Lecture 21 - Hydrogen reactions in stars

What's Important:

- elements in the solar system
- deuterium production
- PP chains

Text: Clayton, Secs. 5.1 and 5.2
Carroll and Ostlie, Sec. 10.3

Elements in the solar system

In the present-day solar system, the distribution of elements is roughly as follows (data quoted from Hoyle; # of atoms relative to silicon)

Element	Relative abundance
H	3.18×10^{4}
He	2.21×10^{3}
Li	4.95×10^{-5}
Be	8.1×10^{-7}
B	3.5×10^{-4}
C	1.18×10^{1}
N	3.64
O	2.14×10^{1}
F	2.45×10^{-3}
Ne	3.44
Na	6.0×10^{-2}
Mg	1.06
Al	8.5×10^{-1}
Si	$\equiv 1$
•	
•	
Fe	8.3×10^{-1}
Co	2.2×10^{-3}
Ni	4.8×10^{-2}
•	
•	*very small*
•	
Pb	4.0×10^{-6}
•	
•	*very very small*
•	
U	2.6×10^{-8}

We have demonstrated earlier that helium is produced in the early universe roughly at 25% by weight, which is not that far from the global ratio still evident today. Where do all these other elements come from?

Deuterium production

As a star is formed, the gravitational potential energy decreases as the (thermal) kinetic energy of the star's gaseous material increases. But H and He cannot undergo any further strong interaction-mediated reactions that produce stable bound states

$p + p \rightarrow 2p$ (unstable)

$p + He \rightarrow {}^5Li$ (unstable)

So, with the exception of a small amount of ^3He produced through the reaction

$p + {}^2H \rightarrow {}^3He$

and ^7Be produced through the Coulomb-inhibited reaction

${}^3He + {}^4He \rightarrow {}^7Be$

there are no strong reactions to create the distribution of heavier elements seen in the solar system. The only reaction that can produce somewhat stable elements are those of the **weak** interaction:

$p + p \rightarrow {}^2H + e^+ + \nu_e$ liberates 0.420 MeV (21.1)

$+ e^- \rightarrow 2\gamma$ annihilation adds another 1.442 MeV

Although this reaction is slow, it provides a mechanism for replenishing the (initial) ^2H lost through burning to ^4He. The overall reaction for "burning" hydrogen is then

$4p \rightarrow {}^4He + 2e^+ + 2\nu_e$ liberates 26.731 MeV when annihilations are included, of which ~3% is carried off by neutrino kinetic energy, depending on the chain

To obtain the rate for reaction (21.1), we need the cross section parameter S_o, which is measured experimentally for this reaction to be

$S_o = (3.78 \pm 0.15) \times 10^{-22}$ keV-barn.

The remaining terms in the rate expression

$$r_{12} = \frac{2.62 \times 10^{29}}{(1+\delta_{12}) A Z_1 Z_2} \rho^2 \frac{X_1 X_2}{A_1 A_2} S_o K^2 e^{-K} \qquad m^{-3}s^{-1} \qquad (21.2)$$

are

$A_1 = 1$ $A_2 = 1$ $A = A_1 A_2 / (A_1 + A_2) = 1/2$

$K = 42.48\, (Z_1^2 Z_2^2 A / T_6)^{1/3} = 42.48 \bullet (1^2 \bullet 1^2 \bullet 0.5)^{1/3}\, T_6^{-1/3} = 33.72\, T_6^{-1/3}$,

so

$$r_{12} = 1.13 \times 10^{11} \rho^2 X_H^2 T_6^{-2/3} \exp(-33.72 T_6^{-1/3}) \qquad m^{-3}s^{-1} \qquad (21.3)$$

For a typical stellar interior, this means that the average proton will live for ~10^{10} years before reacting. This, of course, is what we expect from our knowledge of the age of the Sun - if the lifetime were much shorter than this, the Sun would have burnt itself out some time ago.

Once a source of ^{2}H is available, there are a number of pathways for it to burn through to produce ^{4}He. These reactions are referred to as the PP chains, and we will outline them now.

PP I

The only pathway available in pure H is the PP I chain, which has the following steps:

$^1H + ^1H \rightarrow D + e^+ + \nu_e$	$r_{pp} = \Lambda_{pp} H^2/2$	(21.4a)
$D + ^1H \rightarrow ^3He + \gamma$	$r_{pD} = \Lambda_{pD} HD$	(21.4b)
$^3He + ^3He \rightarrow ^4He + 2\,^1H$	$r_{33} = \Lambda_{33} (^3He)^2/2$	(21.4c)

where the nuclear symbols have been used to represent their number densities. From Eqs. (21.4a) and (21.4b), the production rate of D is,

$$\frac{dD}{dt} = \Lambda_{pp} \frac{H^2}{2} - \Lambda_{pD} HD \, , \tag{21.5}$$

where the different signs arise because the first reaction creates D while the second one destroys it. From some initial state, the concentrations of H, D, *etc*. change until the system comes to a steady state, where $dD/dt = 0$. Solving Eq. (21.5) for this condition gives

$$\left(\frac{D}{H}\right)_{SS} = \frac{\Lambda_{pp}}{2\Lambda_{pD}} \, . \tag{21.6}$$

The same approach can be applied to Eqs. (21.4b) and (21.4c), to give

$$\frac{d\,^3He}{dt} = \Lambda_{pD} HD - 2\Lambda_{33} \frac{(^3He)^2}{2} \tag{21.7}$$

(extra 2 arises because 2 ^{3}He are destroyed simultaneously). After eliminating the steady-state D concentration from Eq. (21.6), the steady state for ^{3}He is

$$\left(\frac{^3He}{H}\right)_{SS} = \left(\frac{\Lambda_{pp}}{2\Lambda_{33}}\right)^{1/2} \tag{21.8}$$

The values of the equilibrium concentrations can be obtained once we have the measured values of S_o:

$^1H + ^1H \rightarrow D + e^+ + \nu_e$	$S_o = 3.78 \times 10^{-22}$ keV-barn	(21.9a)
$D + ^1H \rightarrow ^3He + \gamma$	$S_o = 2.5 \times 10^{-4}$ keV-barn	(21.9b)
$^3He + ^3He \rightarrow ^4He + 2\,^1H$	$S_o = 5 \times 10^{+3}$ keV-barn,	(21.9c)

where the values of S_o reflect the weak, e-m and strong interactions of the processes, respectively.

The numerical values of Λ can be found by substituting into the definition

$$\Lambda = 7.20\times10^{-25}\, S_o\, K^2\, e^{-K} / (AZ_1Z_2) \qquad \text{m}^3/\text{s}$$

where $S_o \equiv S(E_o)$ and is quoted in keV-barns, and

$$K = (3E_o / k_B T) = 42.48\, (Z_1^2 Z_2^2 A / T_6)^{1/3}.$$

At $T_6 = 15$, we obtain

$$\Lambda_{pp} = 1.17\times10^{-49} \quad \text{m}^3/\text{s}$$
$$\Lambda_{pD} = 1.78\times10^{-31} \quad \text{m}^3/\text{s}$$
$$\Lambda_{33} = 3.91\times10^{-40} \quad \text{m}^3/\text{s}.$$

Then, for example,

$$(D/H)_{SS} = 3.3\times10^{-18}$$
$$(^3He/H)_{SS} = 1.22\times10^{-5}.$$

These two numerical values show that ^2H, when produced, quickly reacts to form a ^3He nucleus, because the production step (weak interaction) is much slower than the removal step (electromagnetic interaction). The same applies to ^3He, although the removal step is hindered by the larger Coulomb barrier facing the collision of two $Z = 2$ nuclei.

These rate constants are used in the next lecture to find the mean reaction times, and energy production, in stars.