PHYS 385 FINAL EXAMINATION

Thursday, 7 August, 2003 Name____________________

Time: 3 hours Student #_________________

Calculator and one formula sheet permitted

Please show complete solutions to questions 3 to 6; explain your reasoning.

$e = 1.6\text{x}10^{-19}$ C $\hbar = 1.055\text{x}10^{-34}$ J-s $k = 8.99\text{x}10^{9}$ N-C^2/m^2 $m_{nuc} = 1.67\text{x}10^{-27}$ kg

$\int_{-\infty}^{\infty} \exp(-y^2)\, y^2\, dy = \sqrt{\pi}/2$ $\int_{-\infty}^{\infty} \exp(-y^2)\, dy = \sqrt{\pi}$

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1. For each of the following questions, please circle one selection for your answer. (15 marks)

(i) If the potential energy is independent of time, then the time-dependence of the wavefunction is

(a) $\exp(+iEt/\hbar)$ (b) $\exp(-iEt/\hbar)$ (c) zero (d) $Et/\hbar$ (e) none of [a-d]

(ii) The expectation of the momentum operator $\langle p\rangle$ for a freely propagating particle is

(a) $m\, d\langle x\rangle/dt$ (b) zero (c) $m \langle dx/dt \rangle$ (d) mv (e) $m\, d\langle\psi\rangle/dt$

(iii) In the Bohr model of the hydrogen atom, the orbital radius changes with quantum number n as

(a) $1/n^2$ (b) $1/n$ (c) n^0 (d) n^1 (e) n^2

(iv) For Coulomb wavefunctions with principle quantum number n, the maximum value of the orbital angular momentum quantum number ℓ is

(a) n (b) 0 (c) $n + 1$ (d) $n - 1$ (e) no constraint

(v) A particle with energy E approaches a square barrier with height V_o and width a, such that $E < V_o$. If the width were doubled to $2a$, but all other quantities were held fixed, then the transmission probability would roughly:

(a) decrease exponentially with a (b) decrease by a factor of 2 (c) increase by a factor of 2

(d) remain unchanged (e) increase exponentially with a

2. For each of the following questions, please circle one selection for your answer. (15 marks)

(i) Which of the following approximations is based upon the hierarchy of speeds in a system?

(a) WKB approximation (b) first order perturbation theory

(c) Born-Oppenheimer approximation (d) Debye approximation

(e) variational principle

(ii) Which of the following is an allowed molecular state?

(a) $^1\Pi_1$ (b) $^1S_3$ (c) $^1\Sigma_3$ (d) ^1D_o (e) $2s$

(iii) Which quantum number is automatically conserved if the potential energy is independent of the azimuthal angle ϕ?

(a) ℓ = orbital angular momentum (b) m_ℓ = z-component of ℓ (c) spin angular momentum

(d) n = principal quantum number (e) none of [a-d]

(iv) For a central potential, what is the minimum value of the orbital angular momentum ℓ?

(a) 1 (b) $-\infty$ (c) m_ℓ (d) 0 (e) none of [a-d]

(v) What is the result of operating S_+ on the spin-1/2 state | 1/2, -1/2>?

(a) $\hbar$ |1/2, 1/2> (b) |1/2, 1/2> (c) $\hbar$ | 1/2, -1/2> (d) $(3/4)^{1/2}\,\hbar$ | 1/2, 1/2> (e) 0

3. By explicit substitution for the kinetic energy operator $p_{op}^2/2m$, find the kinetic energy of the ground state wavefunction

$u_o(x) = (\alpha/\pi)^{1/4}\exp(-\alpha x^2/2)$ where $\alpha = m\omega/\hbar$. (18 marks)

4. In two different situations, a gas of non-interacting fermions is confined to move in a line (1D) or plane (2D) with number densities of μ and μ^2, respectively, where μ is a number per unit length. The boundaries of each system are parallel hard walls a distance L apart in a given direction. Find the Fermi energy for each system. To simplify the math, assume that only one fermion is allowed per momentum state. (16 marks)

5. The wavefunctions in a one-dimensional harmonic oscillator with $V(x) = m\omega^2x^2/2$ are given by

$u_n(x) = H_n(\xi)\exp(-\xi^2/2)$

where

$\xi = \sqrt{\alpha}\,x$ and $\alpha = m\omega/\hbar$.

(a) Find the location x_{max} of maximum probability density as a function of the oscillator quantum number n. For the Hermite polynomials, just use the leading order term in ξ, not the exact form!

(b) Find the location of the classical turning points x_{cl} as a function of n.

(b) Determine the ratio x_{cl}/x_{max}, and find its limit as $n \to \infty$. (18 marks)

6. Find the difference in energy between the rotational ground state and first excited state of the three molecules H_2, HD and D_2, where D is the deuterium atom (which has twice the mass of the hydrogen atom). Take the internuclear spacing in all three molecules to be 1.0Å and express your answer in eV. (18 marks)

Answers

1. b, a, e, d, a.
2. c, a, b, d, a.
3. $\langle p^2\rangle/2m = \omega\hbar/2$.
4. 1D: $E_F = \mu^2h^2/8m$; 2D: $E_F = \mu^2h^2/2\pi m$.
5. (a) $x_{max} = (n/\alpha)^{1/2}$; (b) $x_{cl} = ([2n+1]/\alpha)^{1/2}$; (c) $x_{cl}/x_{max} \to \sqrt{2}$ as $n \to \infty$.
6. H_2 is 8.4×10^{-3} eV; HD is 6.9×10^{-3} eV; D_2 is 4.2×10^{-3} eV.