Linear Motion - multiple choice

What is the cross product **A** x **B** of the vectors **A** = (1,0,1) and **B** = (-1,0,1)?
(a) (0,0,2) (b) (0,-1,0) (c) (0,2,0) (d) (0,-2,0) (e) none of [a]-[d]
The two vectors each have length √2 and lie in the xz-plane, making angles of +45^{O} and -45^{O} with respect to the z-axis, respectively. The magnitude of the cross product is then $(\sqrt{2})^2$ sin90º = 2. By the right-hand rule, the direction is in the -y direction. Hence, (0,-2,0)

Consider two vectors **A** = (2,2,0) and **B**(4,-1,0). If a vector **C** = (1,0,0) is added to **B**, what is the change in the cross-product **A** x **B** (*i.e.*, the new value minus the old value).
[a] (0,0,-2) [b] 2 [c] (0,0,2) [d] (-10,0,0) [e] none of [a]-[d]
The difference in the cross products is **A**x(**B** + **C**) - **A**x**B** = **A**x**B** + **A**x**C** - **A**x**B** = **A**x**C**. Thus, the difference is (0, 0, 0 - 1•2) = (0, 0, -2).

Find the vector **C** in the cross product **C** = **A** x **B**, if **A** = (0,1,0) and **B** = (1,0,0).
(a) (1,1,0) (b) (0,0,1) (c) (0,0, -1) (d) (1, -1,0) (e) (0,0,1)
Either use the explicit Cartesian representation, or the following logic:
Vectors **A** and **B** are unit vectors pointing along the *y* and *x* axes, respectively. Thus, the cross product must be perpendicular to the *xy*-plane, pointing along the +*z* or -*z* directions. From the right-hand rule, the direction must be negative. Lastly, the magnitude of **C** equals the product of the magnitudes of **A** and **B** (each of which equals 1) and the sine of the angle between them. Here, the sine equals 1, so the resulting vector is **C** = (0,0, -1).

The displacement of an object for a round trip between two locations
(a) is always greater than zero.
(b) is always less than zero.
(c) is zero.
(d) is not zero.
(e) can have any value.
For a round trip, the displacement must vanish because the initial and final positions coincide.

An object moves with constant speed in a straight line. Which of the following statements must be true?
(a) No force acts on the object.
(b) A single constant force acts on the object in the direction of motion.
(c) The net force acting on the object depends on the value of the speed.
(d) The net force acting on the object is zero.
(e) The net force acting on the object cannot be determined.
The acceleration is determined by the net force; hence, a vanishing acceleration implies that the net force vanishes, but not that there is no force.

As a sky diver falls through the air, her terminal speed

(a) depends on her mass.
(b) depends on her body's orientation.
(c) depends on the local value of the acceleration due to gravity.
(d) depends on the density of air.
(e) is described by all of the above.

All of these features apply to terminal velocity.

A man riding in an elevator has an apparent weight greater than his actual weight. Which one of the following statements could be true?

(a) The elevator moves upward with constant speed.
(b) The elevator moves downward with constant speed.
(c) The elevator moves upward with decreasing speed.
(d) The elevator moves downward with decreasing speed.
(e) The elevator moves downward with increasing speed.

If his apparent weight is less than his real weight, then his acceleration must be upward.
This could arise in two situations:
v is upward and increasing or *v* is downward and decreasing.

A moving box receives an impulse directed to the north. As a result, the box

(a) has a velocity pointing north.
(b) is north of the equator.
(c) stops.
(d) has a velocity towards the south.
(e) accelerates towards the north.

The direction of the acceleration must be the same as the net force, but the velocity may be in any direction.

A lamp of mass *m* hangs from a spring scale that is attached to the ceiling of an elevator. When the elevator is stopped at the fortieth floor, the scale reads mg. What does it read while the elevator decends toward the ground floor at a constant speed?

(a) More than *mg*.
(b) Less than *mg*.
(c) *mg*.
(d) Zero.
(e)This cannot be answered without knowing how fast the elevator is descending.

If the speed of the lamp with respect to the elevator cage is constant, then there is no relative acceleration, and the scale must still read *mg*. Hence: (c).

A falling abject experiences the drag force due to air resistance. Which statement is NOT true?

(a) The drag force depends on the falling speed.
(b) The faster the ball falls, the stronger the air resistance.
(c) The mechanical energy of the object is conserved.
(d) The speed of the object will reach a maximum value and then stop changing.
(e) The net force acting on the object will eventually reach zero.

The mechanical energy of the object must decrease because of the dissipative force.

The displacement of an object during any time interval is always

(a) greater than or equal to the distance it travels during the same time interval.
(b) less than or equal to the distance it travels during the same time interval.
(c) equal to the distance it travels during the same time interval.
(d) greater than the distance it travels during the same time interval.
(e) much greater than the distance it travels during the same time interval.

Distance increases monotonically with time, with the result that displacement can never exceed distance.

The change in the position of a boat is recorded every hour. Viewing the water's surface as an *xy* coordinate system, the change in the (*x*,*y*) coordinates of the boat over two hours is observed to be (0,2) and (1, -1) (quoted in km). What is the average speed of the boat divided by its average velocity?

(a) 4.0 (b) 2.4 (c) 1.00 (d) 2.0 (e) 1.08

The total distance covered in two hours is $2 + (1^2 + [-1]^2)^{1/2} = 2 + \sqrt{2}$.
The total displacement vector is (0,2) + (1, -1) = (1,1), which has a length of $\sqrt{2}$.
Thus, *speed* / *velocity* = *distance* / *displacement* = $(2 + \sqrt{2}) / \sqrt{2} = 1 + \sqrt{2} = 2.4$.

A jogger changes her speed several times during the first four minutes of her run: 50 m/min for the first 30 seconds, 70 m/min for the next 1.5 minutes, and 90 m/min for the last two minutes. What is her average speed in m/min?

(a) 70 (b) 78 (c) 80 (d) 73 (e) 75

The total distance covered by the jogger is 0.5 x 50 + 1.5 x 70 + 2.0 x 90 = 310 m.
Hence, the average speed is 310/4 = 78 m/min.

An automobile is moving towards the east at 50 km/hr, and the wind is from the north at 50 km/hr. Which vector represents the wind velocity as observed by a passenger in the car, where the first component points to the east and the second component to the north?

[a] (50, -50) [b] (-50, 50) [c] (50, 50) [d] (100, 0) [e] (-50, -50)

Subtract the car velocity from the wind velocity to obtain the relative velocity:

$v_{wind} - v_{car}$ = (0, -50) - (50, 0) = (-50, -50).

An object moving along the *x*-axis has a position *x* = +3 m and is moving with a velocity of -4 m/s. The object is slowing down. The sign of its acceleration is

(a) 0 (b) negative (c) positive (d) <0 until it stops, then >0
(e) impossible to tell from the data given.

The initial velocity is negative, but tends to zero as the object stops. Thus, $\Delta v > 0$ and the acceleration must be positive.

Starting at rest, an object falls a height *h* in time *t*. Assuming that the only force on the object is its gravitational attraction to the Earth, how far does the object fall in an elapsed time of 3*t*, starting from rest?

(a) 3*h* (b) 9*h* (c) 27*h* (d) 6*h* (e) *h*/3

The position of an object with zero initial velocity and subject to constant gravitational acceleration $-g$ is $z = -gt^2/2$. Thus, if the elapsed time is $3t$, the height fallen must be $9h$.

An arrow shot vertically into the air rises to a height h before it falls back to Earth. If the speed of the arrow as it leaves the bow is doubled, how high could the arrow rise?

(a) $8h$ (b) h (c) $16h$ (d) $2h$ (e) $4h$

The position of an object with zero final velocity (when the arrow reaches its maximum height) and subject to constant gravitational acceleration $-g$ is

$$h = -(v_f^2 - v_i^2)/2g = v_i^2/2g.$$

Thus, if the initial velocity is doubled, then the maximum height increases by a factor of $2^2 = 4$.

A girl is standing in an elevator traveling upwards at a constant speed v. She observes that it takes a time t for a penny to drop from her hand to the floor of the elevator. If the elevator were traveling downwards at a constant speed of $2v$, how long would it take for the penny to drop?

(a) t (b) $2t$ (c) $4t$ (d) $t/2$ (e) none of [a]-[d]

Because the velocity of the elevator is constant in both situations, then the time for relative motion is unchanged. Hence, it takes t whether the elevator is at rest or moving with constant velocity.

A projectile is fired at an angle of 35° above the horizontal. At the highest point in its trajectory, its speed is 200 m/s. If air resistance is neglected, what is the initial horizontal component of the projectile's velocity (in m/s)?

(a) 0 (b) 200 cos35° (c) 200 sin35° (d) 200/cos35° (e) 200

At the highest point in the trajectory, the velocity is entirely horizontal. Hence, the horizontal component must be 200 m/s throughout the flight of the projectile.

A golfball is shot horizontally from the top of a vertical cliff of height h. If the initial speed of the ball is v, how far away from the bottom of the cliff does the ball land?

(a) $v(2h/g)^2$ (b) $gt^2/2$ (c) $v(2h/g)^{1/2}$ (d) $vg / 2h$ (e) none of (a)-(d)

Resolving the motion into components:

horizontal distance = $R = vt$

vertical distance = $h = gt^2 /2$.

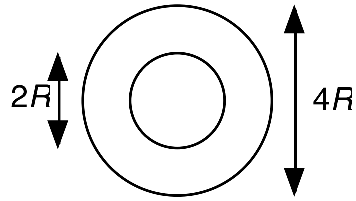
---> $R = v(2h / g)^{1/2}$.

A car moving with constant speed in a circular path experiences a centripetal acceleration a_c. If the speed of the car increases by a factor of three, but the circular path remains the same, what is the new centripetal acceleration in terms of the original a_c?

(a) $3a_c$ (b) $a_c/9$ (c) $a_c/3$ (d) $9a_c$ (e) a_c

Since the centripetal acceleration is proportional to v^2/R, then the acceleration increases by 3^2=9 if the speed increases by 3.

Two cars are travelling in concentric circles of radius R and $2R$, as in the diagram.



Each car completes its own circle in the same time T. What is the centripetal acceleration of the outer car divided by that of the inner car?

(a) 1 (b) 2 (c) 1/2 (d) $4\pi^2$ (e) $8\pi^2$

In general, $a_c = v^2/r$ with $v = 2\pi r/T$, so that $a_c = (2\pi r/T)/r = 4\pi^2 r/T^2$. Hence,

$a_c(\text{outer})/a_c(\text{inner}) = 2R / R = 2$.

A Ferris wheel ride at a circus has a diameter of 20 m. What must be the speed of the perimeter of the wheel to produce a centripetal acceleration (at the perimeter) the same as that of gravity (use $g = 10$ m/s^2; answers quoted in m/s)?

(a) 10 (b) 14 (c) 200 (d) 20 (e) 100

Centripetal acceleration $a_c = v^2/R$. Demanding $a_c = 10$ m/s^2,

$v = (a_c R)^{1/2} = (10 \times 20 / 2)^{1/2} = 10$ m/s.

An object at the end of a string is swung in a circular path at constant speed with a period T. If the period is shortened to $T/2$ without changing the radius of the circle, what is the new centripetal acceleration in terms of the original acceleration a?

(a) $4a$ (b) $a/2$ (c) $a/4$ (d) a (e) $2a$

The centripetal acceleration a is equal to $2\pi v/T$. In turn, the velocity is $2\pi R/T$, so the acceleration is $4\pi^2 R/T^2$. Thus, if the period is decreased by a factor of 2, the acceleration is increased by a factor of 4.

A car travels at constant speed on a circular test track of radius R, completing each lap around the track in time T. The centripetal acceleration of the car, a_c, is at the limit where the tires start to skid. If the test track were three times as large (i.e., had a radius of $3R$), what would be the shortest period in which the car could complete a lap without its acceleration exceeding the same a_c as for the smaller track?

(a) $3T$ (b) $9T$ (c) $T / 3$ (d) $\sqrt{3}\ T$ (e) none of (a - d)

The speed of the car is $2\pi R / T$, so the centripetal acceleration is proportional to R / T^2, or $T^2 \sim R / a_c$. If the centripetal acceleration is to remain unchanged in the new track, then the period must be at least $\sqrt{3}\ T$.

The total linear momentum of a system of particles will be conserved if:

(a) the positions of the particles do not change with respect to each other
(b) one particle is at rest
(c) no external force acts on the system
(d) the internal forces equal the external forces
(e) the particles do not rotate about their axes.

Force is the rate of change of linear momentum. Hence, there must be no external force for the rate of change of momentum to be zero.

An 80 kg man on ice skates pushes a 40 kg boy, also on skates, away from him with a force of 100 N. What is the force exerted by the boy on the man?
(a) 200 N (b) 100 N (c) 50 N (d) 40 N (e) 0
From Newton's third law, action = reaction, so the force of the boy on the man must be 100 N.

A mass travelling with an initial velocity v_i is brought to rest by a force F. If the initial velocity is doubled to $2v_i$, what force is necessary to bring the object to rest in the same distance as the first situation?
(a) F (b) $2F$ (c) $3F$ (d) $4F$ (e) $8F$
Using $a = (v_f^2 - v_i^2) / 2d$, then
$v_i \to 2v_i$ implies $a \to 4a$ and hence $F \to 4F$.

A constant force F is applied for a time T to a body initially at rest. The object covers a distance D during the time T. If the force is doubled and applies for $2T$, what distance is covered by the object, starting from rest?
(a) D (b) $2D$ (c) $4D$ (d) $6D$ (e) $8D$
If the force is doubled, so is the acceleration. Using $d = at^2/2$, the distance becomes
$(2a)\ (2T)^2 / 2 = 8\ (aT^2/2) = 8D$.

Having decided to elope, you construct a rope from nylon stockings, down which you will slide from your second-floor bedroom. The rope can withstand a maximum tension of 300 N without breaking. Your mass is 61.2 kg. What is the smallest acceleration down which you can slide without breaking the rope?
(a) 9.8 m/s^2 (b) 4.9 m/s^2 (c) 0 m/s^2 (d) 2.4 m/s^2 (e) 19.6 m/s^2
The net force on the rope is $mg - ma = 300$N. Thus
$a = g - 300/m = 9.8 - (300/61.2) = 4.9$ m/s^2.

A vertical rope is attached to an object of mass M. What is the tension in the rope in order to give the mass an upward acceleration of $3g$?
(a) $3g$ (b) $4g$ (c) Mg (d) $3Mg$ (e) $4Mg$
A free-body diagram shows that the net force on the mass is $T - Mg$, which must give the object an upward acceleration of $3g$. From $F = Ma$,
$T - Mg = 3Mg$
or
$T = 4Mg$.

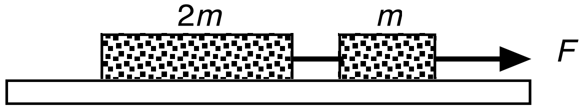
The weight of an object on the Moon is 1/6 the weight of the same object on the Earth. If an object moves with a speed v and kinetic energy K on the Earth, what is the kinetic energy of the same object moving with speed v on the Moon?
(a) $36K$ (b) $6K$ (c) K (d) $K/6$ (e) $K/36$
Kinetic energy is the same, K, as it depends on mass and velocity, not weight.

The acceleration due to gravity on the Moon is only 1/6 that on the Earth. An object with a weight of 60 N on Earth is transported to the Moon. What is its mass as measured on the Moon? Use $g = 10$ m/s^2 on Earth.

(a) 6 kg (b) 1 kg (c) 0.6 kg (d) 60 kg (e) 10 kg

Mass on Earth = $w / g = 60 / 10 = 6$ kg. The same mass must be measured on the Moon.

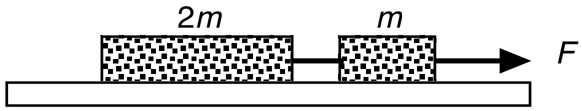
Two masses (m and $2m$) are attached to one another by a string as illustrated. A force F acts on mass m to accelerate the whole system. What is the magnitude of the net force on m?



(a) $F/3$ (b) F (c) $2F/3$ (d) $F/2$ (e) $3F/2$

The overall acceleration of the blocks is $a = F/3m$. But this acceleration is the result of the net force on a given block. Hence, the net force required to produce an acceleration of a on m is $F_{net} = ma = mF/(3m) = F/3$.

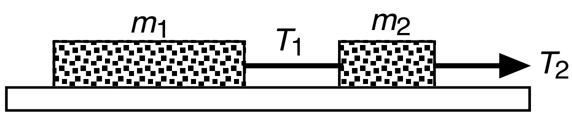
Two masses (m and $2m$) are attached to one another by a string as illustrated. A force F acts on mass m to accelerate the whole system. What is the magnitude of the force on mass $2m$?



(a) $F/3$ (b) F (c) $2F/3$ (d) $F/2$ (e) $3F/2$

The overall acceleration of the blocks is $a = F/3m$. The net force required to produce an acceleration of a on $2m$ is $F = 2ma = 2mF/(3m) = 2F/3$.

Two masses (m_1 and m_2) are connected by a massless string and accelerated uniformly on a frictionless surface, as shown. The ratio of the tensions T_1/T_2 is given by:



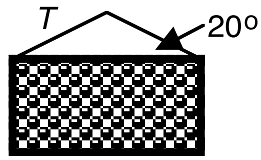
(a) m_1/m_2 (b) m_2/m_1 (c) $(m_1+m_2)/m_2$ (d) $m_1/(m_1+m_2)$ (e) $m_2/(m_1+m_2)$

At m_1, an unbalanced force results in an acceleration $T_1 = m_1 a$.

At m_2, an unbalanced force results in an acceleration $T_2 = T_1 + m_2 a = (m_1+m_2)a$.

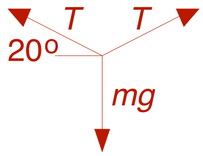
Hence, $T_1/T_2 = m_1/(m_1+m_2)$.

A picture frame of mass 0.5 kg is held in place by a massless wire making an angle of 20^o with respect to the horizontal. What is the tension in the wire (in Newtons)?



(a) 7.2 N (b) 4.9 N (c) 2.5 N (d) 14.3 N (e) 0.84 N

Consider the forces at the nail. Taking components in the vertical direction

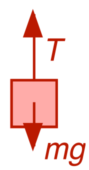


$2T \sin 20^o = mg$ hence $T =$ = 7.2 N.

A vertical rope is attached to an object that has a mass of 40 kg and is at rest. The tension in the rope needed to give the object an upward speed of 3.5 m/s in 0.70 seconds is

(a) 392 N (b) 192 N (c) 532 N (d) 592 N (e) 140 N

The free-body diagram for the mass is



The tension in the string is unbalanced, such that

$T - mg = ma$ or $T = m(g - a)$

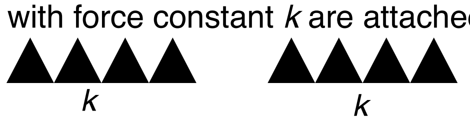
The acceleration of the object is $a = (v_f - v_i)/t = 3.50 / 0.7 = 5$ m/s^2. Hence, $T = 40 \cdot (9.8 - 5) = 192$ N.

The radius of the planet Xion is three times that of the Earth, although the densities of the planets are the same. What is the acceleration due to gravity on the surface of Xion?

(a) $27g$ (b) g (c) $3g$ (d) $g / 3$ (e) $g / 9$

From the gravitational force expression $F = G M_1 M_2/R^2$, the acceleration due to gravity on the surface of a planet is $a_g = G M / R^2$. Since the mass of a planet with constant density grows like R^3, then the gravitational acceleration must grow like $R^3/R^2 = R$. Hence, the acceleration due to gravity on Xion must be $3g$.

Two identical springs, each with force constant k are attached in series

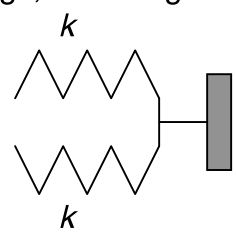


What is the effective force constant of the springs, taken together?

(a) k (b) $2k$ (c) $k / 2$ (d) $4k$ (e) $k / 4$

If a single spring is stretched by a distance x, then the spring system must stretch by $2x$. But the force exerted by the individual spring is kx , and must be constant along the system. If the effective force constant of the system is equal to the applied force divided by the displacement from equilibrium, then the effective force constant is $kx / 2x = k / 2$.

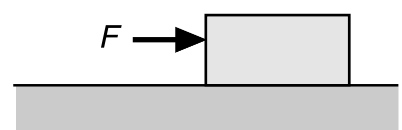
Two identical springs, each with force constant k, are attached in parallel. What is the effective force constant of the springs, taken together?



(a) $2k$ (b) $4k$ (c) k (d) $k/4$ (e) $k/2$

Because the springs are in parallel, it takes twice as much force to displace them a distance x as it does to displace a single spring by an amount x. Hence, the effective spring constant is double that of a single spring: $2k$.

The coefficients of friction between a 10 kg block and a table are $\mu_s = 0.8$ and $\mu_k = 0.6$. If a horizontal force $F = 100$ N is applied to the block, what is the net force that the block experiences? (Use $g = 10$ m/s^2)



(a) 20 N (b) 40 N (c) 60 N (d) 80 N (e) 100 N

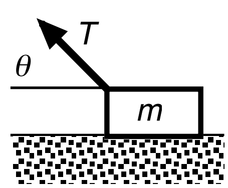
The maximum force from static friction is $0.8\ mg = 0.8 \cdot 10 \cdot 10 = 80$ N, which is less than the applied force. Hence, the block will slide, but is opposed by the force from kinetic friction of $0.6\ mg = 0.6 \cdot 10 \cdot 10 = 60$ N. The net force on the block is then 40 N.

A 10 kg block is at rest on a level surface, where the coefficient of static friction is 0.5 and that for kinetic friction is 0.3. A horizontal force of 40 N is applied to the block. What is the frictional force on the object?

(a) 98 N (b) 40 N (c) 49 N (d) 29 N (e) 0 N

The minimum force required to move the object is $\mu_s mg = 10 \times 9.8 \times 0.5 = 49$ N. This threshold is larger than the applied force, so the frictional force must just be the applied force itself, 40 N.

A block of mass m is pulled across a rough surface with a coefficient of kinetic friction



μ_k, as shown in the drawing. The tension in the rope pulling the block is T. What is the frictional force on the block if it is pulled at a constant non-zero acceleration a?

(a) $\mu_k mg$ (b) $T\cos\theta - ma$ (c) $\mu_k(T - mg)$ (d) $\mu_k T\sin\theta$

(e) none of (a)-(d)

The frictional force reduces the applied force to produce a constant acceleration:

$ma = T\cos\theta - f$ ----> $f = T\cos\theta - ma$.

A spring under a compression x has a potential energy V_o. When the compression is doubled $2x$, the potential energy stored in the spring is

(a) V_o (b) $2V_o$ (c) $3V_o$ (d) $8V_o$ (e) None of [a-d] is correct.

The potential energy of a spring is quadratic in the displacement from equilibrium. Thus, if the compression is doubled, the potential energy must increase by a factor of four.

A car of mass 1000 kg rolls without friction down Burnaby Mountain for a total vertical distance of 300 m. If the car started from rest at the top of the hill, what is its speed at the bottom of the hill in km/hr?

(a) 195 (b) 54.2 (c) 76.7 (d) 2940 (e) 276

Using conservation of energy,

$mgh = mv^2/2$ -----> $v = \sqrt{(2gh)}$.

$= \sqrt{(2 \times 9.8 \times 300)} = \sqrt{5880} = 76.7$ m/s.

Converting to km/hr, $v = 76.7 \times 3600 / 1000 = 276$ km/hr.

A 50 gram piece of cake is dragged across a 2 meter long strip of sandpaper, losing about half of its mass by the time it reaches the end of the strip. Only just enough force is applied to the cake to keep it moving. How much work was done in moving the cake? (Take $g = 10$ m/s^2 and the coefficient of friction μ for cake on sandpaper to be 0.8.)

(a) 60 J (b) 0.3 J (c) 0.6 J (d) 0.4 J (e) 0.8 J

The average weight of the cake is $(mg + mg/2) / 2 = 3mg/4$. Hence, the average frictional force is just $f = \mu(3mg/4)$. Multiplying the average force to obtain the work gives

$W = fD = \mu(3mg/4)D = 0.8 \cdot 0.75 \cdot 0.050 \cdot 10 \cdot 2 = 0.60$ J.

The potential energy experienced by a ball at the bottom of a particular wine glass is ax^4, where $a = 1$ J/m^4. What force is experienced by the ball at $x = -2$ m?

(a) 16 N (b) 32 N (c) -32 N (d) -16 N (e) none of (a) - (d)

The relation between force and potential is $F = -\mathrm{d}U / \mathrm{d}x$. In this problem, $F = -\mathrm{d}(ax^4) / dx$, or

$F = -4ax^3$. At $x = -2$, then $F = -4 \cdot 1 \cdot (-2)^3 = +32$ kg-m/s^2.

A particle is subject to a conservative force that does not depend on distance x. How does its potential energy change with x?

(a) $1/x^2$ (b) x (c) $1/x$ (d) constant (e) x^2

Starting with the definition of work $W = \mathbf{F} \cdot \Delta\mathbf{x}$, the work must be proportional to x if F is constant. The work done on a system raises its potential energy, so we expect $\Delta U \sim x$.

Using a cable, an engine applies a constant force to move a mass m across a frictionless table in time t, starting from rest. The average power generated by the engine during this process is P. If the same engine delivered the same force to a mass $2m$ in moving it across the table, what would be the average power expended by the engine (in terms of the original P)?

(a) P (b) $\sqrt{2}\, P$ (c) $P / 2$ (d) $P / \sqrt{2}$ (e) $2P$

The force and the distance, and hence the work done by the engine, is the same in both situations. The average power then depends upon the time t required for the motion, via

$d = at^2/2$ --> $t = (2d/a)^{1/2}$.

Since the force is constant, the acceleration experienced by $2m$ is half of that experienced by m, and hence $t_{2m} = \sqrt{2}\, t_m$. Therefore,

$P_{2m} = P_m / \sqrt{2}$.

The power required to keep a car moving at a constant speed v against turbulent drag is P. What is the power required to move the car at $2v$?

(a) P (b) $2P$ (c) $4P$ (d) $8P$ (e) none of (a) - (d)

The drag force in the presence of turbulence grows quadratically with the speed

$F_{drag} \sim v^2$.

But power is given by $P = \mathbf{F} \cdot \mathbf{v}$, so the power grows like v^3. Thus, if the velocity is doubled, the power increases by $2^3 = 8$.

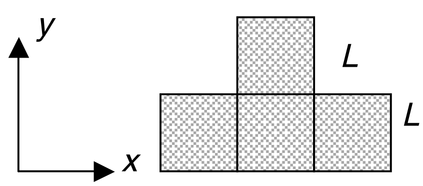
What is the exponent n in the expression [*power*] $\propto$ [*speed*]^n for a drag force that varies with speed as $F_{drag} \propto v^2$?

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

The power P associated with a force **F** acting on an object moving with velocity **v** is $P = \mathbf{F} \cdot \mathbf{v}$. In this problem, $F \propto v^2$, so

$P \propto v^2 \cdot v \propto v^3$.

Four squares of equal mass and dimension L x L are arranged as shown. What is the y-component of their centre-of-mass?



(a) $L/2$ (b) $3L/2$ (c) L (d) $3L/4$ (e) $L/4$

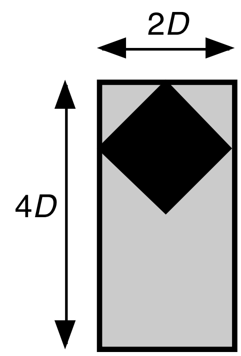
The centre-of-mass of each block lies at its centre, so the cm of the whole system is

$y = (1/4M) \cdot \{3 \cdot (1/2)\, ML + (3/2) ML\}$

$= (L/4) \cdot (3/2 + 3/2)$

$= 3/4\, L$.

A uniform rectangular sheet of height 4 D and width 2 D has a square hole cut in it, as in the diagram. Find the height of the centre-of-mass position of this object, as measured from the bottom of the sheet.



(a) (5/3)D (b) 3D (c) (7/3)D (d) 2D (e) none of (a) - (d)

Consider the problem as a sheet of positive mass plus a hole of negative mass. The sheet has a cm position of 2D, and the hole has 3D. Thus, the position of the cm in the z-direction is

$$r = [8D^2 \cdot 2D - (\sqrt{2}\, D)^2 \cdot 3D] / [8D^2 - (\sqrt{2}\, D)^2]$$

$$= [16D^3 - 6D^3] / [8D^2 - 2D^2] = (10/6)\, D = (5/3)\, D.$$

Two objects have the same mass m and velocities (v,0) and (0,v). What is the magnitude of their centre-of-mass velocity?

(a) $v/2$ (b) 0 (c) $\sqrt{2}v$ (d) 2v (e) $v/\sqrt{2}$

Since the total momentum P_{tot} is equal to $\Sigma_i m_i v_i$, then

$$V_{cm} = P_{tot} / M_{tot} = M_{tot}^{-1} \Sigma_i m_i v_i.$$

With $M_{tot} = 2m$, the centre-of-mass velocity is 1/2 (v, v). The magnitude of this vector must be $1/2 \cdot (v^2 + v^2)^{1/2} = v / \sqrt{2}$.

A car of mass m travels at a speed v towards a stationary truck of mass 3m. What is the speed of the centre-of-mass of the system?

(a) v (b) $v/2$ (c) $3v/4$ (d) $v/4$ (e) $v/3$

The centre-of-mass velocity v_{cm} is given by

$$v_{cm} = m_{tot}^{-1} \Sigma_i m_i v_i = (4m)^{-1} (mv + 3m \cdot 0) = v/4.$$

Two cars leave an intersection at the same time, with the same speed v. Car **A** heads due north, and car **B** heads due east. What is the relative velocity of car **B** as seen from car **A**?

(a) $\sqrt{2}\, v$, south-east (b) $\sqrt{2}\, v$, north-east (c) v, east (d) 2v, south-east (e) 2v, north-east

The velocity vectors for cars **A** and **B** are (0, v) and (v, 0) respectively, for x=east and y=north. Correcting for the motion of car **A**, the velocity of **B** is (v, 0) - (0,v) = (v, -v), which has a magnitude of $\sqrt{2}\, v$ and points to the south-east.

An object of mass m, moving on a frictionless surface, collides with, and sticks to, a second object of mass m, initially at rest. Together, they recoil with a total kinetic energy K. If the stationary object had a mass of $2m$, instead of m, what would be the kinetic energy of the pair?

(a) $K/3$ (b) $2K$ (c) $3K/2$ (d) $2K/3$ (e) $K/2$

By conservation of momentum, the momentum of the pair of objects is p, and this does not depend on the mass of the struck object. After the collision, the kinetic energy K of the first case is

$$K = p^2/2(2m) = p^2/4m.$$

In the second case, the kinetic energy K_2 is

$$K_2 = p^2/2(3m) = p^2/6m.$$

Thus, $K_2/K = 4/6 = 2/3$.

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Object A with mass m is travelling in the positive x direction when it collides inelastically with a body of mass $2m$ and comes to a complete stop. If object A has an initial velocity v, what is the kinetic energy of object B?

(a) $mv^2/2$ (b) $mv^2/4$ (c) mv^2 (d) 0 (e) $2mv^2$

Initial momentum = mv = final momentum = p_B

Kinetic energy of B is therefore $K = p_B^2 / 2m_B = (mv)^2/4m = mv^2/4$.

Two masses M and $5M$ are at rest on a frictionless horizontal table with a compressed (massless) spring between them. When the spring and masses are released, what share of the spring's potential energy is carried off as kinetic energy by mass M?

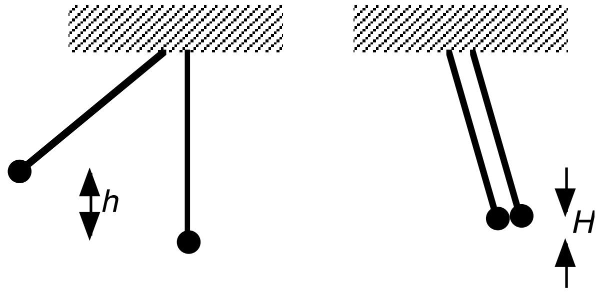
(a) 4/5 (b) 1/5 (c) 1/6 (d) 1/2 (e) 5/6

Each mass recoils with the same momentum p. Hence, the kinetic energies of the two masses are $p^2/2M$ and $p^2/10M$, respectively, for a total of $6p^2/10M = 3p^2/5M$. The fraction of this energy carried by mass M is

$$(p^2/2M) / (3p^2/5M) = 5/6.$$

Two identical masses are hung from massless strings of the same length. One mass is released from a height h above its free-hanging position, and strikes the second mass which is initially at rest. The two masses stick and move off together. To what height H

do they rise?



(a) $3h/4$ (b) $h/4$ (c) h (d) $h/2$ (e) none of (a)-(d)

Before collision, equate energies to find momentum p

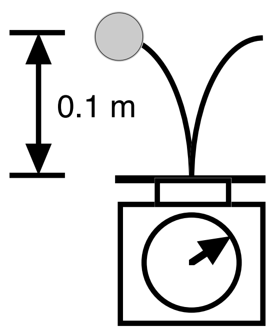
$$p^2 / 2m = mgh \quad \text{-->} \quad p^2 = 2m^2gh.$$

After the collision, the momentum of the pair is still p, but the height is found from

$$p^2 / 2(2m) = 2mgH$$

$$H = p^2 / (8m^2g) = (2m^2gh) / (8m^2g) = h / 4.$$

A steady stream of 0.02 kg balls roll off a table and bounce elastically on a weigh scale as shown in the diagram. If the balls drop through a height of 0.1 m in reaching the scale, and they arrive at 0.5 s intervals, what is the average force measured by the scales (in kg-m/s^2)?



(a) 5.6 (b) 1.4 (c) 0.056 (d) 0.028 (e) 0.11

The kinetic energy of each ball as it hits the scales is mgh, from which the velocity can be found from

$$(1/2)\, mv^2 = mgh \quad \text{or} \quad v^2 = 2gh \quad \text{or} \quad v = (2 \cdot 9.8 \cdot 0.1)^{1/2} = 1.4 \text{ m/s}.$$

Thus, the momentum of the ball is $mv = 0.02 \cdot 1.4 = 0.028$ kg-m/s. The change in momentum at the scale is double the incident momentum, or 0.056 kg-m/s. Since this change occurs every 0.5 seconds, then the force (rate of change of momentum) is 0.056 / 0.5 = 0.11 N.

Linear motion - problems

A student driving up Gaglardi way at 72 km/hr is late for class. When he is 100 m from the Curtis/Gagliardi stoplight, the light turns yellow. He hits the gas and accelerates at 3 m/s^2. But after 2 seconds, he decides that he can't make the light in time and jams on the brakes. He decelerates uniformly, coming to a stop at the light. What is his deceleration?

During acceleration $v_i = 72$ km/hr $= 72 \times 10^3 / 3.6 \times 10^3 = 20$ m/s

$a = 3$ m/s

$t = 2$ s

The distance he covers during acceleration is

$d = v_i t + at^2/2 = 20 \cdot 2 + 0.5 \cdot 3 \cdot 2^2 = 46$ m

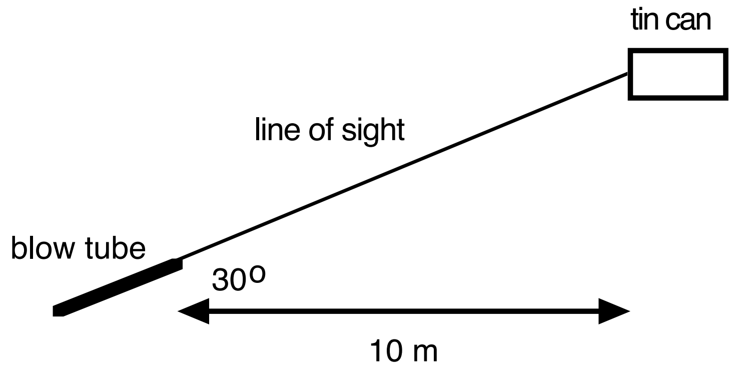
and his final velocity is $v_f = v_i + at = 20 + 3 \cdot 2 = 26$ m/s.

During deceleration the distance he covers is 100 - 46 = 54 m

His deceleration is therefore $a = (v_f^2 - v_i^2)/2a$

$= (0 - 26^2) / (2 \cdot 54) = -6.3$ m/s^2.

The geometry of the "shoot-the-can" demonstration done in class is approximately



The projectile is aimed at an angle of 30^o with respect to the horizontal, and the horizontal distance between the end of the blow tube and the can is 10 m. How far will the can fall before the projectile strikes it, assuming that the can starts to fall as soon as the projectile leaves the tube? The speed of the projectile when it leaves the tube is 24 m/s (neglect air resistance).

The horizontal velocity, v_x, of the projectile as it leaves the tube is

$$v_x = 24 \cos 30^o = 24 \cdot 0.866 = 20.8 \text{ m/s}.$$

Thus, the projectile covers the horizontal distance to the falling can in a time t, given by

$$t = d / v_x = 10 / 20.8 = 0.48 \text{ s}.$$

Since the can starts off with zero velocity, then during time t, the can has fallen a height given by

$$h = -gt^2 / 2,$$

where g is the acceleration due to gravity. Thus,

$$h = - 9.8 \cdot (0.48)^2 / 2 = - 1.13 \text{ m}.$$

The space shuttle orbiter is moving in a circular orbit with a speed of 7 km/s and a period of 80 minutes. In order to return to Earth, the orbiter fires its engines opposite to its direction of motion. The engines provide a deceleration in this direction of 20 m/s^2. What is the magnitude and direction of the total acceleration of the orbiter when the engines are first ignited?

The centripetal acceleration is given by

$$a_c = 2\pi v / T$$
$$= 2\pi \cdot 7\text{x}10^3 / (80 \cdot 60)$$
$$= 9.16 \text{ m/s}^2$$

Because a_c is perpendicular to the direction of motion, the total acceleration can be found from pythagoras theorem to be

$$a_{tot} = (a_c^2 + a_{tan}^2)^{1/2}$$
$$= (9.16^2 + 20^2)^{1/2}$$
$$= 22.0 \text{ m/s}^2.$$

The angle of the acceleration vector is away from the direction of motion, in the general direction of the Earth, making an angle with respect to the vertical obtained from

$$\tan\theta = 20.0 / 9.16 = 2.18$$

or

$$\theta = 65.4^o.$$

An object moves horizontally with a position given as a function of time t by

$$x(t) = at^3 - bt,$$

where a and b are positive constants.

(a) What is the velocity and acceleration of the object?

(b) What is the initial direction of the motion?

(c) At what position (in terms of a and b) will the object reverse direction?

For those students who have not yet encountered this result in their calculus class, the slope of the function at^n is given by $na\,t^{n-1}$. The slope of a polynomial is equal to the sum of the slopes of individual terms.

(a) To find velocity, one takes the slope of the position *vs* time graph. Here, the slope of each contribution gives

$$v(t) = 3at^2 - b.$$

(b) The velocity is initially negative, with

$$v(0) = -b.$$

(c) Solving for the condition $v(t) = 0$, one has

$$3at^2 - b = 0$$

$$3at^2 = b$$

or

$$t = (b / 3a)^{1/2}$$

Substituting back into the position equation gives:

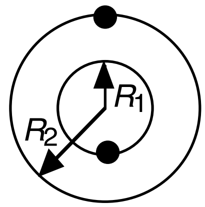
$$x(t) = at^3 - bt$$

$$= (at^2 - b)\ t$$

$$= [a \cdot (b /3a) - b]\ (b / 3a)^{1/2}$$

$$= -(2b /3)\ (b / 3a)^{1/2}.$$

In a binary star system, two stars with masses m_1 and m_2 rotate about their common



centre of mass. Assume that the orbits are circular, with radii R_1 and R_2, such that the distance $D = R_1 + R_2$ between the stars is constant.

(i) Establish that $R_1 = Gm_2T^2 / (4\pi^2D^2)$, where T is the period of the orbit.

(ii) Find the sum of the masses $m_1 + m_2$ in terms of D and T.

(i) Each star experiences an acceleration due to gravity of $F = GMm / D^2$. Equating this force to ma_c, with $a_c = v^2/R$, gives

$$mv^2/R = GMm / D^2 \qquad \text{or} \qquad v^2/R = GM / D^2$$

But the velocity of each star is given by

$$v = 2\pi R / T,$$

so that

$$(2\pi R_1 / T)^2/R_1 = GM_2 / D^2 \text{so that}$$

or

$$4\pi^2 R_1 / T^2 = GM_2 / D^2 \qquad \text{or} \qquad R_1 = Gm_2T^2 / (4\pi^2D^2).$$

(ii) Using the result from part (i), we can write each mass as

$$m_2 = 4\pi^2D^2R_1 / GT^2$$

which gives a total mass of

$$m_1 + m_2 = 4\pi^2D^2R_1 / GT^2 + 4\pi^2D^2R_2 / GT^2$$
$$= 4\pi^2D^3 / GT^2.$$

The viscous drag force exerted by a stationary fluid on a spherical object of radius R is

$F = 6\pi\eta Rv$ at low speed (Stokes' law)

$F = (\rho/2)AC_D v^2$ at higher speeds,

where the symbols are defined in Lec. 9. Let's apply this to a spherical cell one micron in radius, moving in water with $\eta = 10^{-3}$ kg/m•s and $\rho = 10^3$ kg/m^3. Take the cell to have the same density as water, and let its drag coefficient be 0.5.

(a) Plot the two forms of the drag force as a function of cell speed up to 100 μm/s. Quote the forces in pN.

(b) Find the speed at which the linear and quadratic drag terms are the same.

For the linear drag term, the constants are numerically equal to

$$6\pi\eta R = 6\pi \cdot 10^{-3} \cdot 10^{-6}$$
$$= 6\pi \cdot 10^{-9} \text{ kg/s.}$$

For the quadratic drag term, we have

$$(\rho/2)AC_D = 0.5 \cdot 10^3 \cdot \pi\, 10^{-12} \cdot 0.5$$
$$= (\pi/4) \cdot 10^{-9} \text{ kg/m.}$$

(a) Both expressions vanish when $v = 0$. By definition, the linear term increases fastest at small v. However, even when $v = 100$ μm/s, the linear term still dominates:

$$\text{linear drag force} = 6\pi \cdot 10^{-9} \cdot 10^{-6} \text{ kg-m/s}$$
$$= 6\pi \cdot 10^{-15} \text{ N}$$
$$= 6\pi \cdot 10^{-3} \text{ pN} = 0.0188 \text{ pN.}$$

$$\text{quadratic drag term} = (\pi/4) \cdot 10^{-9}\, (10^{-6})^2$$
$$= (\pi/4) \cdot 10^{-21} \text{ N}$$
$$= (\pi/4) \cdot 10^{-9} \text{ pN} = 7.8 \cdot 10^{-10} \text{ pN.}$$

(b) Equating the two drag forces gives

$$6\pi \cdot 10^{-9}\, v = (\pi/4) \cdot 10^{-9}\, v^2$$

or

$$v = 6\pi / (\pi/4) = 24 \text{ m/s.}$$

A stone of weight w is thrown vertically upward into the air with an initial speed v_o. Suppose that the (constant) air drag force f dissipates an amount fy as the stone travels a distance y.

(a) Show that the maximum height reached by the stone is

$$h = v_o^2 / \{2g (1 + f/w)\}.$$

(b) Show that the speed of the stone upon impact with the ground is

$$v = v_o \{(w-f) / (w+f)\}^{1/2}.$$

(a) The initial kinetic energy $mv_o^2/2$ is converted to gravitational potential energy $mgh = wh$ and lost to friction fh by the time the object reaches a height h:

$$mv_o^2/2 = wh + fh = (w+f)h.$$

Thus

$$(w/g) v_o^2/2 = (w+f) h$$

and

$$h = (w / 2g)\ v_o^2 / (w+f)$$
$$= v_o^2 / \{2g (1 + f/w)\}.$$

(b) On the trip back down, the potential energy wh is converted to kinetic energy $mv^2/2$ and lost to friction fh:

$$wh = fh + mv^2/2 = fh + wv^2 / 2g$$

Thus

$$wv^2 / 2g = (w-f)h$$
$$= (w-f)\ v_o^2 / \{2g (1 + f/w)\}.$$

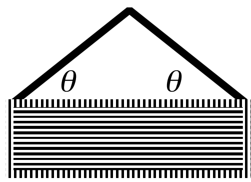
Re-arranging

$$v^2 = (w-f)\ v_o^2 / \{w (1 + f/w)\}$$
$$= \{ (w-f) / (w+f) \}\ v_o^2$$

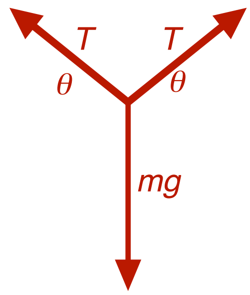
and finally

$$v = v_o \{(w-f) / (w+f)\}^{1/2}.$$

As shown in the diagram, a box of mass 200 kg is hung from a crane hook using a rope that can bear a tension of 2,000 N before it breaks. What is the minimum value of θ required to avoid breaking the rope when the box is lifted? Draw a free-body diagram of the box, and take the acceleration due to gravity to be 10 m/s^2.



Let the tension in the rope be T. The free-body diagram for the box is



In the vertical direction, the forces are balanced by

$$mg = 2T\sin\theta,$$

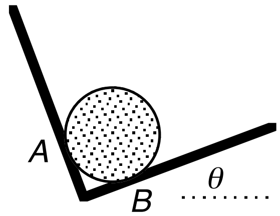
where mg = 10 x 200 = 2000 N.

If the tension is less than 2,000 N, then

$$\sin\theta > mg / 2T = 2000 / 4{,}000 = 0.5.$$

The angle whose sine is 0.5 is 30 degrees.

A spherical ball of mass m sits in a tilted box as shown, where θ is the angle of tilt.



There are two contact points with the box, labelled A and B. In terms of mg, what is the magnitude of the force on the ball at point A?

First, we observe that contact forces are perpendicular to the walls of the box. Resolving the components of the forces at each contact point:

Vertical: $f_A \sin\theta + f_B \cos\theta = mg$ (1)

Horizontal $f_A \cos\theta = f_B \sin\theta$ (2)

Solve (2) for f_B

$$f_B = f_A \cos\theta / \sin\theta \qquad (3)$$

and substitute into (1)

$$f_A \sin\theta + f_A \cos\theta \cdot \cos\theta / \sin\theta = mg$$

$$f_A (\sin^2\theta + \cos^2\theta) / \sin\theta = mg$$

By the trigonometric identity

$$\sin^2\theta + \cos^2\theta = 1$$

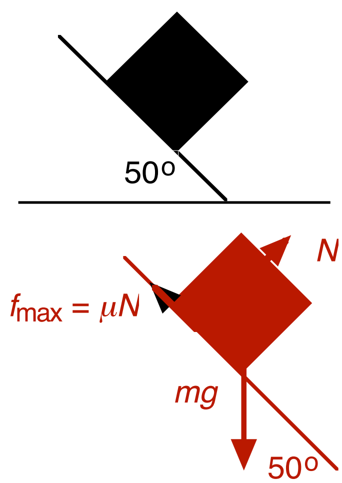
we are left with

$$f_A = mg \sin\theta.$$

A worker applies a horizontal force of 550 N to a 110 kg box which is at rest on a carpet. The coefficients of static and kinetic friction between the box and the carpet are 0.6 and 0,4, respectively. What is the frictional force exerted by the carpet on the box (use g = 10 m/s^2)?

The maximum force that can be generated by static friction is
$f_{max} = \mu_s N$
where N is the normal force mg. In this problem, m = 110 kg, so
f_{max} = 0.6 • 110 • 10 = 660 N.
Since f_{max} is less than the applied force, the box does not move and the frictional force is equal to 550 N.

A 10 kg block slides down an incline plane having an angle of 50º with respect to the horizontal. If the coefficient of friction between the block and the plane is 0.3, what is the acceleration of the block (in m/s^2)?
(9 marks, 00-1M)



Balancing forces perpendicular to the plane gives

$$N = mg \cos 50^\circ$$

Thus, the maximum frictional force is $f_{max} = \mu N = \mu mg \cos 50^\circ$.
The in-plane component of mg, which gives rise to the acceleration, is $mg \sin 50^\circ$.
The acceleration a arises from the net in-plane force:

$$ma = mg \sin 50^\circ - \mu mg \cos 50^\circ$$

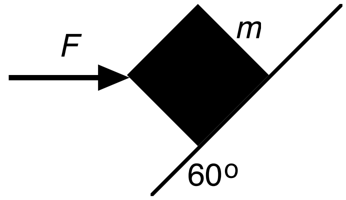
or

$$a = g (\sin 50^\circ - \mu \cos 50^\circ).$$

Substituting the numerical values for this situation gives

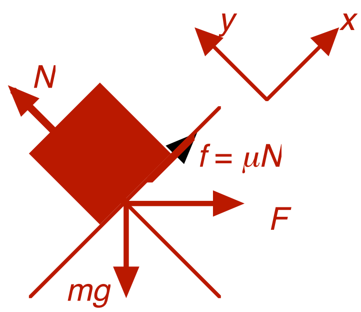
$$a = 9.81 (\sin 50^\circ - 0.3 \cdot \cos 50^\circ) = 5.6 \text{ m/s}^2.$$

A horizontal force is applied to a block of mass *m* sitting on a plane which makes an angle of 60° with respect to the horizontal, as illustrated. The coefficient of static



friction between the block and the plane is 1.0. What is the minimum value of *F* required to stop the block from moving? Draw a free-body diagram and quote your answer in terms of *mg*.

The free-body diagram for the block looks like



Choose a coordinate system aligned with the plane.
In the *x*-direction:

$$f + F\cos 60^\circ = mg\sin 60^\circ. \qquad (1)$$

In the *y*-direction

$$N = F\sin 60^\circ + mg\cos 60^\circ. \qquad (2)$$

Friction equation reads

$$f = N \quad \text{since } \mu = 1. \qquad (3)$$

Substituting (1) and (2) into (3) yields

$$mg\sin 60^\circ - F\cos 60^\circ = F\sin 60^\circ + mg\cos 60^\circ.$$

Solving for *F* gives

$$F\sin 60^\circ + F\cos 60^\circ = mg\sin 60^\circ - mg\cos 60^\circ.$$

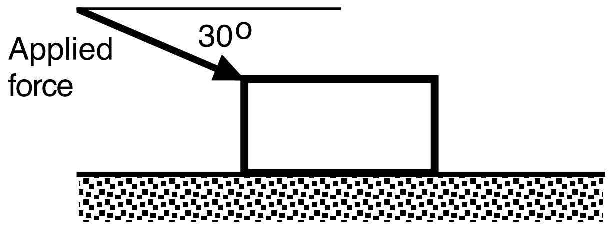
$$F(\sin 60^\circ + \cos 60^\circ) = mg(\sin 60^\circ - \cos 60^\circ)$$

$$F = mg(\sin 60^\circ - \cos 60^\circ) / (\sin 60^\circ + \cos 60^\circ)$$

or finally

$$F = 0.27\ mg.$$

A force F of 100 N is applied to a box of mass 5 kg resting on the floor as shown in the diagram. Both coefficients of friction (static and dynamic) between the box and the floor are 0.95. What is the magnitude and direction of the frictional force between the box and the floor.

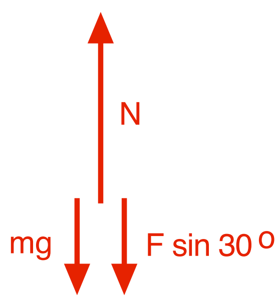


First, resolve the components of the applied force in the vertical and horizontal directions:

x-direction: $F \cos 30^{o} = 100 \cdot 0.866 = 86.6$ N

y-direction: $F \sin 30^{o} = 100 \cdot 0.5 = 50$ N

The normal force in the vertical direction opposes both the weight and the vertical component of F

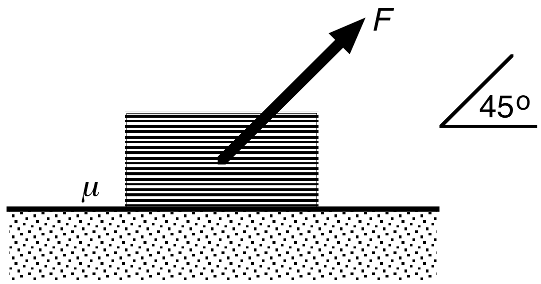


and thus has the value $N = mg + F \sin 30^{o} = 5 \cdot 9.8 + 50 = 99$ N.

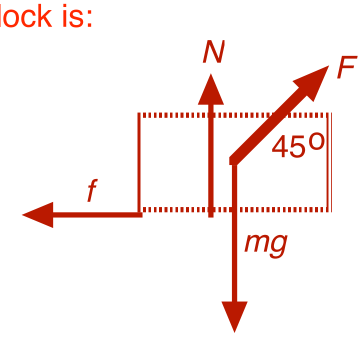
The maximum value of the frictional force is μN, which is $0.95 \cdot 99 = 94.1$ N.

Since this value is larger than the applied force in the x-direction, then the frictional force must be the same as the applied force, 86.6 N. It acts to the left, in the diagram.

A block of mass m is at rest on a horizontal plane, as shown in the diagram, and the coefficient of static friction between the block and the plane is μ. A massless string is attached to the block, making an angle of 45º with respect to the horizontal plane. What minimum force F must be applied through the string in order to move the block horizontally? Draw a free-body diagram of the block, and express F in the simplest expression involving m, μ and g, the acceleration due to gravity.



The free-body diagram of the block is:



Resolving the components of the force:

y-direction: $N + F \sin 45 = mg$ (1)

x-direction: $f = F \cos 45$ (2)

In addition, the maximum frictional force f is related to the normal force N through

$$f = \mu N \quad (3)$$

Solving Eq. (1) for N, we find $N = mg - F \sin 45$ (4)

Equating (2) and (3), $F \cos 45 = \mu N$ (5)

and substituting (4) gives $F \cos 45 = \mu(mg - F \sin 45)$ (6)

Solving for F: $F \cos 45 + \mu F \sin 45 = \mu mg$

$$F = \mu mg/(\mu \sin 45 + \cos 45)$$

$$F = \mu mg/(\mu/\sqrt{2} + 1/\sqrt{2})$$

$$F = \sqrt{2}\mu mg/(1+\mu).$$

A thin rope of mass M hangs over the edge of a table. The fraction of its total length L lying on the table experiences a frictional force with the table governed by a coefficient of friction μ. For a given value of μ, what is the minimum fraction f of the rope that can lie on the table without slipping over the edge? Assume that the edge is smooth and does not contribute to the frictional force.

The fraction of the rope on the table top has a mass of

$$m_{top} = fM$$

and exerts a normal force

$$N = fMg$$

on the table. This results in a frictional force

$$F_{friction} = \mu fMg$$

on the upper part of the rope. The friction acts against the force of gravity pulling the segment of the rope over the edge:

mass of rope over the edge = (1-f)M

$$F_{gravity} = (1-f)Mg.$$

Thus, the forces are balanced at

$$(1-f)Mg = \mu fMg,$$

or

$$1-f = \mu f$$

or

$$f(1+\mu) = 1$$

which gives

$$f = 1 / (1+\mu).$$

An object moves along the x-axis subject to a potential energy $V(x)$ of the form

$$V(x) = V_o(-x^2/2 + x^4/4).$$

(i) (9 marks) At what values of x does the object feel no net force?
(ii) (6 marks) Suppose that the object is displaced very slightly from the position(s) that you calculated in part (i). Describe the motion of the object.

(i) Force is related to potential energy by

$$F = -dV(x) / dx.$$

If the force vanishes, then

$$dV(x) / dx = 0.$$

Here,

$$dV(x) / dx = V_o\, d(-x^2/2 + x^4/4) / dx$$
$$= V_o (-x + x^3).$$

This expression vanishes when

$$(-x + x^3) = x(x^2 - 1) = 0$$

or

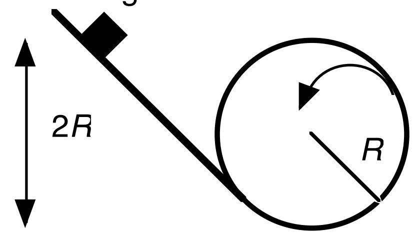
$$x = 0$$
$$x = +/- 1.$$

(ii) The potential energy at the zero force positions is

$$V(0) = 0$$
$$V(+/- 1) = V_o(-1/2 + 1/4) = -V_o/4.$$

V is a minimum at $x = +/- 1$, so that displacements from equilibrium will just oscillate.
V is not a minimum at $x = 0$, so the solution is unstable.

A small block of mass m slides without friction along a loop-the-loop track of radius R, after being released from an initial height of $2R$. To what height h inside the loop will



the block slide before it loses contact with the track?

The initial potential energy is $2mgR$. Hence, at any height h within the loop, the kinetic energy is determined by

$$2mgR = mv^2/2 + mgh$$

or

$$v^2 = 2[2gR - gh]$$

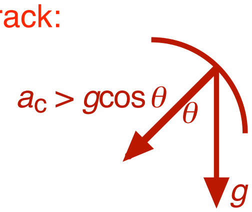
or

$$v^2 = 2g[2R - h].$$

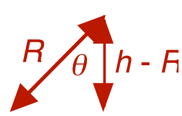
The centripetal acceleration a_c experienced by the block is then

$$a_c = v^2/R = 2g[2 - h/R].$$

When this acceleration is greater than the component of **g** in the radial direction, the block remains in contact with the track:



But $\cos\theta$ is given by



$$\cos\theta = (h - R) / R = h/R - 1.$$

Hence, the condition for contact is

$$a_c > g\cos\theta \quad \text{-->} \quad 2g[2 - h/R] > g(h/R - 1)$$

$$4 - 2h/R > h/R - 1$$

$$3h/R > 5$$

$$h > (5/3)R.$$

A planet of mass m executes a circular orbit of radius R about a very heavy star of mass M.
(a) In terms of M, R and the gravitational constant G, what is the speed v_o of the planet in its orbit?
(b) What is the total energy of the planet?
(c) In terms of v_o, what must its speed v_{esc} be to escape from the orbit to infinity?

(a) The speed can be obtained from the force balance between the gravitational attraction GmM/R^2 and the centripetal force mv^2/R:

$$mv_o^2/R = GmM/R^2$$

or

$$v_o^2 = GM/R$$
$$v_o = (GM/R)^{1/2}.$$

(b) The total energy is the sum of its potential and kinetic parts, which have opposite signs here:

$$E = -GmM/R + mv_o^2/2$$
$$= -GmM/R + GmM/2R$$
$$= -GmM/2R.$$

(c) For the planet to escape, it must overcome the potential energy binding it at radius R; that is

$$mv_{esc}^2/2 = GmM/R$$

or

$$v_{esc}^2 = 2GM/R$$
$$= 2v_o^2.$$

Thus,

$$v_{esc} = \sqrt{2}\, v_o.$$

An object which is restricted to moving horizontally, is subject to a quadratic drag force in the form $F = c_2 v^2$.
(a) Find the power needed to maintain an object at constant speed in the presence of this force.
(b) Find the power (in horsepower) needed to overcome this drag force for a car travelling at 100 km/hr if its cross-sectional area is 2 m^2 and drag coefficient is 0.5.
Your answer in (b) does not include internal friction, tire friction etc.

(a) At a constant speed, the drag force is constant and the resulting power is just

$$P = Fv = c_2 v^3.$$

(b) To do this problem, we need the density of air at $\rho = 1.29$ kg/m^3 and

$$v = 100 \cdot 10^3 / 3600 = 27.8 \text{ m/s}.$$

Thus,

$$P = (\rho /2)\ A\ C_D\ v^3$$
$$= 0.5 \cdot 1.29 \cdot 2 \cdot 0.5 \cdot (27.8)^3$$
$$= 13.9 \times 10^3 \text{ W}$$
$$= 13.9 \times 10^3 / 746$$
$$= 19 \text{ hp}.$$

Show that the terminal speed of a falling spherical object is given by

$$v_{term} = [(mg/c_2) + (c_1/2c_2)^2\ ]^{1/2} - (c_1/2c_2)$$

when both the linear and quadratic terms in the drag force are taken into account.

This question is very easy. Balancing the gravitational force mg against the drag force $c_1 v + c_2 v^2$ gives the terminal velocity. Thus

$$c_1 v + c_2 v^2 = mg$$

or

$$c_2 v^2 + c_1 v - mg = 0,$$

which has the usual quadratic solution

$$v = \{-c_1 \pm [c_1^2 + 4c_2 mg]^{1/2}\} / 2c_2.$$

This can be rearranged to give

$$v_{term} = [(mg/c_2) + (c_1/2c_2)^2\ ]^{1/2} - (c_1/2c_2).$$

Consider three different power-law forms of the drag force with magnitudes:

$F_{1/2} = a\, v^{1/2}$ (square root)

$F_1 = b\, v^1$ (linear)

$F_{3/2} = c\, v^{3/2}$ (3/2 power).

Travelling horizontally from an initial speed v_o, an object subjected to one of these drag forces would come to rest at

$x_{max} = (2m/3a)v_o^{3/2}$ (square root)

$x_{max} = mv_o / b$ (linear)

$x_{max} = 2m\, v_o^{1/2}/c$ (3/2 power).

(a) Determine the coefficents *a*, *b* and *c* for a cell of mass 10^{-14} kg whose drag force is measured to be 5 pN when travelling at 10 microns/second.

(b) Find the maximum displacement that the cell could reach for each force if v_o = 1 μm/s.

(c) Why should x_{max} in part (b) have the order (shortest to longest) that it does?

(a) Calculate the unknown constants for each of the forces, including the units appropriate for each:

square root $5 \cdot 10^{-12} = a\,(10^{-5})^{1/2}$

$a = 1.58 \cdot 10^{-9}$ N(s/m)$^{1/2}$

linear $5 \cdot 10^{-12} = b\, 10^{-5}$

$b = 5 \cdot 10^{-7}$ N•s/m

3/2 power $5 \cdot 10^{-12} = c\,(10^{-5})^{3/2}$

$c = 1.58 \cdot 10^{-4}$ N(s/m)$^{3/2}$.

(b) Calculating the maximum distances travelling is now simple:

square root $x_{max} = (2m/3a)v_o^{3/2}$

$x_{max} = (2\text{x}10^{-14} / [3 \cdot 1.5\text{x}10^{-9}])\,(10^{-6})^{3/2}$

$= 4.2\text{x}10^{-15}$m [kg / N(s/m)]•[m/s]$^{3/2}$ = m

$= 4.2\text{x}10^{-5}$ Å

linear $x_{max} = mv_o / b$

$= 10^{-14} \cdot 10^{-6} / 5\text{x}10^{-7}$

$= 2\text{x}10^{-14}$ m kg (m/s) / (N•s/m) = m

$= 2\text{x}10^{-4}$ Å

3/2 power $x_{max} = 2m\, v_o^{1/2}/c$

$x_{max} = 2\ 10^{-14}\,(10^{-6})^{1/2} / 1.58\text{x}10^{-4-}$

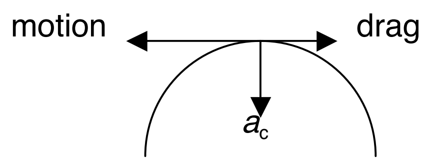
$= 1.26\text{x}10^{-13}$ m kg (m/s)$^{1/2}$ / [N (s/m)$^{3/2}$] = m

$= 1.26\text{x}10^{-3}$ Å.

Of the three forces, the square root force rises fastest with *v*. Conversely, it dies more slowly that the other forces as the cell slows down. Thus, it has a higher average value than the other forces, and brings the cell to rest faster, explaining why x_{max} is smallest for the square root force, and largest for the 3/2-power force.

An object of mass *m* moves in a circular path of radius *R* on a horizontal plane. It is subject to turbulent drag. At what range or values of *R* and instantaneous speed *v* does it experience an acceleration which makes an angle of 45° with respect to a tangent to the circle?

The object experiences a centripetal force ma_c perpendicular to its direction of motion, and a drag force opposite to its direction of motion:



For the acceleration to make an angle of 45° with respect to the direction of motion, the drag force and the centripetal force must have the same magnitude. Thus, in magnitude,

$$F_{DRAG} = F_{CEN}.$$

But

$$F_{DRAG} = c_2 v^2$$

and

$$F_{CEN} = ma_c = mv^2/R$$

so

$$c_2 v^2 = mv^2/R$$

or

$$R = m / c_2.$$

There is no constraint on the velocity for this condition to occur, only on the radius.

An object of mass mass m is attached to one end of a massless string of length L. The top end of the string is attached to the ceiling. Keeping the string straight, the object is drawn to the side and released from rest at an angle θ_o with respect to the vertical. As a function of the angle θ (with respect to the vertical), what is the tension in the string once the mass begins to move?

First, find the kinetic energy of the object by calculating the decrease in the potential energy. The height difference is

$$\Delta h = L\,(\cos\theta - \cos\theta_o)$$

show a diagram

so that

$$\Delta U = -mgL\,(\cos\theta - \cos\theta_o)$$

By conservation of energy

$$\Delta K = -\Delta U$$
$$= mgL\,(\cos\theta - \cos\theta_o).$$

But the change in kinetic energy is just

$$mv^2/2$$

because the object is released from rest. Thus

$$mv^2/2 = mgL\,(\cos\theta - \cos\theta_o)$$

or

$$v^2/L = 2g\,(\cos\theta - \cos\theta_o).$$

Because the motion is in a circle, the centripetal acceleration is

$$a_c = v^2/L = 2g\,(\cos\theta - \cos\theta_o)$$

and the centripetal force is

$$ma_c = 2mg\,(\cos\theta - \cos\theta_o).$$

The tension in the string provides the centripetal force as well as balancing the component of the gravitational force $mg\cos\theta$. Thus, the total force provided by the string is

$$T = 2mg\,(\cos\theta - \cos\theta_o) + mg\cos\theta$$
$$= mg\,(3\cos\theta - 2\cos\theta_o).$$

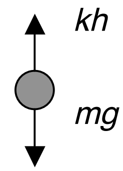
An ideal massless spring hangs motionless, its top end attached to a rigid support as in one of the class demonstrations. The spring has a spring constant k and the bottom end of the unstretched spring, which is free, defines $z = 0$ of a coordinate system. You attach a mass m to the bottom end and carefully lower it with your hand until it reaches a new equilibrium position $-h$, where h is a positive number. In terms of m, g, and k, what non-conservative work, if any, was done by the system in this process?

When the mass is in equilibrium, the net force vanishes. The forces are:

$F = kx$ from the spring

$F = mg$ from gravity

and



Thus

$$kh = mg$$

or

$$h = mg/k.$$

The non-conservative work is governed by

$$W_{\text{non-con}} = \Delta(K + U)$$

In this process, there is no change in kinetic energy, so $\Delta K = 0$.
However, the change in potential energy is

$$\Delta U_{\text{spring}} = +kh^2/2 = +k\,(mg/k)^2 / 2 = +(mg)^2 / 2k.$$

and

$$\Delta U_{\text{grav}} = -mgh = -mg\,(mg/k) = -\,(mg)^2 / k.$$

Thus,

$$\Delta U_{\text{total}} = \Delta U_{\text{spring}} + \Delta U_{\text{grav}}$$
$$= +(mg)^2 / 2k - (mg)^2 / k$$
$$= -\,(mg)^2 / 2k.$$

The non-conservative work is then

$$W_{\text{non-con}} = -\,(mg)^2 / 2k.$$

A bullet travelling at 400 m/s is shot horizontally at a sand-filled tin sitting on a fence 2 m high. The bullet stops inside the can, causing it to fall off the fence.
(a) What is the can's velocity just as it topples off the fence?
(b) How far from the bottom of the fence does it hit the ground?
The masses of the bullet and the can are m_B = 10 g and m_C = 2 kg, respectively. You can ignore the spatial size of the can compared to the height of the fence.

(a) Solve the bullet-can collision problem using conservation of momentum:

$$\mathbf{p}_B + \mathbf{p}_C = \mathbf{p}_{B+C}$$

$$m_B v_B + m_C v_C = m_{B+C} v_{B+C}$$

$$0.01 \cdot 400 + 0 = 2.01 v_{B+C}$$

or

$$v_{B+C} = 4.00 / 2.01 = 2.0 \text{ m/s.}$$

(b) It takes time T for the can to hit the ground. In the vertical (y) direction, the equation of motion reads:

$$d = v_i T + aT^2/2.$$

Here, $v_i = 0$, $d = -2$ m, $a = -g = -9.81$ m/s^2.
Hence,

$$-2 = 0 - 9.8T^2/2 \quad \text{--->} \quad T = \sqrt{(2 \times 2 / 9.8)} = 0.64 \text{ s.}$$

There is no acceleration in the horizontal (x) direction, so

$$x = vT = 2 \times 0.64 = 1.28 \text{ m.}$$

An object of mass m moves on a frictionless surface with a speed v in the $+x$ direction. It explodes into two fragments of equal mass, one of which travels in the $+x$ direction with speed $3v$. What kinetic energy is liberated in the explosion? (8 marks)

Strategy: apply conservation of momentum to determine the velocity of the second fragment.

initial momentum = mv
momentum of forward particle = $(m/2) \cdot 3v = (3/2)mv$
By conservation of momentum, the velocity of the second fragment must be

$$(m/2)v_2 = mv - (3/2)mv$$
$$= -mv/2$$

Thus

$$v_2 = -v.$$

To find the kinetic energy released, we compare:

initial kinetic energy = $mv^2/2$
kinetic energy of fragment #1 = $(1/2) \cdot (m/2) \cdot (3v)^2 = (9/4)mv^2$
kinetic energy of fragment #2 = $(1/2) \cdot (m/2) \cdot (v)^2 = (1/4)mv^2$
final kinetic energy = $\{9/4 + 1/4\}mv^2 = 5/2\ mv^2$

Thus, the kinetic energy released is

$$\Delta K = \{5/2 - 1/2\}mv^2 = 2\ mv^2.$$

(a) Find the centre-of-mass position of the Earth-Jupiter-Sun system in the two configurations shown in the diagram. Choose the centre of the Sun as the coordinate system origin. The centres of the objects lie on a straight line.
(b) Comparing configurations I and II, how much does the position of the Sun change relative to the centre-of-mass of each configuration?

Config I **E ------- S ------- J**

Config 2 **J ------- E -------- S**

Masses: Earth = 5.98×10^{24} kg Jupiter = 1.90×10^{27} kg Sun = 1.99×10^{30} kg
Distances: Earth-Sun = 1.50×10^{8} km Jupiter-Sun = 7.78×10^{8} km

The total mass of the system is essentially the mass of the Sun:

$$m_{TOT} = 5.98 \times 10^{24} + 1.90 \times 10^{27} + 1.99 \times 10^{30}\ \text{kg} = 1.99 \times 10^{30}\ \text{kg}.$$

In configuration I,

$$\Sigma_i m_i \mathbf{r}_i = -(5.98 \times 10^{24} \times 1.50 \times 10^{8}) + 0 + (1.90 \times 10^{27} \times 7.78 \times 10^{8})$$
$$= -8.97 \times 10^{32} + 1.48 \times 10^{36} = 1.48 \times 10^{36}\ \text{kg•km}.$$

Hence,

$$x_{cm} = 1.48 \times 10^{36} / 1.99 \times 10^{30} = 7.42 \times 10^{5}\ \text{km}.$$

In configuration II,

$$\Sigma_i m_i \mathbf{r}_i = -8.97 \times 10^{32} - 1.48 \times 10^{36} = -1.48 \times 10^{36}\ \text{kg•km}.$$

Hence,

$$x_{cm} = -1.48 \times 10^{36} / 1.99 \times 10^{30} = -7.42 \times 10^{5}\ \text{km}.$$

(b) The total shift in the Sun's position is 1.48×10^{6} km.

A physics major skating on the surface of a frozen pond decides to use a flashlight to accelerate himself. The flashlight radiates a beam of light at 2 watts, with an average wavelength of 600 nm.

(a) How many photons leave the flashlight per second?

(b) How much momentum is carried by the photons per second?

(c) If the mass of the skater is 70 kg, what is his acceleration?

(a) The energy of a single photon is:

$E = hc / \lambda = 6.63 \times 10^{-34} \times 3 \times 10^{8} / 600 \times 10^{-9}$

$= 3.32 \times 10^{-19}$ J.

The number of photons leaving per second in 2 watts is

2 J / 3.32×10^{-19} J = 6.0×10^{18} sec^{-1}.

(b) The momentum of each photon is $p = h / \lambda$ or E / c.

Single photon momentum $p = 6.63 \times 10^{-34} / 600 \times 10^{-9} = 1.05 \times 10^{-27}$ kg-m/s.

Total momentum radiated per second is

$1.05 \times 10^{-27} \cdot 6.0 \times 10^{18} = 6.66 \times 10^{-9}$ kg-m/s.

(Shorter way to do this is divide the power by the speed of light c).

(c) Use conservation of momentum to calculate the change in momentum of the skater:

$a = F / m = (\Delta p/\Delta t)/m$

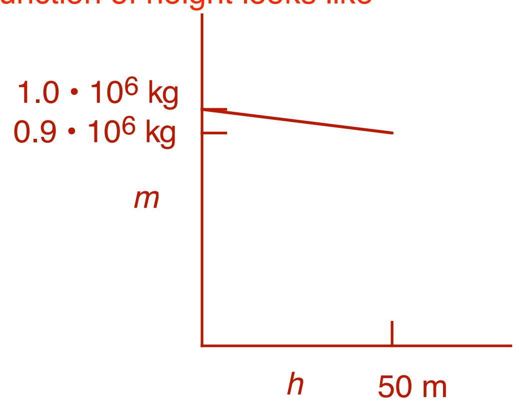
$= 6.66 \times 10^{-9} / 70 = 9.5 \times 10^{-11}$ m/s^2.

Rockets burn up their fuel very rapidly as they accelerate, so that the total mass of a rocket decreases with height above the ground. In this problem, we study a particular rocket with an initial mass of 1.000 x 10^6 kg, which losses 1.0 x 10^5 kg in mass during the first 50 m of its flight.

(a) Assuming that the loss in mass is uniform with height (*i.e.* so many kg per meter) find a numerical expression for the mass m of the rocket as a function of its height h for the first 50 m.

(b) What is the work done by the rocket engines during the first 50 m of flight? Use g = 10 m/s^2.

(a) The mass as a function of height looks like



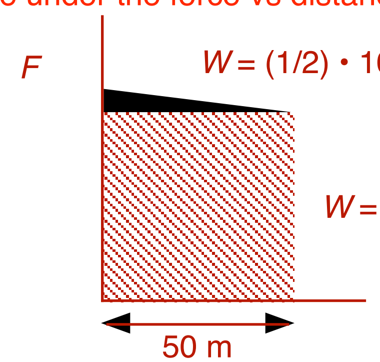
The function is linear, of the form

$$m = 1.00 \times 10^6 - (h / 50) \times 10^5$$
$$= (1.000 - 0.002h) \times 10^6 \text{ kg.}$$

(b) The force due to gravity on the rocket is

$$F = mg.$$

The work done under the force vs distance curve is thus

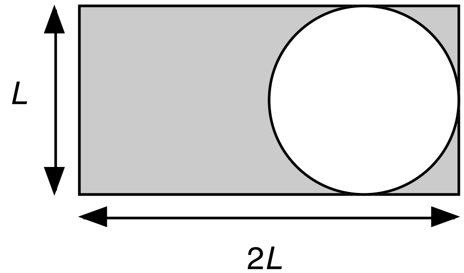


$W = (1/2) \cdot 10^5 \cdot 10 \cdot 50 = 25 \cdot 10^6$ J.

$W = 0.9 \cdot 10^6 \cdot 10 \cdot 50 = 450 \cdot 10^6$ J.

Total work = 475 x 10^6 J.

A uniform rectangular sheet of mass M has dimensions L x $2L$. The coordinate origin is chosen to lie at the geometrical centre of the rectangle. A hole of radius $L/2$ is subsequently drilled in the block, the centre of the hole having coordinates $(L/2, 0)$. What is the x-coordinate of the centre-of-mass position of the block with the hole in it?



First, concentrate on the x-coordinate. We need to know the total mass after the hole is drilled:

$$density = M / 2L^2$$

Hence:

$$m_{after} = \text{density} \cdot \text{area} = (M / 2L^2) \cdot (2L^2 - \pi[L/2]^2) = M(1 - \pi/8).$$

$$m_{hole} = (M / 2L^2) \cdot \pi(L/2)^2 = \pi/8\ M.$$

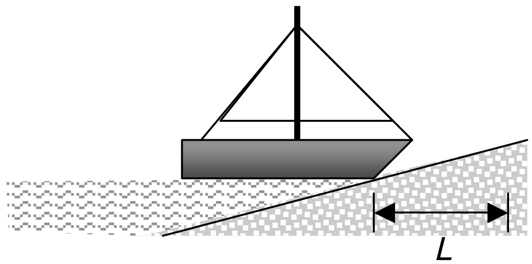
The position of the centre of mass is given by

$$R_{cm} = (m_{sheet}R_{sheet} - m_{hole}R_{hole}) / m_{after}$$
$$= (0 - [\pi/8]\ M[L/2]) / [M(1 - \pi/8)]$$
$$= -(\pi L/16) / (1 - \pi/8)$$
$$= \pi L / (16 - 2\pi)$$
$$= -0.323L.$$

A boat moving straight towards shore plows into a soft sandy beach, coming to a stop in a distance L. As it moves through the sand, it experiences a coefficient of kinetic friction that grows with distance x as

$$\mu_k = \mu_o\, x / L.$$

Find the work done by friction as a function of distance x. Take the normal force on the boat to be mg, where m is the boat's mass.



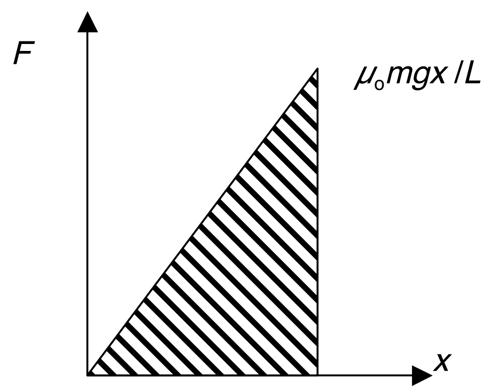
The frictional force is distance-dependent, and given by

$$F = \mu_k\, mg = (\mu_o mg / L)\, x.$$

The work done on the boat is distance-dependent as well

$$W = Fx$$

but is not linear because the force is not constant. To evaluate the work, we look at



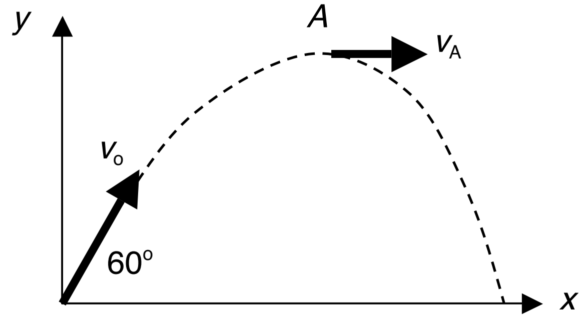
As in the spring problem, the work is equal to the area under the curve, which is

$$[area] = 1/2\, [base]\, [height]$$
$$= 1/2\, x\, (\mu_o mg / L)\, x$$

or

$$W = (\mu_o mg / L)\, x^2 / 2.$$

A cannonball of mass 5 kg is fired into the air at an angle of 60° with an initial velocity of 500.0 m/s. Exactly ten seconds later, it reaches its maximum height of 400 m at point A with a velocity of 100.0 m/s to the right.
(a) Find the average acceleration of the cannonball in the first ten seconds.
(b) Find the average power exerted by air resistance in the first ten seconds.
For numerical simplicity, use $g = 10$ m/s^2.



(a) Acceleration is a vector and its average value must be determined component by component.

$$\bar{a}_x = \frac{v_{f,x} - v_{i,x}}{\Delta t} = \frac{100.0 - 500.0\cos 60^0}{10} = \frac{100.0 - 250.0}{10} = -15.0\ m/s^2$$

$$\bar{a}_y = \frac{v_{f,y} - v_{i,y}}{\Delta t} = \frac{0.0 - 500.0\sin 60^0}{10} = \frac{-433.0}{10} = -43.3\ m/s^2$$

The magnitude of the average acceleration is then $(15.0^2 + 43.3^2)^{1/2} = 45.8$ m/s^2.

The angle of the acceleration vector with respect to the horizontal is

$\tan\theta = a_y/a_x = 43.3 / 15.0 = 2.89$.

Thus

$\theta = \arctan(2.89) = 71°$.

(b) The first challenge is to determine the change in the kinetic and potential energies.

$\Delta K = mv_f^2/2 - mv_i^2/2 = (5 / 2) \cdot (100^2 - 500^2) = -600{,}000$ J

and

$\Delta U = mg\,\Delta h = 5 \cdot 10 \cdot 400 = +20{,}000$ J.

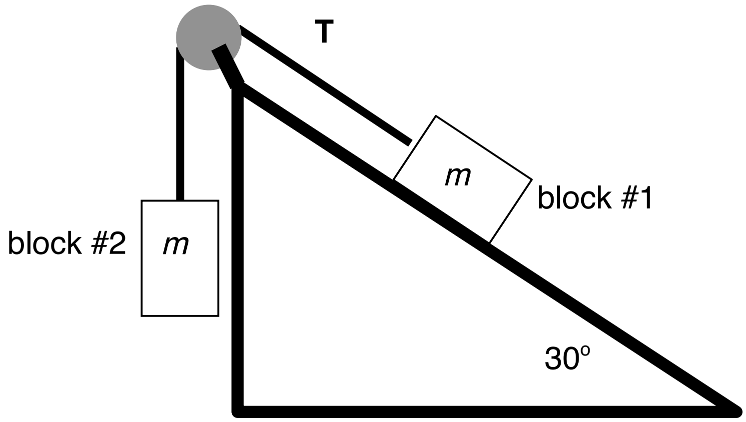
Thus, the change in the total mechanical energy is

$\Delta E = \Delta K + \Delta U = -600{,}000 + 20{,}000 = -580{,}000$ J.

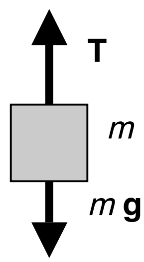
As expected, ΔE is negative because of dissipation.
The average power exerted by the dissipative force is $P = \Delta E / \Delta t$, which becomes

$P = 580{,}000 / 10 = 58{,}000$ watts.

As shown in the figure below, the two blocks have identical masses *m*. Block #1 experiences a sliding friction force with coefficient of friction μ_k. Find the acceleration of the blocks.



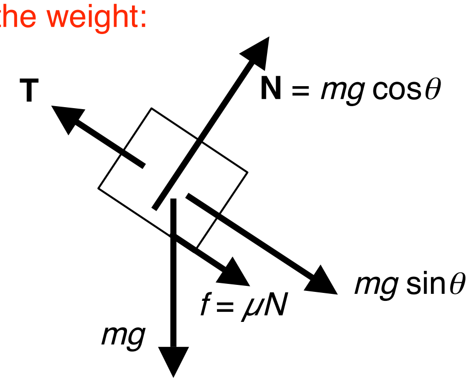
We start by drawing free-body diagrams for both masses. Introducing the acceleration *a*, the FBD for block #2 is



The weight of the mass is only partly compensated by the vertical tension,leaving a net force that accelerates the mass:

$$mg = T + ma. \qquad (1)$$

The force experienced by block #2 is more complex, because of the presence of friction, and the component of the weight:



Here, the tension is opposed by the frictional force as well as $mg\sin\theta$, leaving an unbalanced force that provides the acceleration. That is

$$T = ma + f + mg\sin\theta$$
$$= ma + \mu mg\cos\theta + mg\sin\theta$$

$$= m\,(a + \mu g\cos\theta + g\sin\theta) \qquad (2)$$

Substituting (2) into (1) gives

$$mg = m\,(a + \mu g\cos\theta + g\sin\theta) + ma$$

or

$$g = 2a + \mu g\cos\theta + g\sin\theta$$

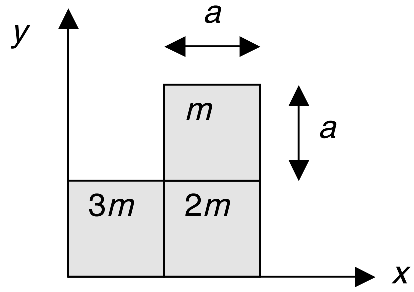
whence

$$2a = g - \mu g\cos\theta - g\sin\theta$$

or

$$a = (g/2) \cdot (1 - \mu\cos\theta - \sin\theta). \qquad (3)$$

Three objects of mass m, $2m$ and $3m$ are glued together in the shape of the letter L:



Each block is a square of length a to the side. Find the position of the centre-of-mass of the objects with respect to the coordinate system shown in the diagram. (8 marks)

Each object acts as if its mass were concentrated at its centre. (mandatory)

Thus, we can regard the system as three point masses, where the coordinates of the masses are:

$3m$	at	$(a/2, a/2)$
$2m$	at	$(3a/2, a/2)$
m	at	$(3a/2, 3a/2)$.

From the definition of the centre of mass

$$\mathbf{R}_{cm} = \Sigma_i m_i \mathbf{r}_i \;/\; m_{total},$$ (mandatory)

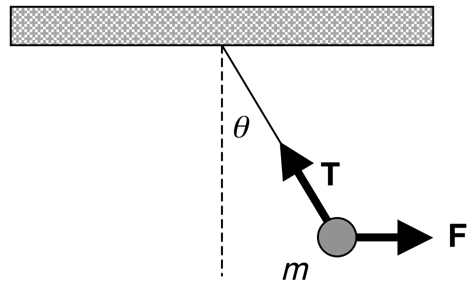
we determine the coordinates to be

$$x_{cm} = [(3m \cdot a/2) + (2m \cdot 3a/2) + (m \cdot 3a/2)] / 6m$$
$$= [\,3ma/2 + 3ma + 3ma/2\,] / 6m$$
$$= 6ma / 6m$$
$$= a$$

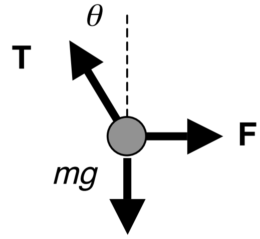
and

$$y_{cm} = [(3m \cdot a/2) + (2m \cdot a/2) + (m \cdot 3a/2)] / 6m$$
$$= [\,3ma/2 + ma + 3ma/2\,] / 6m$$
$$= 4ma / 6m$$
$$= (2/3)a.$$

In the diagram below, a ball hangs from the ceiling, and is held in a steady position by a horizontal force **F** of 8 N. The length of the string is 0.5 meter, and the angle θ is 30°.
(i) Find the magnitude of the tension **T** in the string.
(ii) If the ball is released ($F = 0$), what will be its speed when it reaches the bottom? Set the acceleration due to gravity equal to 10 m/s^2, for numerical simplicity.



(a) The free-body diagram of the ball is



In the z-direction, the force vectors must obey

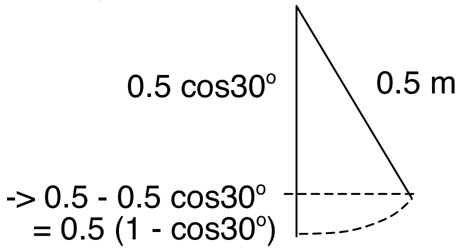
$$F = T \sin\theta = T \sin 30^\circ = T/2.$$

Thus

$$T = 2F = 16 \text{ N}.$$

(b) The change in height experienced by the ball as it falls is

$$\Delta h = 0.5\,(1 - \cos 30^\circ) = 0.067 \text{ m}.$$



By conservation of mechanical energy, the increase in the kinetic energy must equal the decrease in the potential energy, hence (mandatory)

$$mv^2/2 - [\text{initial } K] = mg\Delta h.$$

But the initial kinetic energy is zero, so

$$v^2/2 = g\,\Delta h,$$

or

$$v = [2g\,\Delta h]^{1/2}$$
$$= [2 \cdot 10 \cdot 0.067]^{1/2} = 1.16 \text{ m/s}.$$