Lecture 15 - Stars

What's Important:
- condensation of matter
- distribution of stellar masses and luminosities
- Hertzsprung-Russell diagram

Text: Carroll and Ostlie, Sec. 8.2; Clayton, Sec. 1.6

Condensation of neutral matter

At the end of the first few minutes, the universe was cool enough to permit nuclei to bind ($T \sim 10^9$ K), but nowhere near cool enough to permit electrons to be captured to form neutral atoms ($T \sim 3000$ K). Thus, the universe existed in an ionized state for a considerable length of time as it expanded and cooled. The ionization temperature of 3000 K follows from a detailed calculation, but one can get a rough estimate just from knowing that the ionization energy of a hydrogen atom is of the order 10 eV = 10^{-6} MeV = $10^{-6} \times 10^{10}$ K = 10,000 K.

In the first few minutes, the universe was at least as dense as a nucleus, which is about 10^{15} times as dense as an atom. What was the universe like at $T = 3000$ K? We use the ratio of temperatures, compared to today's density, to answer this question:

$$n(3000) = n(3) \cdot (3000 / 3)^3 = 10^9\, n(3).$$

Taking the easy way out and setting $n(3) = n_{crit} = 5$ nucleons/m^3,

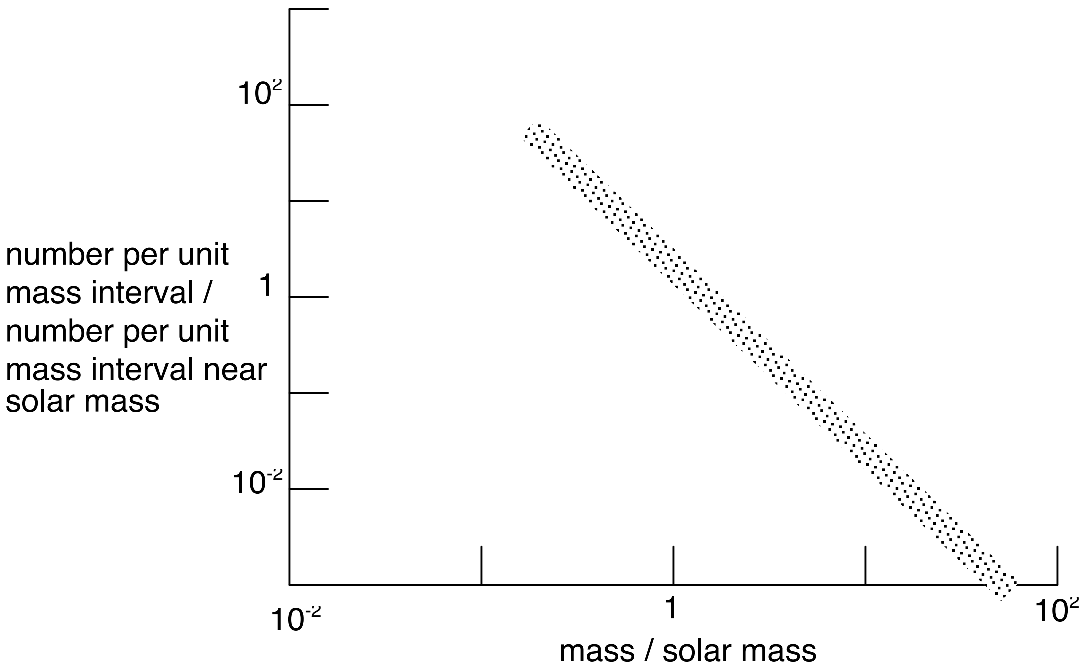
$$n(3000) = 5 \times 10^9 \text{ nucleons/m}^3.$$

Now, this density is MUCH less than even the Earth's atmosphere (10^{25} atoms / m^3), let alone its interior. So, by the time the universe was cool enough to support neutral atoms, it was 10^{17} times **less** dense than a gas at NTP.

It is after the neutralization of ions that density fluctuations in the universe led to the formation of galaxies and stars, as will be shown later. For now, our interest is on the processes which provide a star with its energy and keep it from collapsing.

Distribution of stellar masses

Not all stars have the same mass, nor do all galaxies. Here, we will concentrate on the distribution of stellar masses, and leave galaxies to a later section of the course. The formation of stars from a cloud of gas has parallels with the formation of droplets in a cloud of supersaturated water vapour. Most of the stars probably condensed simultaneously (where this could mean 100 million years!), and produced an initial distribution that has been modified today through the evolution, explosion ... of the original stars. In general, there are fewer heavier stars than there are lighter stars, and the distribution obeys an approximate power-law form called the Salpeter function:



number $M^{-2.35}$ (original fit)
number $M^{-3.27}$ (modern fit in 1 - 10 solar mass range)

Notes:
- the distribution of droplets in the critical region of the vapor -> liquid transition region obeys $M^{-2.3}$.
- the fact that there are so many binary stars may indicate that most stars formed in clusters.

Two stellar characteristics which can be measured directly are:
- temperature (from the colour spectrum)
- luminosity (from the power output)

What do we expect for a plot of luminosity *vs*. temperature for a distribution of stars?

Corresponding to the energy density expression for black-body radiation

$$U = aT^4$$

where

$$a = \frac{8\pi^5 k_B^4}{15c^3h^3} = 7.565 \times 10^{-16}\ \mathrm{J\,m^{-3}\,K^{-4}}$$

This expression provides the theoretical basis for Stephan's Law

$$[flux] = \sigma T^4$$

for which it can be shown that

$$\sigma = \frac{ca}{4} = \frac{2\pi^5 k_B^4}{15c^2h^3} = 5.67 \times 10^{-8}\ \mathrm{W\,m^{-2}\,K^{-4}} = \textit{Stephan – Boltzmann constant}$$

As usual, the luminosity of a star is just the flux integrated over the star's surface area

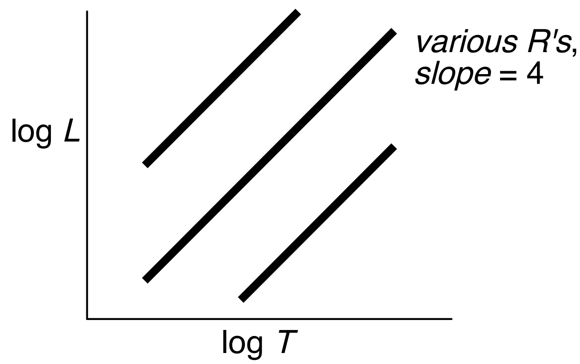
$$L = 4\pi R^2 \sigma T^4.$$

(*recall that* T *is the surface temperature, not the interior temperature*).

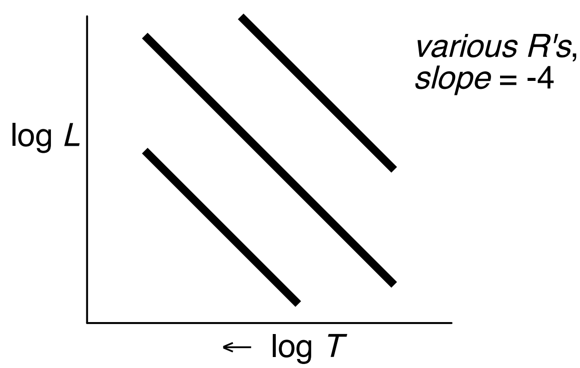
Let's assume that R varies slowly and take a log-log plot of this relationship:

$$\log L = \log(4\pi R^2 \sigma) + 4 \log T.$$

This would predict that (for constant R)



Astronomers usually make this plot against decreasing temperature



This is called the **Hertzsprung-Russell** diagram. In reality, T and R are linked as they represent a steady state condition for a given initial star mass. However, the observed distribution of stars is not that dissimilar from the approximation $L \propto T^4$, and L actually rises as T^5. At their time of formation, then, a distribution of stars looks like