PHYS 390 Final Examination 2008

Time: 3 hours Name______________________________

9 April, 2008 Student number____________________________

Calculator is permitted; a formula sheet is provided.

Show complete solutions to questions 4 to 8.

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1. For each of the following questions, circle one correct answer. (15 marks)

(i) In terms of its radius R, the period of a circular Keplerian orbit is proportional to

(a) R (b) $R^{-1/2}$ (c) $R^{3/2}$ (d) independent of R (e) $R^{2/3}$

(ii) The parallax of the star *Alpha* is θ. If the distance from Earth to the star *Beta* is three times the distance from Earth to *Alpha*, what is the parallax of *Beta*?

(a) $\theta/3$ (b) 3θ (c) θ (d) $\theta/6$ (e) $\theta/9$

(iii) Which of the following statements is true? All other variables being equal, the luminosity of a star

(a) falls as the distance (squared) of the observer

(b) is proportional to its surface temperature (squared)

(c) rises as the radius of the star (squared)

(d) is completely independent of stellar mass

(e) depends on the albedo of the observer

(iv) Kepler's Second Law (equal areas in equal times) was shown by Newton to be based on the conservation of

(a) linear momentum (b) angular momentum (c) kinetic energy (d) mass (e) mechanical energy

(v) When the density of the universe was four times what it is today, what was the value of the Hubble parameter in $V = HR$ in terms of today's value?

(a) $H_{today}/4$ (b) $H_{today}/2$ (c) H_{today} (d) $2H_{today}$ (e) $4H_{today}$

2. For each of the following questions, circle one correct answer. (15 marks)

(i) What is the approximate cross section for the reaction $p + \nu \rightarrow p + \nu$?

(a) 1 fm^2 (b) 10^{-42} m^2 (c) 10^{-24} m^2 (d) 10^{-36} m^2 (e) 10^{-16} m^2

(ii) The pressure of a photon gas:

(a) decreases with temperature

(b) depends on the size of the container

(c) is linearly proportional to its number density

(d) depends on the electromagnetic cross section

(e) is linearly proportional to its energy density.

(iii) In a particular pure gas at a fixed temperature, there are C collisions per second. If the density of the gas is tripled but there is no change in temperature, the number of collisions per second is
(a) $C / 3$ (b) $3C$ (c) $6C$ (d) $9C$ (e) $9C/2$

(iv) The difference between the total energy of a particle and its mass energy is equal to
(a) its kinetic energy (b) its potential energy (c) its spin (d) p^2c^2 (e) $k_B T$

(v) If the mass to light ratio of one region of the Milky Way were 8 times that of the Sun, the typical stellar mass in this region would be
(a) $M_{sun}/8$ (b) $M_{sun}/4$ (c) $M_{sun}/2$ (d) $8M_{sun}$ (e) $2M_{sun}$

3. For each of the following questions, circle one correct answer. (15 marks)

(i) For massless particles in a particular spin state, the mean energy per particle for fermions is what fraction of the mean energy per particle for bosons?
(a) 7/6 (b) 3/4 (c) 6/7 (d) 7/8 (e) 8/7

(ii) In a relativistic gas composed solely of electrons and their associated neutrinos, what fraction of the energy is carried by the electrons?
(a) all (b) 1/3 (c) 2/3 (d) 3/4 (e) 1/2

(iii) The minimum mass of a gas cloud required for it to collapse under gravity:
(a) rises with the temperature of the cloud
(b) rises with the density of the cloud
(c) is independent of density
(d) is linearly proportional to the local gravitational acceleration
(e) is inversely proportional to the mean kinetic energy of its molecules.

(iv) The double peaks in the observed abundances of elements heavier than iron
(a) is a relic of the Big Bang
(b) arises in cool stars at equilibrium
(c) arises from successive addition of ^{4}He nuclei
(d) arises from stellar explosions
(e) is evidence for strong magnetic fields during star formation.

(v) In a matter-dominated universe, the Hubble parameter H scales with the "size" of the universe R as
(a) R^2 (b) $R^{3/2}$ (c) $1/R$ (d) R^{-2} (e) $R^{-3/2}$

4. Calculate the total time in hours taken for an eclipse of the Sun as measured from a specific location on the Earth (from when the shadow first appears to when it disappears). Ignore the rotation of the Earth, assume that the Earth and Moon orbits are co-planar, and take the orbital period of the Moon to be 28 days. Include in your answer a diagram of the process. (10 marks)

5. A “standard” person, Joe, has a surface area of 1.4 m^2 and a surface temperature of 33 °C.
(a) How much power does Joe’s body radiate, assuming that he is an ideal radiator?
(b) How much power does he absorb from his environment at 20 °C, assuming he is an ideal absorber?
(c) What is his net energy production per day in kcal (where 1 kcal = 4186 J), which is the energy equivalent of his food intake? (10 marks)

6. (a) Find the tidal force per unit mass at an equatorial location for the following situations. The “per unit mass” refers to the planet/moon where the force is measured.
(i) on the Moon from the Earth
(i) on Earth from the Sun
(iii) on Mercury from the Sun
(b) Based on your answers to part (a) and your knowledge of the motion of the Moon and the Earth, what do you conclude about the coupling of the rotational motion of Mercury to its orbital motion around the Sun? (12 marks)
Data: M_{Mercury} = 3.3x10^{23} kg R_{Mercury} = 2.44x10^6 m $d_{\text{Mercury-Sun}}$ = 5.79x10^{10} m

7. For a cluster of stars, find the local acceleration due to gravity g_r as a function of distance r from the centre of the cluster. Take the number density to be $n(r) = n_o(r_c/r)^2$, with a sharp cut-off at $r = r_c$ and take all stars to have the same mass m. (10 marks)

8. Imagine a region of stars that obeys an ideal-gas like expression for the pressure P = (2/3) nK, where n is the number density of the stars and K is their mean kinetic energy. The speed of an acoustic wave in such an ensemble should obey $v_{ac} = (P/\rho)^{1/2}$, where ρ is the mass density.
(a) Assuming that all stars have the same mass and travel at the same local speed v_{local} (in random directions), establish that $v_{ac} = v_{local} / \sqrt{3}$.
(b) Suppose that v_{local}, which is the mean *relative* speed, is 10% of the collective mean speed of the star v_{star}, with v_{star} = 200 km/s. Calculate v_{ac}.
(c) Even though a galaxy rotates at v_{star}, the *spiral pattern* could change according to v_{ac}. Find the period of the motion of the pattern in years at a distance of 10 kpc from the centre of the galaxy using v_{ac} as the tangential velocity. How many such periods are there in 10 billion years? (13 marks)

Answers:
1. *c, a, c, b, d*   2. *b/d, e, d, a, c*   3. *a, c, a, d, e*
4. *1.96 hours*   5. *2290 kcal*
6. (a) (i) *2.5x10^{-5} N/kg*; (ii) *5.0x10^{-7} N/kg*; (iii) *3.3x10^{-6} N/kg*;
(b) *The rotational and orbital periods of Mercury should be coupled, but perhaps not as strongly as the Moon is to Earth.*
7. $g_r = 4\pi Gmn_o r_c^2 / r$ 8. (b) *11,500 m/s*; (c) *Period = 5.3 billion years*

Data 1 ly = 9.4×10^{12} km 1 pc = 3.26 ly $G = 6.67 \times 10^{-11}$ N•m^2/kg^2

$c = 3.0 \times 10^8$ m/s $\sigma = 5.67 \times 10^{-8}$ watts / m^{2}K^4 $k_B = 1.38 \times 10^{-23}$ J/K

1 amu = 1.66×10^{-27} kg 1 eV = 1.6×10^{-19} J $(e^2/4\pi\varepsilon_o) = 1.44$ MeV-fm

$M_{Earth} = 6.0 \times 10^{24}$ kg $M_{Moon} = 7.36 \times 10^{22}$ kg $M_{Sun} = 1.99 \times 10^{30}$ kg

$R_{Earth} = 6.4 \times 10^6$ m $R_{Moon} = 1.74 \times 10^6$ m $R_{Sun} = 6.96 \times 10^8$ m

$d_{Earth\text{-}Sun} = 1$ AU $= 1.50 \times 10^{11}$ m $d_{Earth\text{-}Moon} = 3.84 \times 10^8$ m $L_{sun} = 3.9 \times 10^{26}$ watts

Measurements $\tan(p) = R_{es}/d$ $F = L / 4\pi d^2$ $F_{surf} = \sigma T^4$

$m_1 - m_2 = 2.5 \bullet \log_{10}(F_2/F_1)$ $m - M = 5 \bullet \log_{10}(d / 10 \text{ pc})$ $(\lambda' - \lambda) / \lambda = v/c$

Planets $\pi ab/P = L / 2\mu$ $b^2 = a^2 \bullet (1-e^2)$ $P^2 = 4\pi^2 a^3 / GM$

$\Delta F_x \cong 2(GM_p m_m R / r^3)\cos\theta$ $\Delta F_y \cong -(GM_p m_m R / r^3)\sin\theta$ $r < 2.456(\bar{\rho}_p / \bar{\rho}_m)^{1/3} R_p$

$T_{planet} = (1-a)^{1/4}\ T_{star}(R_{star}/2D)^{1/2}$ $v_{esc} = (2GM_{planet}/R_{planet})^{1/2}$ $v_{rms} = (3k_B T/m)^{1/2}$

$T_{esc} > (1/54)\ GM_{planet} m / k_B R_{planet}$

Ensembles $\langle K \rangle = (3/2)k_B T$ $N_\gamma = 2.02 \times 10^7 T^3$ (m^{-3}) $U_\gamma = 7.565 \times 10^{-16} T^4$ (J/m^3)

$P_\gamma = U_\gamma/3$ $N_{fermion} = (3/4)N_{boson}$ $U_{fermion} = (7/8)U_{boson}$

Relativity $E = mc^2 + K$ $E^2 = p^2c^2 + m^2c^4$ $p = hc/\lambda$

$BE = \Sigma_{free} m_i c^2 - m_{bound} c^2$ Q-value $= \Sigma_{initial} m_i c^2 - \Sigma_{final} m_f c^2$

$BE = 16A - 18A^{2/3} - 24(N-Z)^2/A - 1.44(3/5)Z^2/R + \delta$ (MeV)

Nucleosynthesis $R(t) = -\lambda N(t)$ $N(t) = N_o \exp(-\lambda t)$ $P = n_T \sigma$

$t_{av} = 1 / (N_T \sigma v)$ $R = 1.2A^{1/3}$ (fm) $P_0 = (E_c/E)^{1/2}\exp(-bE^{-1/2})$ $b = (\mu/2)^{1/2}(2\pi Z_1 Z_2/)(e^2/4\pi\varepsilon_o)$

$E_c = (e^2/4\pi\varepsilon_o)Z_1 Z_2/[1.4(A_1^{1/3} + A_2^{1/3})]$ $\sigma(E) = S(E)/E \exp(-bE^{-1/2})$ $r_{aX} = N_a N_X \langle \sigma v \rangle / (1+\delta_{aX})$

$\Lambda = 7.20 \times 10^{-25}\ S_o\ K^2\ e^{-K} / (AZ_1Z_2)$ (m^3/s) $K = (3E_o / k_B T) = 42.48\ (Z_1^2 Z_2^2 A / T_6)^{1/3}$

$$r_{12} = \frac{2.62 \times 10^{29}}{(1+\delta_{12})AZ_1Z_2}\rho^2 \frac{X_1 X_2}{A_1 A_2} S_o K^2 e^{-K} \text{ (m}^{-3}\text{s}^{-1}) \qquad X_i = A_i N_i M_u / \rho$$

Stars [*number density*] $\propto M^{-3.27}$ $L_{obs} \propto M^4$ $L_{obs} \propto T^5$ $\Delta U_{grav} = (3/5)GM^2/R$

[*lifetime*] $= (f\varepsilon X_H M_{sun}/L_{sun}) \bullet (M/M_{sun}) \bullet (L_{sun}/L)$ $P_{core}/R_{star} \sim \rho g$

$M_{Jeans} = (5k_B T / G\mu m_H)^{3/2} \bullet (3 / 4\pi\rho_o)^{1/2}$ $t_{ff} = (3\pi / 32G\rho_o)^{1/2}$

$n(z,R) = n_o$ [exp($-z$/0.325 kpc) + 0.02 • exp($-z$/1.4 kpc)] • exp($-R$/3.5 kpc)

$n_{halo}(r) = n_{o,\,halo}\ (r / 2.7 \text{ kpc})^{-3.5}$ $n_{o,\,halo} = 0.2\%$ of $n_{o,\,disk}$ $(M/L)_{disk} = 3(M/L)_{sun}$

Early universe $(\lambda' - \lambda) / \lambda = f - 1$ $T_f = T/f$ $V = HR$

$H_{crit} = (8\pi G\rho/3)^{1/2}$ $H_{crit} = (8\pi GU / 3c^2)^{1/2}$ $U_{matter} \sim 1/R^3$ $U_{radiation} \sim 1/R^4$

$\Delta t = (2/n)\ \Delta(H^{-1})$