CHAPTER 9

THE BIG BANG MODEL

Far away galaxies are receding from us at tremendous speeds that increase with their distance from Earth according to Hubble's law. At face value, this implies that stars and galaxies in the universe were much closer together early in the life of the universe than they are currently. In this chapter, we discover that the universe today is filled with microwave radiation having a temperature of just 3 K^o above absolute zero. Cold as this may seem now, the expansion of the universe implies that at very early times the universe existed as a very hot and dense state with temperatures exceeding 10^{12} ^{o}K and densities throughout the universe of more than 10^{15} times the density of the Earth. This scenario for the birth of the universe is called the Hot Big Bang model.

9.A Temperature and Energy

It has been known for almost two centuries that gases at low density obey the ideal gas equation,

$$PV \quad T, \tag{9.1}$$

where the gas in question is subject to a pressure *P*, occupies a volume *V* and has an absolute temperature *T*. Although pressure is usually introduced as a force per unit area, it equivalently has units of energy density, and the left hand side of Eq. (9.1) is proportional to (but not exactly equal to!) the kinetic energy of the gas. Thus, the ideal gas equation tells us that the kinetic energy of the atoms or molecules in the gas is proportional to its temperature:

$$[kinetic\ energy] \quad T. \tag{9.2}$$

It is not difficult to obtain the relationship between kinetic energy and temperature from a molecular-level argument, but it would divert us far from our pursuit of the Big Bang. Here, we only sketch out the ideas involved, and direct the interested reader to other sources for the proof. Consider the following problem: we place a number of marbles in a box and shake the box back and forth continuously. The marbles start to move, colliding with each other and with the sides of the box. If we could determine the kinetic energy of every marble at every instant in time we would find two things:

(i) not all marbles have the same kinetic energy at the same time. Some marbles are moving slowly, some fast, so the marbles have a *distribution* of kinetic energies, as illustrated in Fig. 9.1.

(ii) a given marble exchanges energy with its surroundings through collisions, so the kinetic energy of each marble changes with time.

Because the marble energies constantly change, then the characteristics of the system should be described by observables that average over marbles and over time. Such quantities include the distribution of kinetic energies and the average kinetic energy.

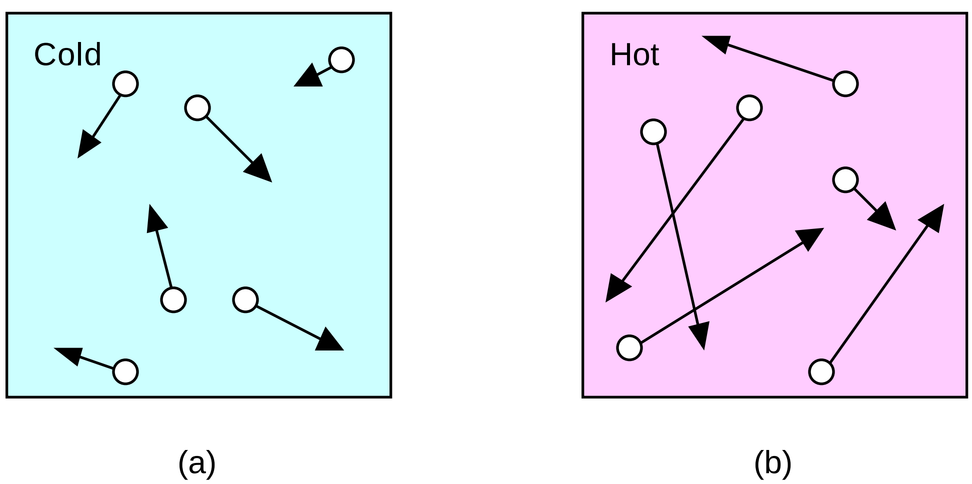

Fig. 9.1 The velocities in a gas of marbles at two temperatures: (a) is cold and (b) is hot. The arrows represent the velocity vectors.

The functional form of the kinetic energy distribution was found by Maxwell in 1859, and the time evolution of such distributions was described by Boltzmann in 1872. For an ideal gas of marbles in a three dimensional box (no internal structure to the marbles) the average kinetic energy per particle is related to the temperature of the system by

$$[\textit{average kinetic energy per particle}] = (3/2)\, k_B T \qquad (9.3)$$

where k_B is Boltzmann's constant (equal to 1.38×10^{-23} J/K^o) and T is the temperature in degrees Kelvin. Eq. (9.3), which applies in three dimensions only, is the same as Eq. (9.2) with the proportionality constant written explicitly. For those readers familiar with the ideal gas equation, k_B is just the gas constant R divided by Avogadro's number N_o. Equation (9.3) provides the link between the temperature of a system and the energies of its constituents. In fact, the temperature tells us not just the average kinetic energy but also the equilibrium distribution of kinetic energies.

Example 9.1: *What is the average kinetic energy per particle for a gas of particles at room temperature* T = *295 ^{o}K?*

From Eq. (9.3),

$$[\textit{average kinetic energy per particle}] = 3/2\, k_B T$$

$$= 3/2 \times 1.38 \times 10^{-23} \times 295 = 6.1 \times 10^{-21} \text{ J.}$$

Now, this may not look like very much energy, but remember that this is the kinetic energy of an individual particle like a molecule. Suppose the particles in question are hydrogen atoms, each with a mass of 1.7×10^{-27} kg? By rearranging the kinetic energy expression $K = (1/2)mv^2$, we can obtain a measure of atomic velocities at room temperature:

$$v = (2K/m)^{1/2} = (2 \times 6.1 \times 10^{-21} / 1.7 \times 10^{-27})^{1/2} = 2.7 \times 10^3 \text{ m/s.}$$

Thus, the velocity of a free hydrogen atom at room temperature will be in the range 3×10^3 m/s.

9.B Photon Gas

Determining the energetics of a photon gas is somewhat different from the box-of-marbles problem because not only can the photons exchange energy, they can also be created and destroyed. That is, the number of photons in the box is not fixed, but can change with time. What is important for photons (or other massless particles like neutrinos) is their *number density* (how many photons per unit volume, on average) and *energy density* (how much energy per unit volume, on average). Fig. 9.2 is a cartoon of a photon gas, illustrating that the number of photons increases with temperature.

The total energy density U_γ of electromagnetic waves in equilibrium is related to the temperature by:

$$U_\gamma = (8\pi^5 k_B{}^4 / 15 h^3 c^3)\ T^4, \qquad (9.4)$$

where k_B is Boltzmann's constant. We put a subscript γ on U to emphasize that this expression is for photons, or electromagnetic radiation. Neutrinos obey a similar, but not identical, expression. Substituting for all the constants in Eq. (9.4), one finds

$$U_\gamma = 7.565 \times 10^{-16}\ T^4 \quad (\mathrm{J/m^3}), \qquad (9.5)$$

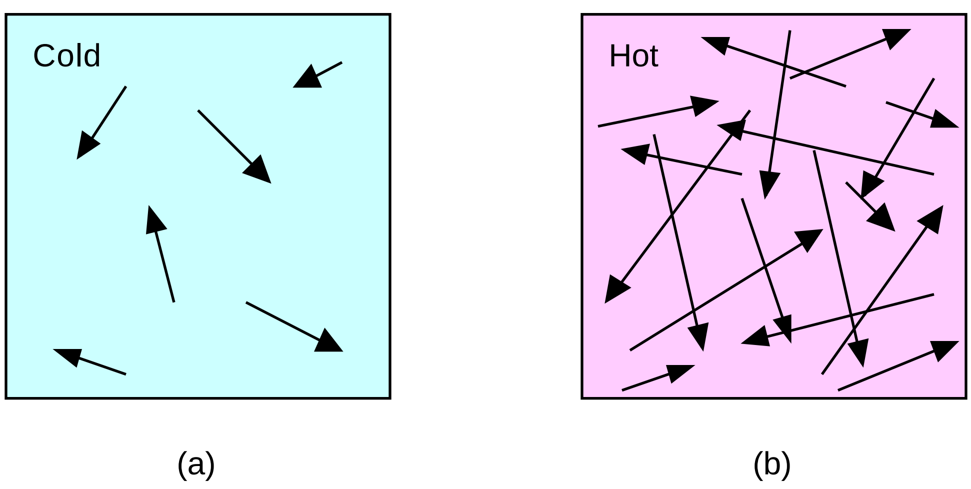

Fig. 9.2 The energies of a photon gas at two temperatures: (a) is cold and (b) is hot. The arrows represent the energies of the photons, *not* their speed. All photons travel at the speed of light!

where the temperature T must be quoted in ^oK, and the resulting U_γ has units of J/m^3.

Similarly, the number density found after summing over all photon wavelengths is

$$N_\gamma = 60.42\ (k_B T / hc)^3 \qquad (9.6)$$

or

$$N_\gamma = 2.02 \times 10^7\ T^3\ (m^{-3}). \qquad (9.7)$$

where the temperature T must be quoted in ^oK, and the resulting N_γ has units of m^{-3}. Dividing (9.5) by (9.7) the average energy per photon is then

$$[\textit{average energy per photon}] = 2.7\ k_B T. \qquad (9.8)$$

Note that for both ideal particles with mass (9.3) and photons (9.8) the typical energy scale per particle is $k_B T$.

Example 9.2: *Find the energy density and the number density of microwave photons at* T = *2.7 ^{o}K*.

We can use Eq. (9.5) to trivially find

$$U_\gamma = 7.565 \times 10^{-16}\ (2.7)^4 = 4.0 \times 10^{-14}\ J/m^3.$$

From Eq. (9.7), the number density is determined to be

$$N_\gamma = 2.02 \times 10^7\ (2.7)^3 = 400 \times 10^6\ m^{-3}.$$

We can use these numbers to perform a consistency check. The average energy per microwave is $4.0 \times 10^{-14} / 400 \times 10^6 = 10^{-22}$ J, according to the two equations above. The wavelength corresponding to this energy from $E = hc / \lambda$ is $(6.63 \times 10^{-34})(3.0 \times 10^8)/10^{-22} = 0.2$ cm. From Table 3.2, this wavelength is in the range expected for microwaves.

9.C Microwave Radiation and the Big Bang

Hubble's Law is one of the general characteristics of the universe on the large scale: it is not an effect that is seen only locally. Another observable that may be a general feature of the universe is the so-called *microwave radiation background*. This radiation was discovered inadvertantly in 1964, and is thought to be relic electromagnetic energy left over from the Big Bang.

We know from the amount of debris hitting the Earth that "empty space" is not empty. The more spectacular objects that come into our atmosphere from space are meteors. There is also a steady stream of elementary particles coming from the Sun (mainly neutrinos) and from other sources (cosmic rays). There are a huge number of neutrinos passing through us from the Sun: about 6×10^{14} solar neutrinos strike a square meter of the Earth directly facing the Sun every second. In comparison, the flux of cosmic rays hitting the surface of the Earth is much lower, a couple of hundred per m^2 per second. About 75% of the cosmic rays that reach sea level are muons, but there are also protons and heavier nuclei.

In 1964, Arno Penzias and Robert W. Wilson were searching for microwave radiation from distant galaxies. Microwaves are electromagnetic radiation having wavelengths approximately in the range 0.1 to 30 cm. Penzias and Wilson were using very sensitive equipment to study microwaves with a 7.35 cm wavelength. What they found was that the universe appeared to be filled with low level microwave radiation. An amusing account of their discovery, and how they determined that the microwave "noise" they were observing was not some artifact of their detector, can be found in Steven Weinberg's book *The First Three Minutes.*

As explained in Sec. 9.B, electromagnetic radiation in equilibrium has a distribution of energies from low to high, and the distribution is characterized by a temperature T. It was the study of this distribution which originally led Planck to propose quantization of emission and absorption of electromagnetic radiation in 1900. Penzias and Wilson were able to examine only one wavelength in their study of microwaves, but from the energy density of the microwaves at that wavelength they were able to determine a temperature. Verification that the microwaves actually have an equilibrium distribution of wavelengths took many more years of observation at different wavelengths. Current observation supports a temperature of 2.73 ± 0.05 ^{o}K. Recent experiments using satellite-based detectors confirm that the 3 ^{o}K microwave radiation is present uniformly in all directions of space; it is not associated with specific stars or the Milky Way.

Let's now take the observations of the expanding universe and of the 3 ^{o}K microwave radiation at face value and construct what is called the Big Bang model of the universe. There are alternative interpretations to these observations than what we are about to make, but only the Big Bang model has been used to make extensive testable predictions. It is the verification of these predictions that gives the Big Bang model its credibility. The Big Bang picture still is not complete: the cause of the Big Bang is not resolved nor has it been determined why the universe is mainly matter rather than an equal mixture of matter and antimatter. These are questions that current research is trying to answer.

So, taking today's value of H allows us to predict that the universe is 7 - 14 billion years old. Let's now join this interpretation of Hubble's law with the 3 ^{o}K microwave background which we assume is present everywhere in the universe today. If the universe was more dense at earlier times then the energy density must have been larger. If the energy density was larger, then the universe must have been hotter at earlier times, according to Eq. (9.4) for U_γ. This is the essence of the Hot Big Bang model: in "the beginning", the universe was very hot and very dense. As time evolved, the immense pressure of the high energy particles (photons, protons, electrons...) caused the universe to expand and cool. In the next chapter, we examine the first few minutes in the life of the universe in more detail, and evaluate the consequences of the Big Bang model for the distribution of chemical elements.

Summary

The temperature of a system is a measure of the average kinetic energy of its constituents. For a gas of objects like marbles or atoms, the average kinetic energy at absolute temperature T is given by Eq. (9.3):

$$[\textit{average kinetic energy per particle}] = (3/2)\ k_B T.$$

For photons, the energy density U_γ and number density N_γ are related to the photon temperature T by Eqs. (9.5) and (9.7)

$$U_\gamma = 7.565 \times 10^{-16}\ T^4 \qquad (\mathrm{J/m^3})$$

$$N_\gamma = 2.02 \times 10^{7}\ T^3 \qquad (\mathrm{m^{-3}})$$

where the temperature T must be quoted in ^{o}K. The average energy per photon is then

$$[\textit{average energy per photon}] = 2.7\ k_B T$$

according to Eq. (9.8).

The Big Bang model unites the picture of the expanding universe with the 3 ^{o}K microwave radiation background. In "the beginning", the universe was very hot and very dense. As time evolved, the immense pressure of the high energy particles (photons, protons, electrons...) caused the universe to expand and cool.

Further Reading

R. L. Armstrong and J. D. King, *Mechanics, Waves and Thermal Physics*, (Prentice-Hall, Englewood Cliffs, N.J., 1970), Chap. 17.

J. Gribbin, *In Search of the Big Bang* (Bantam, New York, 1986), Chaps. 1 - 5 [general reading].

S. Hawking, *A Brief History of Time* (Bantam, New York, 1988), Chaps. 1 - 3.

J. Silk, *The Big Bang* (Freeman, San Francisco, 1980), Chaps. 1 - 3.

S. Weinberg, *The First Three Minutes* (Basic, New York, 1977), Chaps. 1 - 3, Mathematical Supplement.

Problems

1. In a bumper-car ride at an amusement park, padded cars continually collide with each other and the walls of the ride. Suppose that each car weighs 100 kg and has an average speed of 3 m/s. Regarded as an ideal gas, what is the temperature of the collection of bumper cars?

2. When the universe had a temperature triple what it is today, how many photons did it have per unit volume?

3. (a) What must the temperature of a system of photons be such that the number density of photons is equal to the number density of nucleons in the nucleus? (b) What is the average energy per photon of such a collection of particles? (c) How does this compare with the typical nuclear binding energy per particle?

*4. (a) Calculate the *Q*-value for the reaction $4p \rightarrow {}^4He + 2e^+ + 2\nu_e$. Include the reactions $e^+ + e^- \rightarrow 2\gamma$ which occur when the positrons annihilate with electrons in the medium surrounding the protons.
(b) Use the observed stellar luminosity to calculate the number of neutrinos produced in the Sun per second from this reaction.
(c) What would the number density of nuclear-produced ν's be if all the neutrinos produced in the Sun's 5×10^9 year lifetime filled a spherical volume with a radius equal to the Sun-Pluto average distance?
(d) Compare this number with the number density of photons at 2.7 ^oK calculated in Example 9.2.

5. The temperature of the universe today is 2.7 ^oK.
(a) What is the average speed of a *proton* at this temperature?
(b) Find the de Broglie wavelength corresponding to the proton in (a).