PHYS 211 FINAL EXAMINATION

Thursday, 12 August, 2004 Name_____
Time: 3 hours Student # _____

Calculator and one formula sheet permitted.

Please show complete solutions to problems 4-8 and explain your reasoning (this requires words, not just equations). State any principles that you use in your solutions.

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I_{disk} (symmetry axis) = $MR^2/2$ $I_{square} = ma^2/12$ (in plane)

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1. For each of the following questions, please circle one selection for your answer. (8 marks)

(i) An object starts at $t = 0$ with an initial speed v_o and is subject only to a drag force which is quadratic in speed. Which statement correctly describes the motion of the object after the passage of a long period of time?

(a) speed → constant ≠ 0 position → ∞
(b) speed → ∞ position → ∞
(c) speed → 0 position → ∞
(d) speed → 0 position → finite constant
(e) speed → constant ≠ 0 position → finite constant

(ii) Which of the following forces is conserved for all coordinates (x,y):

(a) $\mathbf{F} = y\,\mathbf{i} - x\,\mathbf{j}$ (b) $\mathbf{F} = 3x\,\mathbf{j}$ (c) $\mathbf{F} = x^2\,\mathbf{i} + xy\,\mathbf{j}$ (d) $\mathbf{F} = y\,\mathbf{i} + x\,\mathbf{j}$ (e) none are conserved

(iii) Consider a coordinate system $x'y'z'$ attached to the Earth at 45° south latitude and rotating with it at an angular frequency ω. Which of the following vectors describes the rate of change $d\mathbf{k}'/dt$ of the vector $\mathbf{k}'$?

(a) $\omega(0.707, 0, 0)$ (b) $\omega(0, 0, 0.707)$ (c) $\omega(-0.707, 0, 0)$
(d) $\omega(0, 0.707, 0)$ (e) none of [a-d]

(iv) The eccentricity e of an elliptical orbit is

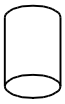
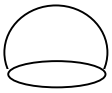
(a) zero (b) one (c) less than one (d) greater than one (e) complex

2. For each of the following questions, please circle one selection for your answer. (6 marks)

(i) Which of the following statements does not apply to a conical pendulum?

(a) angular velocity is constant
(b) angular momentum is not parallel to angular velocity
(c) torque is non-zero
(d) angular acceleration is zero
(e) angular momentum vector is constant

(ii) Order the following shapes according to the distance of their centre-of-mass from the base of the object. All objects have the same height and are uniform solids.

 cone  cylinder  hemisphere

(a) cone > cylinder > hemisphere
(b) cone > hemisphere > cylinder
(c) cylinder > cone > hemisphere
(d) cylinder > hemisphere > cone
(e) none of [a-d]

(iii) For which of the following shapes must the angular momentum vector always be parallel to the angular velocity vector (for rotation about the centre-of-mass)?
(a) sphere (b) cylinder (c) thin disk (d) thin rod (e) none of [a-d]

3. For each of the following questions, please circle one selection for your answer. (**12** marks)

(i) Three springs are coupled in series in a straight line, with one end of the first and third springs attached to a fixed wall. The two masses at the attachment points of the springs have the same value m. The middle spring has a force constant $2k$, while the springs on either side each have the same force constant k. What is the frequency ratio of the anti-symmetric mode compared to the symmetric mode of oscillation?
(a) $\sqrt{5}$ (b) $\sqrt{3}$ (c) 3 (d) 5 (e) 2

(ii) A particular string under tension is plucked, resulting in an amplitude A_o and radiating an average power P_o. If the amplitude of the wave were doubled, what would be the new average power?
(a) P_o (b) $2P_o$ (c) $4P_o$ (d) $8P_o$ (e) $P_o/2$

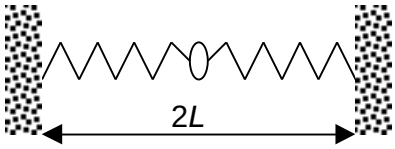
(iii) A string is held under tension between two fixed walls separated by a distance L. What are the angular frequencies of the basic modes of oscillation?
(a) c/L (b) nc/L (c) $n\pi L/c$ (d) $nL/2$ (e) $n\pi c/L$

(iv) A wave on a particular string under tension has a speed c_o. If the tension were reduced by half, the new speed of the wave would be:
(a) c_0 (b) $c_0/2$ (c) $c_0/4$ (d) $c_0/\sqrt{2}$ (e) $2c_0$

4. A damped harmonic oscillator has angular frequency ω. The ratio of the amplitude of two successive maxima as the motion decays is exactly one half. Show that the magnitude of $\gamma = c/2m$ for this oscillator is given by $\omega \ln 2 / 2\pi$. (15 marks)

5. A science technician wishes to set up a Foucault pendulum so that its plane of motion follows the hands of a clock, coinciding with them at the "12 o'clock" position. At what latitude (including hemisphere) can the pendulum be placed so that its period is
(a) 12 hours
(b) 24 hours (6 marks)

6. Two identical springs are attached to each other at one end, and to a fixed wall at the other. The springs have force constant k and unstretched length s, while the distance between the walls is $2L$. A small mass sits between them at the location defining the coordinate origin: x-axis is horizontal and y-axis is vertical.



(a) Find the potential energy as a function of x and y.
(b) From the potential, find the y-component of the force as a function of y at $x = 0$. Describe its behaviour for $s < L$ and $s > L$.
(c) Using $x = y = 0$ as an example, show that the behaviour in part (b) is consistent with the conventional stability condition on the potential energy. (20 marks)

7. Calculate the moment of inertia of a thin sheet with the shape of an equilateral triangle, where M is the mass of the sheet and a is the length of each side. The axis of rotation lies in the plane of the triangle, passing one vertex and the triangle's centre-of-mass. (18 marks)

8. A thin square plate of side a and mass m is thrown into the air so that it rotates freely under zero torque. The rotational period $2\pi/\omega$ is 1 second. If the axis of rotation makes an angle of 45^o with the symmetry axis of the plate, find the period of precession of the axis of rotation about the symmetry axis and the period of wobble of the symmetry axis about the invariable line. (15 marks)

Answers:
1. c, d, a, c
2. e, d, a
3. a, c, e, d
4. $\gamma = \omega \ln 2 / 2\pi$
5. (a) no solution; (b) north pole only
6. (a) $V(x,y) = k\{L^2 + x^2 + y^2 + s^2 - s(D_{left}+D_{right})\}$; (b) $F_y(0,y) = 2ky\{s/(L^2+y^2) - 1\}$ which is always attractive for $s<L$, but repulsive for $s>L$ in the region $y^2 < s^2-L^2$; (c) $(\partial^2 V/\partial y^2)_{y=0}$ is positive for $s<L$ (stable) and negative for $s>L$ (unstable).
7. $I = Ma^2/24$
8. precession period about symmetry axis is $\sqrt{2}$ seconds; wobble period is $(2/5)^{1/2}$ seconds.