Lecture 33 - Temperature and time

What's important:
- relic microwave radiation
- scale-dependence of photon temperature
- time-dependent Hubble parameter

Text: Carroll and Ostlie, Secs. 27.1, 27.2
Weinberg, Chap. III

Thermal distribution of massless particles

In a previous lecture, we described the distribution in energy of a photon gas at finite temperature. The distribution was integrated over energy to give the photon number density and energy density N_γ and U_γ, respectively. These results apply to any massless boson, if the number of polarization states is taken into account. Define the integrated densities:

$$N_{boson} = (8\pi \cdot 1.202)\,(k_B T / hc)^3 \qquad\qquad k_B T >> mc^2$$
$$U_{boson} = (4\pi^5 / 15)\,(k_B T)^4 / (hc)^3.$$

Performing the same calculation for massless fermions with no constraint on the number of fermions, one finds

$$N_{fermion} = (3/4)\; N_{boson}$$
$$U_{fermion} = (7/8)\; U_{boson}.$$

Because photons have two polarization states (a massive boson with $J = 1$ would have an additional state of longitudinal polarization, for a total of three states), our previous results are simply

$$N_\gamma = 2N_{boson}$$
$$U_\gamma = 2U_{boson}.$$

Example Consider a system at a sufficiently high temperature that the masses of particles in the "standard model" of quarks, gluons and leptons can be ignored. Find the energy density in terms of U_{boson}.

Counting the prefactors of U_{boson}, we have

quarks:	7/8 • 6 flavours • 3 colours • 2 anti-q • 2 spins	= 63
photon + gluons	(1 + 8 bosons) • 2 spins	= 18
$W^\pm$, Z (& Higgs)	3 bosons • 3 spins	= 9
massive leptons	7/8 • 3 leptons • 2 anti-L • 2 spins	= 10.5
neutrinos	7/8 • 3 leptons • 2 anti-ν • 1 spin (lefthanded)	= 5.25
Total		105.75

That is, $U_{standard} = 105.75\; U_{boson}$ at high temperatures.

Microwave Radiation

In 1964, Arno Penzias and Robert W. Wilson found that space is filled with low energy microwaves, using an instrument sensitive to 7.35 cm wavelength. Verification that the microwaves actually have an equilibrium distribution of wavelengths took many more years of observation at different wavelengths. Current observation supports a temperature of 2.73 ± 0.05 K. Recent experiments using satellite-based detectors confirm that the 3 K microwave radiation is present uniformly in all directions of space; it is not associated with specific stars or the Milky Way. What is the energy density and the number density of these photons? From the previous lecture:

$U_\gamma = 7.565 \times 10^{-16} (2.7)^4 = 4.0 \times 10^{-14}$ J/m^3.

$N_\gamma = 2.02 \times 10^7 (2.7)^3 = 4.0 \times 10^8$ m^{-3} (or 400 / cm^3)

Note:

- average energy per photon = $4.0 \times 10^{-14} / 4.0 \times 10^8 = 10^{-22}$ J.
- typical wavelength = $hc/E = (6.63 \times 10^{-34})(3.0 \times 10^8)/10^{-22} = 0.2$ cm (microwaves).

Microwaves in an expanding universe

As the universe expands, the energy it contains is spread over an increasing volume, so the energy density decreases, along with the temperature. The following argument tells us how the energy changes with the size of the universe:

Suppose that a ray of light takes a time t to travel between two galaxies. Then the distance D between the galaxies is equal to ct:

$R = ct$.

But, during this time, the galaxies have moved a distance R' further apart, with

$R' - R = vt$.

Thus,

$$\frac{R' - R}{R} = \frac{vt}{ct} = \frac{v}{c}$$

Since any point through which light travels on its way to another galaxy can be regarded as a source, and since any observer is receding from that source, then the light "emitted" from any point will be red shifted by

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$$

Equating the two expressions for v/c shows us how the wavelengths follow the fractional change in the linear dimension of the universe $f = R'/R$:

$$\frac{\Delta\lambda}{\lambda} = f - 1$$

or

$$\lambda'/\lambda = f. \qquad (33.1)$$

In other words, the universe must have been much hotter in its earlier stages, and Eq. (33.1) tells us its temperature history is as function of size.

Radiation dominated or matter dominated?

The energy of an elementary particle can be associated solely with its momentum (as in $E = pc$ of a photon), or with both its kinetic energy and its mass (as in $E = mc^2 + K$). The energy densities of gases of elementary particles have different dependence on the size of the system R, and this is important to understanding how the universe expands.

For a gas of massive particles:
- take the number of particles to be fixed at N.
- their number density is proportional to N / R^3
- their energy density from mc^2 is proportional to Nmc^2 / R^3.

For a gas of photons:
- their number is not fixed
- their energy density is proportional to T^4
- their temperature is inversely proportional to their wavelength λ, which in turn is proportional to the size of the system R. Hence,
- their energy density is proportional to R^{-4}.

In both cases, the energy density obeys

$$U(R) \propto R^{-n}$$

where

$n = 3$ in a *matter-dominated universe*

$n = 4$ in a *radiation-dominated universe*.

What dominates today's universe? Averaged over very large distance scales, there is about one nucleon (and an appropriate number of electrons for charge neutrality) per cubic meter of space. This isn't much, but its energy density from mass is

$$U_{\text{matter}}\,(\text{today}) = 1 \cdot 1.67 \times 10^{-27} (3 \times 10^8)^2 = 1.5 \times 10^{-10} \text{ J/m}^3.$$

This compares with

$$U_\gamma = 4.0 \times 10^{-14} \text{ J/m}^3$$

that we calculated at the beginning of the lecture. Thus, today's universe is matter dominated, by a factor of 10^4.

However, the universe was less matter dominated at earlier times, because of the different scaling behaviour of U. Using today's values for U_{matter} and U_γ, we can write

$$U_{\text{matter}} = f^{-3} \cdot 1.5 \times 10^{-10} \text{ J/m}^3$$

$$U_\gamma = f^{-4} \cdot 4.0 \times 10^{-14} \text{ J/m}^3$$

where f is the fractional "size" of the universe compared to its current value. Let's calculate f when the matter and radiation energy densities were the same:

Start with

$$U_{\text{matter}} = U_\gamma$$

or

$$f^{-3} \cdot 1.5 \times 10^{-10} = f^{-4} \cdot 4.0 \times 10^{-14}$$

or

$$f = 4.0 \times 10^{-14} / 1.5 \times 10^{-10} = 3 \times 10^{-4}.$$

What was the temperature at this point? From the distribution of photon wavelengths,

$$T' = T / f$$

or

$$T' = 3 / 3\text{x}10^{-4} = 10{,}000 \text{ K}.$$

A more accurate calculation (which includes the contribution from relic neutrinos, *etc*) gives a lower value for this cross-over temperature, but the main point is that the universe was radiation-dominated as it cooled down, up to a temperature of a few thousand degrees.