Lecture 34 - Evolution of the early universe

What's important:

- Big Bang model

Text: Carroll and Ostlie, Sec. 28.1
Weinberg, Chaps. IV and V

The Hubble parameter in an expanding universe

Earlier, we established how the Hubble parameter behaves for a system expanding against a gravitational force, namely,

$$H = \left(\frac{8\pi G\rho}{3}\right)^{1/2} = \left(\frac{8\pi G U}{3c^2}\right)^{1/2} \tag{34.1}$$

In the previous lecture, we established that the energy density U varies with the time-dependent length scale $R(t)$ of the universe according to

$$U(R) \propto R^{-n}. \tag{34.2}$$

Substituting Eq. (34.2) into (34.1) yields the scale dependence

$$H \propto \frac{1}{R(t)^{n/2}}.$$

Clearly, $H(t)$ decreases with time if $R(t)$ increases. The explicit functional dependence $H(t)$ permits t to be determined from a given value of H (at a known temperature). We perform the inversion by writing

$$V(t) \propto \frac{1}{R(t)^{n/2}} \bullet R(t)$$

or

$$V(t) = KR(t)^{1-\frac{n}{2}} \tag{34.3}$$

where K is a constant that will disappear momentarily. Invoking $V = dR/dt$, or

$$\frac{dR}{V} = dt$$

the elapsed time can be integrated using Eq. (34.3):

$$\int dt = \int \frac{dR}{V} = \frac{1}{K}\int \frac{dR}{R^{1-\frac{n}{2}}} = \frac{1}{K}\int R^{\frac{n}{2}-1} dR$$

The RHS is just the integral of a polynomial. Inserting the integration limits gives

$$t_2 - t_1 = \frac{1}{K}\frac{1}{\frac{n}{2}-1+1} R^{\frac{n}{2}} \Big|_{R_1}^{R_2} = \frac{2}{n}\frac{1}{K} R^{\frac{n}{2}} \Big|_{R_1}^{R_2} \tag{34.4}$$

From Eq. (34.3), it follows that

$$\frac{1}{K}R^{\frac{n}{2}} = \frac{R^1}{KR^{1-\frac{n}{2}}} = \frac{R}{V} = \frac{1}{H}$$

so Eq. (12.4) becomes

$$t_2 - t_1 = \frac{2}{n}\left(\frac{1}{H_2} - \frac{1}{H_1}\right) \qquad (34.5)$$

or

$$\Delta t = \frac{2}{n}\Delta H^{-1}.$$

This **important** equation allows us to calculate the time difference between two epochs in the universe knowing their energy densities. *Note*: H^{-1} has units of time, and provides a characteristic time for expansion.

Example The measured value of the Hubble parameter is lies in a range centered on 70 km / (s • Mpc). How old is the universe?

In principle, we need to integrate Eq. (34.5) because n changes with time. But we have already established that for most of its history, the universe is matter dominated, so we can put $n = 3$. Further, at very early times, H^{-1} was huge because of the large energy density, so we can set $H_{early}^{-1} \sim 0$. Hence,

$$t_{today} = (2/3)H_{today}^{-1}.$$

We found before that $H_{today}^{-1} = 4.4 \times 10^{17}$ seconds, if H_{today} = 70 km / s•Mpc. Thus,

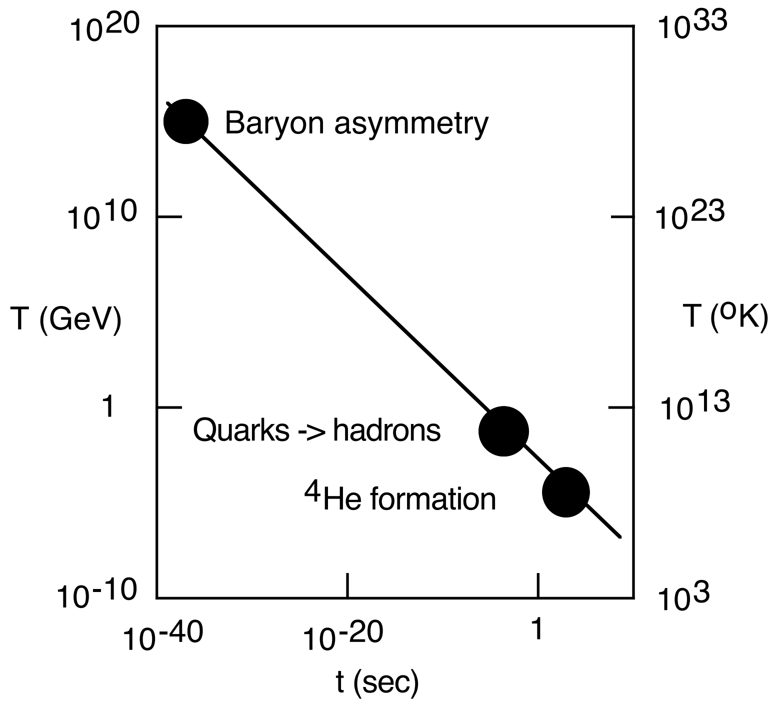
$$t_{today} = (2/3) \cdot 4.4\times10^{17} = 2.9 \times 10^{17} \text{ s} = (2.9\times10^{17} / 3.14\times10^7) = 9.3 \times 10^9 \text{ years.}$$

That is, the universe is just under 10 billion years old according to the measured value of H. The "accepted" value changes with time, and has recently drifted up to about 14±2 billion years.

Scenario for the early universe

Eq. (34.1) can be used to calculate the time evolution of the universe. The procedure is

- determine U at a given T knowing the species of particles present
- obtain H (via 34.1)
- obtain time t via $t \sim (2/n)/H$.



Let's describe what's going on at some important epochs in the early Universe, then treat one temperature in detail as an illustration of how the calculations are done.

Earliest times: the universe is more dense than a nucleus, behaving like a hot gas of quarks, gluons, photons and leptons.

T = 10^{11} K, time = 0.01 sec.

- $k_B T$ = 8.6 MeV, which means that all particles have so much energy that they break apart any nuclei which try to form.
- Protons and neutrons interconvert freely by collisions with high energy electrons.

T = 10^{10} K, time = 1 sec.

- $k_B T$ = 0.86 MeV, still no nuclei
- Protons and neutrons fixed in number, neutrons start to decay.

T = 10^9 K, time = 100+ sec.

- $k_B T$ = 0.086 MeV; nuclei can form, but no atoms
- Any free neutrons around are scooped up to form ^{2}H, which then reacts to form ^{4}He. Very few nuclei heavier than ^{4}He are formed, because no stable A = 5 nuclei exist. Hence, material in planets must have a source other than the early universe.
- Predict: Universe is 24% by weight helium.

T = 3000 K, time = 700,000 years

- $k_B T$ = 0.26 eV, cool enough so that atoms can form.

T = 2.7 K, time = 10 billion years

- $k_B T$ = 2.3 x 10^{-4} eV, today.

Details of $T = 10^{11}$ K = 8.6 MeV

At $T = 10^{11}$ K, the random collisions of nuclei with electrons or protons have sufficient energy to break up any nuclei that might transiently form, given the binding energy per nucleon is about 8 MeV. So, the universe consists of protons, neutrons, electrons, neutrinos and photons (at least) but not nuclei. Further, anti-electrons (positrons) may be produced in collisions, because their mass energy (at 0.511 MeV) is much less than the typical collision energy in reactions such as:

$\gamma + \gamma \rightarrow e^+ + e^-$.

Protons and neutrons may interconvert through reactions with neutrinos

anti-ν_e + p $\rightarrow e^+$ + n and ν_e + n $\rightarrow e^-$ + p.

Their populations are similar at these temperatures because the mass difference between them, about 939.6 - 938.3 = 1.5 MeV, is much less than the typical kinetic energy of a particle. Their equilibrium populations can be maintained if the reaction time is sufficiently small.

So, the species present are:

p,n	same "number" as today by conservation of baryon number
γ	lots, because of the high temperature
e^+ e^-	much larger than today because of collisions with energetic γ's etc.
ν, anti-ν	same numbers as e^+ e^-, although may not be in equilibrium
pions etc.	too massive to be copiously produced, and will have decayed

What is the energy density of these species?

p,n	relatively small because of their small number
γ	$U_\gamma = 2U_{boson} = 7.565 \times 10^{-16}\ T^4$ (J/m^3)
e^+ and e^-	2 (part.) • 2 (spins) • 7/8 U_{boson}
ν_e, anti-ν_e, ν_μ, anti-ν_μ	4 (part.) • 7/8 U_{boson} (left-handed)

The total energy density is then

$$U_{total} = (2 + 7/2 + 7/2)U_{boson} = 9U_{boson} = (9/2)U_\gamma.$$

Now that we have U_{total}, we can calculate H as follows:

$$U_{total} = (9/2)\ 7.565 \times 10^{-16}\ (10^{11})^4 = 3.4 \times 10^{29}\ \text{J/m}^3 \qquad \text{(huge!)}$$

$$H^2 = \frac{8\pi\ 6.67 \times 10^{-11}\ 3.4 \times 10^{29}}{3(3.0 \times 10^8)^2} = 2110\,\text{s}^{-2}$$

or

$$H = 46\ \text{s}^{-1}.$$

The time taken to reach this temperature after the Big Bang is

$$t - 0 = (2/4)\ (H^{-1} - 0)$$

or

$$t = 1 / (2 \cdot 46) = 0.011 \text{ seconds}.$$