Lecture 17 - Nuclei

What's important:

- nuclear sizes and structure
- nuclear binding energies
- fission and fusion

Text: PHYS 120 on-line *Modern Physics: from Quarks to Galaxies*

Nuclear sizes

The strong interaction binds nucleons together to form a nucleus. Because it is a short range force, nucleons like to be close to one another, somewhat like a molecule in a liquid experiences a short range force with its neighbours. Again like a liquid, the volume of a liquid drop is proportional to the number of molecules it contains. For a nucleus, this means

$$[volume] \quad A$$

If the nucleus has the shape of a sphere of radius R, then

$$[volume] = (4\pi/3)R^3$$

or

$$R \quad A^{1/3}$$

Putting in the appropriate units, it is found that

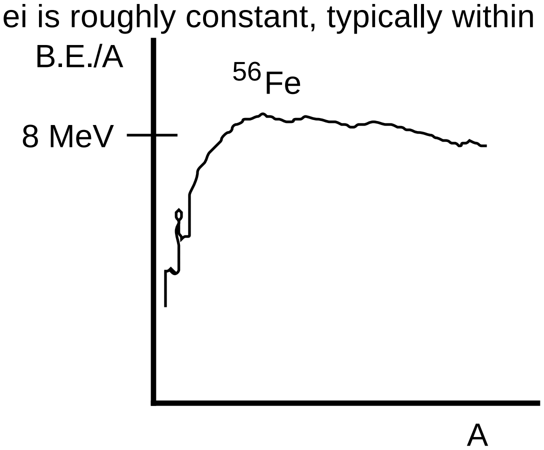
$$R = 1.2\, A^{1/3} \text{ fm.} \qquad (\text{fm} = 10^{-15} \text{ m})$$

Nuclear binding energy

The binding energy *B.E.* of a nucleus with Z protons and N neutrons is given by

$$B.E. = Zm_pc^2 + Nm_nc^2 - m(Z, A)c^2, \qquad (17.1)$$

where $m(Z, A)$ is the nuclear mass. The binding energy per nucleon *B.E.* /A for the most deeply bound nuclei is roughly constant, typically within 10% of 8 MeV.

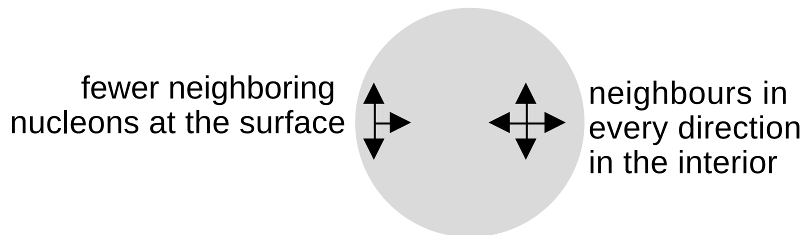


What's the origin of this phenomenon?

(i) The strong interaction binds the nucleons together, but extends over just a short range, so nucleons sense the presence only of their immediate neighbouring nucleons. If a nucleus were so large that surface effects could be ignored, then the strong interaction contribution to its binding energy would be

$C_{vol} A$ $\qquad$ C_{vol} is a constant with units of energy

(ii) Nucleons on the surface of the nucleus are less well bound than those in the interior:



Thus, the $C_{vol} A$ term overestimates the contribution of the surface nucleons. Now, because the nucleons are close-packed, the surface area is proportional to $R^2 \sim A^{2/3}$, and we must reduce the binding energy with a correction term like $C_{surf} A^{2/3}$

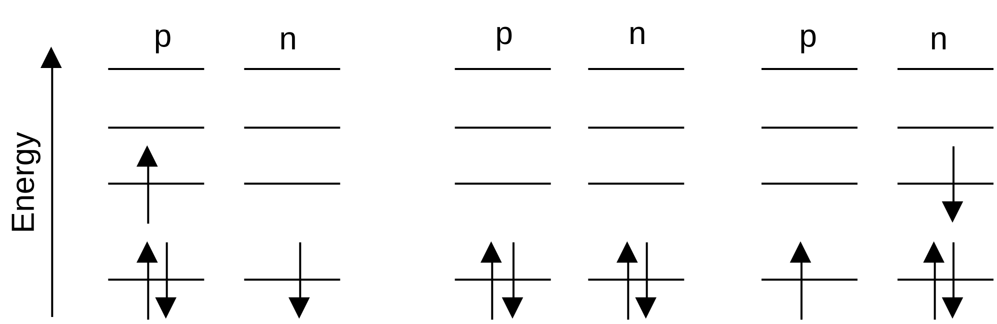
$- C_{surf} A^{2/3}$ $\qquad$ C_{surf} is a constant with units of energy

(iii) Generally speaking, the number of protons in a nucleus rises with the mass number, so that Coulomb repulsion among the protons ultimately starts to drive down *B.E./A*. Ultimately, very large nuclei become unbound because of the Coulomb repulsion. Because the Coulomb energy is proportional to the number of pairs of protons, or Z^2, it reduces the binding energy by

$$- (3/5)\, Z^2/R\, (e^2 / 4\pi\varepsilon_o)$$

where we assume that the protons are spread uniformly through a sphere or radius R. The factor $e^2 / 4\pi\varepsilon_o$ contains all the usual units (electron charge and permittivity of free space) and has the value 1.44 MeV-fm.

(iv) As described above, protons and neutrons occupy separate sets of energy levels, and the two terms that we have introduced so far don't recognize this. As Z or N depart from $A/2$, the binding energy decreases like $(N - Z)^2$:



Thus, we add a symmetry term to the binding energy to lower it when N is unequal to Z

$$- C_{sym} (Z - N)^2/A \qquad C_{sym} \text{ is a constant with units of energy}$$

(v) Lastly, protons and neutrons like to be paired with their mates. This makes a small contribution to the binding energy of the form

$$\delta = \begin{cases} +\Delta & \text{even } Z \text{ \& even } N \\ 0 & \text{odd } A \\ -\Delta & \text{odd } Z \text{ \& odd } N \end{cases} \qquad \Delta = 34 / A^{3/4} \text{ MeV}$$

Summing all of the contributions (i) to (v) gives us the so-called semi-empirical mass formula:

$$B.E. = C_{vol}A - C_{surf}A^{2/3} - C_{sym} (Z - N)^2/A - (3/5)\, Z^2/R\, (e^2 / 4\pi\varepsilon_o) + \delta.$$

A function of A and Z, this expression has **four** explicit parameters, plus one hidden in the form of R. These parameters can be obtained by fitting the formula to a large set of measured nuclear binding energies, to give

$$C_{vol} = 16 \text{ MeV} \qquad C_{surf} = 18 \text{ MeV} \qquad C_{sym} = 24 \text{ MeV}.$$

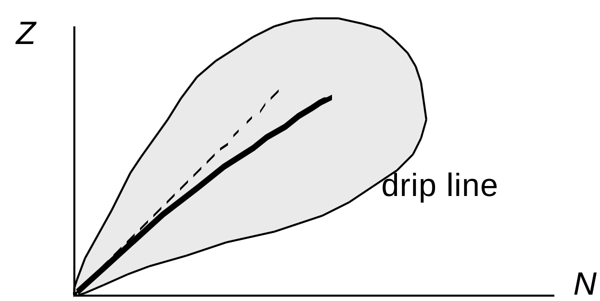
Example Find the binding energy per nucleon for ^{20}Ne, with $Z = N = 20$ using the semi-empirical mass formula. The estimated radius of this nucleus is $R = 1.2\,(20)^{1/3} =$ 3.3 fm.

$$B.E. = [16 \bullet 20] - [18 \bullet 20^{2/3}] - 0 - [0.6 \bullet 1.44 \bullet 10^2 / 3.3] + [34 / 20^{3/4}]$$
$$= 320 - 133 - 26 + 4 = 165 \text{ Mev}$$

This gives $B.E. / A = 165 / 20 = 8.3$ MeV.

Energy landscape

The above formula for the binding energy per nucleon leads to the energy landscape



The most deeply bound nuclei follow the thick line, initially with $Z = N$, but veering towards $N > Z$ as the Coulomb repulsion among the protons increases. Away from $Z \sim N$, the symmetry term tells us that BE / A rises. At the so-called "drip lines", a nucleus has too many protons or neutrons to remain bound, and sheds its extra nucleons. Only those isotopes within the boundary of the drip lines may be stable.

Fission and fusion

Both light and heavy nuclei have lower values of *B.E./A* than do intermediate mass nuclei; *B.E./A* peaks at ^{56}Fe.

Fission: Massive nuclei (*A* > 240) may still be bound (*B.E.* > 0), but their binding energy is so low that they are unstable against breakup into smaller nuclei with a larger *B.E./A*. In general, light nuclei cannot break up into even lighter, because of the low binding energy. Fission occurs in the decay of heavy elements in the Earth.

Fusion: Small-*A* nuclei can join together to produce a heavier nucleus and liberate energy at the same time. In general, heavy nuclei cannot fuse into very heavy nuclei because of the low binding energy. Fusion powers the stars.