thickness along the path that the beam takes through the target. In different words, $n_{\rm T}$ is simply the number of particles which lie in the path

of a beam with unit area. If the target thickness is doubled, then $n_{\rm T}$ must double as well since there are twice as many particles that the beam potentially can strike.

1.C Sizes and Masses

Sections 1.A and 1.B introduce the concepts and mathematical formalism behind the experimental methods used to determine the sizes of very tiny objects. It is shown that scattering experiments, which measure the probability of interaction between a beam and a target, can be described in terms of a quantity called the cross section. While it is demonstrated in Chapter 2 that the geometrical interpretation of the cross section as a direct measure of size has its limitations, nevertheless scattering experiments are the principal means of determining the effective sizes of objects in the atomic and subatomic worlds. Not unexpectedly, it is found that cross sections for the scattering of a particular beam from a particular target depend on the bombarding energy of the beam. Let's see why....

Scattering experiments performed using small low-energy particles (for example, electrons with a speed of 10^6 m/s) can measure atomic dimensions. While 10^6 m/s may be an impressive speed for a car, or even a satellite, such electrons can be deflected by atoms. Experiments with low energy electrons show that atoms typically have a diameter in the range of a few tenths of nm or, equivalently, a few Ångstroms. The apparent radius of an atom depends on its charge state, by which we mean whether the atom is electrically charged or neutral. The systematics of atomic sizes leads us into chemistry which, although an interesting subject in itself, is not the material of these lectures.

If the energy of a beam particle is high enough, it will pass into and perhaps through the atom rather than be scattered by it. Cross sections measured with high energy particles are then much smaller than those found with very low energy particles, since only the target nuclei, rather than the target atoms, scatter the high energy particles. Experiments show that nuclei have radii of the order 10^{-15} m, which is about 10^{-5} times the magnitude of an atomic radius. Because the nucleus contains most of the atom's mass, then the atomic mass must be concentrated in a very small volume of about $(10^{-5})^3 = 10^{-15}$ times the magnitude of the atomic volume

(the cube arises because the volume scales like the radius cubed). Before moving on to describe what happens at high bombarding energies, we pause to examine the nucleus a little further.

Two different particles, *protons* and *neutrons*, are known to inhabit the nucleus. Protons and neutrons are about equal in mass and share many other characteristics in common, so they are referred to collectively as *nucleons*. Each proton carries a particular electrical charge while neutrons are electrically neutral. The rest of the atomic volume outside of the nucleus is occupied by electrons. Each electron carries an electrical charge which is opposite in sign to that of a proton. However, the similarity between electrons and protons ends at the absolute magnitude of their charge: protons are about 2000 times heavier than electrons and have very different scattering properties.

The number of protons in a nucleus is defined as Z and the number of neutrons is N. The mass number A of a nucleus is the total number of protons and neutrons:

$$A = Z + N. \tag{1.4}$$

Experimentally, the nuclear radius increases smoothly with mass number. There is a subtle question here as to how to define the radius of a group of objects (e.g. *nucleons*) in motion about each other. One definition uses the radius R of an imaginary spherical surface within which the moving objects spend 90% of their time. Approximately, it is found that

$$R = 1.2 \ A^{1/3} \ \text{fm}, \tag{1.5}$$

where fm is a metric unit called the femtometer or fermi, and is equal to 10^{-15} m. That is, a nucleus with 125 nucleons (A = 125) has a radius of approximately $1.2 \ge 125^{1/3} = 6$ fm = $6 \ge 10^{-15}$ m.

This $A^{1/3}$ scaling behaviour is what we expect for a system whose constituents each occupy a fixed volume and are closely packed together. For example, suppose nuclei were made of hard cubic building blocks each with the same mass *M* and length *l* to the side. Then eight of these blocks packed in a cube would have a mass of 8*M* and length 2*l*. A cube of 27 blocks would have a mass of 27*M* and length of 3*l*, and so on. We see that the size (2*l*, 3*l*...) of a group of building blocks is proportional to the cube root of the group mass ($[8M]^{1/3}$, $[27M]^{1/3}$...) for close packing in three dimensions. In contrast, if the building blocks were spread out in a straight line, then length of the line would be proportional to its mass, not the cube root of its mass. Hence, the nucleus can be thought of as an object composed of nucleons packed closely together.

According to Eq. (1.5), the nucleons that make up the nucleus are in close contact with each other, but do not all sit at *exactly* the same point in space. What about the individual nucleons: do they really have a radius of about 1 fm as Eq. (1.5) suggests for A = 1? This question can be answered experimentally by scattering electrons from a hydrogen target. The hydrogen nucleus consists of a single nucleon, the proton. Scattering experiments confirm that the proton has a definite size as well: about 1 fm.

The scattering of electrons from electrons shows a different pattern than the scattering of electrons from protons. At all of the electron beam energies that are currently available, the electrons appear to be point-like: they appear to have zero size and no internal structure. Now, it may well be that at some future time we will be able to accelerate electron beams to high enough energy that we probe "inside" the electron in the same way that increasingly energetic beams were used to probe further "inside" the atom earlier in the twentieth century. But for now, electrons appear to be point-like down to a distance of 10^{-16} m. A description of the beam energies and particle produced by the world's most powerful particle accelerators is provided in Appendix B, which also contains more material on the target number density $n_{\rm T}$.

The atom has a measurable size and is made of constituent particles: electrons and a nucleus. The nucleus has a measurable size and is made of constituent particles: nucleons. The proton has a measurable size: is *it* made of constituent particles? The answer to this question can be determined experimentally by raising the beam energy still further so that the beam particles pass into the proton. Unlike nuclei, which will shatter into their constituent nucleons if they are hit hard enough, so far it has not been possible to break up a proton into a set of isolated constituents. However, the scattering data show definite evidence that there are three strong scattering centres within each proton. The most successful model to date for describing nucleons proposes that these scattering centres are *quarks*. It is likely that there is more to a nucleon than just quarks, a subject to which we return in Chapter 5. Before closing out this discussion, we take a brief look at masses. Direct mass measurements may involve injecting a charged particle into a magnetic field. As you will learn in other physics courses, the force that a magnetic field exerts on a charged particle is at right angles to the plane formed by the direction of the field and the direction of the particle's motion. In other words, the force produces an acceleration that is perpendicular to the particle's velocity. As is discussed in Appendix A.III, a perpendicular acceleration does not change the magnitude of a velocity, but does change its direction. The particle executes a circular orbit in the magnetic field, and the radius of the orbit is proportional to the particle's mass. This provides us with an experimental technique for determining the mass of a charged particle.

This and other techniques have been used to determine the masses of a few hundred elementary particles, about a thousand nuclei and several thousand atoms in various charge states. Sample elementary particle masses are given in Table 1.1, while Appendix C contains a more extensive tabulation. Table 1.1 shows that elementary particle masses are very small, typically 10^{-27} kg. The photon, which is the elementary particle of light, may have no mass at all: Table 1.1 only lists the experimental upper bound to the photon mass.

Nuclei are characterized by the number of protons Z and neutrons N that they contain. The mass of a nucleus is generally about 1% less than $Zm_p + Nm_n$, where m_p and m_n are the proton and neutron masses, respectively. This 1% difference in masses is very important, as we demonstrate in Chapter 4. An electrically neutral atom composed of Z electrons and a nucleus of Z protons and N neutrons has a mass very close to, but also slightly less than, the sum of the nuclear mass and the electron masses.

Particle	Symbol	Mass (kg)
photon electron	e⁻	< 5.3 x 10 ⁻⁶³ 9.109 x 10 ⁻³¹
proton neutron	p n	1.6726 x 10 ⁻²⁷ 1.6750 x 10 ⁻²⁷

Table 1.1	Representative ma	sses of elementary	particles.
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Chapter 1

Summary

The sizes of subatomic particles are probed by means of scattering experiments. The experimental probability P that a beam particle is scattered by a target is obtained from

P = [number of beam particles scattered] ÷ [total number of beam particles shot at the target].

The cross section σ , which is the effective area of a target particle in its interaction with a beam particle, can be extracted from *P* via Eq. (1.3),

 $P = n_{\rm T}\sigma$,

where $m_{\rm T}$ is the number of target particles per unit area of the target exposed to the beam. As shown in Appendix B, the areal number density $m_{\rm T}$ is related to the target's atomic mass, thickness (*t*) and mass density (ρ) by $m_{\rm T} = \rho t N_{\rm o}$ / [*atomic mass*] where $N_{\rm o}$ is Avogadro's number, although we emphasize that it is not necessary to know this relationship to understand the remaining material in these lectures.

Scattering experiments show that atomic radii are typically around 0.1 nm, depending on the charge state. Nuclear radii are given approximately by Eq. (1.5),

 $R = 1.2A^{1/3}$ (fm),

where *A* is the mass number of the nucleus. The mass number is the sum of the number of protons *Z* and the number of neutrons *N*. The radii of some elementary particles, like the proton and neutron, are in the 1 fm = 10^{-15} m range, while the radii of a few other particles, such as the electron, is below the detectable limit of current experiments.

Further Reading

F. Close, M. Marten and C. Sutton, *The Particle Explosion* (Oxford, New York, 1987) [general reading].

B. McCuster, The Quest for Quarks (Cambridge, London, 1983), Chap. 1.

W. E. Meyerhof, *Elements of Nuclear Physics* (McGraw-Hill, New York, 1967), Chap, 1.

E. Segre, Nuclei and Particles (Benjamin, New York, 1964), Chaps. 1 and 2.

Chapter 1

Problems

Several of these problems use the surface area $(4\pi R^2)$ and volume $(4\pi R^3/3)$ of a sphere of radius *R*.

1. An oil tanker spills 5 million litres of light oil, which spreads over the ocean until it forms a layer 1 nm thick. (a) What is the area of this film?(b) What fraction of the Earth's surface does it cover?

2. A problem vaguely related to Rayleigh's oil spreading experiment is the melting of the Earth's polar ice caps by global warming. Suppose that an ice sheet 1200 m thick covering an area the size of Greenland (2.2×10^6 km²) melts without a change in volume. By what height would the sea level rise if this amount of water were added to the world's existing oceans? Assume that the oceans cover 71% of the Earth's surface.

3. Okanagan Lake has a surface area of roughly 600 km^2 . How many litres of gasoline are required to cover the lake with a film of thickness 2 nm (i.e. a layer about a molecule thick)?

4. In a particular scattering experiment, particles are sent towards a target at a rate of 10^{16} particles every hour. What is the scattering probability if 10^{10} particles are scattered by the target each second?

5. The scattering probability of a given target is 10^{-3} . How many particles would be scattered per second by the target if the incoming beam contains 10^{13} particles per second?

*6. Suppose that when marbles hit an object, none of them stick and they are scattered equally in all directions in three dimensions? What fraction of the marbles scatter within an angle of 45° of the beam direction?

7. A beam of 100 (absolutely sticky) marbles per second is aimed at an apple. The scattering probability is 0.005. How long will it take before 30 marbles are stuck to the apple?

8. A single layer of atoms in a solid has about 10^{20} atoms/m². If the effective radius of a nucleus in these atoms is 2 fm, what is the probability of a beam particle scattering from a nucleus in the single layer of atoms?

9. Calculate the mass density of a silver nucleus (A = 107) and compare it with the mass density of the Earth and the Sun.

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10. Calculate the approximate radius of helium (A = 4), iron (A = 56) and gold (A = 197). If all of the nucleons in each of these nuclei were within the calculated radii, what would the mass densities of each nucleus be in kg/m³?

11. Greater Vancouver covers an area of approximately 1000 km².

(a) What is the probability that a meteor which strikes anywhere on the surface of the Earth actually strikes Vancouver?

(b) If one such meteor strikes the Earth every week, what is the average time between meteor strikes in Vancouver?

*12. A sphere has the same cross sectional area no matter what angle it is viewed from. In contrast, the cross section of a cube will change with viewing angle. Calculate the cross sections of a cube when it is viewed face on and when it is viewed along a line drawn diagonally through opposite vertices. What is the maximum cross section of the cube?

13. (a) Find the radius of a sphere that has the same volume as a cube of length L to the side.

(b) What is the ratio of the cross section of the cube to the cross section of the sphere in (a)? Assume that the cube is viewed face on.

14. Calculate the cross section of lithium (A = 6), calcium (A = 40) and lead (A = 208) using Eq. (1.5). Quote your answer in m².

15. One mole of atomic hydrogen contains 6.022×10^{23} atoms and occupies 22.4 litres. What fraction of this volume is occupied by the hydrogen nuclei, if each nucleus has a radius of 0.8 fm?

16. From the data in Appendix C.II, calculate the mass density ρ (in kg/m³) of the Earth, Moon and Sun. If a nucleon has a mass of 1.7 x 10⁻²⁷ kg, how many nucleons are there per cubic meter in each of these bodies? Which of the Earth, Moon and Sun would you conclude have a similar physical makeup?

The following questions use material from Appendix B.II for calculating the number density of a target.

17. Calculate the probability of particles scattering from an aluminum (A = 27; $\rho = 2.7$ g/cm³) target 0.01 mm thick and a gold (A = 197; $\rho = 19.3$ g/cm³) target 0.01 mm thick. Assume that the scattering is from the target nuclei and use Eq. (1.5) to determine the nuclear cross section.

*18. Calculate the thickness of an iron (A = 56; $\rho = 7.8$ g/cm³) target such that the total effective area of the iron nuclei is equal to the total target area presented to the beam. Use Eq. (1.5) to determine a cross section for the iron nucleus. [Ignore the situation in which the nuclei are all lined up in a row facing the beam.]

*19. Clouds of gaseous hydrogen present in space tend to obscure our view of stars behind them. Suppose that a cloud composed of H_2 has a density of 10^{-22} g/cc. How thick must the cloud be such that it scatters half of the cosmic ray nuclei passing through it? Take the cross section for scattering of a cosmic ray on H_2 to be 10 fm².

20. a) Find the number of target nuclei per unit area of a rectangular copper (A = 63; $\rho = 8.95$ g/cm³) target 0.1 mm thick.

b) One beam particle in 4000 is observed to scatter from the target. What is the cross section for the scattering process?