

## CHAPTER 8

### THE EXPANDING UNIVERSE

Methods for determining distances to nearby and moderately remote stars and galaxies are reviewed in Chapter 7. In this chapter, we introduce a technique for extracting the velocity of a star or galaxy with respect to the Earth. It is found that the more remote galaxies appear to be moving away from the Earth with a velocity that is proportional to their distance away from us. This velocity/distance relationship, referred to as Hubble's law, is the basis for the Big Bang model in cosmology, and leads to an estimate of the age of the universe.

#### 8.A Doppler Effect

The technique for measuring the velocities of very remote galaxies is based upon the Doppler shift - the shift in the apparent wavelength of a wave due to the relative motion of the emitter of the wave with respect to the observer. We are all familiar with terrestrial examples of the Doppler shift. When an ambulance approaches us, the sound from the siren appears to have a higher pitch (or frequency) than when the ambulance is moving away from us. This is a situation in which the frequency that we hear as a stationary observer changes because of the motion of the wave emitter (the siren). It is easy to see how this effect arises by considering the diagrams in Fig. 8.1.

Diagram (a) of the figure shows the natural wavelength  $\lambda$  of the waves emitted by the source. The waves can be anything: light, sound, water waves.... Now consider what happens when the source  $S$  is moving away from the observer  $O$  with a speed  $v$  as in (c). Say at time  $t = 0$  the crest of a wave emerges from the source. The crest travels towards  $O$  with a speed  $c$  equal to the speed of the wave in whatever medium it is traveling. But the source continues to move away from the observer as

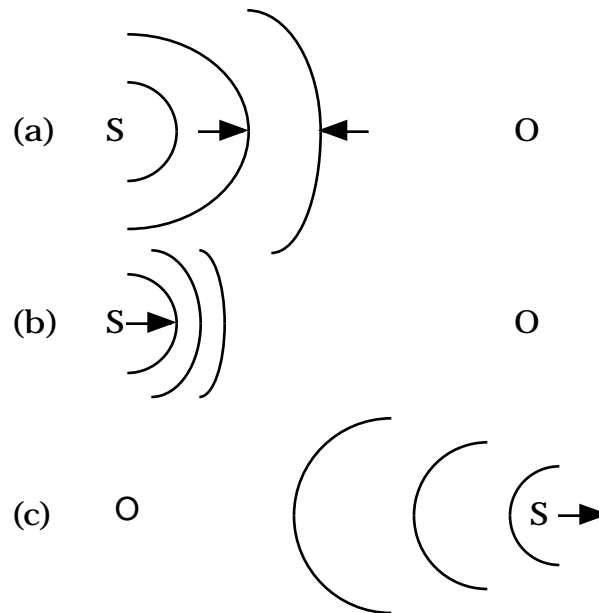


Fig. 8.1  $S$  represents a source of waves, and  $O$  an observer. The natural wavelength  $\lambda$  is shown in (a), in which  $S$  is at rest with respect to  $O$ . In (b),  $S$  moves towards  $O$  and the wavelengths appear shorter as seen by  $O$ . In (c), the source moves away from the observer.

well. At a later time, given by the period  $T$  of the wave, another crest emerges from the source. The period  $T$  is the inverse of the frequency and is equal to  $\lambda/c$ . As seen by  $O$  this crest is *not* at a distance  $\lambda$  from the first crest, because the source has moved a distance  $vT$  since it emitted the first wave crest. According to the observer, the second wave is emitted at a distance  $\lambda$  *plus*  $vT$ . The distance between the crests  $\lambda'$ , which is the wavelength according to the observer, is

$$\lambda' = \lambda + vT = \lambda + v(\lambda/c) = \lambda (1 + v/c). \quad (8.1)$$

The shift in the wavelength from  $\lambda$  at the source to  $\lambda'$  at the observer is referred to as the Doppler shift of the wave. Eq. (8.1) shows that  $\lambda' > \lambda$  for the situation in (c). For sources moving towards the observer, the analysis is the same but the sign of  $v/c$  is reversed and  $\lambda' < \lambda$ . Equation (8.1) applies to a moving source and a stationary observer. Slightly different equations apply to situations in which the source is stationary

and the observer is moving, or in which both the source and observer are moving. These other cases are dealt with in most introductory physics texts. Eq. (8.1) is really an approximation valid at velocities small compared to  $c$ , and it is not accurate as  $v \rightarrow c$ .

*Example 8.1: An ambulance siren is blaring at 3000 cycles per second as the ambulance speeds along at 30 m/s (108 km/hr). What frequency do you hear as the ambulance heads straight towards you, and as the ambulance heads straight away from you? Use  $c = 342$  m/s for the speed of sound in air.*

Eq. (8.1) applies for a source moving away from the observer. For a source moving towards the observer, we have  $\lambda' = \lambda (1 - v/c)$ , or equivalently  $\lambda' / \lambda = (1 - v/c)$ , with  $v > 0$ . Wavelengths can be replaced by frequencies using  $f = c/\lambda$ , so that

$$f' / f = (1 - v/c)^{-1}.$$

For the ambulance siren, the shifted frequency  $f'$  is

$$f' = 3000 / (1 - 30/342) = 3290 \text{ s}^{-1}.$$

Thus, you hear a higher frequency for the siren when the ambulance is heading towards you than when the ambulance is stationary. By similar reasoning, when the siren is moving away, the frequency heard by a stationary observer would be less than that heard inside the ambulance:

$$f' = 3000 / (1 + 30/342) = 2760 \text{ s}^{-1}.$$

The Doppler method can be used to determine the speed of stars or galaxies by measuring light waves. It is known that the emission (and absorption) of light under some circumstances involves waves of a very precise frequency that is characteristic of the element or compound from which the light is emitted. In astronomical applications, light emitted at a specific frequency from an element such as hydrogen is measured both on Earth and from the distant star or galaxy. A comparison of the two frequencies yields the relative velocity between emitter and observer.

Light emitted from atoms on distant stars typically is observed to be shifted to longer wavelengths than are found in experiments performed on Earth. Because the light is shifted towards the red end of the visible spectrum, it is said to be *red-shifted*. If the light is shifted to larger wavelengths by the motion of the stars, then the stars must be moving away from the Earth. As a technical aside, light emitted from stars can be absorbed at specific wavelengths by cool matter surrounding the star but moving with it. It is often more accurate to measure the red-shift of the absorbing wavelengths than to measure the red-shift of a component of the broad spectrum of emitted wavelengths.

Example 8.2: A nebula in the constellation Hydra has a relative redshift of 0.20. Use the Doppler shift expression to find its velocity with respect to Earth.

By the words "relative red-shift", we mean that the change in  $\lambda$  is equal to 0.20 of the value of  $\lambda$ . That is,  $(\lambda' - \lambda)/\lambda = 0.20$ . To find the relative redshift, we rearrange the Doppler shift equation (8.1) for a receding source to read

$$(\lambda' - \lambda)/\lambda = v/c.$$

If  $(\lambda' - \lambda)/\lambda = 0.20$ , then

$$v = 0.20c = 6.0 \times 10^4 \text{ km/s}.$$

Thus, the nebula is receding from Earth at a speed of  $6.0 \times 10^4$  km/s, rather fast by terrestrial standards.

### 8.B Hubble's Law

Using the Doppler shift technique, it is found that distant stars and galaxies appear to be receding from the Earth, just as in Example 8.2. Edwin Hubble sought to find a relation between the velocity of recession  $V$  of a distant object and its distance from the Earth  $R$ . In 1929, he found, within some uncertainty, that

$$V = HR \tag{8.2}$$

where  $H$  is a number called the Hubble parameter. As data from ever distant galaxies became available in the 1930's, Hubble's law was seen to apply over larger and larger distance scales. Clearly, it should not apply at relatively short distance scales (within the Milky Way, for example) where there may be non-uniform relative motion of the stars.

In spite of intense efforts to determine the Hubble parameter accurately, there still is a 50% uncertainty in its value. This should come as no surprise, given the difficulty of obtaining accurate distances over the immense length scales in the universe. The currently "accepted" value of the Hubble parameter is in the range  $40\text{-}100 \text{ km s}^{-1} \text{ Mpc}^{-1}$ , where Mpc is shorthand for megaparsec and is equal to 3.26 million light years or  $3.09 \times 10^{19} \text{ km}$  (1 light-year = 1 ly =  $9.46 \times 10^{12} \text{ km}$ ). Note that the units of  $H$  are actually  $[\text{time}]^{-1}$ , and the reason  $H$  is quoted in  $\text{km s}^{-1} \text{ Mpc}^{-1}$  is for astronomical convenience.

Example 8.3: *Estimate the distance to the nebula in Example 8.2 if  $H = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ .*

In Example 8.2 we estimate that a nebula in the constellation Hydra is receding from Earth at a velocity of  $V = 6.0 \times 10^4 \text{ km/s}$ . To find the distance to the nebula, we rearrange Hubble's law  $V = HR$  to read

$$R = V/H = 6.0 \times 10^4 / 70 = 8.6 \times 10^2 \text{ Mpc.}$$

Using the conversion from Appendix C that  $1 \text{ pc} = 3.26 \text{ ly}$ :

$$R = 3.26 \times 8.6 \times 10^2 \times 10^6 = 2.8 \times 10^9 \text{ ly.}$$

Thus, the nebula is about  $3 \times 10^9 \text{ ly}$ , or  $10^{22} \text{ km}$ , away from the Earth.

Hubble's law tells us that the further away a star is from us, the faster it is receding from us. Hubble's law can be illustrated by considering the relative motion of positions on the surface of a balloon while the balloon is being inflated. A schematic drawing of a balloon is given in Fig. 8.2, where three points are marked on its surface for reference. One can see that as the radius of the balloon increases in drawings (a) to (c) in the diagram, the distance between points scales

uniformly. That is, for all distances between points on the balloon's surface, the distances  $R_i$  in one diagram are related to the distances  $R_i'$  in another diagram by a common scale factor  $s$ :

$$R_i' = sR_i. \quad (8.3)$$

The  $R_i$  are the shortest distances between points as measured along the surface of the balloon. The change in surface distances between any two diagrams is

$$R_i = R_i' - R_i = (s - 1)R_i. \quad (8.4)$$

This means that the velocity of separation between a pair of points is just

$$V_i = R_i / t = [(s - 1)R_i] / t. \quad (8.5)$$

Finally, since  $R_i$  is not a function of time in Eq. (8.5) [it is the set of initial positions], then we can simplify the equation to read

$$V_i = R_i [(s - 1) / t] = R_i \times [\text{common multiplicative factor}]. \quad (8.6)$$

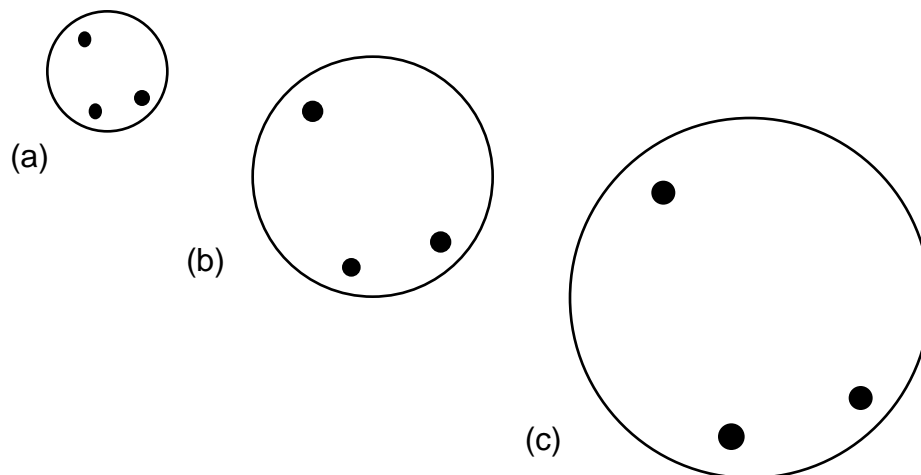


Fig. 8.2 Uniform expansion of a balloon with three positions marked on its surface.

In other words, Hubble's law applies to the motion of particles on the surface of a balloon, or to any system that is expanding according to a scale transformation like Eq. (8.3), and the Hubble parameter is the common multiplicative factor in Eq. (8.6). The balloon example shows another feature, namely that Hubble's law applies to all points in the system. At any point on the balloon's surface it will be found that all other points on the surface are receding with a Hubble law form for their velocities. There is no "special" position on the balloon's surface: a Hubble law for the velocities will be seen by all observers on the surface.

The balloon surface is a two-dimensional situation, and we live in a three-dimensional world. However, the important physical result is that when we apply the scale transformation  $R_i \rightarrow sR_i$  as in Eq. (8.3), a Hubble law form for the relative motion is obtained. This result holds in three dimensions as well as two. That is, if the universe is expanding according to  $R_i \rightarrow sR_i$ , then observers throughout the universe should observe a Hubble law form for the motion of distant galaxies. Just because we on Earth observe that all distant stars are receding from us does *not* mean that the Earth has a special place in the universe.

### 8.C The Age of the Universe

Let's summarize the ideas of Secs. 8.A and 8.B. The distance to a star can be determined from parallax and other techniques. The velocity of the star with respect to the Earth can be found from the red-shift of its light. It is observed that the further a star or galaxy is from the Earth, the faster it is moving away from us:  $V = HR$  [Hubble's law]. Thus, the universe as a whole is expanding. The motion of points on the surface of a balloon illustrates that observers at any point in the universe obtain a Hubble law form for the motion of distant galaxies. The model of the expanding universe is one component of the Big Bang model. At earlier times, the distances between galaxies were much smaller than they are now, and at future times they will be much greater.

The word "expand" does not imply that the universe has a finite size. Rather, it means that the density of the universe decreases with time. What we call the "edge" of the universe are those galaxies and other objects whose light is only now reaching us billions of years after it was emitted. When we look at galaxies ever further away, we look backwards

in time towards the Big Bang. There are more galaxies beyond the visible edge of the universe, and they will become visible to Earth as time evolves.

*Example 8.4: Assume that galaxies always have been receding from each other with the same speed that they are now. How long ago were they all at the same point?*

Suppose that the galaxies started off very close together and then separated by a distance  $R$  in time  $t$ . For motion at constant velocity,

$$R = Vt.$$

The time it took for the galaxies to separate is then

$$t = R/V = R/(HR) = H^{-1}$$

where we have used Hubble's law,  $V = HR$ , to eliminate  $V$  from the equation. The current value of  $H$  is 40-100 km s<sup>-1</sup> Mpc<sup>-1</sup>. Taking the midvalue of 70 km s<sup>-1</sup> Mpc<sup>-1</sup> and using 1 Mpc = 3.09 x 10<sup>19</sup> km,

$$\begin{aligned} H^{-1} &= 3.09 \times 10^{19} \text{ [km/Mpc]} / 70 \text{ [km s}^{-1}\text{/Mpc]} \\ &= 4.4 \times 10^{17} \text{ s} = 14 \text{ billion years.} \end{aligned}$$

Hence, if the galaxies have always receded from one another with the same velocities that they have now, then 14 billion years ago the galaxies were all at the same place. This calculation is subject to a 50% uncertainty in the value of  $H$  plus the assumption that the velocities of recession are constant.

Now, it is incorrect to assume that  $H$  is a constant, but the results of Example 8.4 are not far off. Correcting for the change in  $H$  over time, we find that the age of the universe is approximately  $(2/3)H^{-1}$ , rather than just  $H^{-1}$ . So, *based on the observed value of the Hubble parameter, we predict that the universe is about 7-14 billion years old.* Can we check this? Not directly, but

(i) we can estimate the distance to the furthest visible galaxies and determine how long it would take for light from them to reach us. This calculation just uses  $[distance] = c[time]$  with  $c = 3.0 \times 10^8$  m/s, the speed



of light. The calculated time turns out to be between 10 and 18 billion years.

(ii) we can determine the age of material from meteors *etc.*, as discussed in Chap. 6, and find that the solar system is about 4.5 billion years old. Again, this date is less than our estimated age of the universe, although it does not provide a very stringent test.

### Summary

The distance to a star can be determined from parallax and other techniques. The velocity of the star with respect to the Earth can be found by applying the Doppler effect to the red-shift of the star's light. A wave with wavelength  $\lambda$  emitted by a source moving at velocity  $v$  experiences a Doppler shift to a new wavelength  $\lambda'$  given by Eq. (8.1)

$$\lambda' = \lambda (1 + v/c) \quad [\text{source receding from observer}]$$

$$\lambda' = \lambda (1 - v/c) \quad [\text{source approaching observer}]$$

where  $c$  is the speed of the wave as seen by the observer, and where  $v \ll c$ .

It is found that the further a star or galaxy is from the Earth, the faster it is moving away from us. This is summarized in Hubble's law, Eq. (8.2)

$$V = HR$$

where  $V$  is the recession velocity of the star,  $R$  is the distance of the star from the Earth and  $H$  is the Hubble parameter, which has a value of 40 - 100 km s<sup>-1</sup> Mpc<sup>-1</sup>. Thus, the universe as a whole is expanding.

Motion of points on the surface of a balloon illustrates that observers at any point in the universe will obtain a Hubble law form for the motion of distant objects. The characteristic time for expansion according to Hubble's law is  $H^{-1}$ , and a somewhat more detailed calculation of the age of the universe yields  $(2/3)H^{-1}$ .

Further Reading

J. Gribbin, *In Search of the Big Bang* (Bantam, New York, 1986), Chaps. 1 - 5 [general reading].

S. Hawking, *A Brief History of Time* (Bantam, New York, 1988), Chaps. 1 - 3.

J. Silk, *The Big Bang* (Freeman, San Francisco, 1980), Chaps. 1 - 3.

S. Weinberg, *The First Three Minutes* (Basic, New York, 1977), Chaps. 1 - 3.

Problems

1. An opera singer on a train is singing high C ( $f = 512 \text{ s}^{-1}$ ) when the train passes by an observer stationary on the ground. What frequency does the observer hear if the train is traveling at 140 km/hr (a) towards and (b) away from the observer? (Use 342 m/s for the speed of sound in air)
2. A train blows its whistle at a frequency  $f$  as it approaches a tunnel. An observer standing near the tunnel hears the frequency as  $1.02f$ . What frequency does the engineer on the train hear for the echo?
3. Light from a cluster galaxy in Ursa Major shows a 5% fractional change in wavelength towards the red. (a) Find the velocity of galaxy. (b) If the distance to the galaxy is 700,000,000 light years, find a value for the Hubble parameter (in km / s-Mpc) and compare it with the accepted range of values.
4. A cluster nebula in Virgo is estimated to be 78,000,000 light years away. What is the relative shift  $(\lambda' - \lambda)/\lambda$  of the wavelengths of light emitted from the cluster as we observe it?
5. With better and better telescopes, we can see further into space or, equivalently, look further back in time. What is the apparent velocity of a star that emitted its light 10 billion years ago if  $H = 70 \text{ km / s-Mpc}$ ? Quote your answer in terms of  $c$ .
6. Some theories of cosmology favour a value of 50 km / s-Mpc for the Hubble parameter. What does this value yield for the age of the universe?
7. Three objects emerging from an explosion have positions as a function of time  $[x(t), y(t)]$  given by  $(3t, 0)$ ,  $(-6t, 0)$  and  $(0, 5t)$ , where the positions are in meters and the time in seconds. Using the relative motion of all three objects, show that an observer moving with object #1 concludes that the system obeys a form of Hubble's law. What is the functional dependence on time of the associated "Hubble parameter"? What is the value of this "Hubble parameter" at  $t = 2$  seconds?

