Lecture 22 - Energy production in stars

What's Important:
- energy release in PP I
- other PP chains
- CNO cycle

Text: Clayton, Sec. 5.3

Energy production in PP I

Because ^{3}He production from D is a relatively fast step, the overall reaction

$$3\text{H} \rightarrow {}^3\text{He} + \nu\ (1) + \gamma\text{'s}, \qquad (22.1)$$

has r_{pp} as its rate-determining step. Reaction (22.1) liberates 6.936 MeV in energy, although 0.263 MeV is carried off by the neutrino. So, the amount of "available" energy released per unit volume per unit time by this step is

$$(6.936 - 0.263) \times 10^6 \times (1.6 \times 10^{-19})\, r_{pp}$$
$$= 1.069 \times 10^{-12}\, r_{pp} \qquad \text{(in Joules x [volume•time]}^{-1}\text{ of } r_{pp})$$

This quantity is often written as $\rho\varepsilon$, where ε is the energy liberated per unit mass per unit time.

The last step in the PP I chain,

$${}^3\text{He} + {}^3\text{He} \rightarrow {}^4\text{He} + 2\text{p},$$

liberates 12.858 MeV of energy. Hence, the total rate of energy release is

$$\rho\varepsilon_{PPI} = 1.069 \times 10^{-12}\, r_{pp} + 2.060 \times 10^{-12}\, r_{33} \qquad (22.2)$$

Once ^{3}He has reached a steady state, its time derivative vanishes and [after using Eq. (21.6) for $(\text{H/D})_{SS}$]

$$d\,{}^3\text{He} / dt = 0 = \Lambda_{pp}\, \text{H}^2/2 - 2\Lambda_{33}\, ({}^3\text{He})^2/2. \qquad (22.3)$$

But, $r_{pp} = \Lambda_{pp}\, \text{H}^2/2$ and $r_{33} = \Lambda_{33}\, ({}^3\text{He})^2/2$, so Eq. (22.3) becomes

$$r_{pp} - 2r_{33} = 0$$

or

$$r_{33} = r_{pp}/2, \qquad \text{(at steady state)} \qquad (22.4)$$

allowing (22.2) to be written as

$$\rho\varepsilon_{PPI} = 2.099 \times 10^{-12}\, r_{pp} \qquad \text{(in Joules x [volume•time]}^{-1}\text{ of } r_{pp}) \qquad (22.5)$$

Now, we will perform an incredibly coarse calculation just to show that we are in the right ballpark in taking the source of the Sun's energy to be nuclear. From previous material

$$\Lambda_{pp} = 1.17 \times 10^{-49}\ \text{m}^3/\text{s} \qquad \text{at } T = 15 \times 10^6\ \text{K}$$

and

$r_{pp} = \Lambda_{pp}\, H^2/2$

Assume that all of the Sun's hydrogen (75% of 10^{57} baryons) is available for reaction at the same temperature, and a density of

$$H = \frac{\frac{3}{4} \cdot 10^{57}}{\frac{4\pi}{3} \cdot (6.37 \times 10^8)^3} = 6.93 \times 10^{29}\ nuclei/m^3 .$$

$$\rho\varepsilon_{PPI} = 2.099 \times 10^{-12} \bullet 1.17 \times 10^{-49} \bullet (6.93 \times 10^{29})^2 / 2 = 0.059 \text{ J} / (\text{m}^3 \bullet \text{s})$$

$$\text{Energy released} = 0.059 \bullet (4\pi/3) \bullet (6.37 \times 10^8)^3 = 6.38 \times 10^{25} \text{ J/s}.$$

Now, this can be compared to the observed solar luminosity of 3.9×10^{26} J/s!

Clearly, to do a better job, one would want to work out the temperature, density and concentration gradients in the Sun's interior. However, one can see that, on an order of magnitude basis, the p -> ^{4}He reaction PP I does liberate energy at approximately the correct rate. The one quantity which appears to have been put in by hand is the temperature of 15 million degrees K. This temperature can be estimated by other means (*e.g.* the temperature of a gas needed to balance its gravitational attraction to prevent collapse).

Other PP chains

PP I involves two ^{3}He nuclei colliding to form ^{4}He and releasing two protons. The overall reaction looks like

PP I

$4p \rightarrow 2D + 2e^+ + 2\nu_e$

$2D + 2p \rightarrow 2 \bullet {}^3He + 2\gamma$

$\underline{2 \bullet {}^3He \rightarrow {}^4He + 2p}$

Total $4p \rightarrow {}^4He + 2e^+ + 2\nu_e + 2\gamma$. (energy loss through ν_e = 2%)

Now, more ^{4}He are available for a ^{3}He nucleus reaction than are other ^{3}He. Hence, an alternate sequence could be

PP II

$2p \rightarrow D + e^+ + \nu_e$

$D + p \rightarrow {}^3He + \gamma$

${}^3He + {}^4He \rightarrow {}^7Be + \gamma$

${}^7Be + e^- \rightarrow {}^7Li + \nu_e$ (electron capture)

$\underline{{}^7Li + p \rightarrow 2 \bullet {}^4He}$

Total $4p + e^- \rightarrow {}^4He + e^+ + 2\nu_e + 2\gamma$. (energy loss through ν_e = 4.0%)

A variation of PP II has ^{7}Be capturing a proton, rather than an electron, against a

Coulomb barrier. Clearly, this reaction requires more kinetic energy to proceed than electron capture. The resulting ^{8}Be is unstable and decays to two ^{4}He nuclei.

PP III

$2p \rightarrow D + e^+ + \nu_e$

$D + p \rightarrow {}^3He + \gamma$

${}^3He + {}^4He \rightarrow {}^7Be + \gamma$

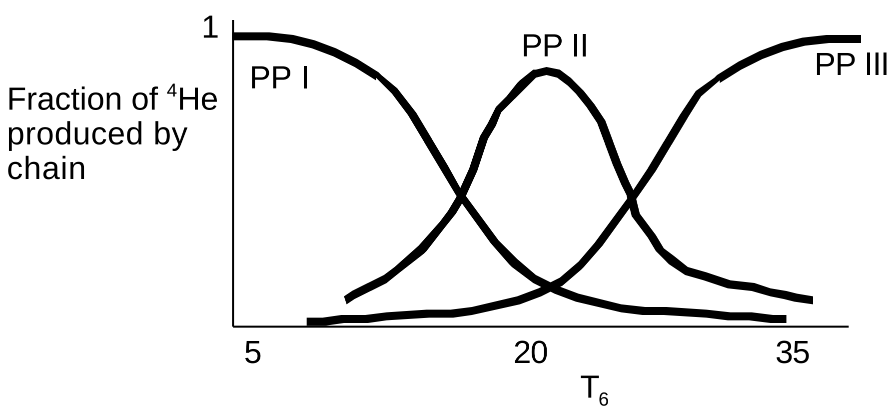
${}^7Be + p \rightarrow {}^8B + \gamma$ (proton capture)

${}^8B \rightarrow {}^8Be + e^+ + \nu_e$

$\underline{{}^8Be \rightarrow 2 \bullet {}^4He}$

Total $4p \rightarrow {}^4He + 2e^+ + 2\nu_e + 2\gamma$. (energy loss through ν_e = 27.9%)

Because of the different Coulomb barriers in the steps of these reactions, their relative importance depends upon the temperature of the star. Detailed calculations yield the following schematic behavior, performed at 50% H and 50% He by weight:



CNO bi-cycle

Another H-burning chain which becomes important at higher temperatures is the CNO bi-cycle, a long series of reactions which requires the presence of C, N or O as a catalyst. In the series below, note how one of these heavy nuclei is removed at the start of a sub-cycle, then reappears at the end.

cycle

${}^{12}C + p \rightarrow {}^{13}N + \gamma$

${}^{13}N \rightarrow {}^{13}C + e^+ + \nu_e$

${}^{13}C + p \rightarrow {}^{14}N + \gamma$

${}^{14}N + p \rightarrow {}^{15}O + \gamma$

${}^{15}O \rightarrow {}^{15}N + e^+ + \nu_e$

${}^{15}N + p \rightarrow {}^{12}C + {}^4He$ (^{12}C is a catalyst) *a second path from ^{15}N occurs 4×10^{-4} of the time*

$^{15}N + p \rightarrow {}^{16}O + \gamma$

$^{16}O + p \rightarrow {}^{17}F + \gamma$

$^{17}F \rightarrow {}^{17}O + e^{+} + \nu_e$

$^{17}O + p \rightarrow {}^{14}N + {}^{4}He$ (^{14}N reappears in the first cycle)

Summing, the total reaction is the same as previously:

$4p \rightarrow {}^{4}He + 2e^{+} + 2\nu_e + \gamma$'s.

This series of reactions is competitive with the PP chains above $T_6 \sim 20$, dominating them at $T_6 \sim 30$ to 50.