

Lecture 21 - Hydrogen reactions in stars

What's Important:

- elements in the solar system
- deuterium production
- PP chains

Text: Clayton, Secs. 5.1 and 5.2

Carroll and Ostlie, Sec. 10.3

Elements in the solar system

In the present-day solar system, the distribution of elements is roughly as follows (data quoted from Hoyle; # of atoms relative to silicon)

Element	Relative abundance
H	3.18×10^4
He	2.21×10^3
Li	4.95×10^{-5}
Be	8.1×10^{-7}
B	3.5×10^{-4}
C	1.18×10^1
N	3.64
O	2.14×10^1
F	2.45×10^{-3}
Ne	3.44
Na	6.0×10^{-2}
Mg	1.06
Al	8.5×10^{-1}
Si	1
•	
•	
Fe	8.3×10^{-1}
Co	2.2×10^{-3}
Ni	4.8×10^{-2}
•	
•	<i>very small</i>
•	
Pb	4.0×10^{-6}
•	
•	<i>very very small</i>
•	
U	2.6×10^{-8}

For a typical stellar interior, this means that the average proton will live for $\sim 10^{10}$ years before reacting. This, of course, is what we expect from our knowledge of the age of the Sun - if the lifetime were much shorter than this, the Sun would have burnt itself out some time ago.

Once a source of ^2H is available, there are a number of pathways for it to burn through to produce ^4He . These reactions are referred to as the PP chains, and we will outline them now.

PP I

The only pathway available in pure H is the PP I chain, which has the following steps:



where the nuclear symbols have been used to represent their number densities. From Eqs. (21.4a) and (21.4b), the production rate of D is,

$$\frac{dD}{dt} = \Lambda_{pp} \frac{H^2}{2} - \Lambda_{pD} HD, \quad (21.5)$$

where the different signs arise because the first reaction creates D while the second one destroys it. From some initial state, the concentrations of H, D, *etc.* change until the system comes to a steady state, where $dD/dt = 0$. Solving Eq. (21.5) for this condition gives

$$\frac{D}{H}_{ss} = \frac{\Lambda_{pp}}{2\Lambda_{pD}}. \quad (21.6)$$

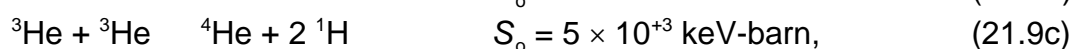
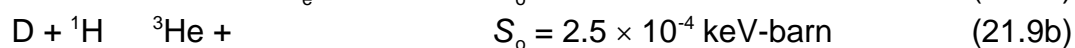
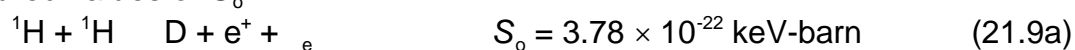
The same approach can be applied to Eqs. (21.4b) and (21.4c), to give

$$\frac{d^3\text{He}}{dt} = \Lambda_{pD} HD - 2\Lambda_{33} \frac{(^3\text{He})^2}{2} \quad (21.7)$$

(extra 2 arises because 2 ^3He are destroyed simultaneously). After eliminating the steady-state D concentration from Eq. (21.6), the steady state for ^3He is

$$\frac{^3\text{He}}{H}_{ss} = \frac{\Lambda_{pp}^{1/2}}{2\Lambda_{33}} \quad (21.8)$$

The values of the equilibrium concentrations can be obtained once we have the measured values of S_0 :



where the values of S_0 reflect the weak, e-m and strong interactions of the processes, respectively.

The numerical values of Λ can be found by substituting into the definition

$$\Lambda = 7.20 \times 10^{-25} S_0 K^2 e^{-K} / (AZ_1 Z_2) \quad \text{m}^3/\text{s}$$

where $S_0 = S(E_0)$ and is quoted in keV-barns, and

$$K = (3E_0 / k_B T) = 42.48 (Z_1^2 Z_2^2 A / T_6)^{1/3}.$$

At $T_6 = 15$, we obtain

$$\Lambda_{pp} = 1.17 \times 10^{-49} \quad \text{m}^3/\text{s}$$

$$\Lambda_{pD} = 1.78 \times 10^{-31} \quad \text{m}^3/\text{s}$$

$$\Lambda_{33} = 3.91 \times 10^{-40} \quad \text{m}^3/\text{s}.$$

Then, for example,

$$(D/H)_{ss} = 3.3 \times 10^{-18}$$

$$(^3\text{He}/H)_{ss} = 1.22 \times 10^{-5}.$$

These two numerical values show that ^2H , when produced, quickly reacts to form a ^3He nucleus, because the production step (weak interaction) is much slower than the removal step (electromagnetic interaction). The same applies to ^3He , although the removal step is hindered by the larger Coulomb barrier facing the collision of two $Z = 2$ nuclei.

These rate constants are used in the next lecture to find the mean reaction times, and energy production, in stars.