CHAPTER 11

ORIGIN OF THE ELEMENTS

The nuclear reactions of the early universe lead to the production of light nuclei like ^{2}H and ^{4}He. There are few reaction pathways leading to nuclei heavier than ^{4}He that can occur during the rapid cooling of the early universe. What process or event produced the moderate mass nuclei that are the main constituents of the Earth? In this chapter, we describe nucleosynthesis in stars.

11.A Elemental Abundances

On the surface of the Earth, direct chemical means can be used to determine what elements are present in what abundance. The abundances in the Earth's lithosphere, which extends from the surface to a depth of 40 km or so, are shown in Table 11.1. But the lithosphere contains less than 1% of the Earth's mass, so the *average* abundances of the Earth as a whole are determined by its interior. Since we do not have access to the Earth's interior, we have little direct knowledge about it and must rely on models based on the Earth's density and magnetic properties. Chemical studies also have been made of the surfaces of the Moon and Mars, as well as meteorites that strike the Earth's surface. Many similarities among the elemental abundances of these bodies have been found and we conclude that the planets and moons of our solar system are characterized by the presence of silicon, oxygen, aluminum, magnesium and iron or nickel.

But we cannot conclude from planetary results that most of the mass of the universe is tied up in moderate mass elements like silicon, magnesium and iron. For example, the *average* elemental abundances within our solar system are largely determined by the material found in the Sun, which is overwhelmingly the most massive object in the solar system. The Sun is about 75% hydrogen, and 25% helium, by weight.

Table 11.1. Abundance of the principal elements in the Earth's lithosphere. Column 3 shows the percent by weight of the element. Column 4 shows the number of atoms of an element compared to the number of silicon atoms.

Element	Average mass number	Percentage abundance by weight	Number of atoms compared to silicon
Oxygen	16	46.6	2.95
Silicon	28	27.7	≡1
Aluminum	27	8.3	0.31
Iron	56	5	0.09
Calcium	40	3.63	0.092
Sodium	23	2.83	0.125
Potassium	39	2.59	0.085
Magnesium	24	2.09	0.087

A selected summary of the estimated elemental abundances in the solar system is given in Table 11.2. The table shows the abundances relative to silicon, which is chosen for convenience to be 1. The abundances shown in Table 11.2 attempt to take into account what is known, or estimated, from the planets. Leaving aside the hydrogen and helium components, one can see that there are a number of differences between what is observed on the surface of the Earth and what is estimated for the solar system as a whole. After hydrogen and helium, the next most abundant elements in the solar system are oxygen, carbon, nitrogen, neon, magnesium, silicon, aluminum and iron. There are very heavy elements such as uranium are present as well, although there is only one uranium atom for every 10^{12} hydrogen atoms. Recalling that the ^{56}Fe nucleus has the greatest binding energy per nucleon, we note that the elemental abundances for $A < 56$ are much greater than for $A > 56$.

Table 11.2. Estimated abundances (by number of atoms) of selected elements in the solar system compared to silicon [from A.G.W. Cameron, *Space Science Reviews* **15**, 121 (1970)].

Element	Symbol	Average mass number	Abundance compared to silicon
Hydrogen	H	1	3.18×10^{4}
Helium	He	4	2.21×10^{3}
Lithium	Li	7	4.95×10^{-5}
Beryllium	Be	9	8.1×10^{-7}
Boron	B	11	3.5×10^{-4}
Carbon	C	12	1.18×10^{1}
Nitrogen	N	14	3.64
Oxygen	O	16	2.14×10^{1}
Fluorine	F	19	2.45×10^{-3}
Neon	Ne	20	3.44
Sodium	Na	23	6.0×10^{-2}
Magnesium	Mg	24	1.06
Aluminum	Al	27	8.5×10^{-1}
Silicon	Si	28	Defined as 1
.		.	.
.		.	.
Iron	Fe	56	8.3×10^{-1}
Cobalt	Co	59	2.2×10^{-3}
Nickel	Ni	59	4.8×10^{-2}
.		.	.
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Lead	Pb	207	4.0×10^{-6}
.		.	.
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Uranium	U	238	2.6×10^{-8}

11.B ^{4}He Production in Stars

The energy that we receive on Earth from the Sun comes from nuclear fusion, the conversion of hydrogen to helium. We use the word "burn" to describe this conversion, although we emphasize that the conversion process is nuclear and *not* chemical, in origin. The reactions leading to ^{4}He production in the Sun are very different from those of the early universe. Just after the Big Bang, the abundance of free neutrons was far higher than it is now, and ^{4}He production involved neutron capture by other nuclei in a set of reactions such as:

$$n + p \rightarrow {}^2H + \gamma \qquad (11.1a)$$

$$n + {}^2H \rightarrow {}^3H + \gamma \qquad (11.1b)$$

$$p + {}^3H \rightarrow {}^4He + \gamma. \qquad (11.1c)$$

These reactions are very rapid in the sense that the reaction participants have strong or at least electromagnetic interactions. The cross sections for such reactions are large and the timescales are short (see Table 2.1). The very short reaction time is the reason that the capture of neutrons to form ^{4}He took place as rapidly as it did in the early universe. If the neutron capture reactions had been slow, then most neutrons would have decayed long before they could have reacted to form ^{4}He.

The fact that the Sun has been shining for about 5 billion years tells us two things. First, the nuclear reactions in the Sun today do not involve free neutrons left over from the Big Bang - all such neutrons have long since decayed or reacted. Second, the reaction sequence important in stars must be slow on the nuclear time scale. If the nuclear reactions in the Sun proceeded on a strong interaction timescale, then there would have been a great burst of energy when the Sun first formed and the Sun's nuclear fuel would have been used up very rapidly.

What reactions could lead to hydrogen burning in the Sun where free neutrons are scarce? Some part of the reaction sequence must involve the conversion of a proton to a neutron, since all stable nuclei with $A > 1$ have at least one neutron. One possibility would involve pion production, say

$$p + p \rightarrow p + n + \pi^+, \qquad (11.2)$$

which is a strong interaction and therefore fast. But the mass energy of the pion produced in reaction (11.2) must come from the kinetic energies of the reacting protons. Equating the pion mass energy of 140 MeV with the average kinetic energy of a particle in thermal equilibrium, $(3/2)k_BT$, shows that protons must have a temperature of about 10^{12} ^oK in order to produce pions frequently. This temperature is about five orders of magnitude larger than the estimated temperature of the Sun, which is 15×10^6 ^oK. Hence, it is very unlikely that reaction (11.2) contributes in any serious way to hydrogen burning.

An alternate possibility for converting protons to neutrons is the weak interaction process

$$p + p \rightarrow {}^2H + e^+ + \nu_e. \qquad (11.3)$$

When the annihilation of the positron (e^+) with plasma electrons is taken into account, this reaction liberates 1.44 MeV of energy. What stops the reaction from occuring rapidly is that it is a weak interaction process and there is Coulomb repulsion between the protons. However, this reaction does liberate kinetic energy rather than consume it as reaction (11.2) does. In spite of its slowness, reaction (11.3) is thought to be the first step of several possible reaction sequences.

One of the main reaction sequences that is thought to occur in the Sun is referred to as the *PPI* chain:

$$p + p \rightarrow {}^2H + e^+ + \nu_e \qquad (11.4a)$$

$${}^2H + p \rightarrow {}^3He + \gamma \qquad (11.4b)$$

$${}^3He + {}^3He \rightarrow {}^4He + 2p. \qquad (11.4c)$$

Reaction (11.4b) is relatively fast so that ^2H produced in (11.4a) is rapidly converted to ^3He by reaction (11.4b). Thus, ^3He is more abundant than ^2H in an equilibrium situation because ^2H is burned up almost immediately. This is why the last step in the chain involves two ^3He nuclei rather than

^{3}He plus ^{2}H: a given ^{3}He nucleus is more likely to run into another ^{3}He rather than a ^{2}H.

An alternate reaction sequence is the *PPII* chain, in which (11.4c) is replaced by a series of steps initiated by ^{3}He reacting with ^{4}He:

$$p + p \rightarrow {}^2H + e^+ + \nu_e \quad \text{(same as } PPI\text{)} \tag{11.5a}$$

$$^2H + p \rightarrow {}^3He + \gamma \quad \text{(same as } PPI\text{)} \tag{11.5b}$$

$$^3He + {}^4He \rightarrow {}^7Be + \gamma \tag{11.5c}$$

$$^7Be + e^- \rightarrow {}^7Li + \nu_e \tag{11.5d}$$

$$^7Li + p \rightarrow 2\ {}^4He. \tag{11.5e}$$

Reaction (11.5e) involves more Coulomb repulsion that the other reactions, so the *PPII* chain is not as important as *PPI* at low temperatures.

Finally, an alternate branch to the capture of an electron by ^{7}Be is proton capture. This results in the *PPIII* sequence

$$p + p \rightarrow {}^2H + e^+ + \nu_e \quad \text{(same as } PPI\text{)} \tag{11.6a}$$

$$^2H + p \rightarrow {}^3He + \gamma \quad \text{(same as } PPI\text{)} \tag{11.6b}$$

$$^3He + {}^4He \rightarrow {}^7Be + \gamma \quad \text{(same as } PPII\text{)} \tag{11.6c}$$

$$^7Be + p \rightarrow {}^8B + \gamma \tag{11.6d}$$

$$^8B \rightarrow {}^8Be + e^+ + \nu_e \tag{11.6e}$$

$$^8Be \rightarrow 2\ {}^4He. \tag{11.6f}$$

Step (11.5e) in *PPII* has less Coulomb repulsion among its participants than step (11.6d) in *PPIII*, so we expect *PPII* to be more important at lower temperatures than *PPIII*. Energy production in the Sun is about half *PPI* and half *PPII*. However, *PPIII* is not a major contributor to the Sun's energy output. The main differences between the chains are their reaction rates and the amount of energy carried off by neutrinos. The neutrinos export 3%, 4% and 28% of the energy in *PPI*, *PPII* and *PPIII* respectively.

In comparing how ^{4}He enters into reactions (11.5c) to (11.5e) and (11.6c) to (11.6f) we see that the sequence is initiated by a ^{4}He that then emerges again in the last step. That is, one ^{4}He goes into the chain and two come out. In that sense, ^{4}He enters the reaction as a *catalyst*. There is another reaction sequence that is important in stars and also uses nuclei as catalysts. This is the *CNO* cycle, proposed independently by Bethe and von Weizsacker in 1938. The word *cycle* refers to the catalytic nature of the reactions (one nucleus *consumed* in the reaction *reappears* by the end of the sequence). There are two distinct subcycles, of which the first is

$$^{12}\text{C} + \text{p} \rightarrow {}^{13}\text{N} + \gamma \qquad (11.7\text{a})$$

$$^{13}\text{N} \rightarrow {}^{13}\text{C} + \text{e}^+ + \nu_e \qquad (11.7\text{b})$$

$$^{13}\text{C} + \text{p} \rightarrow {}^{14}\text{N} + \gamma \qquad (11.7\text{c})$$

$$^{14}\text{N} + \text{p} \rightarrow {}^{15}\text{O} + \gamma \qquad (11.7\text{d})$$

$$^{15}\text{O} \rightarrow {}^{15}\text{N} + \text{e}^+ + \nu_e \qquad (11.7\text{e})$$

$$^{15}\text{N} + \text{p} \rightarrow {}^{12}\text{C} + {}^{4}\text{He}. \qquad (11.7\text{f})$$

In this cycle, it is ^{12}C that is removed in the first step of the sequence and then reappears at the end.

In about 1 time in 2500, reaction (11.7f) takes a different path to form a different subcycle:

$$^{15}\text{N} + \text{p} \rightarrow {}^{16}\text{O} + \gamma \qquad (11.8\text{a})$$

$$^{16}\text{O} + \text{p} \rightarrow {}^{17}\text{F} + \gamma \qquad (11.8\text{b})$$

$$^{17}\text{F} \rightarrow {}^{17}\text{O} + \text{e}^+ + \nu_e \qquad (11.8\text{c})$$

$$^{17}O + p \rightarrow \ ^{14}N + \ ^{4}He. \qquad (11.8d)$$

The ^{14}N that is produced in (11.8d) can then feed into (11.7d).

The rates of the *PP* chains and the *CNO* cycle are known from laboratory-based experiments. The rates are used in model calculations for energy production in the Sun, and the results agree well with the Sun's observed *luminosity*, which is the energy output of the Sun per unit time. Further experimental tests of the calculations involve the predicted number of neutrinos coming from the Sun. The first experiments, done in 1970 by a group headed by an American named Ray Davis, found that the observed number of solar neutrinos was significantly lower than expected. Subsequent experiments show a discrepancy at the factor of two level, although the entire range of neutrino energies has not yet been explored.

Canada is involved in the search for the missing solar neutrinos through the construction of the Sudbury Underground Neutrino Observatory (SNO). Neutrinos have very low cross sections for their interactions with matter and are therefore difficult to detect. The SNO project involves building a large neutrino detector using stockpiled heavy water from Canada's CANDU reactor program. The detector is being placed deep underground in one of INCO's nickel mines in Sudbury, Ontario. The reason for burying the detector is to reduce the number of cosmic rays that can enter the detector and obscure the neutrino count rate. Construction of SNO began in 1990.

11.C Advanced Burning Stages

Many more nuclear reactions can liberate energy than the *PP* chains discussed in Section 11.B. These other reactions involve heavier nuclei and hence require high kinetic energy in order to overcome the Coulomb repulsion between reaction partners. Such high temperatures are usually characteristic of larger or older stars than the Sun. Let's begin a discussion of stellar energetics by estimating the initial temperature of the Sun.

From Eq. (4.7), the gravitational binding energy of an object of mass M and radius R is $(3/5)GM^2R$, which works out to be 2.3×10^{41} J for the Sun. This is the amount of energy released when a dilute cloud of gaseous hydrogen and helium contracts to form a star of uniform density with the current mass and radius of the Sun. Suppose that none of the gravitational binding energy of the Sun escaped during the Sun's formation, but that all of it went to heating up its constituents. Spreading the 2.3 x 10^{41} J of binding energy over the Sun's 1.2 x 10^{57} nucleons (and a similar but not equal number of electrons) gives about 10^{-16} J per particle. Equating this energy with the average kinetic energy per particle in an ideal gas, $3/2 k_B T$, then the initial temperature of the Sun would be about 5 million degrees K. While this estimate is less than the current temperature of the Sun's core, it is high enough to initiate nuclear reactions.

Since the gravitational binding energy scales as a function of a star's mass like M^2, then the binding energy per particle scales like M (ignoring the slow increase in the star's radius with mass). So, a star several times the Sun's mass could have a much higher initial temperature. High temperatures also can be obtained in stars that have been burning for some time. At high temperatures, reactions involving the burning of nuclei like ^{4}He and ^{12}C can occur.

A typical solar mass star would have the following sequence of burning stages. In its "early" life, which lasts for billions of years, the star's energy is mainly produced through the *PPI* and *PPII* chains, with *PPIII* becoming more important as the star's temperature increases. The *PP* chains convert hydrogen to helium, so that at some time the hydrogen "fuel" for the star is strongly diminished. If its temperature is high enough, then the star can begin to burn ^{4}He. Now helium burning is not a weak interaction process, so in principle it can occur rapidly. However, there is significant Coulomb repulsion between ^{4}He nuclei and this slows down the reaction rate.

Helium burning would involve reactions like

$$^4\text{He} + {}^4\text{He} \rightarrow {}^8\text{Be} + \gamma. \qquad (11.9)$$

As it turns out, ^{8}Be is slightly unstable against decay back to two ^{4}He nuclei. The *Q*-value for the decay

$$^8\text{Be} \rightarrow 2 \cdot {}^4\text{He} \qquad (11.10)$$

is very small, only 0.092 MeV, but it is still positive so the decay can occur spontaneously. Now, at face value this is discouraging because it means that two ^{4}He nuclei cannot combine to form a stable nucleus. However, the small *Q*-value means that the decay is slow by strong interaction standards, with a lifetime of 2.6 x 10^{-16} s.

This "long" lifetime of 10^{-16} s means that there is always a trace amount of ^{8}Be present at high temperatures (one ^{8}Be for every 1.5 x 10^9 ^{4}He nuclei at 100 million ^{o}K!), and these ^{8}Be nuclei can react to form ^{12}C:

$$^4\text{He} + {}^8\text{Be} \rightarrow {}^{12}\text{C} + \gamma, \quad (11.11)$$

releasing 7.4 MeV in the process. The other common reaction which ^{8}Be can have is proton capture, but the resulting ^{9}B nucleus is unstable and just leads back to ^{4}He like reactions (11.9) and (11.10):

$$^8\text{Be} + \text{p} \rightarrow {}^9\text{B} \rightarrow 2 \bullet {}^4\text{He} + \text{p}. \quad (11.12)$$

Once ^{12}C nuclei are formed, the bottleneck at ^{8}Be has been passed. Further addition to ^{12}C by ^{4}He nuclei leads to a sequence of products

$$^4\text{He} + {}^{12}\text{C} \rightarrow {}^{16}\text{O} + \gamma \quad (11.13)$$

$$^4\text{He} + {}^{16}\text{O} \rightarrow {}^{20}\text{Ne} + \gamma \quad (11.14)$$

$$^4\text{He} + {}^{20}\text{Ne} \rightarrow {}^{24}\text{Mg} + \gamma \quad (11.15)$$

$$^4\text{He} + {}^{24}\text{Mg} \rightarrow {}^{28}\text{Si} + \gamma. \quad (11.16)$$

All of these reactions release energy, since the binding energy per nucleon increases steadily through this mass region as ^{56}Fe is approached.

It is worthwhile looking at the abundances of medium mass nuclei in the solar system, as summarized in Table 11.2. Looking down the abundance column, one can see that the nuclei in the ^{12}C, ^{16}O, ^{20}Ne, ^{24}Mg, ^{28}Si series are much more abundant than F, Na, Al,... . The abundant nuclei are those whose *Z* and *A* are integral multiples of ^{4}He. Other elements are not absent, of course. They are formed in reactions involving protons, and in reactions that occur at higher temperatures still. At very high temperatures ^{12}C begins to burn with itself, and the photons present have sufficient energy that they can cause protons and neutrons to split off from nuclei. These very high temperature reactions occur at temperatures exceeding 10^9 °K, and occur on a very short time scale, on the order of years in a star's overall lifetime of billions of years.

11.D Explosive Nucleosynthesis

The advanced burning reactions described in Section 11.C occur because it is energetically favourable for light nuclei to combine to form heavier ones. This reflects the fact that the binding energy per nucleon has a maximum at ^{56}Fe. In a very hot star, reactions will continue to liberate energy as the average nucleus in the star becomes heavier and heavier. But reactions involving the *burning* of ^{56}Fe to produce heavier nuclei generally do not liberate energy and are therefore suppressed. Where do the heavy nuclei like gold and uranium that we observe in the planets come from?

To answer this question, we consider the environment of a star at 10^9 °K. There are many intermediate mass nuclei present, and copious high energy photons. The photons have enough energy to pry loose nucleons from Mg, Si *etc* that can then attach themselves to other nuclei. In other words, at these high temperatures there is a rearrangement of nuclear species away from those nuclei that are multiples of ^4He and the gaps in the abundances of odd Z and A nuclei are filled in.

In this hot environment with free protons and neutrons present, it is possible to add neutrons to ^{56}Fe and build up heavier nuclei. Neutron capture is favoured over proton capture in reactions with heavy nuclei because of Coulomb repulsion. Not a huge number of heavy nuclei will be produced this way, but inspection of the abundances in Table 11.2 shows that not many heavy nuclei are observed in the solar system. A number of scenarios for the addition of nucleons to produce heavy nuclei have been proposed, but we describe only two of them:

Slow capture (*s-process*) This involves the (relatively) slow addition of neutrons to heavy nuclei that are very deeply bound. Because the time scale is long, then the nuclei have time to decay or emit particles before they capture another passing neutron and increase their mass number by one. The slow process produces a sequence of nuclei that follow the maximum in the binding energy per nucleon curve at or near $Z = N$. Since typical nuclear decay times of such nuclei are in the 10^{-3} to 10^{3} hour range, then the *s*-process presumably takes of the order years.

Rapid capture (*r-process*) For some stars in their advanced burning stages, there is not enough time for energy to be emitted in an orderly fashion that preserves the integrity of the star. Such stars explode as *supernovae*, perhaps forming neutron stars or black holes (see Fig. 11.1). In a supernova, not only does the star blow apart but also nuclei are broken up and their nucleons react rapidly with whatever is near. A heavy nucleus in such an environment captures neutrons without having time to decay by particle emission to the most deeply bound configuration. Rather, it continues to add neutrons until it is saturated and can hold no more. The *r*-process, then, produces neutron-rich nuclei at their binding limit. Some of the heavy elements that we observe today are the decay products of these neutron rich nuclei.

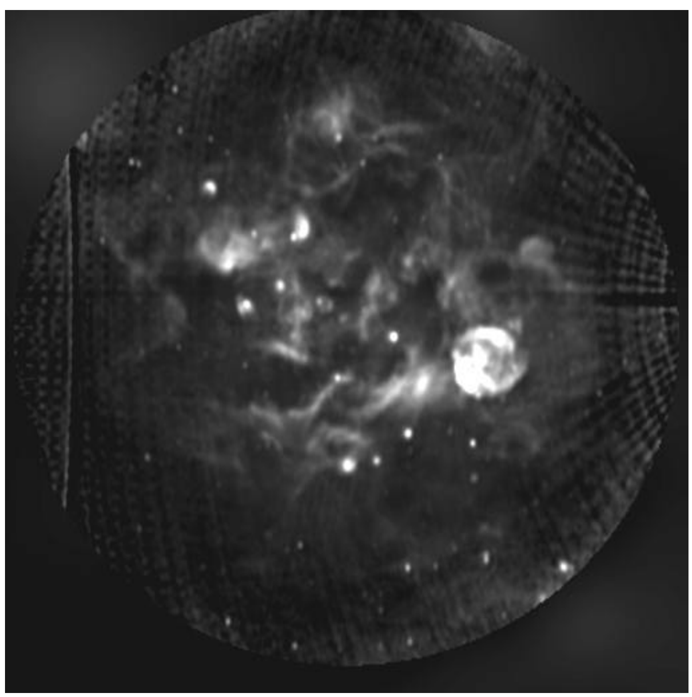

Fig. 11.1 Radiotelescope image of Cygnus X region, long considered a candidate as a black hole (greyscale image from the Dominion Radio Astrophysical Observatory, Penticton, Canada; used with permission).

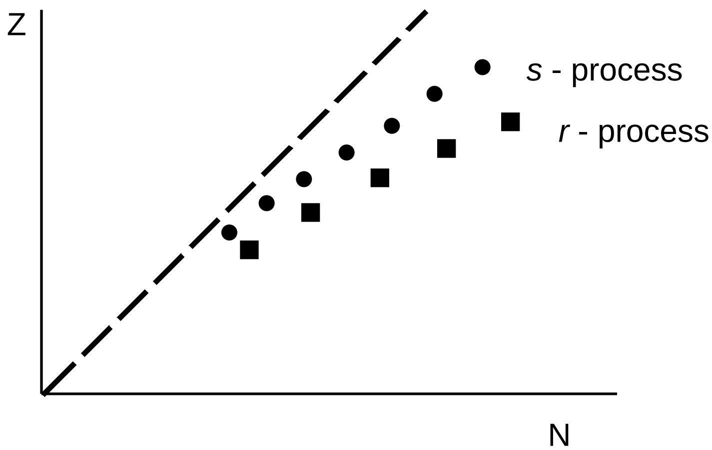

Fig. 11.2. Schematic illustration of the initial nuclei populated by the *s*- and *r*-processes. The neutron-rich *r*-process nuclei subsequently decay.

The nuclei populated by these two processes are shown schematically in Fig. 11.2. Experimentally, one observes evidence for both processes in the elemental abundances. It is likely, then, that the material of which the Earth, and of course ourselves, is made has been recycled at least once. Our atoms contain some residual nuclei from the Big Bang. But we also contain carbon, nitrogen and oxygen originating in a star that met an untimely end, perhaps producing heavy elements. The matter from that star then reformed into subsequent stars and planets, perhaps being recycled again in the process.

Summary

Hydrogen and helium are the most abundant elements in our solar system, just as they are in most stars. Beyond hydrogen and helium, the next most abundant elements in the solar system (by number of atoms) are oxygen, carbon, nitrogen, neon, magnesium, silicon, aluminum and iron. Very heavy elements such as uranium are present as well, although there is only one uranium atom for every 10^{12} hydrogen atoms. In general, the elemental abundances for $A < 56$ are much greater than for $A > 56$.

Helium formation in stars is different from helium formation in the early universe, since there are few free neutrons in stars. There are several main sequences of reactions for the conversion of protons to neutrons in stars such as our Sun (*PPI*, *PPII* and *PPIII*), each of which starts with

$$p + p \rightarrow {}^{2}H + e^{+} + \nu_e$$

$${}^{2}H + p \rightarrow {}^{3}He + \gamma.$$

Energy production in the Sun is largely from these sequences. At higher temperatures, a star burns ^{4}He to form C, O, Ne, Mg and Si.

For temperatures exceeding 10^{9} ^{o}K, high energy photons liberate protons and neutrons from existing nuclei to allow the abundances to adjust to the greatest binding energy per nucleon. Nucleon addition to intermediate mass nuclei such as iron is possible, and ultimately leads to the production of heavy nuclei such as uranium. There are two processes for this addition:
(i) the *s*-process, which is relatively slow and produces isotopes with the largest binding energy per nucleon
(ii) the *r*-process, which is rapid and produces nuclei close to the neutron saturation limit. These nuclei then decay to stable nuclei.

Further Reading

F. Close, *End* (Simon and Schuster, London, 1988) [general reading].

J. Silk, *The Big Bang* (Freeman, San Francisco, 1980), Chaps. 13 - 15.

D. D. Clayton, *Principles of Stellar Evolution and Nucleosynthesis* (McGraw-Hill, New York, 1968), Chap. 5 [advanced reading].

Problems

1. Suppose that all of the Sun's luminosity is generated by H → He conversion and that the neutrinos in the reactions carry off 3% of the total energy per reaction (including e^+e^- annihilation).
(a) What is the average energy per neutrino?
(b) How many neutrinos leave the surface of the Sun per second?
(c) How many neutrinos arrive per second on a cm^2 area of the Earth's surface directly facing the Sun?

2. Calculate the energy released in the main *CNO* cycle (11.7) and in the subcycle (11.7) + (11.8). Compare these results with the energy released in the *PPI* chain.

3. Find *Q*-values for the reactions
(i) $\alpha + {}^{12}C \rightarrow {}^{16}O + \gamma$
(ii) $\alpha + {}^{16}O \rightarrow {}^{20}Ne + \gamma$
(iii) $\alpha + {}^{20}Ne \rightarrow {}^{24}Mg + \gamma$
(iv) $\alpha + {}^{24}Mg \rightarrow {}^{28}Si + \gamma$.
What is the total energy per reacting nucleon liberated in this sequence?

4. In its advanced stages of evolution, a star begins to use reactions such as ${}^{12}C + {}^{12}C \rightarrow {}^{24}Mg + \gamma$.
(a) Calculate the *Q*-value for this reaction.
(b) If the Sun's 10^{57} baryons were entirely tied up in ${}^{12}C$, how much energy would be released in ${}^{12}C \rightarrow {}^{24}Mg$ conversion?
(c) At a luminosity of 3.9 x 10^{26} J/s, for how long could this stage of the star's life last?

*5. Consider the following pairs of nuclei:
(i) ${}^{3}H + {}^{3}He$
(ii) ${}^{4}He$ + p
(iii) ${}^{12}C + {}^{12}C$
(iv) ${}^{4}He + {}^{8}Be$
(v) ${}^{4}He + {}^{12}C$
Assume that the nuclei are hard spheres with radius $R = 1.2A^{1/3}$.
(a) Calculate the magnitude of the repulsive Coulomb force between the nuclei at the minimum sphere separation distance.

(b) Multiply this force by the separation distance to obtain the energy barrier against reaction [this just happens to give the correct value; the proof requires calculus]. Equate this energy to $3/2k_BT$ to estimate the temperature at which the pairs can react easily.
(c) Rank order the pairs according to the temperature at which the reactions occur rapidly.

6. A neutron star may be created when the core of a massive star suddenly collapses from 7 x 10^5 km to 10 km during an explosive event.
(a) Calculate the change in the gravitational binding energy in such an event, if the core has the mass of the Sun.
(b) During the collapse, each proton in the star captures an electron to form a neutron, giving off a neutrino in the process. Assuming that the core was originally composed of protons and electrons in equal numbers, and assuming that all of the gravitational energy released in the collapse is transferred to the neutrinos, calculate the energy released per neutrino. Quote your answer in MeV.