

Lecture 21 - Hydrogen reactions in stars

What's Important:

- elements in the solar system
- deuterium production
- PP chains

Text: Clayton, Secs. 5.1 and 5.2
 Carroll and Ostlie, Sec. 10.3

Elements in the solar system

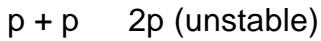
In the present-day solar system, the distribution of elements is roughly as follows (data quoted from Hoyle; # of atoms relative to silicon)

Element	Relative abundance
H	3.18×10^4
He	2.21×10^3
Li	4.95×10^{-5}
Be	8.1×10^{-7}
B	3.5×10^{-4}
C	1.18×10^1
N	3.64
O	2.14×10^1
F	2.45×10^{-3}
Ne	3.44
Na	6.0×10^{-2}
Mg	1.06
Al	8.5×10^{-1}
Si	1
•	
•	
Fe	8.3×10^{-1}
Co	2.2×10^{-3}
Ni	4.8×10^{-2}
•	
•	<i>very small</i>
•	
Pb	4.0×10^{-6}
•	
•	<i>very very small</i>
•	
U	2.6×10^{-8}

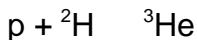
We have demonstrated earlier that helium is produced in the early universe roughly at 25% by weight, which is not that far from the global ratio still evident today. Where do all these other elements come from?

Deuterium production

As a star is formed, the gravitational potential energy decreases as the (thermal) kinetic energy of the star's gaseous material increases. But H and He cannot undergo any further strong interaction-mediated reactions that produce stable bound states



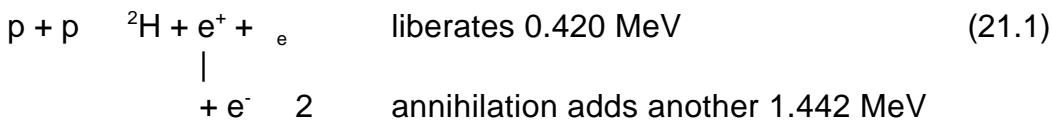
So, with the exception of a small amount of 3He produced through the reaction



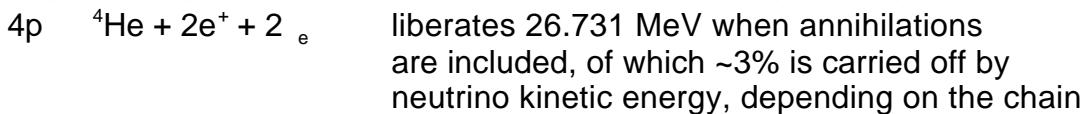
and 7Be produced through the Coulomb-inhibited reaction



there are no strong reactions to create the distribution of heavier elements seen in the solar system. The only reaction that can produce somewhat stable elements are those of the **weak** interaction:



Although this reaction is slow, it provides a mechanism for replenishing the (initial) 2H lost through burning to 4He . The overall reaction for "burning" hydrogen is then



To obtain the rate for reaction (21.1), we need the cross section parameter S_o , which is measured experimentally for this reaction to be

$$S_o = (3.78 \pm 0.15) \times 10^{-22} \text{ keV-barn.}$$

The remaining terms in the rate expression

$$r_{12} = \frac{2.62 \times 10^{29}}{(1 + \delta_{12}) A Z_1 Z_2} \rho^2 \frac{X_1 X_2}{A_1 A_2} S_o K^2 e^{-K} \quad m^{-3} s^{-1} \quad (21.2)$$

are

$$A_1 = 1 \quad A_2 = 1 \quad A = A_1 A_2 / (A_1 + A_2) = 1/2$$

$$K = 42.48 (Z_1^2 Z_2^2 A / T_6)^{1/3} = 42.48 \cdot (1^2 \cdot 1^2 \cdot 0.5)^{1/3} T_6^{-1/3} = 33.72 T_6^{-1/3},$$

so

$$r_{12} = 1.13 \times 10^{11} \rho^2 X_H^2 T_6^{-2/3} \exp(-33.72 T_6^{-1/3}) \quad m^{-3} s^{-1} \quad (21.3)$$

For a typical stellar interior, this means that the average proton will live for $\sim 10^{10}$ years before reacting. This, of course, is what we expect from our knowledge of the age of the Sun - if the lifetime were much shorter than this, the Sun would have burnt itself out some time ago.

Once a source of ^2H is available, there are a number of pathways for it to burn through to produce ^4He . These reactions are referred to as the PP chains, and we will outline them now.

PP I

The only pathway available in pure H is the PP I chain, which has the following steps:

$$^1\text{H} + ^1\text{H} \rightarrow \text{D} + \text{e}^+ + \bar{\nu}_e \quad r_{\text{pp}} = \Lambda_{\text{pp}} \text{H}^2/2 \quad (21.4\text{a})$$

$$\text{D} + ^1\text{H} \rightarrow ^3\text{He} + \gamma \quad r_{\text{pD}} = \Lambda_{\text{pD}} \text{HD} \quad (21.4\text{b})$$

$$^3\text{He} + ^3\text{He} \rightarrow ^4\text{He} + 2 ^1\text{H} \quad r_{33} = \Lambda_{33} (^3\text{He})^2/2 \quad (21.4\text{c})$$

where the nuclear symbols have been used to represent their number densities. From Eqs. (21.4a) and (21.4b), the production rate of D is,

$$\frac{dD}{dt} = \Lambda_{\text{pp}} \frac{\text{H}^2}{2} - \Lambda_{\text{pD}} \text{HD}, \quad (21.5)$$

where the different signs arise because the first reaction creates D while the second one destroys it. From some initial state, the concentrations of H, D, etc. change until the system comes to a steady state, where $dD/dt = 0$. Solving Eq. (21.5) for this condition gives

$$\frac{D}{H_{\text{ss}}} = \frac{\Lambda_{\text{pp}}}{2\Lambda_{\text{pD}}}. \quad (21.6)$$

The same approach can be applied to Eqs. (21.4b) and (21.4c), to give

$$\frac{d^3\text{He}}{dt} = \Lambda_{\text{pD}} \text{HD} - 2\Lambda_{33} \frac{(^3\text{He})^2}{2} \quad (21.7)$$

(extra 2 arises because 2 ^3He are destroyed simultaneously). After eliminating the steady-state D concentration from Eq. (21.6), the steady state for ^3He is

$$\frac{^3\text{He}}{H_{\text{ss}}} = \frac{\Lambda_{\text{pp}}}{2\Lambda_{33}}^{1/2} \quad (21.8)$$

The values of the equilibrium concentrations can be obtained once we have the measured values of S_o :

$$^1\text{H} + ^1\text{H} \rightarrow \text{D} + \text{e}^+ + \bar{\nu}_e \quad S_o = 3.78 \times 10^{-22} \text{ keV-barn} \quad (21.9\text{a})$$

$$\text{D} + ^1\text{H} \rightarrow ^3\text{He} + \gamma \quad S_o = 2.5 \times 10^{-4} \text{ keV-barn} \quad (21.9\text{b})$$

$$^3\text{He} + ^3\text{He} \rightarrow ^4\text{He} + 2 ^1\text{H} \quad S_o = 5 \times 10^{+3} \text{ keV-barn}, \quad (21.9\text{c})$$

where the values of S_o reflect the weak, e-m and strong interactions of the processes, respectively.

The numerical values of Λ can be found by substituting into the definition

$$\Lambda = 7.20 \times 10^{-25} S_o K^2 e^{-K} / (AZ_1 Z_2) \quad \text{m}^3/\text{s}$$

where $S_o = S(E_o)$ and is quoted in keV-barns, and
 $K = (3E_o / k_B T) = 42.48 (Z_1^2 Z_2^2 A / T_6)^{1/3}$.

At $T_6 = 15$, we obtain

$$\Lambda_{pp} = 1.17 \times 10^{-49} \quad \text{m}^3/\text{s}$$

$$\Lambda_{pD} = 1.78 \times 10^{-31} \quad \text{m}^3/\text{s}$$

$$\Lambda_{33} = 3.91 \times 10^{-40} \quad \text{m}^3/\text{s.}$$

Then, for example,

$$(D/H)_{ss} = 3.3 \times 10^{-18}$$

$$(^3He/H)_{ss} = 1.22 \times 10^{-5}.$$

These two numerical values show that 2H , when produced, quickly reacts to form a 3He nucleus, because the production step (weak interaction) is much slower than the removal step (electromagnetic interaction). The same applies to 3He , although the removal step is hindered by the larger Coulomb barrier facing the collision of two $Z=2$ nuclei.

These rate constants are used in the next lecture to find the mean reaction times, and energy production, in stars.