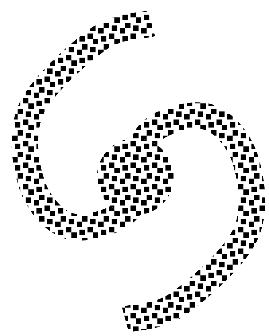
Lecture 31 - Kinematics of galaxies

What's Important:
- Keplerian orbits
- rotation of galaxies
- dark matter

Text: Carroll and Ostlie, Sec. 22.3

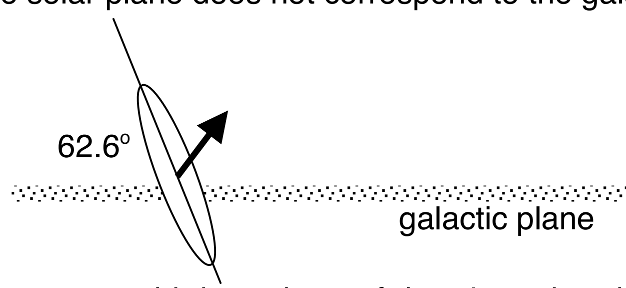
Rotation of a galaxy

Galaxies assume a variety of shapes, but certainly one of the more common shapes is a spiral or pinwheel:



The spiral shape suggests that the galaxy is rotating; in all observed galaxies but one, the arms *trail* the motion of the centre (*i.e.*, ω decreases with increasing radius). For example, the rotation of the Milky Way has been measured to be clockwise, as seen from the north galactic pole.

The measurement of the galactic motion must take into account the motion of Earth-bound observers. It's bad enough that the Earth revolves around its axis, and rotates about the Sun. But the solar plane does not correspond to the galactic plane:



Further, the Sun executes an orbit in and out of the plane, in addition to the motion of the spiral arm in which the Sun resides.

Carroll and Ostlie provide a description of the coordinate systems that properly account for this many-component motion. Here, we are just interested in how to set up the problem, and the rotational velocity that one obtains.

The **local standard of rest** (LSR) is a coordinate frame that is instantaneously centered at the Sun, and moves in a circle at a distance R_o = 8.0 kpc from the galactic center.

The choice of 8.0 is set by averaging over a number of measurements of the distance to the centre, which gives 8.0 ± 0.5 kpc. The motion of a star with respect to the LSR is referred to as its **peculiar velocity**. Even the Sun has a peculiar velocity, because it is moving away from the galactic plane, and towards the galactic centre.

The tangential component of the Sun's motion with respect to the galactic center is about 220 km/s = 2.2×10^5 m/s, about 0.1% of the speed of light. Suppose that this motion were entirely circular and within the galactic plane - what would it tell us about the mass of the galaxy?

First, we obtain the period P_{LSR} corresponding to $V_{tan} = 2.2 \times 10^5$ m/s:

$$P_{LSR} = 2\pi R_o / V_{tan}$$
$$= 2\pi \cdot 8.0 \times 1000 \times 3.09 \times 10^{16} / 2.2 \times 10^5$$
$$= 7.06 \times 10^{15} \text{ seconds}$$
$$= 225 \text{ million years.}$$

In one sense, the motion is relatively slow at 0.2 billion years, but this is still small compared to the age of the galaxy at 10 billion years or more. If the galaxy had been rotating at this rate since its birth, it would have completed 50 revolutions (at the position of the Sun - faster towards the centre, slower further out).

Next, we combine Newton's laws of motion and gravity to obtain

$$ma_c = mV_{tan}^2/R = GM_{enclos}m/R^2$$

or

$$M_{enclos} = RV_{tan}^2 / G.$$

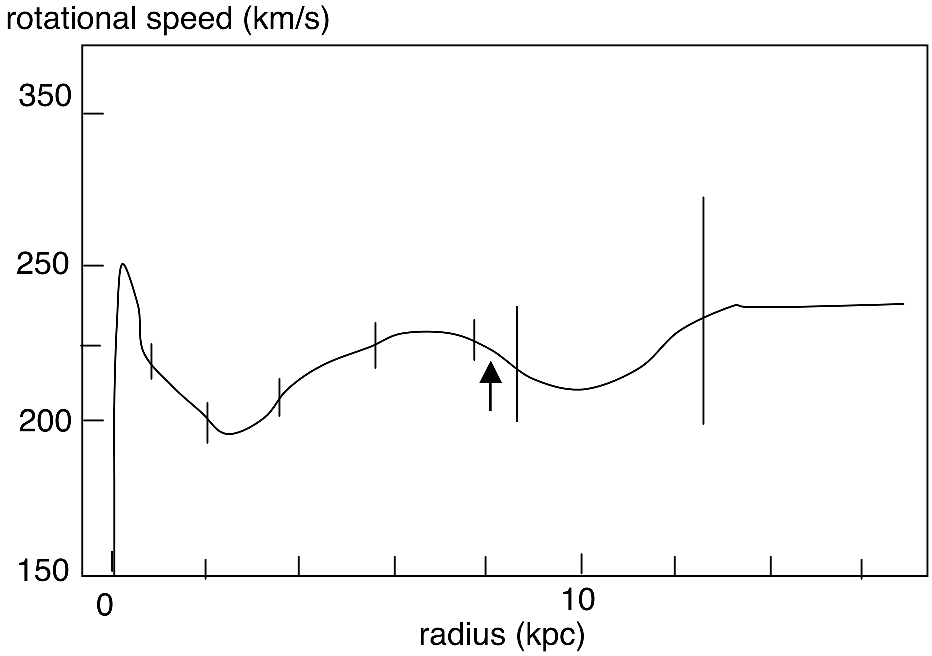
Replacing R by R_o etc., this yields the numerical value

$$M_{enclos} = (2.2 \times 10^5)^2 \cdot 8.0 \times 1000 \cdot 3.09 \times 10^{16} / 6.67 \times 10^{-11}$$
$$= 1.79 \times 10^{41} \text{kg}$$
$$= 9.0 \times 10^{10} \text{ solar masses.}$$

That the mass enclosed within R_o is about 10^{11} solar masses is at once both comforting and disturbing. Comforting, in that it is the correct order of magnitude, as the mass of visible matter in the Milky Way is about 10^{11} solar masses, as previously mentioned. However, not all of this mass lies within R_o: in fact, if the mass were uniformly distributed throughout the disk, only $(8 / 25)^2$ = 10% of it would lie within R_o = 8 kpc. This is our first hint that there is perhaps 10 times as much "dark matter" as there is visible matter.

For the Milky Way as a whole, the measured tangential velocities are drawn below. The vertical bars indicate the typical scatter in the data. The arrow points to the location of the Sun, with a rotational speed of about 220 km/s.

Initially, V rises with R like a rigid body: $V = \omega R$. But after just a few kpc, the distribution is **flat**. Similar rotational curves obtained by Doppler shift measurements of other galaxies show similarly **flat** functions, and also similar values for the tangential speeds of about 200 km/s. How do we interpret this?



Rotational curve for the Milky Way galaxy (redrawn from Clemens, see Carroll and Ostlie, p. 956).

Keplerian orbits

As a star circles the centre of the galaxy, its centripetal acceleration is caused by the gravitational force from the mass enclosed within the star's orbit. If most of the mass M_{gal} is in the galactic nucleus, then a star outside the nucleus would obey

$$GM_{gal}\, m_{star} / R^2 = m_{star}\, v^2 / R$$

=> $$v = (GM_{gal} / R)^{1/2}$$

Thus

Rigid body: $V \propto R^1$

Keplerian: $V \propto R^{-1/2}$ (mass concentrated at core of galaxy)

To obtain a model distribution that generates a flat rotational curve, let's invert the problem. We now assume that the velocity is constant, V = [*constant*]. Then, from two of Newton's laws:

$$mV^2/r = GM_r m / r^2,$$

where M_r is the mass enclosed within radius r, we obtain

$$M_r = (V^2/G)r.$$

The rate at which M_r changes with r can be found by differentiation to be

$$\frac{dM_r}{dr} = \frac{V^2}{G}. \quad (31.1)$$

While this equation tells us that the *mass* rises with r, it does not tell us that the *density* rises with r, or even remains constant. The mass contained within a three-dimensional spherical shell of uniform density ρ is

$$dM_r = 4\pi r^2 \rho \; dr$$

so that

$$\frac{dM_r}{dr} = 4\pi \rho r^2 . \qquad (31.2)$$

Equating Eqs. (31.1) and (31.2) yields

$$V^2 / G = 4\pi r^2 \rho,$$

or

$$\rho(r) = V^2 / 4\pi G r^2. \qquad (31.3)$$

This result isn't bad, in the sense that the density falls like a power law in distance. However, the fall-off observed in the halo of visible matter, as described previously, is

$$n(r) \propto r^{-3.5}.$$

Well, perhaps that's the way it is. A model with uniform distribution in 2Ddisk is worse - $\rho(r) \propto r^{-1}$. Now, (31.3) needs some modification to be physically acceptable:

*small r*We don't want the density to become singular at small r, so we modify the parametrized form to read

$$\rho(r) = C_o / (a^2 + r^2)$$

$\rho(r) \to C_o/a^2$ at small r
$\rho(r) \to C_o/r^2$ at large r

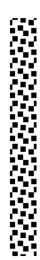
large r Even the modified form leads to infinite mass, as it implies dM_r/dr is constant, according to Eq. (31.2). Thus, one must impose a cut-off at large r.

A fit to the rotational curve yields (integrate to verify)

$C_o = 4.6 \times 10^8$ solar masses / kpc
$a = 2.8$ kpc.

Spiral arms

The rotational speed of stars, as described above, immediately tells us something about the nature of the spiral arms. If the galaxy starts out as a bar:



then (unless the bar is rigid!) outer stars have a larger period than inner stars in either scenario:

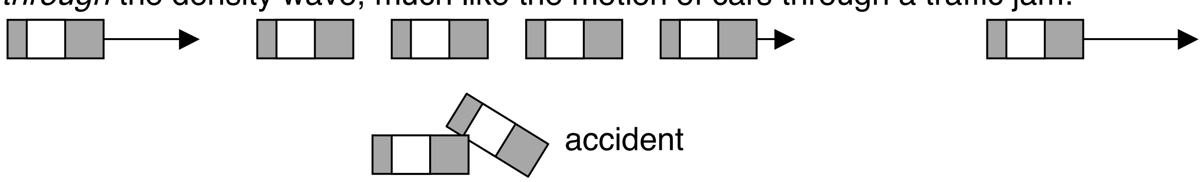
constant V:	period = $2\pi R / V$	or	period $\propto R^1$ (not constant ω)
Keplerian:	period$^2 \propto R^3$	or	period $\propto R^{3/2}$.

In both cases, the period grows with distance. Thus, a group of freely orbiting stars would evolve from a bar to a trailing arm spiral very fast:

The trouble is that the evolution is *too* fast, and the arms would wind up after just a few revolutions of the inner stars to look like

For example, we showed above that the Sun would have completed 50 revolutions at its current speed and position. This is referred to as the **winding problem**.

One possible resolution to this problem is the regard the spirals as density waves, rather than a sequence of stars in a fixed relative order. Then, the motion of the stars is *through* the density wave, much like the motion of cars through a traffic jam:



The mathematics behind this idea is treated in more detail in Carroll and Ostlie, Sec. 23.3.