

Online Appendix:

Property Rights Over Marital Transfers

Siwan Anderson* Chris Bidner†

February 5, 2015

Abstract

This online appendix accompanies our paper “Property Rights Over Marital Transfers”.

APPENDIX

A Non-Linear Version

The main model utilizes the simplifications that follow from assuming that an offspring’s consumption is linear in their property and in the property of their spouse. This section outlines the more general argument where this relationship need not be linear.

Let z_k denote the property of a gender k offspring, where $z_f \equiv w_f$ and $z_m \equiv w_m + t$. This offspring consumes $c_k(z_f, z_m)$, where c_k is strictly increasing in both arguments but need not be linear. Importantly, property rights matter in the sense that consumption is more sensitive to one’s own property:

$$\frac{dc_f}{dz_f} > \frac{dc_f}{dz_m} \text{ and } \frac{dc_m}{dz_m} > \frac{dc_m}{dz_f}. \quad (1)$$

All other aspects of the model are retained. To help analyze this more general case, we use the insight of [Peters and Siow \(2002\)](#) and draw connections to the literature on hedonic equilibrium ([Rosen \(1974\)](#)). To this end, it is useful to let p denote a marriage payment amount and to pose the problem facing families as one of choosing characteristics and a marriage payment subject to the marriage payment coinciding with that required by the market for the chosen characteristics. Thus, for female families the problem is:

$$\max_{p, (w_f, w_m) \in \mathcal{W}} V\left(W - \frac{1}{\theta_f} \cdot w_f - p, c_f(w_f, w_m + p)\right), \text{ s.t. } p = t(w_f, w_m), \quad (2)$$

and for male families is:

$$\max_{p, (w_f, w_m) \in \mathcal{W}} V\left(W - \frac{1}{\theta_m} \cdot w_m, c_m(w_f, w_m + p)\right), \text{ s.t. } p = t(w_f, w_m). \quad (3)$$

*Vancouver School of Economics and CIFAR. siwan.anderson@ubc.ca

†Department of Economics, Simon Fraser University. cbidner@sfu.ca

Starting with the female problem, and letting ℓ^f be the Lagrange multiplier associated with the marriage payment constraint, the first-order conditions associated with the optimal choices of w_f , w_m , and p respectively, are:

$$V_C \cdot \frac{dc_f}{dz_f} - V_c \cdot \frac{1}{\theta_f} = \ell^f \cdot \frac{dt}{dw_f} \quad (4)$$

$$V_C \cdot \frac{dc_f}{dz_m} = \ell^f \cdot \frac{dt}{dw_m} \quad (5)$$

$$V_C - V_c \cdot \frac{dc_f}{dz_m} = \ell^f. \quad (6)$$

The first and third of these together imply

$$\frac{dt}{dw_f} = -\frac{\frac{V_C}{V_c} \cdot \frac{1}{\theta_f} - \frac{dc_f}{dz_f}}{\frac{V_C}{V_c} - \frac{dc_f}{dz_m}}. \quad (7)$$

The left side is the slope of the marriage market transfer function and the right side is the slope of the female indifference curve in (w_f, p) space (i.e. holding w_m constant). This optimality condition is illustrated in the left panel of Figure 1 where the curve I^f is a typical female indifference curve. Points ‘under’ the indifference curve are preferred, so optimality requires that females choose the w_f that puts them on the lowest indifference curve subject to lying on the marriage market price schedule. This is analogous to the Rosen (1974) model whereby females are the ‘suppliers’ of w_f (accounting for the fact that the ‘price’ received is *minus* p).

Similarly, the second and third female first-order condition tells us

$$\frac{dt}{dw_m} = \frac{\frac{dc_f}{dz_m}}{\frac{V_C}{V_c} - \frac{dc_f}{dz_m}}. \quad (8)$$

The left side is again the slope of the marriage market transfer function, and the right side is the slope of the female indifference curve in (w_m, p) space. This optimality condition is illustrated in the right panel of Figure 1 where the curve I^f is a typical female indifference curve. Points to the right of the indifference curve are preferred, so optimality requires that females choose the w_m that puts them on the right-most indifference curve subject to lying on the marriage market price schedule. This is analogous to the Rosen (1974) model whereby females are the ‘consumers’ of w_m .

Turning to the male problem, and letting ℓ^m be the Lagrange multiplier associated with the marriage payment constraint, the first-order conditions associated with the optimal choices of w_f , w_m , and p respectively, are:

$$V_c \cdot \frac{dc_m}{dz_f} = -\ell^m \cdot \frac{dt}{dw_f} \quad (9)$$

$$V_c \cdot \frac{dc_m}{dz_m} - \frac{1}{\theta_m} \cdot V_C = -\ell^m \cdot \frac{dt}{dw_m} \quad (10)$$

$$V_c \cdot \frac{dc_m}{dz_m} = \ell^m. \quad (11)$$

The first and third of these together imply

$$\frac{dt}{dw_f} = -\frac{\frac{dc_m}{dz_f}}{\frac{dc_m}{dz_m}}. \quad (12)$$

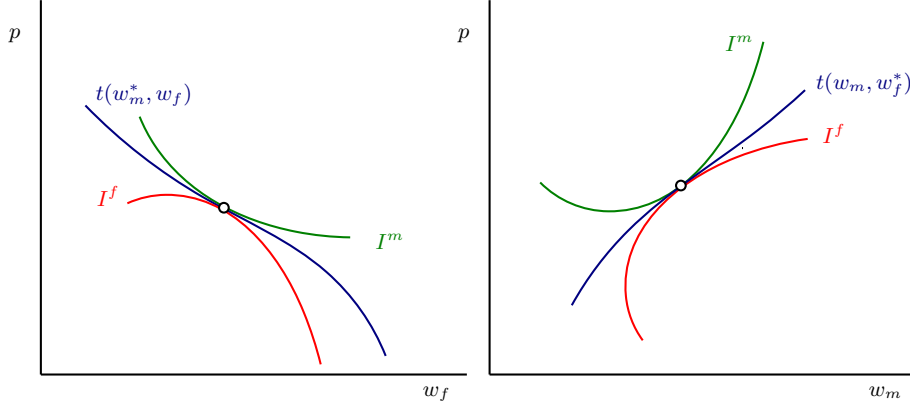


Figure 1: Hedonic Pricing Interpretation

The left side is again the slope of the marriage market pricing function and the right side is the slope of the male indifference curve in (w_f, p) space. This optimality condition is illustrated in the left panel of Figure 1 where the curve I^m is a typical male indifference curve. Points above the indifference curve are preferred, so optimality requires that males choose the w_m that puts them on the highest indifference curve subject to lying on the marriage market price schedule. This is analogous to the Rosen (1974) model whereby males are the ‘suppliers’ of w_m .

Similarly, the second and third male first-order condition tells us

$$\frac{dt}{dw_m} = \frac{V_C}{V_c} \cdot \frac{1}{\theta_m} \cdot \frac{1}{\frac{dc_m}{dz_m}} - 1 \quad (13)$$

The left side is again the slope of the marriage market transfer function, and the right side is the slope of the male indifference curve in (w_m, p) space. This optimality condition is illustrated in the right panel of Figure 1 where the curve I^m is a typical male indifference curve. Points above the indifference curve are preferred, so optimality requires that males choose the w_m that puts them on the highest indifference curve subject to lying on the marriage market price schedule. This is analogous to the Rosen (1974) model whereby males are the ‘suppliers’ of w_m .

To solve for equilibrium prices, one computes male and female demand for each characteristic pair and adjusts the marriage market transfer function until male and female demand coincide at each characteristic pair. Explicitly doing this requires a specific form for the c_k functions. But even if a form were chosen, it is well-known from the hedonic pricing literature (e.g Heckman *et al.* (2010) and Ekeland *et al.* (2004)) that analytical solutions for hedonic price functions generally do not exist. Indeed, establishing the existence and uniqueness of equilibrium beyond the linear consumption case we consider is an interesting area of future research but at present is hindered by the absence of results in the literature beyond special settings that are fundamentally inappropriate for our purposes (specifically, for quasi-linear payoff functions; see Chiappori *et al.* (2010) and Ekeland (2010)).

Nevertheless, we can use the first-order conditions to gain some insight into the structure of marriage market prices in equilibrium. As we shall see, these insights are qualitatively

identical to those produced in the linear case of our main model. First, analysis of the male problem has already indicated in (12) that each family perceives the price of w_f at their optimal choices as

$$\frac{dt}{dw_f} = - \left\{ \frac{\frac{dc_m}{dz_f}}{\frac{dc_m}{dz_m}} \right\}. \quad (14)$$

Similarly, the female optimality conditions (7) and (8) together imply that

$$\frac{dt}{dw_m} = \frac{\frac{1}{\theta_f} + \frac{dt}{dw_f}}{\left\{ \frac{\frac{dc_f}{dz_f}}{\frac{dc_f}{dz_m}} \right\} - \frac{1}{\theta_f}}.$$

Using (14) in this condition indicates that each family perceives the price of w_m at their optimal choices as

$$\frac{dt}{dw_m} = \frac{\frac{1}{\theta_f} - \left\{ \frac{\frac{dc_m}{dz_f}}{\frac{dc_m}{dz_m}} \right\}}{\left\{ \frac{\frac{dc_f}{dz_f}}{\frac{dc_f}{dz_m}} \right\} - \frac{1}{\theta_f}}. \quad (15)$$

Conditions (14) and (15) are simple generalizations of the prices that we identify in the main model. In the linear case the terms in braces are constants, so that these prices are the same across the entire market. In the general setting these prices will generally not be constant across agents since their equilibrium choices will differ.

Nevertheless, this analysis gives a glimpse into the more general features of the consumption functions, c_k , that affect local marriage prices perceived by families at their equilibrium choices. Specifically, prices are only affected by the ratios $\frac{dc_m}{dz_f} / \frac{dc_m}{dz_m}$ and $\frac{dc_f}{dz_f} / \frac{dc_f}{dz_m}$. More specifically, a decrease in $\frac{dc_m}{dz_f} / \frac{dc_m}{dz_m}$ (e.g. as was caused by an increase in β in the linear model) raises the price of both w_m and w_f . Similarly, a decrease in $\frac{dc_f}{dz_f} / \frac{dc_f}{dz_m}$ raises the price of w_m .

B A Symmetric Version

In order to simplify the presentation, our main model contained an inherent asymmetry: we allow transfers between female families and grooms, but ignore the possibility of transfers between male families and brides. This section outlines the symmetric counterpart to the model and demonstrates that doing so has no effect on the main results. In fact, some results are strengthened if a more symmetric model were used.

Suppose that female families can make a transfer to grooms, denoted t_m , and that male families can make a transfer to brides, denoted t_f . The property of females is $z_f \equiv w_f + t_f$, and of males is $z_m \equiv w_m + t_m$. Both transfers are non-negative: $t_m, t_f \geq 0$. As in the main model, offspring consumption is given by $c^k(z_f, z_m) = a_k \cdot z_f + b_k \cdot z_m$ and parent consumption is $C^k = W - \frac{1}{\theta_k} \cdot w_k - t_{-k}$ for $k \in \{m, f\}$.

For given characteristics, $\mathbf{w} = (w_m, w_f)$, the marriage market specifies a multi-valued function, $\mathbf{t}: \mathbb{R}_+^2 \rightarrow \mathbb{R}^2$, which indicates a pair of prices $\mathbf{t}(\mathbf{w}) = \{t_m(\mathbf{w}), t_f(\mathbf{w})\}$ for each characteristic pair. Given that transfers must be non-negative in this setting, let $\mathcal{W} \equiv \{\mathbf{w} \mid \mathbf{w} \in \mathbb{R}_+^2, \mathbf{t}(\mathbf{w}) \in \mathbb{R}_+^2\}$ denote the implied equilibrium set of feasible characteristics.

The equilibrium concept remains as in the main model: briefly, an *equilibrium* is a marriage market pricing function, \mathbf{t} , such that (i) agents take \mathbf{t} as given and choose their most preferred \mathbf{w} on \mathcal{W} , and (ii) for each $A \subseteq \mathbb{R}_+^2$, the measure of males choosing $\mathbf{w} \in A$ equals the measure of females choosing $\mathbf{w} \in A$ (i.e. market clearing).

We start with the claim that the only interesting equilibria are those in which the market requires at most one side to make a positive payment within a given match. Specifically, when $\theta_k > 1$ there is never an equilibrium in which families choose characteristics which require a positive groom-price and a positive bride-price. This is because of a type of ‘no arbitrage’ pricing condition. To explain, consider a bride and groom with characteristics (w_m, w_f) . Suppose the bride’s family transfers t_m to the groom and the groom’s family transfers t_f to the bride, where $t_f, t_m \geq d > 0$. Now consider an alternative arrangement whereby both families lower their cross-family payment by d units and instead give the d units to their offspring. That is, the bride’s family transfers $\tilde{t}_m = t_m - d$ to the groom and diverts d to the bride. Since families always have the option of raising their offspring’s quality by making a cash transfer to them, this means that the bride in effect has a quality of $\tilde{w}_f = w_f + d$. At the same time, the groom’s family transfers $\tilde{t}_f = t_f - d$ to the bride and diverts d to the groom, effectively generating a quality of $\tilde{w}_m = w_m + d$. Since all individuals have the same allocations across the two arrangements, the families treat them as equivalent. This equivalence is reflected in equilibrium prices: $\mathbf{t}(\tilde{w}_f, \tilde{w}_m) = (\tilde{t}_f, \tilde{t}_m)$. More generally, suppose characteristics $\mathbf{w} = (w_f, w_m)$ require transfers of $\mathbf{t}(\mathbf{w}) = (t_f, t_m) \in \mathbb{R}_+^2$. Then, for any $d \in [0, \min\{t_f, t_m\}]$, the qualities $\mathbf{w}' = (w_f + d, w_m + d)$ require transfers $\mathbf{t}' = (t_f - d, t_m - d)$. This is summarized by the ‘no-arbitrage’ condition.

Definition 1 (No Arbitrage). *For each \mathbf{w} , we have $\mathbf{t}(\mathbf{w} + d \cdot \mathbf{1}) = \mathbf{t}(\mathbf{w}) - d \cdot \mathbf{1}$ for all $d \in [0, \underline{\mathbf{t}}(\mathbf{w})]$, where $\mathbf{1} \equiv (1, 1)$ and $\underline{\mathbf{t}}(\mathbf{w}) \equiv \min\{t_f(\mathbf{w}), t_m(\mathbf{w})\}$.*

Under the no arbitrage condition, it is straightforward to see that no family would ever choose characteristics that required a positive groom-price *and* a positive bride-price: by instead choosing characteristics that were d units higher in each dimension, the family can keep their consumption and that of their offspring unchanged by the no-arbitrage condition. But they can do even better than that since they can deliver the additional d units via investment rather than via cash. This therefore allows them to keep their offspring’s consumption the same while increasing their own consumption by $(1 - (1/\theta_k)) \cdot d > 0$, thereby making them strictly better off.¹ Given this, equilibria in which a groomprice and a brideprice are not simultaneously paid within the same marriage are the only interesting ones in the sense that by allowing the market to require $\mathbf{t}(\mathbf{w}') \in \mathbb{R}_{++}^2$ for some characteristics, \mathbf{w}' , we are only giving ourselves freedom in constraining equilibrium outcomes by specifying off-equilibrium prices.²

¹This result can be demonstrated formally by supposing that a family did choose characteristics such that $t_f > 0$ and $t_m > 0$. By differentiating the no arbitrage condition with respect to d we get $\frac{dt_f}{dw_m} + \frac{dt_f}{dw_f} = \frac{dt_m}{dw_m} + \frac{dt_m}{dw_f} = -1$. This can be used in the system of first-order conditions to derive the contradiction that $\frac{dc_f}{dw_m} / \frac{dc_f}{dw_f} = \frac{dc_m}{dw_m} / \frac{dc_m}{dw_f}$ (which does not hold since the former is strictly less than unity whereas the latter is strictly larger.)

²As an extreme case, consider an equilibrium with $\mathbf{t}(\mathbf{w}) = (0, 0)$ if $\mathbf{w} = \mathbf{w}^*$ and $\mathbf{t}(\mathbf{w}) = (\bar{W}, \bar{W})$ otherwise (where \bar{W} is the highest wealth among all families). All families would choose the arbitrary bundle \mathbf{w}^* , but for completely artificial reasons.

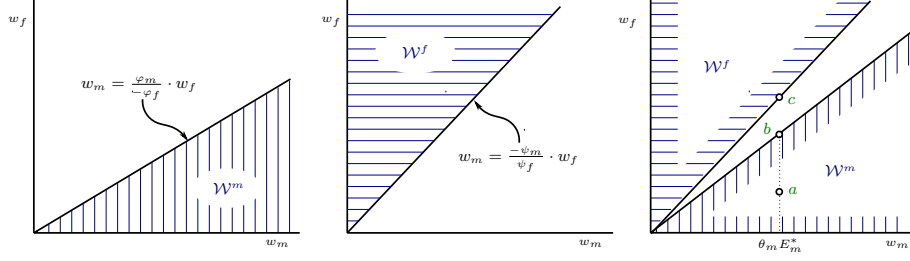


Figure 2: Symmetric Model

Given the above discussion, we search for three type of equilibria: (i) a *groom-price equilibrium* whereby $t_f(\mathbf{w}) = 0$ for all \mathbf{w} , (ii) a *bride-price equilibrium* whereby $t_m(\mathbf{w}) = 0$ for all \mathbf{w} , and (iii) a *hybrid equilibrium* whereby $t_f(\mathbf{w}) = 0$ for some characteristics whereas $t_m(\mathbf{w}) = 0$ for others.

In general terms, the problem facing families of gender $k \in \{f, m\}$ is:

$$\max_{(w_f, w_m) \in \mathcal{W}} V \left(W - \frac{1}{\theta_k} \cdot w_k - t_{-k}(w_f, w_m), a_k \cdot [w_f + t_f(w_f, w_m)] + b_k \cdot [w_m + t_m(w_f, w_m)] \right). \quad (16)$$

In calculating a groom-price equilibrium, we derive optimal characteristic choices using $t_f(\mathbf{w}) = 0$. This is essentially the exercise undertaken in the main model, the only difference being that only characteristics that lead to a groom-prices of at least zero (as opposed to $-w_m$ in the main model) are permitted. That is, in such an equilibrium the marriage market sets a groom-price function identical to that derived in the main model: $t_m(w_f, w_m) = \varphi_f \cdot w_f + \varphi_m \cdot w_m$, where

$$\varphi_f \equiv -\frac{\frac{dc_m}{dw_f}}{\frac{dc_m}{dw_m}}, \text{ and } \varphi_m \equiv \frac{\frac{1}{\theta_f} - \frac{\frac{dc_m}{dw_f}}{\frac{dc_m}{dw_m}}}{\frac{dc_f}{dw_f} - \frac{1}{\theta_f}}. \quad (17)$$

The feasible characteristics set, denoted \mathcal{W}^m for the groom-price case, consists of all characteristics that command a non-negative groom-price. That is:

$$\mathcal{W}^m = \left\{ (w_f, w_m) \mid w_f \leq \frac{\varphi_m}{-\varphi_f} \cdot w_m \right\}. \quad (18)$$

This set is illustrated in the left panel of Figure 2. As in the main model, each family has families have an optimal total expenditure but are indifferent as to how this is allocated (i.e. males have a most-preferred w_m but are indifferent to w_f , and females are indifferent to (w_m, w_f) pairs that keep $\frac{1}{\theta_f} \cdot w_f + t_m(w_f, w_m)$ constant). The existence of this sort of equilibrium again requires that we can find a matching of male and female families such that each pair has a ‘compatible’ optimal expenditure (in the sense that there exists a characteristic that induces the optimal expenditure for both sides), and this arises most easily when matching is positive assortative on wealth.

In calculating a bride-price equilibrium, we derive optimal characteristic choices using $t_m(\mathbf{w}) = 0$. Taking the analogous steps as in the previous case, one can show that manipulation of the first-order conditions yields an equilibrium marriage market bride-price function:

$t_f(w_f, w_m) = \psi_f \cdot w_f + \psi_m \cdot w_m$, where

$$\psi_f \equiv \frac{\frac{1}{\theta_m} - \frac{\frac{dc_f}{dw_m}}{\frac{dc_f}{dw_f}}}{\frac{\frac{dc_m}{dw_m}}{\frac{dc_m}{dw_f}} - \frac{1}{\theta_m}}, \text{ and } \psi_m \equiv -\frac{\frac{dc_f}{dw_m}}{\frac{dc_f}{dw_f}}. \quad (19)$$

The set of feasible characteristics, denoted \mathcal{W}^f for the bride-price case, consists of all characteristics that command a non-negative bride-price. That is:

$$\mathcal{W}^f = \left\{ (w_f, w_m) \mid w_f \geq \frac{-\psi_m}{\psi_f} \cdot w_m \right\}. \quad (20)$$

This set is illustrated in the centre panel of Figure 2.

The sets \mathcal{W}^m and \mathcal{W}^f are always disjoint when $\theta_k > 1$ (they share a common boundary when $\theta_f = \theta_m = 1$). It can be shown that if a pair makes compatible expenditures in a groom-price equilibrium then they can not make compatible expenditures in a bride-price equilibrium (and vice-versa). As such, the existence of a groom-price equilibrium precludes the existence of a bride-price equilibrium.³

It can also be shown that there does not exist a ‘hybrid’ equilibrium in which regions like \mathcal{W}^m and \mathcal{W}^f coexist. The argument is as follows. Suppose that some pair chose a point in \mathcal{W}^m , such as point a in the right panel of Figure 2. Since families only care about their total expenditure in equilibrium, the male family is indifferent to the points on the vertical dotted line (each of which corresponds to an equal expenditure of E_m^* as indicated). Specifically, the male is indifferent between their equilibrium choice, point a , and choosing a higher quality female at the expense of a lower groomprice. Specifically, they are indifferent to point b where no groomprice (or brideprice) is paid. But then they must strictly prefer a higher point, such as c , since this point also involves no brideprice (or groomprice), but it does involve a strictly larger w_f with no change in w_m . This contradicts the original choice being optimal.

Of course, the existence of a groom-price or a bride-price equilibrium is not guaranteed (as in the main model). What this analysis does reveal however is the process through which marriage payments disappear. As in the main model, a groom-price equilibrium fails to exist when θ_f becomes too great. Unlike the main model though, we should not expect bride-prices to arise once groom-prices are pushed to zero. The inefficiency underlying the decline in bride-prices is also present on the groom’s side—i.e. male families too would find that paying brides prohibitively inefficient (indeed *more so* to the extent that $\theta_m > \theta_f$).

In summary, the qualitative conclusions from the main model are not driven by the inherent asymmetries. Specifically, the natural symmetric version permits a groom-price equilibrium with features that are essentially identical to the main model. It also permits a bride-price equilibrium in which the roles of males and females are switched. The symmetric version requires far greater set-up, but does deliver a clearer insight into the process through which marriage payments disappear.

³The argument is that if there exists a groom-price equilibrium, then there exists one with positive assortative matching on wealth. But then there does not exist a bride-price equilibrium with positive assortative matching on wealth, and therefore no bride-price equilibrium exists.

References

- CHIAPPORI, P.-A., MCCANN, R. J. and NESHEIM, L. P. (2010). Hedonic price equilibria, stable matching, and optimal transport: equivalence, topology, and uniqueness. *Economic Theory*, **42** (2), 317–354.
- EKELAND, I. (2010). Existence, uniqueness and efficiency of equilibrium in hedonic markets with multidimensional types. *Economic Theory*, **42** (2), 275–315.
- , HECKMAN, J. J. and NESHEIM, L. (2004). Identification and Estimation of Hedonic Models. *Journal of Political Economy*, **112** (S1), S60–S109.
- HECKMAN, J. J., MATZKIN, R. L. and NESHEIM, L. (2010). Nonparametric Identification and Estimation of Nonadditive Hedonic Models. *Econometrica*, **78** (5), 1569–1591.
- PETERS, M. and STOW, A. (2002). Competing premarital investments. *Journal of Political Economy*, **110** (3), 592–608.
- ROSEN, S. (1974). Hedonic prices and implicit markets: Product differentiation in pure competition. *Journal of Political Economy*, **82** (1), 34–55.