Lecture 25 - Star formation

What's Important:

- interstellar gas clouds
- gravitational collapse
- Jeans mass

Text: Carroll and Ostlie, Secs. 12.1, 12.2

Star formation

The average density of the universe today is rather low: from Lec. 13, the critical number density of protons is 6 m^{-3}, and the number density of visible matter is ~1 m^{-3}. This is a far cry from the 10^{30} m^{-3} in the Sun. This density, if uniform, is just too low to account for star formation. Where, then, do stars come from?

Even leaving aside the stars themselves, matter is not uniformly distributed in the universe. There exist large clouds of hydrogen and helium gas that have been identified in a number of ways:

- absorption of light from background stars
- reflection of light from other sources
- their own emission spectra.

The size and density of the clouds varies. A typical cloud might be:

- 10^8 - 10^{10} atoms/m^3
- 1 - 100 solar masses
- T ~ 30 - 80 K

As we will show momentarily, these regions may not be dense enough to collapse under gravity.

But there also exist Giant Molecular Clouds (GMCs), within which are found more dense regions, typically

- 10^{13} - 10^{15} atoms/m^3
- 10 - 1000 solar masses
- T ~ 100 - 200 K.

There are thousands of such GMCs in our own Milky Way.

The elemental composition of the clouds is not that far from primordial. However, they also contain material that has been processed through the nuclear reactions of stars - the material then being returned to a local region of the galaxy through explosions.

Energetics of gravitational collapse

As a first step to understanding the energetics of gravitational collapse, we evaluate the gravitational potential energy of a spherical distribution of matter with uniform density ρ. The sphere has radius R and mass M.

We break up the sphere into a succession of concentric shells, evaluating the change in potential energy as each shell is brought from infinity to a radius r surrounding a spherical mass of the same radius. The mass dm of the shell is

$$dm = \rho \, 4\pi r^2 \, dr,$$

and the mass M_r of the enclosed region is

$$M_r = (r/R)^3 M,$$

Thus, the change in the gravitational potential energy of the shell is

$$dU = -G M_r \, dm / r.$$

Integrating over all shells gives the expression

$$\int dU = -G \int_0^R \left(\frac{r}{R}\right)^3 M \, \rho 4\pi r^2 \, \frac{1}{r} \, dr = -\frac{4\pi G M \rho}{R^3} \int_0^R r^4 dr$$

The integral itself yields $R^5/5$, permitting terms to be regrouped as

$$\Delta U = -\frac{GM}{R} \frac{3}{5} \left(\rho \frac{4\pi}{3} R^3\right) = -\frac{3}{5} \frac{GM^2}{R} . \qquad (25.1)$$

Some of the change in potential energy is radiated away, but some must also go into the kinetic energy of the cloud's constituents as they contract. In fact, the constituents *must* be moving if they are to have stable orbits. For a two-body system, the kinetic energy of a stable orbit is half the change in potential energy, ignoring signs. A quick proof using an object of mass m moving at a radius R about a fixed center of mass M starts with the change in potential energy

$$\Delta U = -GMm/R$$

which corresponds to a centripetal acceleration

$$a = F/m = GM/R^2 \qquad \text{(from Newton's second law)}$$

The kinetic energy of the moving partner is

$$K = (m/2)v^2 = (m/2)(v^2/R)\,R = (m/2)\,a\,R = (m/2)(GM/R^2)\,R$$

after substituting for the centripetal acceleration a. Cleaning up

$$K = \frac{1}{2} \frac{GMm}{R} = -\frac{1}{2} \Delta U \qquad (25.2)$$

For a collection of particles, the kinetic and potential energies K and U are not constant, although their time averages $\langle K \rangle$ and $\langle U \rangle$ are. The result has the same general appearance as the two-body problem

$$\langle K \rangle = -\langle \Delta U \rangle / 2, \qquad (25.3)$$

the proof of which is given in Sec. 2.4 of Carroll and Ostlie [under the virial theorem, Eq. (2.45)].

In other words, the mean kinetic energy of the ions in the gas of a stable star is

$$< K >= \frac{3}{10}\frac{GM^2}{R} \quad (25.4)$$

Jeans mass

Whether a gas cloud is stable against gravitational collapse depends upon the kinetic energy of its constituents. If the kinetic energy is high, meaning its temperature is high, then the material may not form a bound system at all:

$$\text{if } E = K + U > 0 \text{ the system is unbound.} \quad (25.5)$$

We can find the criteria for stability of a cloud at temperature T by using the equipartition theorem result for the kinetic energy

$$K = (3/2)\, Nk_B T, \quad (25.6)$$

where N is the number of particles. Then the condition

$$2K < |\Delta U| \qquad \text{(for bound system)}$$

is

$$3Nk_B T = (3/5)\, GM^2/R.$$

Now, the number of atoms in the cloud is equal to its mass M divided by the mean molecular mass μm_H, where μ is the mean molecular weight and m_H is the molecular mass of a hydrogen atom, 1.67×10^{-27} kg:

$$N = M / \mu m_H. \quad (25.7)$$

Thus,

$$k_B T / \mu m_H < (1/5)\, GM /R,$$

after eliminating a common factor of $3M$. For a gas cloud of uniform initial density ρ_o, the radius R can be replaced by $(3M / 4\pi\rho_o)^{1/3}$, such that

$$5k_B T / G\mu m_H < M^{2/3} / (3 / 4\pi\rho_o)^{1/3}. \quad (25.8)$$

This expression gives a minimum mass for collapse at a given ρ_o and T, which is called the Jeans mass M_J:

$$M_J = (5k_B T / G\mu m_H)^{3/2} \bullet (3 / 4\pi\rho_o)^{1/2}. \quad (25.9)$$

This tells us that the lower the initial density, the higher the mass must be in order for the cloud to collapse.

Example Carroll and Ostlie present two examples of the Jeans mass. The first is a dilute gas cloud, showing that the required mass for condensation is *very* high. The second is for a more dense region of a giant molecular cloud, which we reproduce here. Representative conditions in such a cloud might be:

T = 150 K
$n = 10^{14}$ m^{-3} → $\rho_o = m_H n = 1.67 \times 10^{-27} \bullet 10^{14} = 1.7 \times 10^{-13}$ kg/m^3

Hence

$$M_J = \left(\frac{5 \cdot 1.4 \times 10^{-23} \cdot 150}{6.67 \times 10^{-11} \cdot 1 \cdot 1.7 \times 10^{-27}}\right)^{3/2} \left(\frac{3}{4\pi \cdot 1.7 \times 10^{-13}}\right)^{1/2}$$

or

$$M_J = 3.3 \times 10^{31} \text{ kg.}$$

Now, one solar mass is 2×10^{30} kg, so the Jeans mass here corresponds to 17 solar masses, easily within the range of masses stated at the beginning of the lecture.