Physical data

Constants:
$G = 6.67 \times 10^{-11}$ Nm^2/kg^2
$g = 9.81$ m/s^2

Masses:
proton = 1.67×10^{-27} kg
electron = 1.67×10^{-27} kg
Earth = 5.98×10^{24} kg
Sun = 1.99×10^{30} kg
Jupiter = 1.90×10^{27} kg

Radii:
Earth = 6.37×10^{3} km
Sun =

Distances:
Earth-Sun = 1.50×10^{8} km
Jupiter - Sun = 7.78×10^{8} km

Moments of inertia through the c.m.
Ring = MR^2
Disk = MR^2 / 2
Rod = MR^2 / 12
Solid sphere = 2/5 MR^2

Multiple choice

Two targets have the same number of scattering centres per unit area, but are made from different metals - zinc (**A** = 64) or aluminum (**A** = 27). What is the ratio of the scattering probability of the zinc target compared to the aluminum target for the scattering of a beam of strongly interacting particles?

(a) 1 (b) 4/3 (c) 64/27 (d) 16/9 (e) 27/64

Since the radii are proportional to $A^{1/3}$, then the ratio of the radii is 4/3. But the cross section is proportional to R^2, so the ratio of the cross sections is 16/9. The scattering probability is proportional to the cross section, if n_T is the same for both targets.

Nucleus *A* has 8 times the mass number of nucleus *B*. Approximately what is the ratio of cross sections for strongly interacting particles scattered from nucleus *A* compared to nucleus *B*?

(a) 8 (b) 4 (c) 1 (d) 1/4 (e) 1/8

Since the nuclear radius scales like $A^{1/3}$, the cross section must scale like $A^{2/3}$. Thus, for the nuclei in question, the cross section ratio is $(A_A/A_B) = (8)^{2/3} = 4$.

The cross section for the strong interaction scattering from ^{27}Al nuclei is how many times that of ^{8}Li nuclei?

(a) 3/2 (b) 27/8 (c) 2/3 (d) 9/4 (e) 4/9

Since the nuclear radius scales like $A^{1/3}$, the cross section must scale like $A^{2/3}$. Thus, for the nuclei in question, the cross section ratio is $(A_A/A_B) = (27/8)^{2/3} = 9/4$.

The radius of a nucleus scales with mass number like $A^{1/3}$. If the mass number of a nucleus is doubled, by what factor does the density change?

(a) 1 (b) 2 (c) 8 (d) 1/8 (e) 1/2

Density ~ mass / volume ~ $A / (A^{1/3})^3$ ~ constant. Thus, no change.

Order the following reactions from slowest to fastest (Δ^{++} has quark content *uuu*)

reaction X: $\pi \rightarrow \gamma + \gamma$
reaction Y: $\Delta^{++} \rightarrow p^+ + \pi^+$
reaction Z: $n \rightarrow p + e + \nu$

(a) XYZ (b) ZYX (c) YXZ (d) ZXY (e) none of [a]-[d]

The reactions are dominated by:
X: e-m, because of γ Y: strong, all have quarks Z: weak, because of ν
Hence, slow to fast order is ZXY.

What is the approximate cross section for the reaction $p + \gamma \rightarrow p + \gamma$?

(a) 1 fm^2 (b) 10^{-42} m^2 (c) 10^{-24} m^2 (d) 10^{-36} m^2 (e) 10^{-16} m^2

The reaction has a photon, but no neutrino, so it must be dominated by the electromagnetic interaction. Typical electromagnetic cross sections are 10^{-36} m^2.

What is the momentum of a particle of mass m and kinetic energy mc^2?

(a) mc (b) $\sqrt{2}mc$ (c) $(3/2)mc$ (d) $3mc$ (e) $\sqrt{3}mc$

Since the total energy is $E = K + mc^2$, then if $K = mc^2$, $E = 2mc^2$.

But $E^2 = p^2c^2 + m^2c^4$, so $pc = (E^2 - m^2c^4)^{1/2}$.

$pc = (4m^2c^4 - m^2c^4)^{1/2} = (3m^2c^4)^{1/2} = \sqrt{3}mc^2$

$p = \sqrt{3}mc$

What is the kinetic energy of a particle of mass m and momentum mc?

(a) $(\sqrt{2} - 1)mc^2$ (b) mc^2 (c) $mc^2/2$ (d) $\sqrt{2}\ mc^2$ (e) $(\sqrt{2} + 1)mc^2$

Since $E^2 = p^2c^2 + m^2c^4$, and we are given $pc = mc^2$.

$E = (m^2c^4 + m^2c^4)^{1/2} = (2m^2c^4)^{1/2} = \sqrt{2}mc^2$

But the total energy $E = K + mc^2$; hence if $K = E - mc^2$, $E = (\sqrt{2} - 1)mc^2$.

Particle **A** decays at rest into two identical particles **B**. If one of the **B** particles has kinetic energy $m_Ac^2 / 3$, what is its momentum?

(a) $\sqrt{10}\ m_Ac / 6$ (b) $\sqrt{2}\ m_Ac / 3$ (c) $m_Ac / 3$ (d) $m_Ac / 2$ (e) $m_Ac / 6$

Since **A** is at rest, and decays into 2 identical particles, each **B** must have the same kinetic energy $m_Ac^2 /3$ and the same total energy $E = m_Ac^2/2$. Hence, the mass energy of each **B** particle must be $m_Ac^2 /6$.

Using $E^2 = (pc)^2 + (mc^2)^2$,

$pc / m_Ac^2 = [(E / m_Ac^2)^2 - (m_Bc^2 / m_Ac^2)^2]^{1/2}$

$= [(1/2)^2 - (1/6)^2]^{1/2}$

$= [1/4 - 1/36]^{1/2}$

$= \sqrt{(8/36)}$

$= \sqrt{2} / 3.$

You wish to probe a nucleus with radius 5 fm by bombarding it with a particle whose wavelength is not significantly larger than the nuclear radius. Which one of the following particles will not be able to probe the nucleus?

(a) photon with $\lambda = 10^{-17}$ m (b) photon with momentum = 10^{-17} kg-m/s

(c) proton with velocity = 10^6 m/s (d) neutron with momentum = 10^{-18} kg-m/s

(e) electron with energy = 10^{-7} J

The momentum and wavelength of each particle is

(a) $\lambda = 10^{-17}$ m

(b) $\lambda = h/p = 6.63 \times 10^{-34} / 10^{-17} \sim 7 \times 10^{-17}$ m

(c) $v << c$, so apply $p = mv = 1.67 \times 10^{-27} \times 10^6 = 1.67 \times 10^{-21}$ kg-m/s

and $\lambda = h/p = 6.63 \times 10^{-34} / 1.67 \times 10^{-21} \sim 4 \times 10^{-13}$ m

(d) $\lambda = h/p = 6.63 \times 10^{-34} / 10^{-18} \sim 7 \times 10^{-16}$ m

(e) $E >> mc^2$, so apply $p = E/c = 10^{-7} / 3 \times 10^8 \sim 3 \times 10^{-16}$ kg-m/s and $\lambda = h/p = 6.63 \times 10^{-34} / 3 \times 10^{-16} \sim 2 \times 10^{-18}$ m

Only (c) exceeds the desired length scale of 5×10^{-15} m.

The energy released due to gravitational binding when the Earth formed is *E*. What is the energy released in the formation of a planet with the same density as the Earth, but three times the radius of the Earth?

(a) 27*E* (b) *E*/3 (c) 3*E* (d) 81*E* (e) 243*E*

The energy released in gravitational binding of a spherical object with uniform density is $(3/5)GM^2/R$, or $E \sim M^2/R$. Now, the mass of a planet is equal to the density of the planet times its volume, or $M \sim R^3$. Hence, for planets of constant density, $E \sim (R^3)^2/R$, or $E \sim R^5$. If the radius of the planet increases by a factor of 3, the binding energy increases by a factor of $3^5 = 243$.

How many neutrons are there in the beryllium nucleus produced in the reaction

^{4}He + ^{4}He -> Be + n?

(a) 7 (b) 8 (c) 3 (d) 4 (e) none of (a)-(d)

Because each ^{4}He nucleus contains 2 neutrons, there must be 4 neutrons in the reaction products. But one neutron is emitted as a free particle, leaving 3 in ^{7}Be.

How many nucleons are there in the unknown nucleus *X* produced in the reaction

p + ^{3}He -> X + γ?

(a) 1 (b) 2 (c) 3 (d) 4 (e) none of (a)-(d)

There are 4 nucleon in the reactants p + ^{3}Be, so there must be 4 in the products by conservation of baryon number. The gamma has zero baryon number; hence *X* has 4 nucleons.

Which of the following pairs of nuclei are isotones?

(a) p, n (b) ^{6}Be, ^{7}Li (c) ^{6}Li, ^{7}Li (d) ^{3}H, ^{4}He (e) p, ^{2}H

Isotones implies equal numbers of neutrons: ^{3}H, ^{4}He have 2 neutrons each.

What quantum number is not conserved in the reaction $p + \pi^- \rightarrow n + \gamma$?

(a) all are violated (b) L_e (c) Q (d) B (e) all are conserved.

All quantum numbers are conserved:

	Before	After
L_e	0 + 0 = 0	0 + 0 = 0
Q	+1 - 1 = 0	0 + 0 = 0
B	1 + 0 = 1	1 + 0 = 1

The quark content of the particle π^+ is:

(a) *d* anti-*d* (b) *u* anti-*d* (c) *d* anti-*u* (d) *u* anti-*u* (e) *uud*

u anti-*d* has Q=+1, B=0 and L_e=0.

The "third-life" is the time it takes for a sample to decay to 1/3 of its original activity (not too useful, we'll admit). What is the "third-life" in terms of the decay constant λ?
(a) $\lambda/3$ (b) $\ln 3 / \lambda$ (c) $\ln 2 / 3\lambda$ (d) $3 / \lambda$ (e) $\ln(1/3) / \lambda$

The decay law must satisfy:

$$e^{-\lambda t} = 3^{-t/t_{1/3}}$$

Taking logs of both sides

$$\lambda = (1/t_{1/3})\ln 3$$

or $t_{1/3} = \ln 3 / \lambda$.

A sample of radioactive material is observed to decrease to 1/4 of its initial activity in 16 seconds. What is the lifetime of the nuclei in the material (in seconds)?
(a) 11.5 (b) 8 (c) 16 (d) 4 (e) 5.5

The lifetime τ is equal to $1/\lambda$, and the decay constant λ is $\ln 2 / t_{1/2}$, where $t_{1/2}$ is the half life. Hence $\tau = t_{1/2} / \ln 2$. The sample decays to 1/2 of its initial activity in one half-life, and 1/4 of its activity in 2 half-lives. Thus, the half-life is 8 seconds. This means that the lifetime is

$$\tau = 8 / \ln 2 = 11.54 \text{ seconds.}$$

The activity of a radioactive source drops from R to $R/4$ in 4 seconds. After a further 8 seconds, the activity of the source is:
(a) $R/16$ (b) $R/64$ (c) $R/12$ (d) $R/24$ (e) $R/8$

If R decreases to $R/4$ in 4 seconds, then it decreases to $(R/4)/4 = R/16$ after 8 seconds, and to $(R/16)/4 = R/64$ after 12 seconds.

A Geiger counter is used to measure the radioactivity of a source with a half-life of 8 hours. At noon today, the counter measured 480 counts per minute. How many counts per minute will be read at noon tomorrow?
(a) 480 (b) 30 (c) 120 (d) 60 (e) 240

Using the decay law $R(t) = R_o\, 2^{-t/t_{1/2}}$,

$$R(24 \text{ hours}) = R_o\, 2^{-24/8} = R_o\, 2^{-3} = R_o /8.$$

Thus, $R(24 \text{ hours}) = 480/8 = 60$ counts per minute.

The parallax of the star *Alpha* is θ. If the distance from Earth to the star *Beta* is twice as far as the distance from Earth to *Alpha*, what is the parallax of *Beta*?
(a) $\theta/2$ (b) 2θ (c) θ (d) 1 (e) $\theta/4$

The parallax of a star is inversely proportional to its distance from Earth. Hence, an object which is twice as far away has half of the parallax: $\theta/2$.

Having been pulled over by the police for running a red light, a driver claims that the light appeared green to him because of the Doppler shift. How fast would he have to be travelling to make such a claim? Take λ_{red} = 700 nm and λ_{green} = 500 nm.

(a) (5/7)c (b) (2/7)c (c) c (d) (7/5)c (e) none of (a)-(d)

The Doppler shift expression for an approaching source is $\lambda' / \lambda = (1 - v/c)$.
If λ' / λ = 5/7, then v / c = 2/7.

An oven at a temperature of 150° C resides in a kitchen with a temperature of 25° C. What is the ratio of the photon number density in the oven's interior compared to the number density in the kitchen?

(a) 2.9 (b) 6^3 (c) 6^4 (d) 4.1 (e) none of [a]-[d]

The photon number density is proportional to T^3. For the two temperatures in the problem,

$$N_{oven} / N_{kitchen} = ([273+150] / [273+25])^3 = 2.9$$

By what factor does the photon energy density change when the interior of a furnace increases from 400 K to 800 K?

(a) 2 (b) 32 (c) 8 (d) 16 (e) none of [a]-[d]

The energy density of a photon gas is proportional to T^4. So, if the temperature increases by a factor of 2, then the energy density increases by a factor of 2^4=16.

Consider a gas of nuclei in a star. Which of the following nuclei would have the highest average speed?

(a) ^{1}H at 10^6 K (b) ^{16}O at 10^6 K (c) ^{2}H at 3x10^6 K (d) ^{12}C at 10^7 K (e) ^{238}U at 10^6 K

The average kinetic energy of a particle in a gas at temperature T is of the order $k_B T$, where k_B is Boltzmann's constant. Equating this to the kinetic energy $mv^2/2$ gives

$$v \sim (T / m)^{1/2}.$$

Thus, the nuclei with the highest average speed will have the highest T / m.

(a) ^{1}H at 10^6 K $T / m = 10^6$
(b) ^{16}O at 10^6 K $T / m = 6.2 \times 10^4$
(c) ^{2}H at 3x10^6 K $T / m = 1.5 \times 10^6$
(d) ^{12}C at 10^7 K $T / m = 8.3 \times 10^5$
(e) ^{238}U at 10^6 K $T / m = 4.2 \times 10^3$

The Universe today is about 13 billion years old and has a mass density ρ. In another 13 billion years, what will be the mass density?

(a) 2ρ (b) $\rho / \sqrt{2}$ (c) $\rho / 2$ (d) $\sqrt{2}\,\rho$ (e) $\rho / 4$

The age of a system expanding according to Hubble's law is approximately H^{-1}. But the Hubble parameter H is proportional to 1 / $\rho^{1/2}$. Hence, ρ is proportional to [age]$^{-2}$, and in another 13 billion years, the density will be $\rho / 2^2 = \rho / 4$.

What was the value of the Hubble parameter when the universe was 1/4 of the age that it is today (in terms of today's value of H)?

(a) $4H$ (b) H (c) $H/4$ (d) H^{-1} (e) $2H$

The age of the universe is proportional at H^{-1}.
Hence, if $T = T_{today} / 4$, then $H = 4H_{today}$.

If the age of the universe today is T, how old was the universe when its density was four times larger than it is today?

(a) $T/2$ (b) $T/4$ (c) $4T$ (d) $2T$ (e) $T/16$

The age of the universe at any given time is proportional to the reciprocal of the Hubble parameter H. In turn, the Hubble parameter is proportional to the square root of the density. Hence, [age] ~ $[\text{density}]^{-1/2}$. So, if the density increases by a factor of four, the age decreases by a factor of √4 = 2.

What is the approximate ratio of the energy density of relic microwave radiation from the early universe ($T = 3$K) compared to infrared radiation at room temperature?

(a) 10^{-8} (b) 10^{-2} (c) 10^{6} (d) 10^{8} (e) 10^{-6}

The energy density U_γ scales like T^4. Hence, the ratio of the energy densities scales like $(10^{-2})^4 \sim 10^{-8}$ if the ratio of the temperatures is $3/300 = 10^{-2}$.

Problems

On a particularly cloudy day, about one-third of the light from the Sun is scattered back into space over a rainy BC city. Let's assume that the water droplets in the clouds have a radius of 2×10^{-6} m. What is the number of droplets per square meter in the clouds, as seen from the Earth?

We use the scattering expression $P = n_T\sigma$, where P is the scattering probability, n_T is the areal density of the target, and σ is the scattering cross section.

Since one third of the light is scattered by the clouds

$$P = 1/3.$$

If the droplets have a radius of 2×10^{-6} m, they have a cross section of

$$\sigma = \pi(2 \times 10^{-6})^2 = 4\pi \times 10^{-12} \text{ m}^2.$$

Hence,

$$n_T = P / \sigma = (1/3) / 4\pi \times 10^{-12} \text{ m}^2 = 2.65 \times 10^{10} \text{ m}^{-2}.$$

Most of the energy in the Sun is generated by a sequence of reactions whose overall form is $4p \rightarrow {}^4He + 2e^+ + 2\nu_e$.

(i) (7 marks) Find the Q-value of the reaction in MeV.

(ii) (4 marks) Most of the e^+ produced in the reaction annihilate almost immediately with nearby electrons generating a pair of photons. What is the total Q-value of the reaction including these annihilations (in MeV)?

(i) Q-value = initial mass energies - final mass energies

$$= 4m_pc^2 - m_{He}c^2 - 2m_ec^2 - 2m_\nu c^2.$$

Substituting $m_{He}c^2 = 2m_pc^2 + 2m_nc^2 - \text{BE(He)}$

and $m_\nu c^2 = 0$,

$$Q\text{-value} = 4m_pc^2 - [2m_pc^2 + 2m_nc^2 - \text{BE(He)}] - 2m_ec^2$$
$$= 2m_pc^2 - 2m_nc^2 + \text{BE(He)} - 2m_ec^2$$
$$= 2 \times 938.3 - 2 \times 939.6 + 28.296 - 2 \times 0.511$$
$$= 24.67 \text{ MeV}.$$

(ii) In each annihilation, we have $e^+ + e^- \rightarrow 2\gamma$ for which the Q-value is

Q-value = $2m_ec^2 - 0 = 2 \times 0.511 = 1.022$ MeV.

But there are two annihilations for each He produced, so the total Q-value is

Q-value = $24.67 + 2 \times 1.022 = 26.72$ MeV.

Show that the speed of a massive particle must always be less than the speed of light.

The general definition of velocity is

$$v = pc^2/E,$$

where the energy of a particle E is

$$E^2 = (pc)^2 + (mc^2)^2,$$

or

$$E = [(pc)^2 + (mc^2)]^{1/2}.$$

Substituting into the expression for the velocity gives

$$v = pc^2/ [(pc)^2 + (mc^2)]^{1/2},$$

$$= c/ [1 + mc^2 / p^2c^2]^{1/2}.$$

Since mc^2 / p^2c^2 is always greater than 1, then $v < c$.

Suppose that the work function of a metal is negligibly small compared to the energy of an incoming photon and outgoing electron in a photoelectric experiment.

(a) What is the maximum kinetic energy of a photoelectron emitted when yellow light ($f = 5 \times 10^{14}$ sec^{-1}) strikes this particular metallic surface?

(b) What is the de Broglie wavelength of the emitted electron?

(c) Is the de Broglie wavelength less than the incident photon wavelength?

(a) Using $E = hf$ for the incident photon

$$E = 6.63 \times 10^{-34} \times 5 \times 10^{14} = 3.32 \times 10^{-19} \text{ J}.$$

That is, the maximum kinetic energy of the emitted electron must be 3.32×10^{-19} J.

(b) The electron kinetic energy is $p^2/2m$, so the electron momentum p must be

$$p = (2mE)^{1/2}$$

$$= (2 \times 9.11 \times 10^{-31} \times 3.32 \times 10^{-19})^{1/2}$$

$$= (6.04 \times 10^{-49})^{1/2} = 7.77 \times 10^{-25} \text{ kg m / s}.$$

The de Broglie wavelength of the electron is

$$\lambda = h/p$$

$$= 6.63 \times 10^{-34} / 7.8 \times 10^{-15} = 8.5 \times 10^{-10} \text{ m}.$$

(c) The wavelength of the incident photon can be obtained from $c = f\lambda$,

$$\lambda = c/f = 3.0 \times 10^8 / 5 \times 10^{14} = 6 \times 10^{-7} \text{ m}.$$

Hence, the outgoing electron wavelength is much less than the incoming photon wavelength.

A beam of unstable particles is directed towards a target, with 2×10^{10} particles arriving at the surface of the target each second. Each particle decays completely into gamma rays with a decay constant of 5×10^{16} sec^{-1}. The mass energy of each particle is 140 MeV, and it travels at 10^8 m/s.

(a) What interaction dominates the decay? Quote three pieces of evidence.
(b) If the target is 4 mm thick, how many particles decay per second in the target?
(c) What is the energy carried off by the gamma rays per second, in J/s?

(a) The electromagnetic interaction dominates the decay because:

- the are only gamma rays in the decay products --> EM or weak
- there are no neutrinos in the decay products --> EM + others
- the lifetime is $\tau = \lambda^{-1} = (5 \times 10^{16})^{-1} = 2 \times 10^{-17}$ secs, which is a typical electromagnetic lifetime

(b) If the target is 0.4 mm thick, it takes $d / v = 4 \times 10^{-12}$ seconds for a particle to pass through the target, where d is the target thickness and v is the particle velocity. After this time, almost all particles have decayed, as can be seen from the expression

$$N(t) = N_o \exp(-\lambda t) = 2 \times 10^{10} \exp(-5 \times 10^{16} \times 4 \times 10^{-12})$$
$$= 2 \times 10^{10} \exp(-2 \times 10^5),$$

which is essentially zero for our purposes. Hence, all particles, 2×10^{10} per second, decay in the target.

(c) Each decay liberates 140 MeV = $140 \times 10^6 \times 1.6 \times 10^{-19} = 2.24 \times 10^{-11}$ J of energy. If there are 2×10^{10} decays per second, then the amount of energy released is $2 \times 10^{10} \times 2.24 \times 10^{-11} = 0.448$ J/s.

Emergency exit signs are sometimes powered by the decay of radioactive tritium, which has a half-life of 12.3 years and emits an electron with a kinetic energy of 0.0186 MeV. Light is emitted when the electron is captured by the surrounding plastic in the sign. Suppose that you want to construct a radioactive Christmas tree light with tritium. How many tritium atoms would you need for the light to shine with a power of 5 watts?

The tritium half-life of 12.3 years is $t_{1/2} = 12.3 \cdot 365 \cdot 24 \cdot 3600 = 3.88 \times 10^8$ seconds.

The corresponding decay constant is $\lambda = \ln 2 / t_{1/2}$ or

$$\lambda = \ln 2 / 3.88 \times 10^8 = 1.79 \times 10^{-9}\ \text{sec}^{-1}.$$

The energy released per decay is

$$E = 0.0186\ \text{MeV} = 0.0186 \cdot 10^6 \cdot 1.6 \times 10^{-19} = 3.0 \times 10^{-15}\ \text{J}.$$

Since the power (or energy released per unit time) is $P = E\,\lambda\,N$, then

$$N = P / E\lambda$$
$$= 5 / (3.0 \times 10^{-15} \cdot 1.79 \times 10^{-9}\ \text{sec}^{-1})$$
$$= 9.4 \times 10^{23}\ \text{atoms}.$$

It takes 120 days for a radioactive source to decay to 1/4 of its initial activity. What is the decay rate of the whole source (in Becquerels) if it contains 10^{20} radioactive nuclei?

For radioactive decay, we can use

$$N(t) = N_o\, 2^{-t/t_{1/2}}$$

If $N(t) / N_o = 1/4$ at $t = 120$ days, then

$$t_{1/2} = 60\ \text{days}.$$

Converting this to seconds gives

$$t_{1/2} = 60 \times 24 \times 3600 = 5.18 \times 10^6\ \text{secs}.$$

We need the decay constant λ, which is

$$\lambda = \ln 2 / t_{1/2} = \ln 2 / 5.18 \times 10^6 = 1.34 \times 10^{-7}\ \text{s}^{-1}.$$

Thus, the decay rate of the whole souce is

$$R = \lambda N_o = 1.34 \times 10^{-7} \times 10^{20} = 1.34 \times 10^{13}\ \text{s}^{-1}.$$

To an observer on the Earth, a distant star with the same luminosity as the Sun is observed to have an apparent brightness only 10^{-12} that of the Sun. What is the parallax of the star, in arc-seconds?

The apparent brightness of a star, f, is related to its luminosity L by

$$f = L / (4\pi d^2),$$

where d is the distance to the star.

Compared to the Sun a distance R_{es} away from the Earth, the brightness of the star is

$$f_{star} / f_{Sun} = [L / (4\pi d^2)] / [L / (4\pi R_{es}^2)]$$

$$= (R_{es}/d)^2.$$

Thus, the distance to the star is

$$d = R_{es} / (f_{star} / f_{Sun})^{1/2}$$

$$= R_{es} / 10^{-6}$$

$$= 10^6 R_{es}.$$

Now the parallax $\theta/2$ is given by

$$d = R_{es} / (\theta/2)$$

so that

$$\theta/2 = R_{es} / d.$$

Substituting,

$$\theta/2 = R_{es} / 10^6 R_{es},$$

or

$$\theta/2 = 10^{-6} \text{ radians}.$$

To convert this to arc seconds, we note that

$$\pi \text{ radians} = (60 \times 60 \times 180) = 648000 \text{ arc seconds}.$$

Thus,

$$\theta/2 = 10^{-6} \times 648000 / \pi = 0.206 \text{ arc seconds}.$$

A star with the luminosity of the Sun displays a parallax of 0.01 arc seconds.
(i) (4 marks) How far away is the star in parsecs?
(ii) (5 marks) If the light from this star can reach the Earth without being absorbed along the way, how bright would the star appear to be compared to the Sun?

(i) Distance d is related to parallax $\theta/2$ through

$$d = R_{es} / \theta/2.$$

By definition, $\theta/2$ = 1 arc second corresponds to a distance of 1 parsec.
Hence, $\theta/2$ = 0.01 arc seconds must be 100 parsecs.
(ii) The flux of light f from a star a distance d away is

$$f = L / (4\pi d^2),$$

where L is the stellar luminosity. Thus

$$f_{star} / f_{sun} = (R_{es} / d)^2.$$

Converting the distance from parsecs to m involves

$$d = 100 \times 3.26 \times 9.46 \times 10^{15} \text{ m} \quad (= 3.08 \times 10^{18} \text{ m})$$

Thus,

$$f_{star} / f_{sun} = (1.5 \times 10^{11} / 3.08 \times 10^{18})^2$$
$$= 2.37 \times 10^{-15}.$$

A bare 100 watt light bulb hangs in the middle of a room of height 2.5 m and floor dimensions 3 m by 3 m. If the light is evenly distributed over all of the surfaces in the room, what flux is measured on one of those surfaces (*e.g.* a wall)? What is the ratio of this flux to that of sunlight at 1400 watts/m^2.

The total area of the surfaces in the room is

walls: 4 x 3 x 2.5 = 30 m^2
floor and ceiling: 2 x 3 x 3 = 18 m^2
total = 48 m^2.

Distributing the light over this area gives a flux of

flux = power / area = 100 / 48 = 2.1 watts/m^2.

The ratio of this to the power of sunlight is

$2.1 / 1400 = 1.5 \times 10^{-3}$.

A police car radar set operates at a frequency of 3.00×10^9 Hz. What is the frequency shift obtained by the radar as it measures a car travelling at 30 m/s? Use $c = 3 \times 10^8$ m/s for the speed of the radar wave.

This is a problem involving both a moving observer and a moving source. Define the speed of the car to be c.
Radar wave emitted by set at a frequency f.
Radar wave observed by moving vehicle at $f' = [c / (c - v)]\ f$.
The radar is re-emitted by the car as a moving source, and observed by the apparatus again at $f'' = [(c + v) / c]\ f'$.
Thus,

$$f'' = [(c + v) / c][c / (c - v)]\ f = [(c + v) / (c - v)]\ f.$$

In this question, we wish to compare f'' for a car at 30 m/s with one at 30.5 m/s.
The expressions in brackets give

$$f'' = [(1 + v/c) / (1 - v/c)]\ f = (1 + 2v/c)\ f.$$

Thus

$$f'' - f = 2\ v/c\ f.$$

At 30 m/s, $f'' - f = 2\ v/c\ f = 2\ (30 / 3.0 \times 10^8)\ 3 \times 10^9 = 600$ Hz.

A galaxy in Ursa Major is 215 Mpc away, and its light shows a 5% fractional change in wavelength towards the red.
(a) Find the velocity of the galaxy relative to the Earth in km/sec.
(b) Deduce a value of the Hubble parameter from this observation.

(a) The Doppler shift expression reads

$$(\lambda' - \lambda) / \lambda = v/c.$$

Hence,

$$v/c = 0.05$$

$\Rightarrow$ $v = 0.05 * c = 0.05 * 3.0 \times 10^8 = 1.5 \times 10^7$ m/s $= 1.5 \times 10^4$ km/s.
(b) The Hubble formula reads

$$v = HR$$

so that

$$H = v/R = 1.5 \times 10^4 / 215 = 70 \text{ km / (s-Mpc)}.$$

Blue light (λ = 400 nm) from a nearby galaxy is observed to be shifted to shorter wavelengths by 1 nm. What is the galaxy's apparent velocity (including direction) with respect to the Earth?

We use the Doppler shift expression for a receding object

$$(\lambda' - \lambda) / \lambda = v/c.$$

But $\lambda' - \lambda = -1$ nm (shorter wavelength)

$\Rightarrow$ $v/c = -1 / 400$

$\Rightarrow$ $v = - c / 400 = - 3.0 \times 10^8 / 400 = -7.5 \times 10^5$ m/s.

Therefore, the star is moving towards the Earth at 7.5×10^5 m/s.

Three objects emerging from an explosion have positions as a function of time [$x(t)$, $y(t)$] given by ($5t$, 0), ($-3t$, 0) and (0, $4t$). Verify that an observer on object 1 concludes that the other two objects obey a Hubble's Law type motion with respect to object 1. What is the value of the Hubble parameter for these objects at t = 2 sec?

The distance between object #1 and object #2 is

$$d_{12} = 5t - (-3t) = 8t$$

The distance between object #1 and object #3 is

$$d_{13} = [(5t - 0)^2 + (0 - 4t)^2]^{1/2} = [25t^2 + 16t^2]^{1/2} = 6.4t.$$

Therefore, the distances increase linearly with time as

$$d = vt.$$

This is of the Hubble law form $v = HR$ with $H = 1/t$.

For t = 2 secs, $H = 0.5$ sec^{-1}.