CHAPTER 10

THE EARLY UNIVERSE

Two observations that characterize the universe on large length scales, the Hubble Law expansion (Chap. 8) and the 3 ^{o}K microwave radiation background (Chap. 9) are joined together in the Big Bang model. Another observation that appears to apply across the visible universe is the abundance of the lightest chemical elements: the universe appears to be approximately 75% hydrogen and 25% helium by weight. In this chapter, the predictions of the Big Bang model for the distribution of the chemical elements is presented.

10.A Universal Helium Abundance

Direct chemical knowledge exists of the surfaces of the Earth, Moon and Mars, which are characterized by their abundance of silicon, oxygen, aluminum and iron. But these elements are not necessarily the most common ones within our solar system. Since the Sun accounts for more than 99% of the solar system's mass, then it is the Sun that dominates the elemental composition of the solar system. The Sun's surface can be examined spectroscopically: sunlight is broken into its component wavelengths and these wavelengths are scanned for signatures of elements whose light emission and absorption wavelengths are known from terrestrial experiments. Using this spectroscopic technique, the Sun is found to be about 75% by weight hydrogen, and 25% by weight helium. As an historical footnote, the element helium was first identified on the Sun in 1868 by J. Norman Lockyer, before it was found on the Earth (the name is derived from *helios*).

Stars and galaxies beyond our own also can be studied spectroscopically to determine their elemental abundances. As with the Sun, other stars appear to be overwhelmingly hydrogen and helium. A

sample of results given by D. N. Schramm and R. V. Wagoner in *Annual Review of Nuclear Science* **27**, 37 (1971) for the helium mass abundance is:
(i) interstellar medium and young stars: 26 - 32%
(ii) large Magellanic cloud: 24 - 27%
(iii) small Magellanic cloud: 21 - 28%.
Each of these measurements has an error associated with it at the several percent level. The main point is that the helium abundance is remarkably similar for all of those systems that have been studied.

It is appealing to take this data to be a universal feature of the elemental abundances, that the weight fraction of ^{4}He in the universe is about 25%. However, ^{4}He is also produced by nuclear reactions in stars, as discussed in Chapter 11. Hence, a correction must be made to the observed abundances in order the determine what the "primordial" helium abundance in a star was. Such calculations give a value of 23 ± 2% [S. M. Austin, *Progress in Particle and Nuclear Physics* **7**, 1 (1981)] for the percent by weight ^{4}He. In this chapter, we show how the helium abundance is *predicted* by the Big Bang model.

10.B Scenario for the Early Universe

The first investigations of *nucleosynthesis* (the synthesis of nuclei) in the early universe were made by George Gamov and collaborators in the 1940's. Within the scientific community, widespread support for the idea of Big Bang nucleosynthesis did not occur until the 1960's with the discovery of the microwave radiation background and the accurate prediction of the ^{4}He abundance. Our discussion of these calculations begins by constructing a scenario for the time evolution of the early universe.

First, we study the behaviour of the Hubble parameter H as the universe expands. To see how H should behave qualitatively, consider the motion of two particles that can interact gravitationally and are moving away from one another. The attractive gravitational force between the particles acts in a direction opposite to their direction of motion and reduces their speed of recession. Suppose that at some initial time, the relative velocity V of the particles is equal to a constant C times their relative separation R:

$$V = CR. \tag{10.1}$$

At some later time, their relative velocity V' is less than V because of the action of their mutual gravitational attraction, while the separation R' between the particles has increased. Thus, if we again write

$$V' = C'R' \tag{10.2}$$

then C' (later time) must be less than C (earlier time).

This analysis can be applied to a group of particles under mutual gravitational attraction as discussed in Chapter 8. Initially, all particles at moderate separations obey Eq. (10.1) with C replaced by H, a Hubble parameter appropriate to the initial conditions. If the particles initially have a large average kinetic energy, then the interparticle separation increases with time while the relative velocities are reduced by gravity as the gas of particles expands. Thus, the apparent Hubble parameter H must be reduced as time goes on.

It is not a very complicated problem in Newtonian mechanics to find the behaviour of H for a gas of energetic particles expanding against a gravitational force (see the Appendix in S. Weinberg's *The First Three Minutes* cited in Further Reading at the end of this chapter). If the initial kinetic energy is such that the gas ultimately stops expanding when the particles are infinitely separated, then

$$H = (8\pi G\rho / 3)^{1/2} \tag{10.3}$$

where G is the gravitational constant (6.67 x 10^{-11} m^3 kg^{-1} s^{-2}) and ρ is the mass density (kg/m^3) of the system at any given time in its evolution. Clearly H is proportional to $\rho^{1/2}$ and decreases with time if ρ decreases with time. Further, consistent with the assumption leading to Eq. (10.3), H goes to zero as ρ goes to zero at infinite separation. That is, the speeds of the particles vanish at infinite separation.

Now, if there are photons present in the system, the mass density ρ should include the mass equivalent U_γ / c^2 of the photon energy density U_γ (that is, U_γ / c^2 has the dimensions of mass per unit volume and is the photons' contribution to the mass density of the system). Neutrinos and anti-neutrinos also contribute to the energy density with a temperature dependence of T^4, just like U_γ in Eq. (9.5). The T^4 dependence of U_γ for massless particles implies that H scales with temperature like T^2. But, from Example 8.4, H^{-1} at any given time is crudely equal to the age of the universe t, so the temperature of the universe should decrease with its age like

$$T \approx t^{-1/2}. \quad (10.4)$$

We now construct an explicit temperature history of the universe by using Eq. (10.3) along with the energy density of photons and neutrinos. Beginning with a temperature of 10^{11} °K at 0.01 sec after the Big Bang, the kinetic energy scale, $k_B T$, is about 9 MeV per particle, an energy that is much larger than the mass energy of the electron of 0.51 MeV. Hence, the large number of high energy photons present at 10^{11} °K can interconvert into electron-positron pairs through reactions like $\gamma + \gamma \rightarrow e^+ + e^-$, although the photon energies are not high enough to produce muons and anti-muons. The dominant particle species at 10^{11} °K are then γ, e^+, e^-, assorted neutrinos and antineutrinos. There are also protons and neutrons present, but few antiprotons or antineutrons, a fact that can be deduced from the observation that the universe today is mainly matter, not antimatter.

Table 10.1. Temperatures and times in the Big Bang model. The temperatures are determined from Eq. (10.3) using the energy densities of all the particles present.

Temperature	$k_B T$	Time	Event
10^{11} °K	8.6 MeV	0.01 s	Universe is too hot for bound atoms or nuclei
10^9 °K	0.086 MeV	3 min	^{4}He nuclei form
3,000 °K	0.26 eV	700,000 yr	Electrons and nuclei combine to form atoms
2.7 °K	2.3×10^{-4} eV	13×10^9 yr	Present day

At 10^{11} ^{o}K, the universe is simply too hot for nuclei to exist. The presence of high energy photons, electrons *etc*. leads to the rapid breakup of any nuclei that happen to form. As the universe cools, the average kinetic energy per particle decreases as well, so that at some point nucleons can coalesce to form nuclei. The temperature for this to occur is about 10^9 ^{o}K, which occurs about 3 minutes after the Big Bang. However, at 10^9 ^{o}K particles still have far too much kinetic energy for atoms to form. Protons, ^{4}He nuclei and electrons are present throughout the high temperature epoch, but they form a *plasma* of electrically charged ions. The temperature at which nuclei and electrons can bind to form atoms is much lower - 3000 ^{o}K - and the universe does not reach this temperature for 700,000 years.

The temperature/time history of the early universe is summarized in Table 10.1 and is shown graphically in Fig. 10.1. The first thing to note about Fig. 10.1 is that it is a logarithmic plot: the spacing on the axes is linear in the *exponent* of the quantity of interest. In other words, the spacing along the x-axis is not 1 x 10^{-40}, 2 x 10^{-40}, 3 x 10^{-40}... but is rather 10^{-40}, 10^{-30}, 10^{-20}... The reason why the temperature appears to be a straight line on this plot is that the temperature is a power law function of time: from Eq. (10.4) $T = Ct^{-1/2}$ where C is a constant. Hence, $\log T = -(1/2)\cdot\log t + \log C$, which is a function of the form $y = mx + b$. The slope of the $\log T$ vs. $\log t$ plot confirms that the temperature falls like $t^{-1/2}$.

Several important events have been marked on the plot.

Baryon asymmetry generation. The excess of matter over antimatter is thought to have occurred at very early times, around 10^{-40} s, although there are several model calculations that yield a later time. For temperatures above 10^{30} ^{o}K, it is thought that the universe had a net baryon number of zero - equal numbers of baryons and antibaryons. As the temperature cooled, it appears that the net baryon number, and net lepton number, became non-zero. This is just another way of saying that there were more protons than antiprotons, and more electrons than antielectrons. This change in the net baryon number did not seem to affect the net charge in the universe - overall the universe remained electrically neutral.

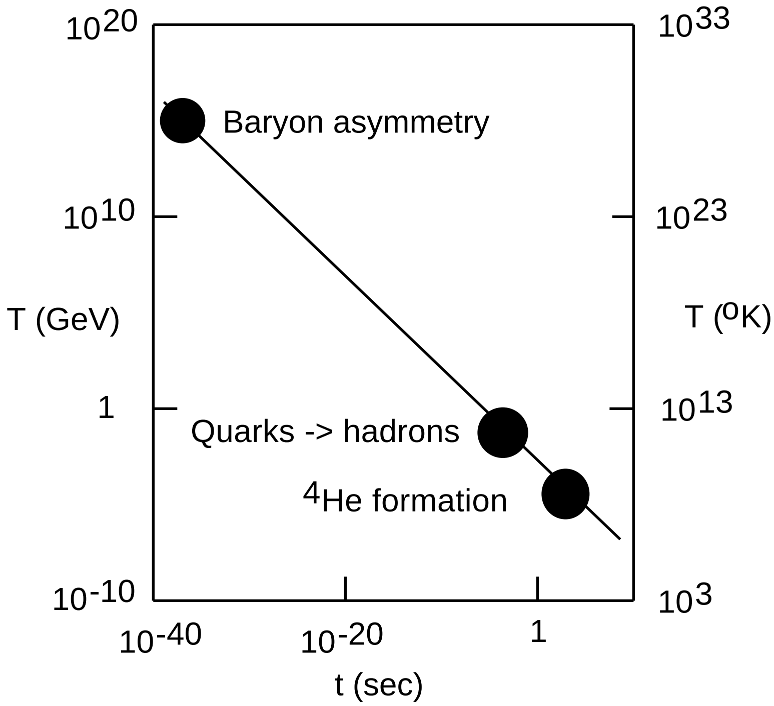

Fig. 10.1 Temperature *vs*. time plot of the early universe. The temperature is given in ^{o}K and as $k_B T$ in energy units (1 GeV = 10^9 eV).

Quark → Hadron Transition. At very high temperatures, it is predicted that hadrons are broken apart into their constituent quarks and gluons. This is a *plasma* state similar to the plasma formed when electrons are stripped from atoms to form ions. The quark-gluon plasma has not been observed in the laboratory, but efforts are now underway to produce it at a large accelerator complex called RHIC (Relativistic Heavy Ion Collider) under construction on Long Island, New York. The plasma state is predicted to exist above temperatures of 10^{12} ^{o}K or so, below which the quarks and gluons are combined in the form of hadrons. From Fig. 10.1, the quark-to-hadron transition took place about 10^{-5} s after the Big Bang.

^{4}He Formation. Finally, at a temperature of about 10^9 ^{o}K the universe had cooled sufficiently that nuclei could condense from the hot gas of protons and neutrons. The reactions at this time were very fast, and any free neutrons left over from the Big Bang rapidly reacted with protons and more complex nuclei. The dominant reaction product was ^{4}He, although small amounts of other nuclei were formed as well.

The description of the early universe presented above is a theoretical one based upon the Big Bang model and the theory of elementary particle interactions. There are still many gaps in our understanding of elementary particle interactions, and so the picture has an element of uncertainty to it. In particular, one question of religous importance - why did the Big Bang happen - is left unanswered. But for Big Bang times later than a hundredth of a second, the universe was cool enough that the relevant physics can be determined in laboratory experiments. This leads us to a prediction for ^{4}He abundance.

10.C Helium Synthesis (optional)

Let's describe the early universe at 10^{11} ^{o}K. Because of the high temperature, the densities and energies of e^+, e^-, γ, ν_e, anti-ν_e, ν_μ and anti-ν_μ are large, and nuclei are torn apart as soon as they form. There are protons and neutrons left over from the earlier matter-antimatter annihilation. The mass difference between the protons and neutrons is 1.29 MeV which is much smaller than the average nucleon kinetic energy, $(3/2)k_BT$, of 13 MeV at 10^{11} ^{o}K. Hence, collisions between protons or neutrons and other particles have sufficient energy that protons and neutrons are rapidly interconverted. Typical reactions that result in protons converting to neutrons would be:

$$e^- + p \rightarrow n + \nu_e \qquad (10.5)$$

and

$$e^+ + n \rightarrow p + \text{anti-}\nu_e \qquad (10.6)$$

as well as their inverse reactions. Neutrons decay at this temperature by the usual decay mode,

$$n \rightarrow p + e^- + \text{anti-}\nu_e \qquad (10.7)$$

but no sooner does a neutron decay than another reaction like (10.5) produces a new neutron.

The high temperature reactions keep the protons and neutrons in equilibrium. If there were no mass difference at all between protons and neutrons, then they would be present in equal numbers. But because the neutron has slightly more mass energy than the proton, there are fewer neutrons than protons. At very high temperatures, the neutrons are only slightly outnumbered by the protons. But as the temperature drops so does the neutron abundance. It is a straightforward problem in equilibrium statistical mechanics to determine the ratio of the neutron abundance, which we shall denote by [*n*], to the proton abundance, [*p*]:

$$[n]/[p] = \exp\,(-\Delta mc^2/k_B T) \qquad (10.8)$$

where exp(-x) is just e^{-x} and Δmc^2 is the difference in the mass energies

$$\Delta mc^2 = m_n c^2 - m_p c^2 = 1.29 \text{ MeV}. \qquad (10.9)$$

For high temperatures, $k_B T >> \Delta mc^2$ and the argument of the exponential in Eq. (10.8) is roughly zero. Hence Eq. (10.8) predicts [*n*]/[*p*] must be close 1 at high temperatures. Conversely, at low temperatures the argument of the exponential is a large negative number, and [*n*]/[*p*] goes to zero.

Example 10.1: *At what temperature are there only half as many neutrons as protons?*

We must solve Eq. (10.8) for $k_B T$:

$0.5 = \exp(-1.29/k_B T)$ or $k_B T = -1.29/\ln(0.5) = 1.86$ MeV.

To express this temperature in ^{o}K, first convert $k_B T$ from MeV to Joules

$$k_B T = 1.86 \times 10^6 \times 1.6 \times 10^{-19} = 2.98 \times 10^{-13} \text{ J}$$

and then find T using $k_B = 1.38 \times 10^{-23}$ J/K^o:

$$T = 2.98 \times 10^{-13}\,[J] \,/\, 1.38 \times 10^{-23}\,[J/K^o] = 2.2 \times 10^{10}\ ^oK.$$

Thus, the temperature at which [n]/[p] = 1/2 is about 2×10^{10} ^{o}K.

In general, then, we expect that the neutron to proton ratio decreases as the temperature decreases. But it will *not* follow Eq. (10.8) for all temperatures. This is because Eq. (10.8) is an *equilibrium* expression. The reactions that interconvert protons to neutrons have to be rapid for equilibrium to be maintained and for Eq. (10.8) to hold. A reaction goes out of equilibrium when the rate of the reaction is slow compared to the rate at which the universe is expanding.

A *reaction rate* is a number of reactions per unit time and it is a function the density of particles and the frequency with which they hit each other. Now, as the universe expands, the density decreases so that the particles are not as close to each other. Further, because their kinetic energies decrease as the temperature decreases, then the particles do not run into each other as frequently. The net result of these two effects is that at some point the reactions no longer support the equilibrium. It is not difficult to calculate reaction rates, although it is beyond what we have time for here. The calculation shows that the equilibrium is destroyed at about 1 second after the Big Bang, or 10^{10} ^oK.

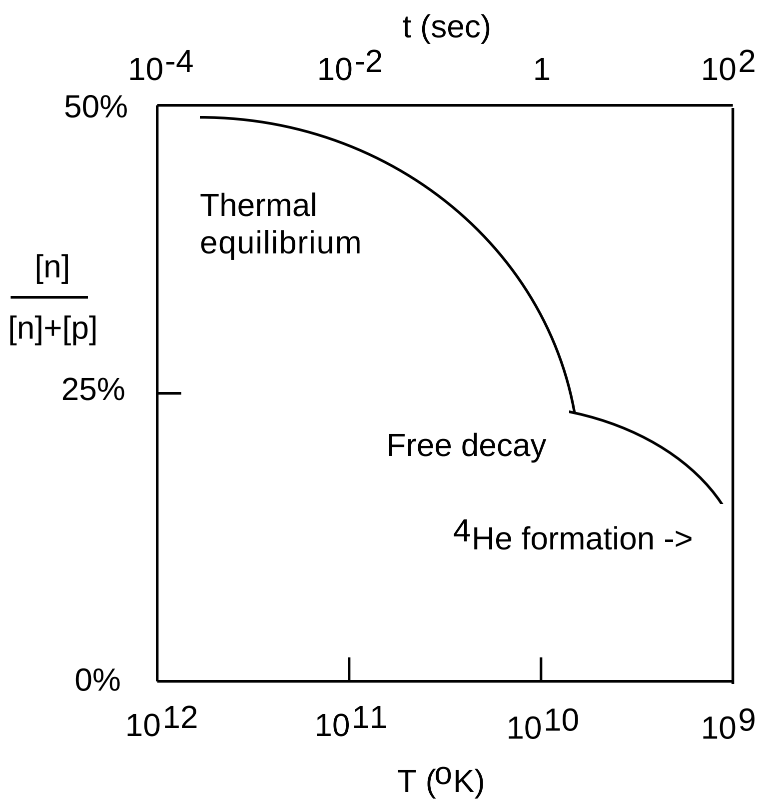

Fig. 10.2 Schematic dependence of the neutron abundance $[n]/([n]+[p])$ on temperature (bottom scale) and time (top scale) in the early universe.

What happens once the equilibrium is destroyed? Since there are few reactions to restore the neutron balance, then the neutrons simply decay freely through the decay mode Eq. (10.7) with a lifetime τ of 14.8 minutes and follow the usual decay law $N(t)=N_0\exp(-t/\tau)$. A schematic plot of the neutron abundance based upon reaction rates and neutron decay is shown in Fig. 10.2, which illustrates the smooth decrease in the neutron abundance in equilibrium up to a time of roughly 1 second, followed by free neutron decay as the reactions go out of equilibrium.

Nuclear reactions have been omitted from the above discussion because the temperature is still too hot at 10^{10} ^oK to allow nuclei to form. But once the temperature starts to fall into the 10^9 range, it is possible for nuclear reactions to occur. The first reaction is the formation of a deuteron (D or ^2H), which is a hydrogen isotope with one proton and one neutron:

$$n + p \rightarrow {}^2H + \gamma. \qquad (10.10)$$

As at higher temperatures, energetic photons tend to reverse this reaction, and maintain an equilibrium between protons, neutrons and deuterons. A straightforward calculation in statistical mechanics shows that few deuterons are broken up by photons at temperatures below 8×10^8 ^oK. Hence, most neutrons which are present in the early universe at 8×10^8 ^oK bind rapidly with protons to form deuterons.

What happens to the deuterons once they are formed? They react fairly rapidly through one of two reaction sequences

$$^2H + n \rightarrow {}^3H + \gamma \qquad (10.11a)$$

$$^3H + p \rightarrow {}^4He + \gamma \qquad (10.11b)$$

or

$$^2H + p \rightarrow {}^3He + \gamma \qquad (10.12a)$$

$$^3He + n \rightarrow {}^4He + \gamma. \qquad (10.12b)$$

Both reaction sequences occur rapidly because of the energy liberated. The binding energy per nucleon for ^2H is only an MeV, whereas

^{4}He has a binding energy per nucleon of more than 7 MeV. Basically, then, as soon as a deuteron is formed it reacts with other particles to form ^{4}He. Why doesn't ^{4}He react with protons or neutrons to form ^{5}Li or ^{5}He? Neither of these nuclei are stable, so if they do form they immediately decompose. The only reactions available to ^{4}He would be to add ^{2}H, ^{3}H or ^{3}He. But these other species are present in only very small numbers (they were used up to produce ^{4}He!) so the ^{4}He nuclei essentially do not react.

Nuclear physics tells us that most free neutrons present at 8 x 10^8 ^{o}K react to form ^{4}He. We calculate the weight fraction of ^{4}He as follows. Define the abundances of protons and neutrons just before ^{4}He formation to be $[p]$ and $[n]$ respectively. Just after the protons and neutrons have reacted to form ^{4}He,

(i) the net abundance of protons drops to $[p]$ - $[n]$ since each neutron has taken a proton with it to form ^{4}He

(ii) the abundance of ^{4}He must be $[^4He] = [n]/2$, since there are two neutrons in each ^{4}He nucleus.

To a good approximation, ^{4}He has four times the mass of ^{1}H, so the mass fraction of ^{4}He is

$$4[^4He]/([p] - [n] + 4[^4He]) = 2[n]/([p] + [n]). \qquad (10.13)$$

If we compare with Fig. 10.2, then $[n]/([n]+[p])$ at 8 x 10^8 ^{o}K is about 12%. Hence, we predict that the weight fraction of ^{4}He is 24%.

This is a remarkable prediction of the Big Bang model and uses only nuclear physics that is known from laboratory-based experiments. Further predictions for the light nuclei using reactions such as (10.11) - (10.12) have been made, and general agreement is found with experiment. Because the Big Bang model is capable of making testable predictions, it has become the standard against which other cosmological models are compared.

Summary

The Hubble parameter for a gas of particles expanding against their mutual gravitational interaction is given by Eq. (10.3)

$$H = (8\pi G\rho / 3)^{1/2},$$

where G is the gravitational constant and ρ is the mass density which includes the mass equivalent U_γ / c^2 of the photon energy density U_γ. Hence, the temperature of the universe T decreases with its age t like

$$T \approx t^{-1/2}.$$

according to Eq. (10.4).

Several important events in the early universe are:
(i) baryon asymmetry generation at 10^{28} ^oK and 10^{-37} s (conjectured).
(ii) helium formation at 10^9 ^oK and 10^2 s.
(iii) atom formation at 10^3 ^oK and 10^6 yr.

At temperatures T greater than 10^{10} ^oK, protons and neutrons interconvert, and their equilibrium abundances follow

$$[n]/[p] = \exp(-\Delta mc^2 / k_B T), \qquad (optional)$$

where $\Delta mc^2 = m_n c^2 - m_p c^2 = 1.29$ MeV. In the early universe, neutron-proton interconversion goes out of equilibrium at 10^{10} ^oK (1 sec after the Big Bang), after which neutrons decay freely until the temperature is 8 x 10^8 ^oK (3 minutes). At 3 minutes, any neutrons present are captured by protons to form deuterons, ^2H, which in turn react rapidly with other nuclei to form ^4He. Very few nuclei heavier than ^4He are produced in the early universe because there are no stable $A = 5$ nuclei.

The Big Bang model predicts that the early universe should be about 1/4 helium and 3/4 hydrogen by weight. This prediction is one of the successful tests of the Big Bang model, since the visible material in the universe today is observed to have this mass fraction.

<u>Further Reading</u>

R. A. Carrigan, Jr. and W. P. Trower, *Particle Physics in the Cosmos* (Freeman, New York, 1989), Secs. I, II and V.

J. Gribbin, *In Search of the Big Bang* (Bantam, New York, 1986), Chaps. 6 - 10 [general reading].

S. Hawking, *A Brief History of Time* (Bantam, New York, 1988), Chaps. 8 - 10 [general reading].

S. Weinberg, *The First Three Minutes* (Basic, New York, 1977), Chaps. 4 - 7, Mathematical Supplement.

Problems

1. Using $U\gamma$ for the average energy per particle for a photon gas at temperature T (see Sec. 9.B), find the threshold temperatures for the following reactions to occur readily:
(a) $\gamma + \gamma \rightarrow \pi^+ + \pi^-$
(b) $\gamma + \gamma \rightarrow e^+ + e^-$
(c) $\gamma + \gamma \rightarrow$ p + anti-p.

2. Suppose that today's value of H is 70 km s^{-1} Mpc^{-1}.
(a) What is the corresponding value of ρ from Eq. (10.3)?
(b) What is U_γ / c^2 for the microwave radiation background at T = 2.7 ^{o}K?
(c) Comparing your results from (a) and (b), does the microwave radiation contribute significantly to the expansion rate of the universe today?

3. When the temperature of the universe was $T = 10^9$ ^{o}K,
(a) what was the value of U_γ?
(b) what was the corresponding value of H from Eq. (10.3)?
(c) what was the corresponding value of t, if $t = H^{-1}$? Compare your answer with Table 10.1.

These questions use optional material from Section 10.C.

4. If the reactions to interconvert protons and neutrons were rapid enough in interstellar space at T = 3 ^{o}K to establish equilibrium abundances, what would be the ratio of neutrons to protons?

5. If the reactions to interconvert protons and neutrons were rapid enough to establish equilibrium abundances in the Sun at $T = 5 \times 10^6$ ^{o}K, what would be the ratio of neutrons to protons?

6. Suppose that ^{2}H did not react to form heavier nuclei. If the neutrons in the early universe had formed ^{2}H when $[n]/([n]+[p]) = 0.5$, what would be the mass fraction of ^{2}H?