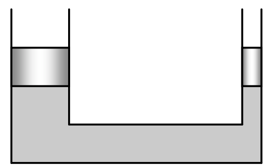
Materials - multiple choice

A solid object of density ρ_S is lowered slowly into a liquid of density ρ_L obeying $\rho_S = (2/3)\rho_L$. What fraction of the object's volume will be immersed?
(a) 3/2 (b) 1/3 (c) 1/2 (d) 2/3 (e) 1
The volume of liquid displaced by the object must be equal to the mass of the object:
$$m_{object} = \rho_S V_{object} = \rho_L V_{immersed}$$
Hence
$$V_{immersed} / V_{object} = \rho_S/\rho_L = 2/3.$$

A solid cylinder of height h floats upright in a liquid of density ρ_L, with $h/3$ of its length above the surface of the liquid. What is the density of the solid?
(a) $2\rho_L / 3$ (b) $\rho_L / 3$ (c) $3\rho_L$ (d) $3\rho_L / 2$ (e) ρ_L.
For a floating object, the weight of the object is equal to the weight of the fluid displaced:
$$\rho_S V_{object} g = \rho_L V_{fluid} g \quad \text{or} \quad \rho_S V_{object} = \rho_L V_{fluid}.$$
But $V_{fluid} = (2/3) V_{object}$
so $\rho_S V_{object} = \rho_L (2/3) V_{object}$ or $\rho_s = (2/3)\rho_L$.

Pistons in the hydraulic press have radii $2R$ and R, respectively. If the fluid in the press is incompressible, what distance does piston #2 move if piston #1 moves by h?
(a) $h/4$ (b) $h/2$ (c) $4h$
(d) $16h$ (e) $2h$



Volume must be conserved if the fluid is incompressible. The volume change at piston #1 is $4\pi R^2 h$, so piston #2 obeys $\pi R^2(4h)$. Hence, #2 moves $4h$.

Blood flows from a large artery of radius 0.3 cm, where its speed is 8 cm/s, into a region where the radius has been reduced to 0.2 cm because of thickening of the walls. Assuming the blood to be incompressible, what is its speed in the narrower region in cm/second?
(a) 18 (b) 0.06 (c) 12 (d) 5.3 (e) 3.6
For inconpressible fluids, vA is a constant; that is $v_1A_1 = v_2A_2$. Thus, $v_2 = v_1(A_1/A_2)$. In this question, we have $v_2 = 8 \cdot (0.3/0.2)^2 = 18$ cm/s.

Two balls of the same radius but densities $\rho_1 = 2\rho_L$ and $\rho_2 = 3\rho_L$ are placed in a liquid of density ρ_L. What is the ratio of the effective weight w_1 / w_2 of the balls?
(a) 2/3 (b) 3/2 (c) 3/4 (d) 4/3 (e) none of [a-d]
The effective weight of an object of density ρ_S in a medium of density ρ_L is
$$w_{eff} = Vg(\rho_S - \rho_L) \quad \to \quad w_1/w_2 = (\rho_1 - \rho_L) / (\rho_2 - \rho_L),$$
whence
$$w_1/w_2 = (2\rho_L - \rho_L) / (3\rho_L - \rho_L) = 1/2.$$
None of the displayed answers is correct.

The heat flow per unit time through a rectangular slab of material is *H*. If the slab is made twice as thick, but the area and temperature difference are unchanged, what is the rate of heat flow?

(a) *H* (b) 2*H* (c) *H*/2 (d) 4*H* (e) *H*/4

The heat flow through a material is given by

$$H = KA (\Delta T / d)$$

where *K* is the thermal conductivity, *A* is the area, ΔT is the temperature difference and *d* is the thickness. If the thickness is doubled, then the temperature gradient is halved and the heat flow drops by a factor of 2.

A solid cube has length *L* to the side. If the length of a side increases by 1% upon heating, what is the relative change in volume?

(a) 1% (b) 10^{-1} (c) 30% (d) 10^{-6} (e) 3×10^{-2}.

For small changes in volume, $\Delta V / V = 3\ \Delta L / L$. Thus, if $\Delta L / L = 10^{-2}$, then $\Delta L / L = 3\text{x}10^{-2}$.

In an ideal gas, what is the ratio of the mean speed of a nitrogen molecule compared to an unbound nitrogen atom?

(a) 2 (b) √2 (c) 1 (d) 1/2 (e) 1/ √2

The mean kinetic energy of a particle in an ideal gas is $3k_B T/2$, independent of its mass. Thus, the mean speed is of a particle is

$$v = (3k_B T / m)^{1/2}.$$

The mass of a nitrogen molecule is double that of a nitrogen atom, so the speed of the molecule must be 1/ √2 that of the atom.

Two stars, *Alpha* and *Beta*, produce the same amount of energy per unit time, but have different radii: *Alpha* is twice as large as *Beta*. What is the ratio of their surface temperatures, T_{Alpha} / T_{Beta}?

(a) $2^{1/2}$ (b) $2^{-1/4}$ (c) $2^{1/4}$ (d) $2^{-1/2}$ (e) none of [a-d]

The power radiated per unit area of the stars equals the energy output per unit time divided by the surface area. Having double the radius, the power per unit area of *Alpha* is $1/2^2$ that of Beta, since area scales like R^2. However, the temperature is proportional to the fourth root of the power per unit area, so

$$T_{Alpha} / T_{Beta} = (2^{-2})^{1/4} = 2^{-1/2} = 1/\sqrt{2}.$$

A light bulb contains a fine tungsten wire that is heated to 3000K by electric current. At this temperature, the emissivity of tungsten is 0.34. The surface area of the tungsten wire is 1.0cm^2. The room temperature is 300K. What is the radiated power of the light bulb in watts? (Note: The Stefan-Boltzmann constant is 5.67×10^{-8} J/(s-m^2-K^4))

(a) 1.56 (b) 0.00567 (c) 56.7 (d) 0.0156 (e) 156

$P = \varepsilon\sigma A T^4 = (0.34)\bullet(1.0\times10^{-4})\bullet(5.67\times10^{-8})\bullet(3000)^4 = 156$ watts.

As it forages for food, a particular species of animal randomly explores an area of radius *R* in the daylight hours of a winter's day. In the summer, when there are twice as many hours of daylight, what is the radius of the region that the animal can explore, all other things being equal?

(a) 2*R* (b) *R* (c) 4*R* (d) *R*/2 (e) √2 *R*

In a random walk, the mean end-to-end displacement of a path is proportional to the square root of the elapsed time *t*, or the number of steps *N*. The radius of the walk is proportional to the end-to-end displacement, so

$$R \sim \sqrt{t}.$$

Thus, if the available time increases by a factor of two, the radius of the path increases by a factor $\sqrt{2}$.

When inspecting the heating system of an old building, an engineer notices that many small pipes flow in parallel towards the furnace. She proposes that the small pipes be replaced by a larger pipe of twice the diameter. How many small pipes can the large one replace, and still maintain the same flow of fluid? Assume that the fluid is viscous, and that the pressure drop per unit length of pipe is the same in both cases.

(a) 16 (b) 8 (c) 4 (d) 32 (e) 2

According to Poiseuille's Law, the volume rate of flow Q in a cylinder of radius R is given by

$$Q = \pi R^4 (\Delta P / 8\eta L)$$

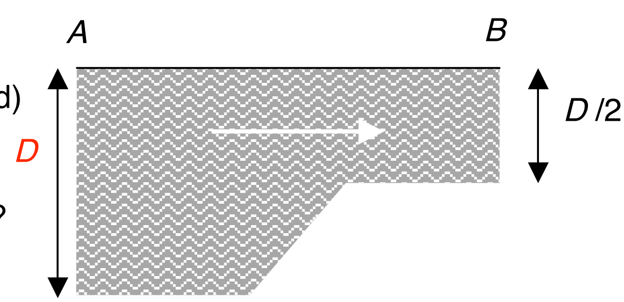
where η is the fluid viscosity and $\Delta P / L$ is the pressure gradient. Thus, if the radius is doubled, the flow increases by a factor of 2^4, and 16 pipes can be replaced.

Water is deliver to a single house on a very large lot through a pipe of radius R. A developer tears down the house and replaces it with a cluster of 81 townhouses. What must be the radius of the main water pipe to service the new homes so that each has the same flow rate as the original house?

(a) $81R$ (b) R (c) $9R$ (d) $27R$ (e) $3R$

According to Poiseuille's Law, the flow rate is proportional to the pipe radius to the fourth: R^4. Thus, to give each home the original flow rate, the pipe radius has to increase to $2R$, so the flow increases by a factor of $3^4 = 81$.

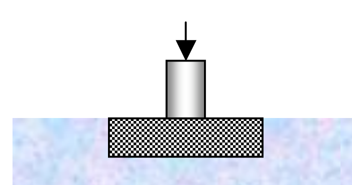
Water flows along a canal of depth D with a flow rate Q (volume per second) at point A. If the depth of the canal changes to $D/2$ at point B, what is the new flow rate in terms of the original Q? Treat the water as incompressible.



(a) $Q/2$ (b) Q (c) $2Q$ (d) 0 (e) $4Q$

For an incompressible fluid, the flow rate is given by $Q = Av$, where A is the cross sectional area and v is the speed. If the fluid is incompressible, Q does not change, although v does.

A rectangular block of mass m is placed in a fluid and held by a rod such that the top face of the block lies at the surface of the fluid. If the density of the block is 1/3 that of the fluid, what



force must the rod exert on the block, ignoring direction?

(a) $3mg$ (b) $mg/3$ (c) $2mg/3$ (d) $3mg/2$ (e) $2mg$

The bouyant force from the water on the object is

$$F_{bouyant} = \rho_{fluid} Vg,$$

where V is the volume of the object and ρ_{fluid} is the density of the fluid. This is balanced by the applied force $F_{applied}$ and the weight of the object $mg = \rho_{solid} Vg$. That is

$$F_{applied} + mg = F_{buoyant} = \rho_{fluid} Vg$$

or

$$F_{applied} = \rho_{fluid} Vg - mg.$$

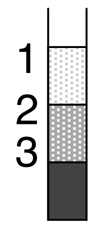
But

$$\rho_{fluid} = 3\rho_{solid},$$

so

$$F_{applied} = 3\rho_{solid} Vg - mg = 3mg - mg = 2mg.$$

Water, mercury and oil have densities 1.0×10^3, 13.6×10^3 and 0.9×10^3 kg/m^3, respectively. Knowing that these fluids do not mix with each other, what is their order (1-2-3) if equal amounts are placed in a cylinder?



(a) water-merc-oil (b) oil-merc-water
(c) oil-water-merc (d) merc-water-oil (e) merc-oil-water

The liquids separate with the most dense at the bottom, and the least dense at the top. Thus the order from top to bottom is oil-water-mercury.

Air flowing horizontally with a speed v over the flat roof of a building reduces the pressure on the roof by an amount P_v. What is the pressure reduction if the speed of the air is $3v$?
(06-2F)

(a) 0 (b) $9P_v$ (c) $3P_v$ (d) $P_v/3$ (e) $P_v/9$

The pressure difference in Bernoulli's equation is proportional to v^2. Thus, if the speed increases by a factor of 3, the pressure reduction increases by a factor of 9.

Materials - problems

A bubble rises from the bottom of a glass of water (density = 1.0 x 10^3 kg/m^3) from a depth of 12 cm below the surface. At the bottom, the initial volume of the bubble is V_{bot}, growing to V_{top} just as it reaches the surface. Treating the bubble as an ideal gas, what is V_{top}/V_{bot}? The external pressure on the fluid is one standard atmosphere.

The amount of gas in the bubble does not change as it rises, so the **ideal gas law**,

$$PV = Nk_B T \qquad \text{or} \qquad PV = nRT$$

tells us that the product PV is constant. Thus

$$P_{bot} V_{bot} = P_{top} V_{top}$$

or

$$V_{top}/V_{bot} = P_{bot}/P_{top}.$$

Now, the pressure at the bottom of the glass is

$$P_{bot} = P_{top} + \rho gh,$$

where ρ is the density of water, g is the acceleration due to gravity and h is the depth of the glass.

Substituting,

$$P_{bot}/P_{top} = (P_{top} + \rho gh)/P_{top} = 1 + \rho gh/P_{top}.$$

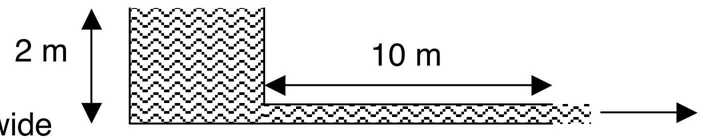
Numerically,

$$P_{bot}/P_{top} = 1 + 1.0 \times 10^3 \bullet 9.8 \bullet 0.12 / (1.01 \times 10^5) = 1.012.$$

Thus,

$$V_{top}/V_{bot} = 1.012.$$

A long garden hose is attached to the base of an open tank filled to the top with water. With a height of 2 meters, the tank is sufficiently wide that water flows through it without dissipation. The hose, however, is both long (10 meters) and narrow (radius = 0.5 cm).



(a) What is the pressure at the bottom of the tank arising from the weight of the water?
(b) What is the speed of the water as it sprays from the hose? Take the viscosity of water to be 10^{-3} kg / m•s, and ignore the difference in air pressure between the top and bottom of the tank. (Use $g = 10$ m/s^2)

(a) The gauge pressure at the bottom of the tank is ρgh, or

$$P_{bottom} = \rho gh = 10^3 \cdot 10 \cdot 2 = 2 \times 10^4 \text{ N/m}^2.$$

(b) The volume rate of flow from Poiseuille's Law is

$$\Delta V / \Delta t = \Delta P \pi R^4 / 8\eta L.$$ (mandatory)

But the change in volume for an incompressible fluid is

$$\Delta V = Av\, \Delta t = \pi R^2 v\, \Delta t,$$ (mandatory)

so

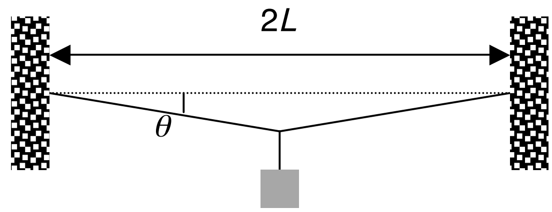
$$\pi R^2 v\, \Delta t / \Delta t = \Delta P \pi R^4 / 8\eta L$$

and

$$v = \Delta P\, R^2 / 8\eta L.$$

From (a), the pressure along the garden hose drops by 2×10^4 N/m^2 along its length. Substituting,

$$v = (2 \times 10^4) \cdot (10^{-2}/2)^2 / [8 \cdot 10^{-3} \cdot 10] = (1/16) \cdot 10^2 = 6.2 \text{ m/s}.$$



Let's revisit the demonstration done in class where a horizontal wire of length 2L has a small weight hung from it. Upon heating, the wire expands and the weight drops, as in the diagram. For small displacements, establish that $\theta = (2\alpha\,\Delta T)^{1/2}$, where α is the linear expansion coefficient and ΔT is the temperature difference. You may use without proof the small x approximations $\sin x \cong x$, $\cos x \cong 1 - x^2/2$ and $1/(1+x) \cong 1-x$.

Consider just half of the wire, of length L when it is in the horizontal position. When heated, its length increases to $L + \Delta L$, where

$$\Delta L\,/L = \alpha\,\Delta T. \qquad (1)$$

Here, α is the linear expansion coefficient and ΔT is the temperature difference.

By trigonometry,

$$\cos\theta = L\,/\,(L + \Delta L)$$
$$= 1\,/\,(1 + \Delta L\,/L).$$

If $\Delta L\,/L$ is small, then we have

$$1\,/\,(1 + \Delta L\,/L) \cong 1 - \Delta L\,/L.$$

Applying the small angle approximation to $\cos\theta$, we find

$$1 - \theta^2/2 = 1 - \Delta L\,/L$$

or

$$\theta^2/2 = \Delta L\,/L.$$

Invoking Eq. (1), we obtain

$$\theta^2/2 = \alpha\,\Delta T,$$

from which we find

$$\theta = (2\alpha\,\Delta T)^{1/2}.$$

A thin, square plate is made from a material with a coefficient of linear expansion α. Obtain an approximate expression for the coefficient of area expansion (define it as γ) for this plate in terms of α.

The area expansion is parametrized as

$$\Delta A / A = \gamma \, \Delta T.$$

But

$$\begin{aligned} \Delta A &= (L + \Delta L)^2 - L^2 \\ &= L^2 + 2L \cdot \Delta L + (\Delta L)^2 - L^2 \\ &= 2L \cdot \Delta L + (\Delta L)^2 \end{aligned}$$

Thus,

$$\begin{aligned} \Delta A / A &= \{ 2L \cdot \Delta L + (\Delta L)^2 \} / L^2 \\ &= 2\Delta L / L + (\Delta L)^2 / L^2. \end{aligned}$$

The second term is negligible with respect to the first, so it can be dropped. Then

$$\begin{aligned} \Delta A / A &= 2 (\Delta L / L) \\ &= 2\alpha \, \Delta T \end{aligned}$$

and

$$\gamma = 2\alpha.$$

You have just struggled to the top of a scenic valley in the mountains, out of breath in part because you have carried a barometer in your backpack. Your barometer indicates an air pressure of 8.5×10^4 Pa. What is your altitude compared to sea level, where the pressure is one atmosphere? Take the density of air in the atmosphere to be a constant 1.0 kg/m^3, independent of altitude.

The pressure difference between two locations in a fluid is given by

$$\Delta P = \rho g h$$

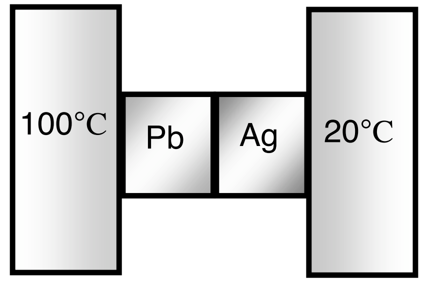
where h is the height difference. Compared with atmospheric pressure at sea level, the pressure in the valley is

$$\Delta P = 1.01 \times 10^5 - 0.85 \times 10^5 = 0.16 \times 10^5 \text{ Pa}$$

In terms of the pressure difference, the height is given by

$$h = \Delta P / \rho g$$

$$= 0.16 \times 10^5 / (1.0 \cdot 9.8) = 1630 \text{ m.}$$



Two metal cubes, with 2-cm sides, are held between two walls, one with a temperature of 100°C and the other at 20°C. The cubes are lead (Pb) and silver (Ag), whose thermal conductivities are 353 W/m•K and 429 W/m•K, respectively.

(a) Find the temperature at the lead-silver junction.
(b) Find the energy transfer through the cubes in 1.0 s.

Let T = the Temperature at the Pb-Ag junction in °C.

(a) The rate of heat flow through the lead cube:

$$H_1 = k_1 A(100°\text{C} - T) / L$$

The rate of heat flow through the silver cube:

$$H_2 = k_2 A(\text{T} - 20°\text{C}) / L$$

Since the energy arriving an departing per unit time must be equal,

$$H_1 = H_2 \qquad \text{(mandatory)}$$

or

$$k_1 A(100°\text{C} - T) / L = k_2 A(T - 20°\text{C}) / L$$

That is,

$$k_1(100°\text{C} - T) = k_2(T - 20°\text{C})$$

Solve for T:

$$T = [(100°\text{C})k_1 + (20°\text{C})k_2] / (k_1 + k_2)$$

$$T = (100\times353 + 20\times429) / (353 + 429)$$

$$T = 56°\text{C}$$

(b) The rate of energy transfer can obtained with a knowledge of T:

$$H = H_2 = H_1 = k_1 A(100°\text{C} - T) / L.$$

Substituting

$$H = 353 \cdot (0.02\times0.02) \cdot (100°\text{C} - 56°\text{C}) / 0.02 = 310 \text{ W}$$

The amount of energy transfer through the blocks in 1.0 s is then

$$Q = Ht = 310\text{W} \times 1.0\text{s} = 310 \text{ J}.$$

A spherical ball is placed sequentially in two fluids *A* and *B*, with densities ρ_A and $\rho_B = 1.3\rho_A$. In fluid *B*, the ball floats with none of its surface exposed, but in fluid *A* it sinks. What is the acceleration of the ball in fluid *A* in terms of the acceleration due to gravity *g*?

Because the ball is suspended in fluid *B* (floats with no exposed volume), it must have the same density:

$$\rho_{ball} = \rho_B.$$

The effective weight of the ball in fluid *A* is then

$$w_{eff} = w_{real}\,(1 - \rho_L/\rho_S) = w_{real}\,(1 - \rho_A/\,1.3\rho_A) = w_{real}\,(1 - 1/\,1.3) = 0.23\ w_{real}$$

The effective weight determines the acceleration experienced by the ball:

$$w_{eff} = m_{ball}a$$

so

$$m_{ball}a = 0.23\ w_{real} = 0.23\ m_{ball}g$$

or

$$a = 0.23\ g.$$

A passenger jet plane with a pressurized cabin (atmospheric pressure of 1.00×10^5 Pa) is travelling at 720 km/hr.
(a) If the air outside the plane has a density of just 0.2 kg/m^3 and one-tenth atmospheric pressure, what is the pressure difference across a window on the side of the plane?
(b) If the window is a 20 cm by 30 cm rectangle, what force does it experience?
(c) For a given air speed, how does the pressure difference change with altitude?

(a) Inside $P = 1.0 \times 10^5$ Pa
Outside atmospheric $P = 1.0 \times 10^4$ Pa

If the plane were at rest (say on Mt. Everest) the pressure difference would be

$$\Delta P = P_{atm,in} - P_{atm,out}$$
$$= 100{,}000 - 10{,}000 = 90{,}000 \text{ Pa}$$

However, because the plane is moving, the outside pressure is reduced by

$$\rho v^2/2 = 1.0 \times 10^4 + 0.2\,(720/3.6)^2/2 \qquad (v = 200 \text{ m/s})$$
$$= 1.0 \times 10^4 + 0.2\,(200)^2/2$$
$$= 4000 \text{ Pa}$$

Hence, the total pressure difference is

90,000 + 4,000 = 94,000 Pa. (wrong sign gives 86,000 Pa)

(b) The force on the window is equal to the pressure difference times the area, which yields

$$F = \Delta P \cdot A$$
$$= 0.2 \cdot 0.3 \cdot 9.4 \times 10^4$$
$$= 5640 \text{ N.}$$ (wrong sign gives 5160 Pa)

(c) Factors effecting the pressure difference are:
1. Atmospheric pressure decreases with altitude as there is less air above a given location. As a result, ΔP ***increases***.
2. The density drops with increasing altitude, so the $\rho v^2/2$ term drops, as a result of which ΔP ***decreases***.
3. The $\rho g h$ term is unchanged by the difference in density, because $\Delta h = 0$.

The smog that rises over Vancouver every day originates from emission gases and dust. Let's model a dust particle as a cube 10 μm to the side composed of material with a density of 2.5×10^3 kg/m^3.

(a) What is the mean speed of a dust particle at $T = 20$ °C?

(b) To what height h above the ground could a dust particle travel at room temperature before its thermal energy is lost to gravity? Assume $g = 10$ m/s^2 = constant.

(a) First, calculate the mass m of the dust particle with a volume of (10 μm)^3:

$$m = (10\ \mu\text{m})^3 \cdot 2.5\text{x}10^3$$
$$= 10^{-15} \cdot 2.5\text{x}10^3$$
$$= 2.5\text{x}10^{-12}\ \text{kg}.$$

Then, knowing that the mean kinetic energy of a particle in thermal equilibrium is

$$mv^2/2 = (3/2)k_B T$$

so

$$v^2 = 3k_B T / m$$
$$= 3 \cdot 4\text{x}10^{-21} / 2.5\text{x}10^{-12}$$
$$= (1.2 / 2.5)\ \text{x}\ 10^{-8}\ \text{m}^2/\text{s}^2$$

or

$$v = 6.9\ \text{x}\ 10^{-5}\ \text{m/s}.$$

(b) To find the height h to which the dust particle can move from thermal energy alone, equate

$$(3/2)k_B T = mgh$$

or

$$h = (3/2)k_B T / mg$$
$$= (3/2) \cdot 4\text{x}10^{-21} / (2.5\text{x}10^{-12} \cdot 10)$$
$$= 2.4\text{x}10^{-10}\ \text{m}.$$

Trivial – 2Å.