Physics 415 - Quantum mechanics

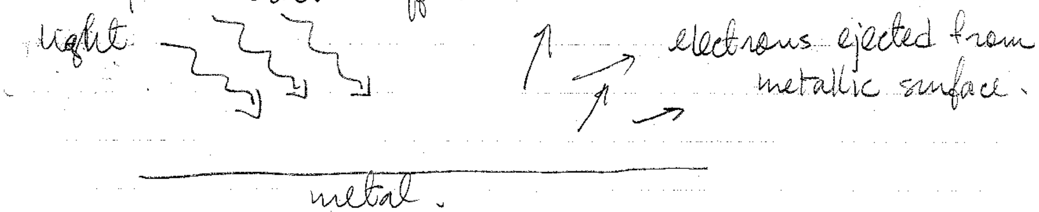
Textbook:
varies by instructor, but in the 1980s was *The Foundations of Quantum Mechanics* by Sol Wieder

Course outline:
1. Review of experimental evidence and old quantum theory
2. Classical mechanics
3. Quantum mechanics formalism
4. Motion in one dimensions, including free particles and square well potentials
5. Motion in three dimensions, including free particles, coulomb potential, harmonic potential and rotations
6. Path integral formalism
7. Perturbation theory: ground states
8. Perturbation theory: excited states and transitions
9. Scattering theory
10. Many-body problems: non-interacting systems
11. Many-body problems: interacting systems

Review

Historically, the first problem to which the idea of quantization was applied was that of the frequency spectrum of radiation in a cavity (Black body radiation) The student is directed to the text for a further discussion of this topic; the mathematics requires too long an excursion into statistical mechanics to be justified here.

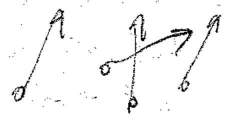
A relatively straightforward experiment for seeing the necessity for the need for quantization comes from the photo electric effect:



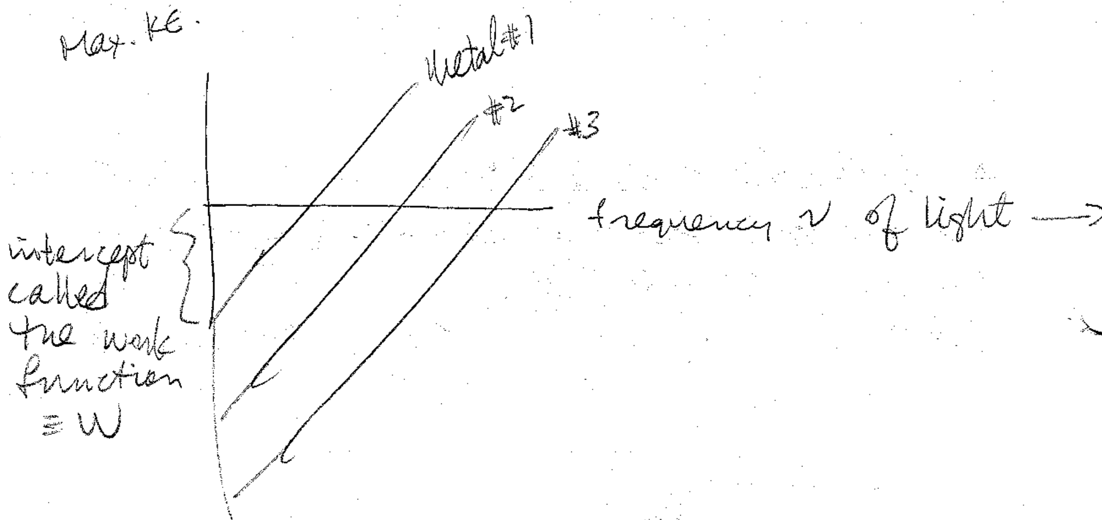
There were a number of observations associated with this effect which contradicted what was expected classically:

Observation	Classical expectation
1. Kinetic energy of electrons does not depend on intensity of incident light. There were more electrons as intensity inc.	1. Intensity $\propto \mathcal{E}_0^2$ (electric field) $F = q\mathcal{E}_0 \Rightarrow$ higher velocity expected.
2. Kinetic energy did depend on the frequency of the light.	2. $\mathcal{E}_0$ does not depend on frequency.
3. No time delay at low intensity.	3. Time delay expected since $\mathcal{E}_0$ small at low intensity.

To quantify things a little more, we look at an experimental measurement of the kinetic energies.



distribution of kinetic energies, but there is a well defined maximum K.E.
(caused by rescattering etc.)



$$K.E._{max} = h\nu - W$$

h: proportionality constant (common to all metals) found to be numerically equal to Planck's constant found in Black body radiation problem. $h = 6.63 \times 10^{-34}$ J-sec.

Einstein (1905) proposed that h was associated with the light, and that $h\nu$ was the energy associated with the elementary unit of light, the photon.

The work function is then the measure of the binding of electrons in the metal.

That the photons were "particles" was a novel idea. Their energy depended neither on their mass ($m=0$) nor their speed (c = constant). This resulted in the predictions that (something of a tautology here).

1, 2. KE_{max} increased with frequency of light, not intensity. Intensity $\propto$ # of photons. So more intensity, more ejected electrons.

3. No time delay at low intensities.

Energy - momentum relations.

Einstein's general relation between energy & momentum reads

$$E^2 = m^2c^4 + p^2c^2$$

Which has the usual non-relativistic reduction when $pc \ll mc^2$

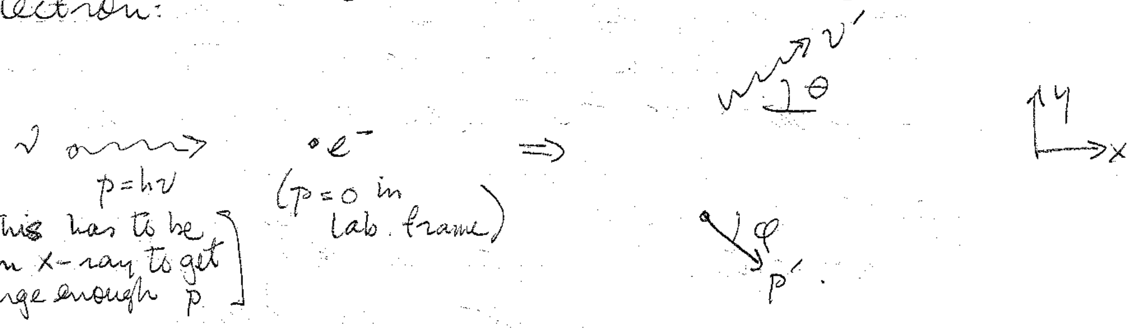
$$E = (m^2c^4 + p^2c^2)^{1/2} = mc^2\left(1 + \frac{p^2}{m^2c^2}\right)^{1/2}$$

$$\simeq mc^2 + \frac{1}{2}\frac{p^2}{m}.$$

If the photon is a particle, then we expect that it will have a momentum given by

$pc = E = h\nu \Rightarrow p = \frac{h\nu}{c}$ or since $\lambda\nu = c \Rightarrow p = \frac{h}{\lambda}$.

It was not for a considerable number of years until the momentum of the photon could be tested. The Compton effect (Compton, 1923) measures the energy change of a photon scattering from an electron:



[This has to be an x-ray to get large enough p]

Resolving components in x, y direction,

$$\begin{cases} \text{x-dir.} & \dfrac{h\nu}{c} = \dfrac{h\nu'}{c}\cos\theta + p'\cos\varphi \\ \text{y-dir.} & \dfrac{h\nu'}{c}\sin\theta = p'\cos\varphi \end{cases}$$

E cons. $h\nu = h\nu' + \text{KE of electron}$

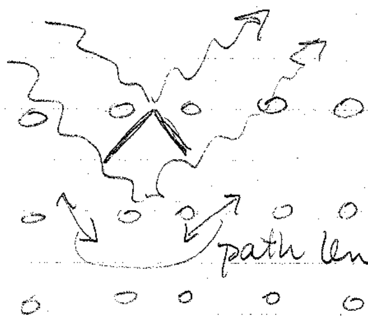
3 equ's, 4 unknowns $\theta, \varphi, \nu', p'$. So specify one variable, all the rest are determined.
By looking at the angular distributions, $p = \frac{h\nu}{c}$ can be verified.

<u>Wavelike nature of particles</u>

If waves had a particle-like characteristic, [p depending on ν]; then deBroglie (1924) reasoned that particles could have wavelike properties. The wavelength of the particles (referred to as the deBroglie wavelength) obeys the same relationship

$$p = \frac{h\nu}{c} = \frac{h}{\lambda}.$$

This wavelike nature results in interference, for example, in the scattering of neutrons from crystals

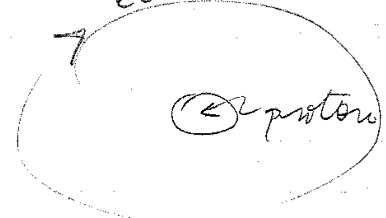


Bohr Theory of Hydrogen

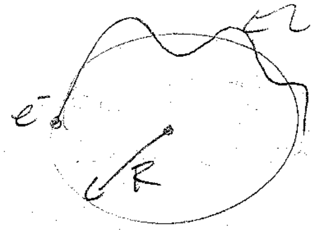
Long before deBroglie's hypothesis, Bohr had proposed that angular momentum

$$\vec{L} = \vec{r} \times \vec{p}$$

was quantized in units of $h/2\pi \equiv \hbar$. In the hydrogen atom



this implies that only certain orbits of the electron are allowed. This idea fits in well with deBroglie's hypothesis, if one looks upon the quantization of L as arising from interference:



wave "interferes" during repetetive motion.

Constructive interference if

circumference = integer # of wavelengths

$\Rightarrow \quad 2\pi r = n\lambda \qquad n = 1, 2, 3 \cdots$

$\Rightarrow \quad r \times \frac{1}{\lambda} = \frac{n}{2\pi}$

$r \times \frac{p}{h} = \frac{n}{2\pi}$

$r \times p = n \, \frac{h}{2\pi}$

$\Rightarrow \quad L = n\hbar$, same as Bohr.

This quantization condition allows us to solve for the radii and energy levels of the "Bohr" atom:

Use:

$$F = ma = m\frac{v^2}{r}.$$

and $F = k\frac{e^2}{r^2}$ from Coulomb's law.

$$\Rightarrow \quad m\frac{v^2}{r} = k\frac{e^2}{r^2} \Rightarrow mv^2 = ke^2/r \quad (1)$$

($k = 8.99\times10^9$ Nm^2/C^2 or absorb into e)

But $L = mvr = n\hbar \Rightarrow v = \frac{n\hbar}{mr}$.

$$\Rightarrow m\left(\frac{n\hbar}{mr}\right)^2 = ke^2/r$$

$$\text{or} \quad r_n = \frac{n^2\hbar^2}{kme^2} \quad (2).$$

The corresponding energy is

$$\epsilon_n = \underbrace{\tfrac{1}{2}mv_n^2}_{\text{kinetic}} \underbrace{- k\frac{e^2}{r_n}}_{\text{coulomb}} = -\frac{1}{2}\frac{e^2}{r_n} \quad \text{(from (1))}$$

$$= -\frac{1}{2}ke^2\left(\frac{kme^2}{n^2\hbar^2}\right) \quad \text{(from (2))}$$

$$= -\frac{k^2me^4}{2n^2\hbar^2}.$$

Transitions may occur between these states,

$\epsilon = 0$

ϵ_3

ϵ_2

$\epsilon_1 = -\frac{me^4}{2\hbar^2}k^2$ ($r_1 = a_0 \equiv$ Bohr radius)