

Lecture 3 - Stellar luminosities

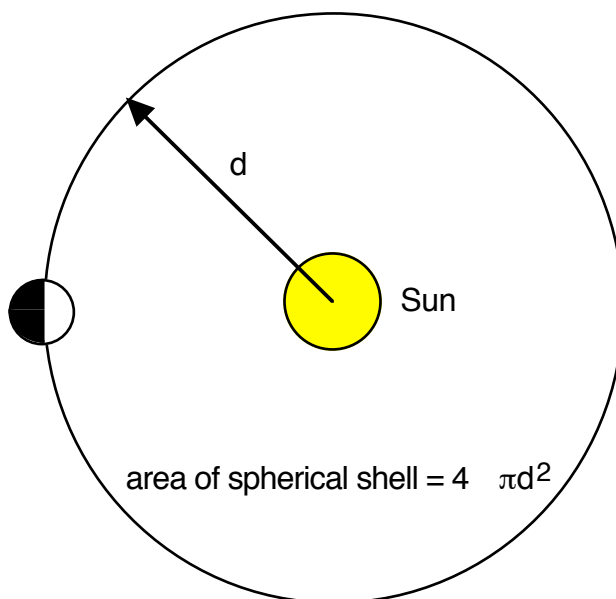
What's important:

- stellar luminosities
- standard candles

Text: Carroll and Ostlie, Secs. 3.2 and 3.4

Luminosities and distances to stars

The total energy emitted from the surface of a star per unit time (which is the total power of the star) is referred to as its **luminosity** L . The luminosity of the Sun, for example, is 3.9×10^{26} J/s (or watts). The amount of energy from the Sun that reaches a particular planet depends on the distance of the planet from the Sun, since solar energy is emitted in all directions and spreads throughout space. By the time the solar radiation reaches a distance d from the Sun, it has been spread over an area of $4\pi d^2$, as illustrated:



The amount of energy per unit time crossing an element of area facing the Sun, but a distance d away from it, is referred to as the flux F (may also be called the apparent brightness in astronomy). In terms of the luminosity, the flux is given by:

$$F = L / 4\pi d^2$$

and has units of energy per unit area per unit time. Further, there is nothing special about the Sun in this equation, it applies to all stars.

Example

The solar luminosity is 3.9×10^{26} J/s, and the corresponding energy flux from the Sun as seen on the Earth (a distance of 1.5×10^{11} m away) is 1380 watts / m².

If one can determine the luminosity of a star WITHOUT knowing d , then a measurement of the flux F on Earth can be inverted to find d . That is:

- (i) extract L from some observable characteristic of the star
- (ii) measure F on Earth
- (iii) use $F = L / 4\pi d^2$ to solve for d .

The problem with this approach is that dust and gas between Earth and the star in question tend to reduce F and give a calculated distance that is longer than the true distance. One may be able to correct for part of this using the spectrum of light from the star, which will be depleted in the blue because of scattering.

Methods for finding L generally involve selecting a particular observable (*e.g.* surface temperature) of nearby stars whose distance is known from parallax, then measuring the same observable on a distant star. For example

- (i) L is related to surface temperature for many stars

Light emitted from a hot object has a distribution of wavelengths that is specific to the object's temperature. For example, most stars have surface temperatures in the 2,500 to 30,000 K range, with the Sun having a surface temperature of 5,800 K. The luminosity of young stars is observed to increase steadily with their temperature, so a measurement of temperature (from wavelength) provides a measure of L . Knowing T by itself doesn't tell us L unless the radius is known.

- (ii) L is related to the pulsation period of Cepheid variables

Originally discovered by amateur astronomer John Goodricke in 1794, Cepheids are stars whose luminosity oscillates with periods of roughly 1 to 50 days. With large luminosities in the 10^2 to 10^4 times that of the Sun, they can be seen at distances of millions of parsecs, well beyond our galaxy. Astronomer Henrietta Leavitt established that Cepheid luminosity is a unique function of the oscillation period, through her studies of nearby Cepheids (discussed further in Carroll and Ostlie, Sec. 14.1).

Apparent magnitude m

Efforts to quantify the brightness of stars date back to the Greeks. Hipparchus assigned an apparent magnitude m to a selection of visible stars, starting with $m = 1$ for the brightest stars visible in his part of the world. The apparent magnitude *increases* as the brightness *decreases*, with the faintest stars having the largest values of m .

Brightness, according to our eyes or a photometer, is determined by the flux F on Earth, not the luminosity L at the source. Because the response of the human eye is more logarithmic than linear,

$$[magnitude] \propto \log[flux]$$

In the modern definition of m , a difference of 5 magnitudes is assigned to a factor of 100 in flux. Thus, if we compare two stars 1 and 2,

$$\frac{F_2}{F_1} = 100^{(m_1 - m_2)/5} = 10^{(m_1 - m_2)/2.5}$$

Taking \log_{10} of both sides

$$m_1 - m_2 = 2.5 \log_{10}(F_2/F_1)$$

or

$$m_1 - m_2 = -2.5 \log_{10}(F_1/F_2)$$

Aside: the minus sign confirms that m increases as F decreases.

The reference point of m is fixed by convention; the brightest star is Sirius (now known to be a binary - see below) with a negative apparent magnitude of $m = -1.43$. Many familiar stars have m in the +3 to +8 range. As seen from Earth, the Sun is much brighter than any other star and has

$$m_{\text{sun}} = -26.81.$$

Absolute magnitude M

Other brightness scales are more useful in certain circumstances. One approach is to define an "absolute magnitude" M that eliminates the distance dependence of the apparent magnitude:

$M = m$ if the observer were 10 pc from the star

M is a better reflection of the luminosity.

Imagine two observers measuring a distant star

from Earth: m and F

from 10 pc: M and F_{10} .

Thus,

$$100^{(m-M)/5} = \frac{F_{10}}{F} = \left(\frac{d}{10\text{pc}} \right)^2$$

where the latter equality follows from $F = L / 4\pi d^2$. Taking logarithms again,

$$2(m - M) / 5 = 2 \log_{10}(d / 10 \text{ pc})$$

or

$$m - M = 5 \log_{10}(d / 10 \text{ pc}).$$

Stellar radii

Although the structure of galaxies can be seen with a telescope, stars appear to be pointlike. What techniques are available to us for determining their sizes? Here, we concentrate on two approaches - the Hanbury-Brown Twiss effect, and stellar luminosities.

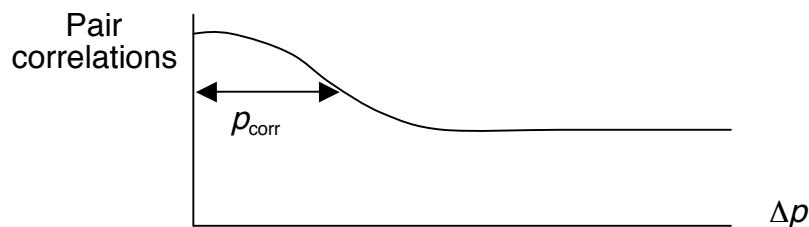
Hanbury-Brown / Twiss

Photons are bosons (to be reviewed in a few lectures), which means that two of them can occupy the same state. This is in contrast to fermions, no two of which can share the same quantum numbers. In a thermal distribution of particles, we will show later that compared to a distribution of classical particles (*i.e.*, ignoring the effects of spin), at low relative momenta:

bosons pairs have a higher probability of occurrence

fermion pairs have a lower probability of occurrence.

Consider, then, pairs of photons emitted from a star and observed some time later on Earth. Defining as 1 the number of photon pairs in a random distribution having a given relative momentum Δp , we expect



Crudely speaking, the region of momentum p_{corr} over which the pairs are enhanced is related to the size of the star by

$$R \sim \hbar / p_{\text{corr}}$$

Only the brightest stars can be analyzed with this technique. To date, more than 30 stars in the southern hemisphere have been analyzed, using an interferometer in Australia.

Luminosity and Stephan-Boltzmann

This technique is a little less direct, and its implementation requires several pieces of data. The strategy is to measure the star's

- surface temperature T
- luminosity L

and then use the Stephan-Boltzmann equation for the flux

$$F_{\text{surf}} = \sigma T^4$$

to find the star's radius via

$$L = 4\pi R^2 F_{\text{surf}}$$

That is

$$R = \left[\frac{L}{4\pi\sigma T^4} \right]^{1/2}$$

where σ = Stefan-Boltzmann constant = 5.67×10^{-8} watts / m²K⁴.

Example

Sirius is the brightest star, and is actually a binary. Located just 2.6 pc from Earth, its distance is accurately known from parallax. Thus, measuring the flux of its light at the Earth allows the determination of its luminosity from $4\pi d^2 F_{\text{at Earth}}$. The surface temperature can be obtained from the spectrum of light from each member of the binary pair. The resulting values are

<i>Sirius A</i>	$L = 23.5$ solar luminosity	$T = 9910$ K
<i>Sirius B</i>	$L = 0.03$ solar luminosity	$T = 27000$ K

where the solar luminosity is 3.9×10^{26} watts. Let's find the radii of these two stars.

Sirius A

$$R = \left[\frac{23.5 \cdot 3.9 \times 10^{26}}{4\pi \cdot 5.67 \times 10^{-8} (9910)^4} \right]^{1/2} = 1.15 \times 10^9 \text{ m}$$

Sirius B

$$R = \left[\frac{0.03 \cdot 3.9 \times 10^{26}}{4\pi \cdot 5.67 \times 10^{-8} (27000)^4} \right]^{1/2} = 5.6 \times 10^6 \text{ m}$$

Compared to the radius of the Sun at 6.960×10^8 m,

Sirius A is twice as large

Sirius B is a hundred times smaller!

In fact, Sirius B is slightly smaller than the Earth, whose radius is 6400 km. Initially resolved as a star in 1862, the temperature of Sirius B was determined in 1915, demonstrating that it was completely different from conventional stars – the first example of a white dwarf.