

A Spillover-Based Theory of Credentialism*

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Abstract

I propose a model in which credentials, such as diplomas, are instrumentally valuable to workers; a situation described as *credentialism*. The model avoids an important criticism of standard job market signaling models by tying a worker's wage to their output. A worker's productivity is influenced by the skills of their coworkers, where such skills arise from an ability-augmenting investment that is made prior to matching with coworkers. A worker's credentials allow them to demonstrate their investment to the labor market, thereby allowing workers to match with high-skill coworkers in equilibrium. Despite the positive externality associated with a worker's investment, I show how over-investment is pervasive in equilibrium.

Keywords: Credentialism, Matching, Spillovers, Signaling

JEL Codes: D80, I20, J24, C78

1 Introduction

There are at least two interpretations of the dramatic increase in educational attainment experienced in the post-war period in many countries. The first is that it represents a natural response to the changing nature of work, including the implementation of new technologies that require high skilled labor. A second perspective asserts that educational attainment is fueled, in part, by *credentialism*, whereby credentials are instrumentally valuable to workers. Proponents of this perspective assert that “credentialing, not educating,

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has become the primary business of North American universities”, that a “resume without one or more degrees from a respected institution will not be taken seriously enough even to be considered, no matter how able or informed the applicant may be”, and that economic forces have made “credentials the object rather than the byproduct of educational achievement”.¹

These two interpretations are by no means mutually exclusive, and it is of great policy relevance to understand the relative contribution of each. Despite decades of intensive research, the literature is yet to reach a solid consensus on the relative magnitudes of each explanation. This is due, in no small part, to a tension that exists between the plausibility of credentialism on the one hand, and the absence of a compelling underlying theory of credentialism on the other. The development of such a theory is the primary objective of this paper.

Economists have typically used models of signaling (and/or screening) to understand credentialism.² These models focus on the worker-firm relationship, where credentials are valuable because they affect the beliefs, and therefore the willingness to pay wages, of the firm. The relevance of this mechanism has been placed in serious doubt in recent years, as criticisms have been levied on signaling theory’s heavy reliance on an unrealistic assumption: that a worker’s wages are largely insensitive to their performance. The criticism is summed up nicely as the idea “that companies rather quickly discover the productivity of employees who went to college, whether a Harvard or a University of Phoenix. Before long, their pay adjusts to their productivity rather than to their education credentials” (Becker (2006)).³ The assumption that wages are insensitive to performance is not only intuitively unpalatable, it is not in accordance with the evidence regarding the prominence of explicit performance pay (Lemieux *et al.* (2009)), nor with the estimates of the speed of employer learning (Lange (2007), Arcidiacono *et al.* (2008), Lang and Siniver (2011)).

Although this type of argument is damaging for the practical relevance of traditional signaling theory in the labor context, it raises the issue of what we are to make of the widely expressed credentialist sentiment⁴, and of the empirical evidence that has amassed over the decades in support of signaling.⁵ Rather than dismiss the practical relevance of

¹The first two quotes are from Jacobs (2004) [pp. 44-45], and the third is from Labaree (1997), [p. 253].

²Seminal work of course includes Spence (1973), Spence (1974), Rothschild and Stiglitz (1976) and Arrow (1973).

³See Alós-Ferrer and Prat (2012) for a model of signaling in which employers learn their worker’s type over time by observing noisy output. That paper focuses on new signaling equilibria that arise, and does not have skill spillovers across workers as stressed here.

⁴For example, as expressed in the sociological literature (e.g. Labaree (1997), Brown (1995), Collins (1979) and Berg (1970)) and in the popular press (e.g. Jacobs (2004)).

⁵For example, Tyler *et al.* (2000), Lang and Kropp (1986), Bedard (2001), as well as others surveyed in Weiss (1995), Ferrer and Riddell (2002), and Lange and Topel (2006).

credentialism altogether, I propose an alternative model that addresses the above criticism.

Specifically, I model an economy in which workers with heterogeneous abilities make skill-enhancing investments prior to joining N -worker firms. A worker's marginal productivity is shaped in part by this skill, but also in part by the skill of their coworkers. Once in a firm, each worker exerts labor effort in order to produce a verifiable, individual output. That is, there are no effort externalities, such as team production. In order to most cleanly address the above criticism of signaling models, I assume that a worker's wage is mechanically tied to their output. Such output-contingent wage contracts effectively removes any asymmetric information between a firm and worker, and therefore, in contrast to the standard signaling mechanism, credentials play no role in influencing beliefs within a *worker-firm* relationship. Instead, credentials play a role in shaping the composition of coworker groups, thereby exerting influence on the *worker-coworker* relationship.

The assignment of workers to firms - i.e. to other coworkers - unfolds on the basis of workers' investments (as opposed to skills). This feature reflects a situation in which skills are 'soft' - prohibitively difficult to describe, quantify, and communicate - relative to investments. For example, it is much easier to accurately communicate one's educational history on a resume than it is to communicate one's creativity, punctuality, or honesty. Given this, workers tend to be hired, and therefore exposed to coworkers, on the basis of their quantifiable investment history rather than on loose claims of being 'skilled'.

Since higher types make higher investments in equilibrium, all workers find those with higher investments more desirable as coworkers. As such, I following the literature (e.g. [Peters \(2007b\)](#), [Hoppe et al. \(2009\)](#), [Peters and Siow \(2002\)](#)) in modeling the matching process via the imposition of positive assortative matching. This approach disciplines the analysis; not only does it produce a stable matching outcome in equilibrium, it also imposes concrete consequences associated with making off-equilibrium investments.

Given these basic ingredients, the mechanism is simple: credentials are valuable because they grant access to groups of higher-skilled coworkers. Unlike standard signaling models then, it is interaction within firms - and skill spillovers in particular - that produces credentialism. The first result shows that pooling equilibria fail to exist in this setting - another point of contrast to standard signaling models. The next result establishes the existence and uniqueness of separating equilibria. I show how over-investment is pervasive in equilibrium, and then explore how changes in the degree of spillovers influences the level and distribution of investment, output, and payoffs. In the discussion that follows, I demonstrate the robustness of the conclusions by considering a generalization of the function describing spillovers, as well as by analyzing a very simple dynamic extension that captures the reality that a "credential is not a passport to a job, as naive graduates sometimes suppose. It is more basic and necessary: a passport to consideration for a job",

Jacobs (2004). Specifically, the extension allows workers to replace undesirable coworkers prior to the commencement of production. I then briefly discuss how the theory could be tested against related theories, and draw connections between the theory proposed here and standard signaling models.

1.1 Relationship to the Literature

The possibility that education infers positive externalities has long been recognized, but standard treatments are ‘global’ in the sense that all agents in a given region benefit from some measure of aggregate educational investment (e.g. Lucas (1988), and Moretti (2004)). In contrast, spillovers in this model are ‘local’ in that they occur within the boundaries of the firm.⁶ This approach opens the possibility that individuals influence who they match with by choosing appropriate investments. This is a central feature of the model and one that distinguishes it from models of matching with exogenous characteristics (e.g. static models; Becker (1973), Sattinger (1993), Kremer (1993), Legros and Newman (2007), and dynamic models; Shimer and Smith (2000), Smith (2006), and Burdett and Coles (1997)) and models with endogenous characteristics where potential partners are encountered at random (Burdett and Coles (2001), and de Meza and Lockwood (2010)). I show how this ‘competition for coworkers’ reverses the standard intuition that positive spillovers imply under-investment and that greater spillovers reduce investment.

The model is designed to capture the feature that there is often a disconnect between the characteristics used to form matches and the characteristics that potential partners are interested in. This aspect distinguishes the model from models of pre-marital investment, where an agent’s attractiveness as a partner is completely captured by their observable investment (e.g. Peters (2007b), Peters and Siow (2002), Cole *et al.* (2001), Felli and Roberts (2002), and Gall *et al.* (2006)).

An extreme form of this feature is prominent in a class of models in which the investment is unproductive and acts purely as a signal of an underlying characteristic (Hoppe *et al.* (2009), Rege (2008), Damiano and Li (2007), and Bidner (2010)). The analysis here complements these models by exploring the consequences of productive investment. Although this type of extension is largely trivial in standard signaling models, it produces a number of additional insights and subtleties in the present context. For instance, productive investment allows one to drop the assumption of complementary interaction between workers, and therefore removes the central trade-off studied in the case of unproductive investments (investments are wasteful but facilitate efficient matching). However, a new trade-off emerges in which the over-investment inherent in a separating equilibrium is

⁶In this way, firms take on some key properties of ‘clubs’ in the sense of Buchanan (1965).

compared with the under-investment inherent in random matching. In addition, having productive investment allows one to study the impact of spillovers on aggregate variables such as the level and distribution of income, whereas such impacts are absent when investment is unproductive since they are determined by the exogenous distribution of types.⁷

In a similar setting, Hopkins (2011) studies an economy in which workers make potentially productive investments prior to being paired with a firm. The papers share the feature that workers invest in order to secure a job in a ‘good’ firm, but differ in the account of why firms are heterogeneous. There, firms are exogenously heterogeneous, whereas here firms are ex-ante homogeneous and are only differentiated in equilibrium because of the human resources they house. Both models can be thought of as workers investing in order to compete for ‘prizes’ (i.e. coworker skills). Spillovers have a more nuanced effect in the model presented here because spillovers not only influence the benefit derived from a given ‘prize’, but also influence the distribution of ‘prizes’, and therefore have secondary effects on investment incentives.

Given that spillovers form the basic source of credentialism identified here, a primary concern of the analysis is understanding the comparative statics related to the spillover parameter. None of the above papers are centrally concerned with this (or other technological parameters).

1.2 Foundations of Key Assumptions

Far from ignoring the relevance of the ‘changing nature of work’ interpretation of recent educational attainment, the theory developed here is built upon salient features of the modern workplace. Technical progress has not only increased the physical proximity of workers, but has also changed the types of tasks performed, the ways in which labor is managed, and the nature of the relationship a worker has with their coworkers. These changes have altered the types of skills that are valuable in the labor market. For instance, in compiling a list of these ‘new basic skills’, Murnane and Levy (1996) write:

A surprise in the list of New Basic Skills is the importance of soft skills. These skills are called “soft” because they are not easily measured on standardized tests. In reality, there is nothing soft about them. Today more than ever, good firms expect employees to raise performance continually by learning from each other through written and oral communication and by group problem solving.

The earlier part of this quote inspires the assumption that skills are ‘soft’, whereas the latter part inspires the assumption of skill spillovers. Extending this theme, Autor *et al.* (2003)

⁷Although, see Bidner (2008) and Cole *et al.* (1995) for models in which the underlying characteristics being signaled are endogenous.

and Levy and Murnane (2004) argue that computers can not easily perform non-routine tasks, and document the growing importance of tasks related to *expert thinking* (“solving problems for which there are no rule-based solutions”), and *complex communication* (“interacting with humans to acquire information, to explain it, or to persuade others of its implications for action”). A worker’s ability to perform such tasks is reasonably sensitive to the skills of their coworkers: expert thinking is often a collaborative process that involves input from multiple people and perspectives, whereas complex communication is an inherently inter-personal process.

Skill spillovers may also arise as the result of changes in the way a workforce is managed. Ichniowski and Shaw (2003) describe recent trends in Human Resource Management, such as the importance of “pay-for-performance plans like gain-sharing or profit-sharing, problem-solving teams, broadly defined jobs, cross-training for multiple jobs, employment security policies and labor-management communication procedures”. Many of these practices enhance the interconnectedness of employees. For example, Gant *et al.* (2002) provide evidence that worker productivity is improved following the introduction of innovative work practices because of stronger social capital developed between workers, and Drago and Garvey (1998) find that ‘helping effort’ is more readily expended by workers engaged in a large variety of tasks.

The model is presented in section 2, and analyzed in section 3. Various aspects relating to robustness are discussed in section 4. Empirical implications are outlined in section 5, and the theoretical connections to standard job market signaling models are presented in section 6. Conclusions are drawn in section 7. Proofs not in the text are contained in the appendix.

2 Model

2.1 Fundamentals

The economy is populated by a continuum of workers, indexed by $i \in [0, 1]$. Each worker has an ability $\theta_i \in \Theta \equiv [\theta_0, \theta_1]$, where $0 \leq \theta_0 < \theta_1$. The distribution of abilities is given by F , where F has a strictly positive density on Θ . The economy unfolds in three stages. First is the *Investment Stage* where workers choose a skill-enhancing investment, $x \geq 0$. Second is the *Matching Stage* where workers are matched into N -worker ‘firms’ on the basis of their first-stage investment. Finally, in the *Production and Payoff Stage*, workers produce output with a productivity that depends on their skill and the skill of their coworkers and get paid a wage, w . A worker that invests x and gets paid w simply obtains a payoff of $w - c \cdot x$, where $c > 0$ is the marginal cost of investment. The three stages are now described more fully in

turn.

2.1.1 Investment Stage

Having observed their ability, workers simultaneously and publicly choose an investment, $x \geq 0$. This investment produces a *skill* of:

$$s(x, \theta) = \theta \cdot g(x), \quad (1)$$

where g is a strictly increasing and concave function that satisfies $g(x) \geq 0$, $\lim_{x \rightarrow 0} g'(x) = \infty$, and $\lim_{x \rightarrow \infty} g'(x) = 0$. That is, workers have diminishing, but heterogeneous, marginal returns to investment. The feature that higher types have a higher marginal return arises naturally when ability is interpreted as an ‘aptitude for comprehension’ or a ‘capacity to learn’ as opposed to an endowment of knowledge, and is commonly employed in the empirical literature.⁸ For notational clarity, let $s_i \equiv s(x_i, \theta_i)$ denote the skill of worker i .

2.1.2 Matching Stage

The matching market determines the relationship between a workers’ investment and the average skill of the coworkers that they are matched with. Suppose workers invest according to the function $\sigma : \Theta \rightarrow \mathbb{R}_+$, so that $\sigma(\Theta)$ is the resulting set of investments. Following the literature (e.g Peters (2007b), Hoppe *et al.* (2009) and Peters and Siow (2002)) I assume that matching is positive assortative on investment. In this setting, this means that workers will end up being segregated on the investment dimension for any σ . That is, all workers expect to be matched to coworkers that have each made a common investment. This simplifies matters since we can formalize the workings of the matching market by means of a *matching function*, $m : \mathbb{R}_+ \rightarrow \sigma(\Theta)$. Here $m(x)$ is the common investment made by the coworkers that a worker rationally expects to be matched with as a consequence of making any given investment $x \in \mathbb{R}_+$. Note that the range of a matching function is required to be the set of realized investments; i.e. $m(x) \in \sigma(\Theta)$ for all $x \in \mathbb{R}_+$. This requirement disciplines the analysis by ruling out the ability of m to propose matches that do not exist (Peters (2007b)). Positive assortative matching requires the matching function to satisfy two additional intuitive conditions. The first is segregation:

$$m(x) = x \text{ for all } x \in \sigma(\Theta). \quad (2)$$

The second is that m is weakly increasing:

$$x \geq x' \Rightarrow m(x) \geq m(x') \text{ for all } x, x' \in \mathbb{R}_+. \quad (3)$$

⁸For example, multiplicative separability is implicitly assumed whenever a log wage equation is additively separable in schooling and ability. Evidence of investment-ability complementarity (i.e. a demonstration that returns to education are increasing in ability) is surveyed in Weiss (1995).

Since segregation implies that this automatically holds for equilibrium investments, condition (3) (along with the fact that $m(x) \in \sigma(\Theta)$) places a restriction on how m treats off-equilibrium investments.⁹ A matching function, m , is said to be *positive assortative relative to σ* if it satisfies (2) and (3).

2.1.3 Production and Payoff Stage

Production occurs within N -worker ‘firms’, but each worker produces a verifiable, individual output.¹⁰ This is produced by exerting effort, where i ’s marginal productivity is denoted y_i . Importantly, there are *no* effort externalities across workers; i.e. there is no team production, sabotage, etc.¹¹ I assume that effort is inelastically supplied at one unit to avoid making the analysis needlessly complicated, so that y_i is also i ’s total output. A worker’s skill contributes to their productivity, but critically, so too does the skill of their coworkers. Specifically, letting ω_k be the set of N workers belonging to firm k , I assume that worker i ’s productivity in firm k is determined by

$$y_i = y(s_i, \bar{s}_i) \equiv (1 - \phi) \cdot s_i + \phi \cdot \bar{s}_i, \quad (4)$$

where $\phi \in (0, 1]$ parameterizes the degree of skill spillovers, and

$$\bar{s}_i \equiv \frac{1}{N-1} \cdot \sum_{j \in \omega_k / \{i\}} s_j \quad (5)$$

is the average skill of i ’s coworkers.¹² Letting the vector of worker skills in firm k be denoted $\mathbf{s}_k \equiv \{s_i\}_{i \in \omega_k}$, the total output in firm k is given by $Y(\mathbf{s}_k) \equiv \sum_{i \in \omega_k} y_i$.

The linear specification adopted in (4) is convenient but inessential. Specifically, linearity ensures that spillovers are *socially neutral* in the sense that, for a fixed set of skills, a firm’s total output is independent of the spillover parameter regardless of the specific

⁹For instance, if x_0 is the smallest equilibrium investment then $m(x) = x_0$ for all $x < x_0$, and if x_1 is the largest equilibrium investment then $m(x) = x_1$ for all $x > x_1$. See section 6.2 for further discussion of how (3) and $m(x) \in \sigma(\Theta)$ discipline the treatment of off-equilibrium investments, as well as how this compares with standard signaling models.

¹⁰Here a ‘firm’ should be understood as simply the shell that houses the N workers, rather than a separate entity making strategic choices. This is for clarity only - I show that equilibrium outcomes are such that there do not exist any profitable opportunities for re-matching workers or offering them more complicated wage contracts (such as those in which wages are sensitive to coworker output) in section A.1.

¹¹Although, team production (whereby workers earn an equal share of the group’s total output) can be thought of as a special case in which $\phi = (N-1)/N$.

¹²If a match has less than N workers, say $N-n$ workers, then I assume that this is equivalent to having an N -worker match in which n workers have a skill of zero. The substance of this assumption is that all workers weakly prefer to have a coworker fill a slot to having the slot remain empty.

manner in which workers are sorted into firms:

$$Y(\mathbf{s}_k) = \sum_{i \in \omega_k} (1 - \phi) \cdot s_i + \phi \cdot \bar{s}_i = \sum_{i \in \omega_k} (1 - \phi) \cdot s_i + (N - 1) \cdot \phi \frac{1}{N - 1} \cdot s_i = \sum_{i \in \omega_k} s_i. \quad (6)$$

Thus, spillovers are parameterized in such a way that any effect of ϕ on output arrives purely via the effect on investment incentives. To show that linearity is inessential, I offer a generalization of (4) that allows for strict skill complementarity in section 4. I show that separating equilibria under the more general formulation are the same as that arising in the linear setting.

Finally, I assume that each worker is simply paid their output. As such, a worker that invests x and produces y obtains a payoff of $y - c \cdot x$. This extreme form of performance pay is assumed in order to confront the criticism of standard job market signaling models (based on Spence (1973)) as cleanly and transparently as possible.¹³ The underlying mechanism in the standard job market signaling model relies on such output-contingent wage contracts being infeasible (since this would allow firms to ‘contract around’ the ex-ante information asymmetry).¹⁴

2.2 Benchmark Investments

2.2.1 Efficient Investment

Given (6), the net social benefit for a type θ worker is

$$V(x, \theta) \equiv s(x, \theta) - c \cdot x. \quad (7)$$

The *efficient investment* for a type θ worker, denoted $\sigma^*(\theta)$, is the unique maximizer of V and is characterized by the first-order condition $s_x(\sigma^*(\theta), \theta) = c$.

When skills are strictly complementary (as in section 4.1), the way in which workers are matched has important additional implications for efficiency. Specifically, the efficient outcome arises when workers are segregated *and* when investments are optimal conditional on being in a segregated firm. A planner who faces the same information constraints as agents in the economy (i.e. only workers’ investments are observed) can achieve the efficient outcome only if the efficient investment is separating. When the investment is a pure signal (i.e. $s_x(x, \theta) = 0$ for all x and θ), as in Hoppe *et al.* (2009), the fact that the efficient investment is zero for all types means that a planner can never implement the efficient outcome.

¹³I add to the justification for this assumption in appendix section A.1, where I verify that in equilibrium there are no incentives for any set of N workers to form a group and divide their output among themselves in some other manner.

¹⁴Of course, in the standard model wages *coincide* with expected output in equilibrium. This distinction is discussed further in section 6.1, where I compare the model with the standard job market signaling model.

This motivates an interesting analysis of constrained efficiency whereby investments are wasteful but can facilitate the efficient matching pattern. However, when investment is productive, as it is here, the impossibility of implementing the efficient outcome disappears - indeed the efficient investment in this model always varies by type.¹⁵ As such, this notion of efficiency does not overlook any additional ‘match-facilitating’ social value of investment (as it necessarily must if investment were unproductive).¹⁶

2.2.2 Match-Insensitive Investment

The value of a worker’s output, y , depends upon their investment, x , both directly and indirectly. The direct channel is seen by writing the payoff to a worker of type θ that invests x and has coworkers of average skill \bar{s} as:

$$u(x, \bar{s}, \theta) \equiv (1 - \phi) \cdot s(x, \theta) + \phi \cdot \bar{s} - c \cdot x. \quad (8)$$

These preferences are depicted in (x, \bar{s}) space as U-shaped indifference curves in Figure 1(a). The U-shape comes from the fact that at relatively low investment levels the marginal return to investment is greater than the marginal cost, implying that a marginally higher investment raises the payoff - which requires a lower average coworker skill to retain indifference. The reverse is true for relatively high investment levels. In addition, the fact that the marginal return to investment is higher for higher types implies a single-crossing property (also indicated in the Figure 1(a)).¹⁷

A type θ worker’s *match insensitive investment*, denoted, $\bar{\sigma}(\theta)$, is the unique value of x that maximizes $u(x, \bar{s}, \theta)$, and is characterized by the first-order condition $(1 - \phi) \cdot s_x(\bar{\sigma}(\theta), \theta) = c$. This is the investment that would arise if there were no connection between a worker’s investment and the quality of the coworkers they end up matching with. A comparison of the first order conditions reveals that this represents an under-investment: $\bar{\sigma}(\theta) \leq \sigma^*(\theta)$ with strict inequality for all $\theta > 0$. Intuitively, when making the match insensitive investment workers do not take into account that their investment raises the productivity of their coworkers.

¹⁵This is apparent in the linear case since $s_{x\theta}, s_{xx} \neq 0$, and as will become clear, the exact same argument applies in section 4.1 where I allow for skill complementarities.

¹⁶This is of course not to say that observable investment has no ‘match-facilitating’ social value in the presence of skill complementarities - rather, that any such value is automatically achieved by adopting the efficient investments. For instance, relative to some set of benchmark investments, the additional welfare gained by adopting the efficient investments could be decomposed into a part that reflects changes in welfare within existing matches and a part that reflects changes in welfare due to the establishment of more efficient matches. Note however that the latter ‘match-facilitating’ component is zero when the original benchmark set of investments are separating (as when we analyze the efficiency of separating equilibria below).

¹⁷The U-shapedness and single-crossing properties are easily verified by consulting the explicit expression for an indifference curve: $I(\theta) = (1/\phi) \cdot [u + c \cdot x - (1 - \phi) \cdot s(x, \theta)]$.

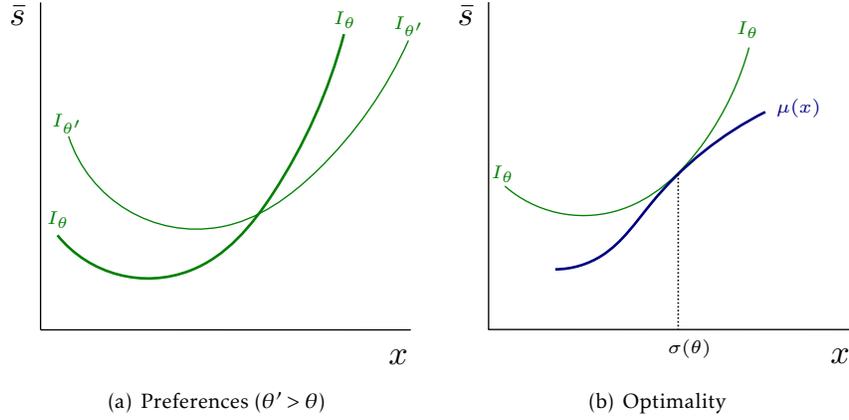


Figure 1: Preferences and Optimality

Workers will generally not find it optimal to make their match insensitive investment however, since their matching prospects are generally sensitive to their chosen investment level (as outlined in the matching stage). Still, the match-insensitive investment represents a natural benchmark and plays an important role in the analysis of separating equilibria below.

2.3 Equilibrium

When deciding on their investment level, workers anticipate a relationship between their investment and the average skill of coworkers that they will be matched with. Let this relationship be described by a *return function*, $\mu : \mathbb{R}_+ \rightarrow \mathbb{R}_+$, where $\mu(x)$ is interpreted as the average skill of coworkers that a worker expects to be matched with as a consequence of investing x . Using rational expectations, the return function is determined by the matching function, m , along with the investment function, σ . Specifically, since investing x leads a worker to match with coworkers that invest $m(x)$, a worker that invests x anticipates that they will be matched with coworkers that have an expected skill equal to the average skill among workers that invest $m(x)$. To this end, a return function μ is said to be *consistent with m and σ* if $\mu(x) = \mathbb{E}[s(m(x), \theta) \mid \sigma(\theta) = m(x)]$.¹⁸

Given μ , the payoff to a type θ worker when investing x is given by

$$v(x, \theta) \equiv u(x, \mu(x), \theta). \quad (9)$$

¹⁸To be more specific, let $\Omega(x) \subseteq \Theta$ be the set of types that invests $m(x)$. If $\Omega(x)$ is an interval with positive length, then $\mu(x) = \int_{\Omega(x)} s(m(x), \theta) \cdot f(\theta) \cdot d\theta / \int_{\Omega(x)} f(\theta) \cdot d\theta$. If $\Omega(x)$ consists of a single point, say θ_x , then $\mu(x) = s(m(x), \theta_x)$. While μ can be defined for other possible forms of Ω , the above two possibilities will suffice because any equilibrium investment function must be weakly increasing in type (see corollary 2).

An investment rule σ is *optimal with respect to μ* if $\sigma(\theta)$ maximizes $v(x, \theta)$ for each $\theta \in \Theta$. The notion of optimality is illustrated in Figure 1(b), where $\sigma(\theta)$ is the value of x that places type θ workers on their highest indifference curve subject to lying on μ .

An investment function σ is an *equilibrium* if there exists a matching function, m , and a return function, μ , such that: (i) σ is optimal relative to μ , (ii) m is positive assortative relative to σ , and (iii) μ is consistent with m and σ .

2.3.1 Pooling Equilibria

We begin by exploring pooling equilibria. In such equilibria, all workers investing the same amount, say x_p . This implies that workers are indistinguishable in the matching market. As a result, any investment leads a worker to be matched with coworkers that each invest x_p and have types that are randomly drawn from the population. That is, if $\sigma(\theta) = x_p$ for all θ then $\mu(x) = \mathbb{E}[\theta] \cdot g(x_p)$ for all x . Since workers find that their matching prospects are unaffected by their investment (i.e. $\mu(x)$ is a constant), each makes their match insensitive investment. But this investment differs across types, contradicting the supposition that workers make the same investment.

Proposition 1. *A pooling equilibrium does not exist.*

Proof. Suppose to the contrary that all agents invest x_p , so that $\sigma(\Theta) = \{x_p\}$. Since m has the property that $m(x) \in \sigma(\Theta)$, it must be that $m(x) = x_p$ for all x . Therefore the consistency of μ requires that $\mu(x) = \mu_p \equiv \int_{\theta_0}^{\theta_1} s(x_p, \theta) dF(\theta) = \mathbb{E}[\theta] \cdot g(x_p)$, which is a constant. Each worker must therefore optimally choose their match-insensitive investment; i.e. optimality requires that type θ workers invest $\tilde{\sigma}(\theta)$, where $\tilde{\sigma}(\theta)$ is implicitly defined by the first-order condition $u_x(\tilde{\sigma}(\theta), \mu_p, \theta) = 0$. The fact that $u_{x\theta} > 0$ (since $u_{x\theta} = (1 - \phi) \cdot s_{x\theta} > 0$), along with the fact that $u_{xx} < 0$ (since $u_{xx} = (1 - \phi) \cdot s_{xx} < 0$), implies that $\tilde{\sigma}(\theta)$ is strictly increasing. This contradicts the supposition that all workers invest the same amount. \square

This result highlights a key contrast between this model and a standard job market signaling model (where pooling equilibria generally exist). The underlying difference is that here positive assortative matching imposes concrete implications for making off-equilibrium investments, whereas workers can be dissuaded from deviating from the pooling outcome in standard signaling models by firms offering wages that reflect extreme beliefs off the equilibrium path. This, and related points, are elaborated upon in section 6.

Proposition 1 also highlights a contrast with related models in which agents are matched in a positive assortative manner on the basis of *unproductive* investments, again, since pooling equilibria exist in such models (e.g. Hoppe *et al.* (2009)). The reason for this is that

proposition 1 relies on the fact that the match insensitive investment varies by type.¹⁹ This, in turn, requires that investment complements ability in skill formation - i.e. $s_{x\theta} > 0$. This is impossible when investment is unproductive, even if skills are complements in the determination of productivity - i.e. when $y_{\bar{s}} > 0$. Specifically, all types share a match insensitive investment of zero when investment is unproductive and therefore a pooling equilibrium in which all agents invest zero always exists in such models.

Finally, it is important to note that the result does not rule out equilibria with partial pooling. The possibility of partial pooling equilibria and related issues, including a slightly stronger version of the above result, are discussed further in section A.2 of the appendix.

2.3.2 Separating Equilibria

We now explore separating equilibria - i.e. equilibria in which σ is one-to-one. Using the results from [Mailath and von Thadden \(2012\)](#) (Theorem 2.1), all separating equilibrium investment functions in this environment must be differentiable on the interior of Θ , denoted $\text{int}(\Theta)$, with $\sigma'(\theta) > 0$ for all $\theta \in \text{int}(\Theta)$. Such investment functions permit a strictly increasing and differentiable inverse function, $t : \sigma(\text{int}(\Theta)) \rightarrow \text{int}(\Theta)$.

If a type $\theta \in \text{int}(\Theta)$ worker invested $x \in \sigma(\text{int}(\Theta))$, then they would match with type $t(x)$ workers that each have invested x . Therefore the payoff to doing this for the type θ worker is

$$v(x, \theta) = y(s(x, \theta), s(x, t(x))) - c \cdot x. \quad (10)$$

Since any such x can always be chosen, a consequence of type θ optimizing is that $\sigma(\theta)$ maximizes this payoff. Since t is differentiable at $x = \sigma(\theta)$, so too is v . As such, the first order condition $v_x(x, \theta)|_{x=\sigma(\theta)} = 0$ must be satisfied. That is, for all $x \in \sigma(\text{int}(\Theta))$, the marginal return

$$v_x(x, \theta) = y_s \cdot s_x(x, \theta) + y_{\bar{s}} \cdot [s_x(x, t(x)) + s_{\theta}(x, t(x)) \cdot t'(x)] - c, \quad (11)$$

where y_s and $y_{\bar{s}}$ are evaluated at $s = s(x, \theta)$ and $\bar{s} = s(s, t(x))$, must equal zero at the equilibrium investment of type $\theta \in \text{int}(\Theta)$ workers. Using the identity $\theta = t(x)$ at $x = \sigma(\theta)$, and rearranging (11), indicates that t must satisfy the following differential equation:

$$t'(x) = T(t, x) \equiv \frac{c - [y_s + y_{\bar{s}}] \cdot s_x(x, t)}{y_{\bar{s}} \cdot s_{\theta}(x, t)}, \quad (12)$$

¹⁹The assumption that $\lim_{x \rightarrow 0} g'(x) = \infty$ is useful in this regard because it ensures that match insensitive investments are interior, not only permitting a characterization via the first-order condition, but also ruling out the possibility that a set of relatively low types all have match insensitive investments on the boundary at zero. However, the assumption is not crucial since the result only relies on two types having different match insensitive investments. As such, the result only requires that $g'(0) > (1/\theta_1) \cdot c/(1 - \phi)$ so that the highest type's match insensitive investment is interior.

where y_s and $y_{\bar{s}}$ are evaluated at $s = \bar{s} = s(s, t(x))$. Using the given functional forms for y and s , we have

$$T(t, x) = \frac{c - t \cdot g'(x)}{\phi \cdot g(x)}. \quad (13)$$

The inverse investment function can be derived from this differential equation once an initial condition is imposed. The initial condition is $t(x_0) = \theta_0$ where x_0 is interpreted as the limit of $\sigma(\theta)$ as θ approaches θ_0 . To derive x_0 , we begin by identifying the equilibrium behavior of those of the lowest type.

Lemma 1. *In any separating equilibrium, the investment of the lowest types satisfies:*

$$\sigma(\theta_0) \in \arg \max_x u(x, \mu_0, \theta_0), \quad (14)$$

where $\mu_0 \equiv s(\sigma(\theta_0), \theta_0)$. That is, workers of the lowest type make their match insensitive investment in any separating equilibrium.

Proof. In any separating equilibrium, type θ_0 workers will match with coworkers that each have a skill of $\mu_0 \equiv s(\sigma(\theta_0), \theta_0)$. The fact that θ_0 types optimize implies $u(\sigma(\theta_0), \mu_0, \theta_0) \geq u(x, \mu(x), \theta_0)$ for all $x \geq 0$. Since σ is increasing, $m(x) \in \sigma(\Theta)$, and m is non-decreasing, we have $\mu(x) \geq \mu_0$ for all $x \geq 0$. Since $u_{\bar{s}} > 0$, we have $u(x, \mu(x), \theta_0) \geq u(x, \mu_0, \theta_0)$. Together, we have that $u(\sigma(\theta_0), \mu_0, \theta_0) \geq u(x, \mu_0, \theta_0)$. That is, $\sigma(\theta_0)$ maximizes $u(x, \mu_0, \theta_0)$ as claimed. \square

This result is in contrast with standard signaling models, since the lowest types make the efficient (full-information) investment in such models. See section 6 for an elaboration on this point.

The initial condition need not coincide with the investment of the lowest types, since σ need not be continuous at θ_0 . Nevertheless, identifying the investment of the lowest types will pin down their equilibrium payoff and this in turn will pin down the initial condition. To see this, note that the equilibrium payoff to type θ workers if they invest x in equilibrium is equal to the net social benefit $V(x, \theta)$ as defined in (7): since workers are segregated we have $\mu(x) = s(x, \theta)$ (equivalently, $t(x) = \theta$) at $x = \sigma(\theta)$, so that type θ has an equilibrium payoff of $v(x, \theta) = y(s(x, \theta), s(x, \theta)) - c \cdot x = s(x, \theta) - c \cdot x = V(x, \theta)$. Note that V is strictly concave in x , reaching a global maximum at some positive investment, $\sigma^*(\theta)$. From lemma 1, the equilibrium payoff to type θ_0 workers is $V_0 \equiv V(\bar{\sigma}(\theta_0), \theta_0)$. Furthermore, since $\bar{\sigma}(\theta_0) \leq \sigma^*(\theta_0)$, it follows that $V_0 \leq V(\sigma^*(\theta_0), \theta_0)$ (strictly so when $\theta_0 > 0$). As such, the investment

$$\sigma^+(\theta_0) \equiv \min\{x \mid V(x', \theta_0) \leq V_0 \ \forall x' \geq x\} \quad (15)$$

is well-defined. Since V is strictly concave, an alternative characterization of $\sigma^+(\theta_0)$ is the maximum value of x for which $V(x, \theta_0) = V_0$. That is, the investment such that, if all type

θ_0 workers made it and they remained segregated, then they would get a payoff equal to their equilibrium payoff. The fact that $\bar{\sigma}(\theta_0) \leq \sigma^*(\theta)$, along with the concavity of V , implies that $\sigma^+(\theta_0) \geq \sigma^*(\theta_0)$. These points are illustrated in Figure 2 which depicts V as a function of x for θ_0 types, the match-insensitive investment $\bar{\sigma}(\theta_0)$, the efficient investment $\sigma^*(\theta_0)$, and $\sigma^+(\theta)$.

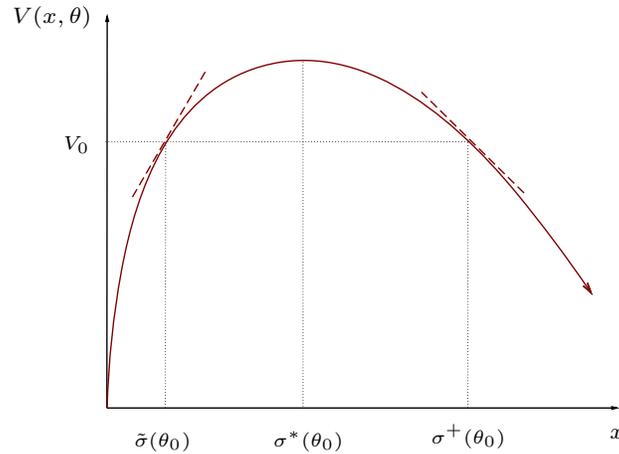


Figure 2: Identifying $\sigma^+(\theta_0)$ In Investment-Payoff Space

Figure 3 provides an alternative perspective on $\sigma^+(\theta_0)$ by considering the investment-skill space. If there were no relationship between investment and coworker skill, then μ would be flat. As such, the match insensitive investment places the worker on the highest indifference curve subject to lying on the flat μ curve. The equilibrium payoff for θ_0 types is that associated with the indifference curve that goes through the point $(\bar{\sigma}(\theta_0), \mu_0)$, where $\mu_0 \equiv s(\bar{\sigma}(\theta_0), \theta_0)$, as indicated. The efficient investment places the worker on the highest indifference curve subject to lying on the skill production function as indicated. Finally, $\sigma^+(\theta_0)$ is the investment above the efficient investment that lies on the intersection of the skill production function and the original indifference curve. The significance of $\sigma^+(\theta_0)$ is identified in the following result.

Lemma 2. *In any separating equilibrium, the initial condition is given by $x_0 = \sigma^+(\theta_0)$.*

Proof. Recall $x_0 \equiv \lim_{\theta \searrow \theta_0} \sigma(\theta)$. In any separating equilibrium, all types greater than θ_0 must prefer their equilibrium investment to masquerading as a θ_0 type. That is, for all $\theta > \theta_0$ we must have $V(\sigma(\theta), \theta) \geq y(s(\bar{\sigma}(\theta_0), \theta), s(\bar{\sigma}(\theta_0), \theta_0)) - c \cdot \bar{\sigma}(\theta_0))$. The right side is greater than or equal to V_0 . Therefore, we must have $V(\sigma(\theta), \theta) \geq V_0$ for all $\theta > \theta_0$. By taking θ to θ_0 , noting that V is continuous in θ at each $\theta \in \text{int}(\Theta)$, it must be the case that $V(x_0, \theta_0) \geq V_0$. It follows that $x_0 \leq \sigma^+(\theta_0)$ by definition of $\sigma^+(\theta_0)$. Similarly, type θ_0

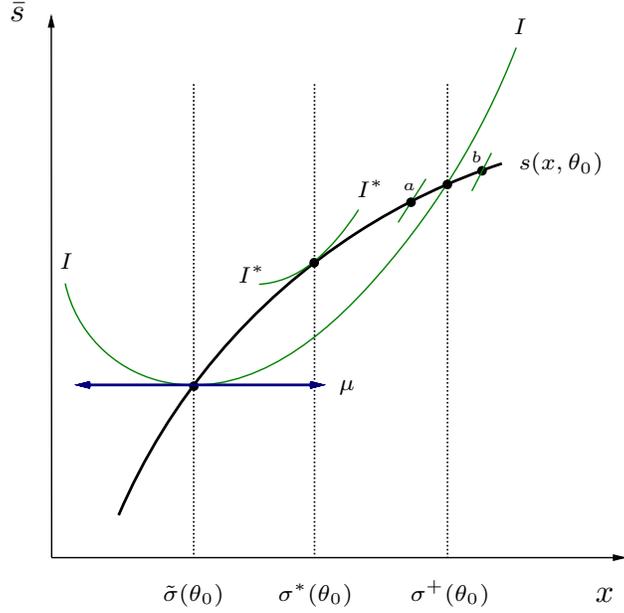


Figure 3: Identifying $\sigma^+(\theta_0)$ In Investment-Skill Space

workers must prefer their equilibrium investment to that of some higher type. For $\theta > \theta_0$, we must have $V_0 \geq y(s(\sigma(\theta), \theta_0), s(\sigma(\theta), \theta)) - c \cdot \sigma(\theta)$. The right side is no smaller than $V(\sigma(\theta), \theta_0)$, so it must be that $V_0 \geq V(\sigma(\theta), \theta_0)$ for all $\theta > \theta_0$. Since σ is increasing (and continuous on $\text{int}(\Theta)$), it follows that $x_0 \geq \sigma^+(\theta_0)$ by definition of $\sigma^+(\theta_0)$. Since we have shown $\sigma^+(\theta_0) \leq x_0 \leq \sigma^+(\theta_0)$, we conclude that $x_0 = \sigma^+(\theta_0)$. \square

The intuition behind this can be seen using figure 3. If x_0 were less than $\sigma^+(\theta_0)$, then there would be feasible alternatives arbitrarily close to a point such as “a”, since types marginally above θ_0 , when investing x , would have a skill marginally above $s(x, \theta_0)$. This can not be an equilibrium since θ_0 types would prefer such an alternative. Similarly, if x_0 were greater than $\sigma^+(\theta_0)$, then types arbitrarily above θ_0 would be choosing points arbitrarily close to a point such as “b”. This can not be an equilibrium either, since such types, having indifference curves arbitrarily close to those of θ_0 types, would prefer to mimic θ_0 types.

It is worth noting that this result has implications for the continuity of the equilibrium investment function. Specifically, when $\theta_0 > 0$ we have $\sigma^+(\theta_0) > \tilde{\sigma}(\theta_0)$, implying that the investment function is *discontinuous* at θ_0 ; it jumps up. The investment function is continuous only in the case of $\sigma_0 = 0$.

Bringing together the above results, we have that if t is an inverse investment function

it must solve the initial values problem:

$$\langle t'(x) = T(t, x), t(x_0) = \theta_0 \rangle, \quad (16)$$

where $x_0 = \sigma^+(\theta_0)$. In the appendix I show that this problem has a unique solution given by

$$t(x) = \left[\theta_0 \cdot g(x_0)^{\frac{1}{\phi}} + \frac{c}{\phi} \int_{x_0}^x g(z)^{\frac{1-\phi}{\phi}} dz \right] \cdot g(x)^{-\frac{1}{\phi}}, \quad (17)$$

where $x_0 = \sigma^+(\theta_0)$, and that this solution is strictly increasing and unbounded on $(x_0, \infty]$. It follows that t has a strictly increasing inverse defined on $\text{int}(\Theta)$. If a separating equilibrium exists, then it coincides with this inverse on $\text{int}(\Theta)$. Once it is shown that the investment of the highest type is $\sigma(\theta_1) = t^{-1}(\theta_1)$, and that no type has a profitable deviation, we have the following.

Proposition 2. *A unique separating equilibrium exists. In this equilibrium, the investment function is $\sigma(\theta_0) = \bar{\sigma}(\theta_0)$ and $\sigma(\theta) = t^{-1}(\theta)$ for $\theta > \theta_0$, where t is given by (17).*

The unique separating equilibrium is illustrated from two perspectives in Figure 4. Figure 4(a) illustrates the jump discontinuity at θ_0 . The implications for the equilibrium matching return function, μ , are depicted in Figure 4(b). To see how this works, note that the discontinuity at θ_0 implies that the set of equilibrium investments, $\sigma(\Theta)$, is of the form $\{\bar{x}_0\} \cup (x_0, x_1]$ where $\bar{x}_0 \in [0, x_0]$. First consider equilibrium investments. Since there is segregation, each worker must find that $\mu(x) = s(x, \theta)$ when evaluated at their equilibrium investment. Thus, when workers choose the investment that places them on the highest indifference curve subject to lying on μ , they find that such a point lies on their skill production function. The value of μ at off-equilibrium investments is determined by positive assortative matching - specifically, we have $m(x) = x_1$ for $x > x_1$ and $m(x) = \bar{x}_0$ for all $x \in [0, x_0]$.²⁰ As such, μ is flat on the intervals $[0, x_0]$ and $(x_1, \infty]$, equaling $s(\sigma(\theta_0), \theta_0)$ on the former and $s(\sigma(\theta_1), \theta_1)$ on the latter.

3 Analysis

3.1 Efficiency

A higher investment not only raises a worker's productivity, but also the productivity of their coworkers. In this sense, investment entails a positive externality. If workers' exposure to spillovers were insensitive to their investment, as in models of global spillovers

²⁰The fact that $m(x) = x_1$ for $x > x_1$ is straightforward, but perhaps $m(x) = \bar{x}_0$ for all $x \in [0, x_0]$ is less so. To see this, suppose that this was not the case. Then for some $x \in [0, x_0]$ we have $m(x) = \sigma(\theta)$ for some $\theta > \theta_0$. But then for any $\theta' \in (\theta_0, \theta)$ we have (i) $\sigma(\theta') > x$, and (ii) $m(\sigma(\theta')) = \sigma(\theta') < \sigma(\theta) = m(x)$. This contradicts m being non-decreasing.

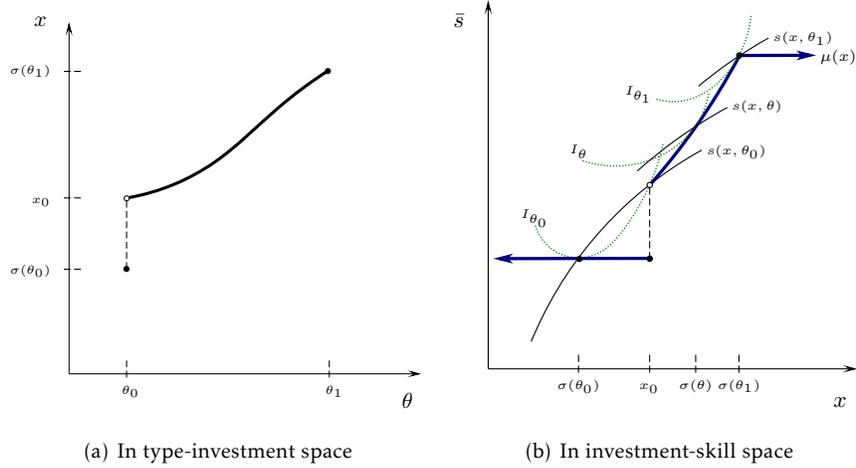


Figure 4: Two Perspectives on Equilibrium

such as Lucas (1988), then this naturally leads to underinvestment. Arguments along these lines support the public subsidization of education. However, when investments allow workers to join higher skilled firms, this implication no longer holds (for all but the lowest type workers).

Proposition 3. *Almost all workers over-invest in equilibrium: $\sigma(\theta) > \sigma^*(\theta)$ for $\theta \in \Theta/\{\theta_0\}$.*

Workers of the lowest type under-invest since they make their match-insensitive investment.²¹ Since workers of the lowest type exist with zero measure, we say that over-investment is *pervasive* in equilibrium. This result highlights the importance of considering ex-ante heterogeneity among workers; if there was only one type, then all workers would be of the lowest type and under-investment would be pervasive.

The fact that workers over-invest can be seen in figure 4(b). Each type (except the lowest) finds that if all workers of that type were to cut their investment, then the resulting (x, \bar{s}) pair (which will lie on their $s(x, \theta)$ curve) would lie on a higher indifference curve than that experienced in equilibrium. However, workers are dissuaded from cutting their investment since they realize that this would also involve them matching with coworkers of a lower type (in addition to such workers having a similarly lower investment).

The over-investment result does not rely on the linearity or separability of y . To see this, note that if type θ workers invest x in any separating equilibrium, then their payoff will be $\tilde{V}(x, \theta) \equiv y(s(x, \theta), s(x, \theta)) - c \cdot x$. Retaining the assumption that worker skills are com-

²¹If $\theta_0 = 0$, then workers of the lowest type happen to invest efficiently as a consequence of their match-insensitive, equilibrium, and efficient investments all equalling zero. Nevertheless, the point remains that over-investment is not being driven by the over-investment of the lowest types.

plements (so that segregation remains efficient), the type θ workers' efficient investment maximizes $\tilde{V}(x, \theta)$. The marginal social return is

$$\tilde{V}_x(x, \theta) = y_s \cdot s_x(x, \theta) + y_{\bar{s}} \cdot s_x(x, \theta) - c, \quad (18)$$

where y_s and $y_{\bar{s}}$ are evaluated at $s = \bar{s} = s(x, \theta)$. In equilibrium, workers perceive a payoff function of $\bar{v}(x, \theta) \equiv y(s(x, \theta), \mu(x)) - c \cdot x$. The marginal private return is therefore

$$\bar{v}_x(x, \theta) = \{y_s \cdot s_x(x, \theta) - c\} + \{y_{\bar{s}} \cdot \mu'(x)\}, \quad (19)$$

where y_s and $y_{\bar{s}}$ are evaluated at $s = s(x, \theta)$ and $\bar{s} = \mu(x)$. The first bracketed term is the 'private' component and the second is the 'credential' component. The private component alone clearly produces too little incentive to invest, since the return suffers a shortfall of $y_{\bar{s}} \cdot s_x(x, \theta)$. This arises because the private return does not take into account the external benefit on coworkers. To explore the extent to which the credential component provides added incentive to invest, note that the fact that $\mu(x) = s(x, t(x))$ at each equilibrium investment implies that

$$\mu'(x) = s_x(x, t(x)) + s_\theta(x, t(x)) \cdot t'(x). \quad (20)$$

Evaluating this at the equilibrium investment of a type θ worker (so that $t(x) = \theta$) and using this in (19), we see that the credential component contributes the required shortfall in the net marginal return, $y_{\bar{s}} \cdot s_x(x, \theta)$. However, it contributes even more than this when $y_{\bar{s}} \cdot s_\theta(x, t(x)) \cdot t'(x)$ is positive. But, this term is positive whenever (i) there are skill spillovers, (ii) skill depends on ability, and (iii) higher types make higher investments. Thus, the model not only captures the feature that workers find credentials valuable, but also verifies that this leads to over-investment.

The fact that agents invest efficiently in models of pre-marital investment with complete information (such as Peters and Siow (2002) and Cole *et al.* (2001)) can be seen partly as a consequence of the fact that 'skill' in those models depends only on investment, and not on type (so that $s_\theta = 0$).

3.2 The Effect of Spillovers

Technological change not only makes skills more productive, but also shapes the extent of spillovers. For instance, if a worker's productivity is given by $y = \varphi_1 \cdot s + \varphi_2 \cdot \bar{s}$, where $\varphi_1 > 0$ and $\varphi_2 \geq 0$ are general productivity parameters, then technological change will likely increase both φ_1 and φ_2 . We can always decompose productivity as a general productivity term, Φ , and a spillover term, ϕ , so that

$$y = \Phi \cdot [(1 - \phi) \cdot s + \phi \cdot \bar{s}]$$

by letting $\Phi \equiv \varphi_1 + \varphi_2$ and $\phi \equiv \varphi_2/[\varphi_1 + \varphi_2]$. It is unsurprising if investment is increased when the general productivity term increases, which is why the analysis has focused on the spillover component. In terms of the underlying parameters, the spillover component increases when φ_2/φ_1 increases. The following result indicates how the spillover component of technological change influences investment incentives.

Proposition 4. *An increase in ϕ raises the investment of almost all workers (i.e. those with $\theta \in \Theta/\{\theta_0\}$).*

Intuitively, greater spillovers make workers more concerned about finding skilled coworkers to match with. As such, workers are more willing to invest in order to achieve this. There is a secondary effect when $\theta_0 > 0$: since workers of the lowest type make their match-insensitive investment, their equilibrium investment is decreasing in spillovers. This further lowers their equilibrium payoff, and therefore *increases* $\sigma^+(\theta_0)$ (and therefore x_0). Thus, for $\theta > \theta_0$, higher spillovers cause the equilibrium investment function to rise more steeply *and* start from a higher base.

Recall that spillovers are ‘neutral’ in equilibrium in the sense that if we changed ϕ while holding investment fixed, no worker’s productivity would change. Given this, the result points to a mechanism through which spillovers raise productivity without making existing skills more productive *per se*.

Propositions 3 and 4 together indicate that, for types $\theta \in \Theta/\{\theta_0\}$, greater spillovers leads to higher investment (and therefore productivity and income), but lower welfare (since ϕ does not affect payoffs directly and exacerbates over-investment). We therefore have the following implication which points to a *negative* correlation between income and welfare.

Corollary 1. *Almost all workers (i.e. $\theta \in \Theta/\{\theta_0\}$) find that an increase in the spillover parameter lowers their welfare despite raising their income.*

Workers of the lowest type also find that their welfare decreases in ϕ , strictly so if $\theta_0 > 0$, however this drop in welfare is accompanied by a drop in income.

3.3 An Illustration

By imposing a functional form on the skill production function, one can derive closed-form solutions for equilibrium outcomes. This not only allows one to illustrate the above results, but also to produce some further results of interest. To this end, let $\theta_0 = 0$ and let the skill production function be given by $s(x, \theta) = \theta \cdot x^\eta$, where $\eta \in [0, 1)$ is the elasticity of skill with respect to investment. It is straightforward to verify (details in section C in the appendix) that the equilibrium outcomes – investments, incomes (or productivities), and

payoffs – are given by

$$\sigma(\theta) = z^{\frac{1}{1-\eta}} \cdot \theta^{\frac{1}{1-\eta}}, \quad y(\theta) = z^{\frac{\eta}{1-\eta}} \cdot \theta^{\frac{1}{1-\eta}}, \quad \text{and} \quad u(\theta) = [z^{\frac{\eta}{1-\eta}} - c \cdot z^{\frac{1}{1-\eta}}] \cdot \theta^{\frac{1}{1-\eta}}, \quad (21)$$

where $z \equiv [\phi + (1 - \phi) \cdot \eta]/c$.

3.3.1 Distributional Consequences of Spillovers

Since investment is productive, spillovers will have an impact on the income (productivity) distribution. A consequence of proposition 4 is that higher spillovers produce income distributions that first-order stochastically dominate those produced by lower spillovers. A more interesting question is that of how spillovers affect *inequality*. One measure of income inequality is the difference between the incomes at the p th and p' th percentiles, where $p' < p$. The income at the p th percentile is the income of type $\theta_p \equiv F^{-1}(p)$ workers, so the inequality measure, denoted $\Delta(p, p')$, is just the difference in income of types θ_p and $\theta_{p'}$:

$$\Delta_y(p, p') \equiv z^{\frac{1}{1-\eta}} \cdot [\theta_p^{\frac{1}{1-\eta}} - \theta_{p'}^{\frac{1}{1-\eta}}].$$

The analogous measure for payoff inequality is

$$\Delta_u(p, p') \equiv \left[z^{\frac{\eta}{1-\eta}} - c \cdot z^{\frac{1}{1-\eta}} \right] \cdot [\theta_p^{\frac{1}{1-\eta}} - \theta_{p'}^{\frac{1}{1-\eta}}].$$

Proposition 5. *An increase in spillovers increases income inequality but reduces payoff inequality: $\frac{d}{d\phi} \Delta_y(p, p') > 0$ and $\frac{d}{d\phi} \Delta_u(p, p') < 0$ for all p and $p' < p$.*

The result highlights the point that increases in observed income inequality need not indicate increases in the inequality of underlying payoffs. If greater income inequality is fueled by more extensive over-investment, then underlying payoffs in fact become more equal.

3.3.2 Comparisons to Benchmarks

Producing explicit solutions also allows us to compare equilibrium outcomes to a benchmark case. Specifically, I consider compare equilibrium outcomes to those that would arise if workers were exogenously segregated by type²², as well as efficient outcomes. It straight-

²²An alternative benchmark would be that of random matching. The additive separability of y implies that the investments are the same in either case, and so the average payoff would be the same in both benchmarks (although the distribution of payoffs will differ, with low types faring better under random matching and vice versa). The ‘exogenous segregation’ benchmark is useful because the simplicity of the calculation of the benchmark investment extends effortlessly to the case where y is not additively separable, as in Section 4.1 below. In fact, the benchmark investments derived here are exactly the same for the class of production functions that are considered in that section.

forward to verify (details in section C in the appendix) that the exogenous-segregation outcomes are given by

$$\tilde{\sigma}(\theta) = \tilde{z}^{\frac{1}{1-\eta}} \cdot \theta^{\frac{1}{1-\eta}}, \quad \tilde{y}(\theta) = \tilde{z}^{\frac{\eta}{1-\eta}} \cdot \theta^{\frac{1}{1-\eta}}, \quad \text{and} \quad \tilde{u}(\theta) = [\tilde{z}^{\frac{\eta}{1-\eta}} - c \cdot \tilde{z}^{\frac{1}{1-\eta}}] \cdot \theta^{\frac{1}{1-\eta}}, \quad (22)$$

where $\tilde{z} \equiv (1 - \phi) \cdot \eta/c$, and the efficient outcomes are given by

$$\sigma^*(\theta) = z^*{}^{\frac{1}{1-\eta}} \cdot \theta^{\frac{1}{1-\eta}}, \quad y^*(\theta) = z^*{}^{\frac{\eta}{1-\eta}} \cdot \theta^{\frac{1}{1-\eta}}, \quad \text{and} \quad u^*(\theta) = [z^*{}^{\frac{\eta}{1-\eta}} - c \cdot z^*{}^{\frac{1}{1-\eta}}] \cdot \theta^{\frac{1}{1-\eta}}, \quad (23)$$

where $z^* \equiv \eta/c$.

It is apparent that spillovers have different effects in equilibrium compared to the benchmark: we see from z being increasing in ϕ that equilibrium investment and income are *increasing* in spillovers, whereas the reverse is true in the benchmark case since \tilde{z} is decreasing in spillovers (and the efficient outcomes are independent of spillovers). In contrast to proposition 5, greater spillovers decrease inequality of both income and payoffs (see proof of proposition 5).

The ‘competition for coworkers’ in this economy drives inefficient over-investment. However, outcomes are also inefficient in the benchmark case where this competition is absent, since all workers *under*-invest. This can be seen clearly by noting that $\tilde{z} < z^* < z$. To ascertain the extent to which one source of inefficiency dominates the other, we can compare equilibrium payoffs to the benchmark payoffs. Hopkins (2011) shows how workers are always better off in the benchmark case when they are to be paired with exogenously heterogeneous firms. The key difference here is that the ‘quality’ of firms lies in the skill of the coworkers it houses and therefore endogenously responds to changes in investment incentives. As a result, there is no unambiguous welfare ranking: competition for coworkers can either increase or decrease welfare relative to the case with exogenous segregation, depending on the sensitivity of skill to investment. Intuitively, when skill is highly sensitive to investment, over-investment is less problematic than under-investment (and vice versa).

Proposition 6. *Equilibrium payoffs are higher than in the benchmark case when the elasticity of skill with respect to investment is sufficiently large. Specifically, $u(\theta) < \tilde{u}(\theta)$ for $\eta \in [0, 1/2)$, $u(\theta) = \tilde{u}(\theta)$ for $\eta = 1/2$, and $u(\theta) > \tilde{u}(\theta)$ for $\eta \in (1/2, 1)$.*

This result allows us to compare the average welfare in equilibrium to the average welfare that arises under a hypothetical policy whereby matching on credentials is banned. In the latter case matching is random, and we know that the average welfare under random matching is the same as the average welfare in the benchmark case (see footnote 22). Proposition 6 indicates that active credentialism is socially beneficial relative to the complete absence of credentials when the sensitivity of skill to investment is relatively high. The assumed additive separability is useful here because we know that this conclusion

has nothing to do with the usual argument of credentials facilitating superior matching patterns.

4 Robustness

4.1 Skill Production Function Generalization

One may be concerned that the linear specification of the skill production function is responsible for many of the above results. To explore this, consider a generalization of the skill production function (4) to allow for skill complementarities. Specifically, suppose that y belongs to the constant elasticity of substitution (CES) class; for $\rho \in (-\infty, 1]$, the output of worker i in firm k is

$$y_i = y(s_i, \bar{s}_i) = \left[(1 - \phi) \cdot s_i^\rho + \phi \cdot \bar{s}_i^\rho \right]^{\frac{1}{\rho}}, \quad (24)$$

where \bar{s}_i is a generalized mean coworker skill:

$$\bar{s}_i \equiv \left[\frac{1}{N-1} \cdot \sum_{j \in \omega_k / \{i\}} s_j^\rho \right]^{\frac{1}{\rho}}. \quad (25)$$

The linear case considered above corresponds to $\rho = 1$, and this specification also covers the Cobb-Douglas and Leontief cases ($\rho \rightarrow 0$ and $\rho \rightarrow -\infty$, respectively). The CES class is useful because it retains the property that $y(s, s) = s$ so that any effect of changing parameters (ϕ or ρ) on outcomes arises purely through investment incentives.

To analyze how separating equilibria under this generalization compare with that derived above, suppose that agents invest according to some strictly increasing function, σ , that is differentiable on $\text{int}(\Theta)$.²³ In deriving necessary conditions, equations (10)-(12) (and related discussion) continue to hold. Using the CES functional form in the general differential equation (12), noting the fact that all CES functions have the property that $y_s(s, s) = 1 - \phi$ and $y_{\bar{s}}(s, s) = \phi$ for any s , we see that we continue to arrive at (13).

The generalization does not change the fact that θ_0 types make a match insensitive investment. In the linear case we could speak of *the* match insensitive investment since the investment that maximized $u(x, \bar{s}, \theta)$ was independent of \bar{s} , but this is no longer true under this generalization. Nevertheless, lemma 1 still applies, and as such, $\sigma(\theta_0)$ satisfies the first-order condition $y_s \cdot s_x(\sigma(\theta_0), \theta_0) = c$, where y_s is evaluated at $s = \bar{s} = s(\sigma(\theta_0), \theta_0)$. Again, the fact that $y_s(s, s) = 1 - \phi$ for any s implies that $\sigma(\theta_0)$ is independent of ρ and therefore

²³In the case where $\theta_0 = 0$, any separating equilibrium strategy must be differentiable on $\Theta / \{\theta_0\}$ (with a non-zero derivative) by Theorem 2.2a of Mailath and von Thadden (2012). The fact that the lowest types invest zero, along with separation, implies that the function must also be strictly increasing.

coincides with that used in the linear case. Furthermore, the fact that $y(s, s) = s$ implies that $V(x, \theta) = s(x, \theta) - c \cdot x$ remains unchanged, and therefore $\sigma^+(\theta)$ is also unchanged. The result in lemma 2 continues to hold, and therefore the initial condition, x_0 , remains unchanged. The investment of the highest type is unchanged from the linear case (see the proof of proposition 3), as is the fact that no worker has a profitable deviation from the proposed equilibrium strategy (see the proof of proposition 11).

These observations lead to the conclusion that, under the CES generalization, there exists a unique strictly increasing and differentiable (on $\text{int}(\Theta)$) separating equilibrium investment function. More interestingly, *the equilibrium investment function is independent of the substitution parameter ρ , and therefore coincides with the separating investment function arising in the linear case derived above.*

This result points to a strong sense in which the linearity assumption is not driving any of the separating equilibrium results; separating equilibria are not just ‘qualitatively unaffected’ by this generalization, but rather, are *identical* to that derived in the special case. As noted in section 3.1, the over-investment result holds more generally. In the CES context, we learn that skill complementarities as captured by ρ are unimportant relative to the spillover parameter ϕ .

A major convenience of focusing on the linear case is that it generates an additive separability between own- and coworker skill. This allows for a simpler analysis of the matching market since matching outcomes can be summarized by a single variable - expected coworker skill. A more general formulation would have the matching market allocate a distribution of coworker skills for each possible investment. This more elaborate set-up would add little if anything, since (i) the essential argument for why pooling equilibria fail to exist goes through unchanged, and (ii) when analyzing separating equilibria, such distributions would be degenerate – and therefore using the expected coworker skill will produce identical conclusions.

4.2 The Observability of Skill

The criticism of job market signaling models addressed here is based on the claim that a worker’s productivity is ex-post observable (i.e. after the match is formed) to the firm, even if it is not ex-ante observable. In this model, the key is that *skill* is ex-ante unobservable. To what extent are the results sensitive to the ex-post observability of skill?

To begin to address this, note that each worker’s skill is revealed to their group ex-post under the plausible assumption that the output of each worker is observed by their coworkers. But, this alone has no effect on the results, since the static nature of the model means that there is no means by which this information can be utilized to influence ex-ante incentives (by punishing masquerading workers, say). Therefore, if the ex-post observability

of skill is to have any effect on the results, a dynamic extension of the model must be used.

To this end, consider a very simple dynamic extension of the model in which matching occurs in two stages. Potential firms are formed in the first stage according to positive assortative matching on investment as before. These are only *potential* firms because of two reasons - i) a small proportion of potential firms are exogenously dissolved at the end of the first stage, and ii) workers can decide to reject potential coworkers. A worker is rejected from the firm if all of their coworkers decide to reject them, in which the rejected worker incurs a utility cost of $k > 0$ and all non-rejected workers in the firm incurs a utility cost of $\kappa > 0$.²⁴ In the second stage, the workers from dissolved firms plus all rejected workers are once again matched positive assortative on investment. This will mean that some workers from dissolved firms will fill the positions left by rejected workers. All other workers remain with their first-stage matches. A worker's coworkers are those that they are matched with at the end of the second stage.

There are weaker incentives to over-invest in this environment. Masquerading as a higher type may be ineffective since it will lead to rejection by such types when κ is sufficiently low. This rejection will be prohibitively costly to the masquerading worker when k is sufficiently high, and therefore the worker will be dissuaded from masquerading. While this is true, it turns out that separating equilibrium outcomes are completely unaffected by these dynamic considerations – even when κ is very small and k is very large. A simple argument establishes that the separating equilibrium outcomes in the static setting analyzed above will continue to arise in this dynamic setting: incentives to raise investment are weakly lower than in the static case (because of the possibility of rejection) and incentives to lower investment are unaffected (since such a deviation would be welcomed by the resulting coworkers, implying no rejection occurs). But, more than this, *any separating equilibrium in this dynamic environment will coincide with that derived in the static environment of the main analysis.*

To see this, suppose σ is a separating equilibrium strategy with associated inverse, t . Workers will not reject a co-worker if their experienced reduction in productivity does not warrant the switching cost, κ . Given that workers are segregated, this is equivalent to the coworker's type not being too low. Specifically, a type θ worker will not be rejected when investing $x \in \sigma(\Theta)$ if:²⁵

$$s(x, \theta) \geq s(x, t(x)) - \frac{(N-1)\kappa}{\phi}. \quad (26)$$

²⁴All the results in this section go through if there is discounting rather than explicit utility costs.

²⁵The following comes from finding the critical skill that would make workers investing $x \in \sigma(\Theta)$ indifferent between accepting and rejecting a deviator. Let this be denoted \hat{s} and note that it satisfies $\phi \cdot s(x, t(x)) + (1 - \phi) \cdot \left[\frac{N-2}{N-1} \cdot s(x, t(x)) + \frac{1}{N-1} \cdot \hat{s} \right] = s(x, t(x)) - \kappa$. Rearranging for \hat{s} gives the result.

In other words, θ cannot be too far below $t(x)$. In addition, no worker has to reject a coworker in equilibrium, since (26) is satisfied at the equilibrium investment of type θ for any $\kappa > 0$. Given these observations, the payoff to investing $x \in \sigma(\Theta)$ must be modified to

$$v^*(x, \theta) = \begin{cases} v(x, \theta) & \text{if (26) is satisfied} \\ v(x, \theta) - k & \text{otherwise,} \end{cases} \quad (27)$$

where $v(x, \theta)$ is given by (10). But since (26) is satisfied at the equilibrium investment of type θ for any $\kappa > 0$, the necessary first-order condition remains as stated in (11). That is, it still must be the case that type θ workers prefer their equilibrium investment to that made by marginally higher and lower types, but these types will not reject the worker and therefore the incentive compatibility condition remains unchanged. The sufficiency of this condition is also maintained since $v^*(x, \theta) \leq v(x, \theta)$ and $v^*(x, \theta) = v(x, \theta)$ at $x = \sigma(\theta)$ (i.e. if $x = \sigma(\theta)$ maximizes $v(x, \theta)$ then it will maximize $v^*(x, \theta)$). It follows that any separating equilibrium in this dynamic setting must coincide with the unique separating equilibrium in the static setting. The central conclusion from analyzing this extension is that *the results of the main analysis go through unchanged when workers are able to act upon the ex-post observability of coworker skill by rejecting undesirable coworkers at an arbitrarily small cost.*

Another way that workers could potentially act upon the ex-post observability of coworker skill is via side payments. Such payments play no role in the static analysis since such payments are not forthcoming until the end of the game and therefore can not be enforced. Side payments may play a role in the dynamic setting however, as the possibility of leaving the match creates a bargaining problem. The resolution of this bargaining will generally involve transfers that influence ex-ante investment incentives. However, as long as there are search frictions, a worker's total income (post-bargaining) will be sensitive to the skill of their co-workers, and the qualitative results of the static section go through.

While the above broadly indicates that it does not matter if skill is ex-post observable within the firm, it matters somewhat more whether skill is ex-post observable to *the market*. If skill were ex-post observable to the market, and the market exhibits few frictions, then it seems more reasonable to take the approach of Peters and Siow (2002), Peters (2007b), or Cole *et al.* (2001), and assume that matching occurs on basis of skill. However, the information requirements that would allow the market to infer a worker's skill are highly demanding. Even if a worker's output history could be perfectly communicated to, and costlessly verified by, the market, the impact of an array of individual-level and firm-level factors, unobserved by the market, would weaken the connection between a worker's output and their skill. If investment choices are less subject to such unobserved factors, then investment becomes a stronger signal of skill and therefore the dominant dimension along which matches are formed. In any case, there are a variety of frictions associated with

changing jobs and replacing workers, and therefore the impact of a worker's initial placement can be long-lasting (e.g. see Oyer (2006)).

5 Empirical Aspects

There are many existing explanations for the observed positive relationship between education and earnings. For this reason, it is important to be clear about the empirical implications of this model, and how these differ from existing explanations.

In the model, all investment earns an associated credential. But in principal, an investment could be made without receiving the associated credential (e.g. taking a university course without registering). A central prediction of the model is that, for any given actual investment level, there is a positive *causal* relationship between the possession of the credential and productivity. This is because credentials determine which group of coworkers a worker belongs to, thereby influencing the worker's productivity via spillovers. This prediction is not shared by standard human capital models where productivity depends only a worker's actual investment.²⁶ The relationship is also not present in signaling models; although credentials influence beliefs and therefore wages, productivity is unaffected by credentials (conditional on investment). This suggests an approach that can, in principal, allow us to discriminate between theories: take two groups of workers that have made the same investment, and endow one with credentials.²⁷ An examination of the resulting differences in productivity between the groups will provide evidence for or against the theory relative to standard human capital and signaling theories.²⁸

This empirical prediction is also shared by models i) in which investment is unproductive, as in Hoppe *et al.* (2009), and ii) in which firms are *ex-ante* heterogeneous, as in Hopkins (2011). These explanations can potentially be disentangled by using worker-firm matched data, as this allows for the estimation of firm fixed effects. The issue of whether education is productive is illuminated by examining the extent to which educational at-

²⁶Similarly, the relationship is not present in assignment models with observed skill (e.g. such as Kremer (1993), Sattinger (1993), Peters and Siow (2002)) for similar reasons.

²⁷This is the type of variation used in Tyler *et al.* (2000), who exploit the fact that U.S. states have different test score requirements in order to be awarded the GED credential. Their key finding - that the GED credential causes higher wages - is supportive of standard signaling models, but is also consistent with the mechanism proposed here. Data on productivity were not available, so it is difficult to distinguish the two interpretations of the data. However, it is interesting to note that the credential effects do not appear until five years after the test was taken. This weighs against the signaling interpretation to the extent that one finds it implausible that discrepancies between productivity and wages can last five years.

²⁸An examination of the resulting differences in *wages* will be insufficient since it will not allow for discrimination between this model and the standard signaling model. It will be sufficient to discriminate between these models and standard human capital models however.

tainment remains significant after controlling for firm fixed effects. The issue of whether firms are ex-ante heterogeneous is illuminated by examining the extent to which coworker characteristics can explain the firm fixed effects (as opposed to characteristics such as capital per worker).

Observing a panel of matched worker-firm data allows us to go one step further and identify worker fixed effects. In this setting, [Abowd *et al.* \(1999\)](#) demonstrate how average worker effects are much more important than average firm effects in explaining industry effects. Explicitly analyzing the effect of coworkers, [Lopes de Melo \(2008\)](#) reports that workers' fixed effects are not correlated with firm fixed effects, but are positively correlated with the average fixed effect of their coworkers. This type of evidence is supportive of the notion that it is the quality of employees that really sets firms apart. The latter result in [Lopes de Melo \(2008\)](#) provides valuable evidence of the type of labor market sorting stressed in this model.

6 Theoretical Connections to a Standard Signaling Model

Both the spillover-based matching model presented here and a standard signaling model help us understand 'credentialism'. This section compares and contrasts the approaches by drawing out some implications of two key differences between the approaches; first, the relationship between wages and productivity, and second, the treatment of off-equilibrium investments. I conclude that the two frameworks are complementary, but qualitatively distinct, approaches to understanding credentialism.

6.1 Relationship Between Wage and Productivity

The key conceptual difference between the spillover-based matching approach and the standard signaling approach lies in the relationship between a worker's productivity and the wage they earn. The spillover model has wages *identically equal* to output whereas the signaling model has wages that *coincide* with expected output in equilibrium. That is, wages respond to changes in a worker's output in the spillover model but are set at a fixed level in the signaling model.

Given this fundamental distinction, I contrast the two models by considering a modified version of the spillover-based matching model in which workers instead earn a wage that coincides with expected output in equilibrium. That is, I use the "signaling wage assumption" in a matching model. The matching and signaling approaches are qualitatively distinct to the extent that this modified matching model produces results that are qualitatively distinct from the original matching model analyzed above.

Starting with separating behaviour, consider a one-to-one investment function σ . Given segregation, the ‘reduced form’ wage schedule at equilibrium investments must satisfy

$$w(x) = y(s(x, \sigma^{-1}(x)), s(x, \sigma^{-1}(x))) = s(x, \sigma^{-1}(x)). \quad (28)$$

Workers choose their investment optimally taking this wage schedule as given. The investment function is an equilibrium only if $\sigma(\theta)$ coincides with the optimal investment of type θ workers. It is straightforward to see that *a separating equilibrium does not exist in this modified setting* by noting that the problem facing each worker is to maximize $w(x) - c \cdot x$. This problem is independent of θ and therefore any optimum will be shared by all types, contradicting σ being one-to-one. This logic immediately implies that *any equilibrium in this modified setting must be fully pooling*. There will generally be multiple pooling equilibria because of the freedom that the signaling approach permits in terms of imposing off-equilibrium beliefs. These results are in direct contrast to those arising in the spillover-based model presented here, where separating but not pooling equilibria exist. These key differences alone demonstrate that the spillover-based model is not simply a re-labeling of a standard signaling model.

Of course, separating behaviour could be supported in this modified version by making the ad hoc assumption that investment costs vary by type. However, this would not change the fact that the resulting separating behaviour would be driven by forces that are qualitatively different from those arising in the spillover-based version. This is most clearly seen by noting that investment would be insensitive to the strength of spillovers in the signaling version since the equilibrium wage schedule (28), and thus the incentive to invest, is *independent* of the spillover parameter. In contrast, the strength of spillovers is a central determinant of investment behaviour in the spillover-based matching model, as much of the above analysis has demonstrated. Being able to identify spillovers as an exogenous factor driving investment (and over-investment) is a strength of the spillover-based matching approach since it allows us to connect credentialism with characteristics of the workplace, such as the production technology, remuneration policies, and human resource practices. Identifying plausible key drivers of credentialism is less obvious when viewed through the lens of a standard signaling model, since such drivers would have to be related to a weakened ability and/or willingness to adopt performance-based pay.

Despite fundamental differences, the two approaches to credentialism are clearly not unrelated. Indeed, the specific way in which the model is presented - e.g. the geometric approach, including the use of a return function, μ - is a deliberate attempt to draw parallels between the approaches and to leverage the familiarity of signaling’s analytical framework. To draw connections between the approaches, I offer an interpretation of the formal structure of the spillover-based matching model (albeit a far-fetched one) which can be de-

scribed as a ‘non-standard’ signaling model. To this end, suppose that there are firms that offer fixed wages conditional on investment as in a standard signaling model. In contrast to a standard signaling model, workers only spend a fraction ϕ of their time working for such firms and the remaining fraction engaging in self-production (using a technology identical to that used by firms). For any equilibrium investment, the wage schedule announced by firms satisfies (28) in any separating equilibrium, and satisfies $w(x_p) = \int_{\Theta} s(x_p, \theta) dF(\theta)$ in any pooling equilibrium. But these equilibrium conditions on the wage function are *precisely* those imposed on the return function μ from the spillover-based matching model. Workers invest to maximize $(1 - \phi) \cdot s(x, \theta) + \phi \cdot w(x) - c \cdot x$, which is clearly the same as in the spillover model. Separation *is* possible here, despite investment costs being independent of type, because of the fact that workers also benefit directly from their higher productivity due to self-production.

This ‘non-standard’ signaling model gives us a sense of the ways in which a standard signaling model needs to be augmented in order to deliver results qualitatively similar to the spillover-based matching model presented above. The benefit of adopting the spillover interpretation is relatively clear given that the interpretation given above is highly awkward and implausible in many respects. For instance, this interpretation leaves open various questions such as why self- and firm-production use the same technology, why firms are unable to observe a worker’s self-production levels, and why the fraction of time spent working for firms is not a choice variable. Furthermore, the interpretation becomes even less compelling when the separable case is generalized to allow for complementarities (i.e. why would a worker treat income from self-production and firm-production as complements?).

6.2 Treatment of Off-Equilibrium Investments

A second difference between the spillover-based matching approach and the standard signaling approach is the consequences of off-equilibrium investments. In signaling, firms have great freedom in the beliefs they hold about the type of the worker making the deviation. Workers can therefore be dissuaded from making an off-equilibrium deviation purely because of the beliefs held by firms.²⁹ As such, a worker is primarily concerned with how a deviation will be perceived by firms. In contrast, under the matching approach a worker is primarily concerned with who they will be able to match with following a deviation. The approach taken here is sympathetic to the arguments in Peters (2006; 2007b; 2007a) which asserts that if we view the continuum case as the limit of an economy with a finite number of agents, then, since each agent prefers some partner to no partner, all agents should

²⁹This observation motivates the large literature motivating equilibrium refinements in signaling games, e.g. Cho and Kreps (1987).

expect that they will end up matched with some set of agents. The requirement that the matching function maps the set any possible investment to some *equilibrium investment* - i.e. $m(x) \in \sigma(\Theta)$ - as well as the imposition of positive assortative matching on investment, is the way in which I have attempted to retain this discipline in the continuum case.

The differences in the treatment of off-equilibrium investments has consequences for the equilibrium behaviour of the lowest types in separating equilibria: workers of the lowest type invest efficiently under signaling but under-invest under matching.³⁰ This difference is not qualitatively significant in the sense that all the results of the spillover-based matching model would go through if the lowest types were to invest efficiently for some reason (as is the case when $\theta_0 = 0$). All that would change is the initial condition (note that the investment function would become continuous). The point is that over-investment is pervasive in the spillover-based matching model *even if* workers of the lowest type under-invest.

In contrast, the difference in how off-equilibrium investments are treated has qualitatively important implications for pooling equilibria. The reasoning behind non-existence in the spillover model (see proposition 1) relied heavily on the feature that any worker in a pooling equilibrium can change their investment without changing the set of coworkers that they are matched with. Pooling would obviously be possible if we allowed workers to believe that they will be matched with (non-existent) suitably undesirable coworkers if they were to make an off-equilibrium investment. However, note that allowing this would not have any impact on the properties (including existence) of the separating equilibrium derived above.

6.3 Summary

To summarize, the spillover-based matching model presented here and the standard signaling model are complementary, but qualitatively distinct, approaches to understanding credentialism. The spillover-based matching model generates over-investment even if wages are sensitive to performance, generates separating behavior even if investment costs are insensitive to type, and rules out pooling behaviour without recourse to restrictions on off-equilibrium beliefs. The standard signaling model highlights the importance of non-contractible output and can be applied to the analysis of a single worker. In contrast, the

³⁰To see this, first consider signaling and let $u(x, w(x), \theta_0)$ be the payoff to a type θ_0 worker that invests x can receives a wage of $w(x)$. If x^* is their equilibrium investment, then $u(x^*, w(x^*), \theta_0) \geq u(x, w(x), \theta_0)$ for all x . Workers of the lowest type know that they will get paid at least $w(x) = s(x, \theta_0)$ for any x because this corresponds to the most pessimistic beliefs that firms can hold. Since u is increasing in w , we have $u(x, w(x), \theta_0) \geq u(x, s(x, \theta_0), \theta_0)$. But together, these imply $u(x^*, w(x^*), \theta_0) \geq u(x, s(x, \theta_0), \theta_0)$ for all x . Thus, if x^* is the equilibrium investment, then it maximizes $u(x, s(x, \theta_0), \theta_0)$ and therefore is the efficient investment. On the other hand, in matching, the lowest type makes the match-insensitive investment (see Lemma 1), and therefore under-invests.

spillover-based matching model stresses the importance of coworker interaction and can be applied even if output is contractible. The relative merits of the approaches depend upon the extent to which the spread of credentialism can be attributed to a weakened capacity for firms to offer performance-based pay relative to a growing degree of spillovers in the workplace.

7 Conclusions

The model of credentialism developed here is based on two key assumptions regarding skill. First, that there skill spillovers in the workplace; a worker's productivity is influenced by the skill of their coworkers. Second, that skills are soft information; it is infeasible for workers to match on the basis of their skill. Instead, by allowing matches to form on the basis of investment, I propose that investment inherits a credential quality as it acts as a ticket to desirable coworkers.

Despite similarities to standard signaling models, I show how credentialism arises in a setting where firms offer output-contingent wage contracts and therefore effectively observe worker productivity. Despite the positive spillover, I show that over-investment is pervasive in equilibrium. Despite the neutral nature of spillovers, I show that investment and productivity are increasing in the degree of spillovers (but welfare is decreasing).

The model is of course stylized in many respects, and although I have shown robustness in a number of dimensions, there remains many more interesting possibilities that are beyond the scope of this paper. Among these are making the model 'two-sided' by having firms make pre-match investment in capital, introducing strategic interaction between credentialing bodies, allowing multiple sectors with different spillover parameters, introducing strategic interaction between coworkers via team production, allowing for randomness in investment outcomes, deviating from perfect assortative matching (perhaps via an explicit search process), and exploring outcomes with a finite number of workers. Each of these avenues will likely shed further light on the phenomenon of credentialism and are left for future research.

APPENDIX

A Supporting Details

A.1 Demonstrating Stability in Separating Equilibrium

The key assumptions of the model, most notably (i) positive assortative matching on investment and (ii) wage equals output, ensures that in the separating equilibrium workers are

segregated and are paid their segregated output. Since worker skills are revealed by their investment and workers are presumably able to allocate their group's output in any way,³¹ these assumptions are reasonable only if the resulting outcome is *stable* in the sense that there does not exist N workers that together could produce a total output that is greater than the sum of their individual equilibrium (i.e. segregated) outputs.

To explore this, first consider a group of N workers that are matched in equilibrium. Since effort is supplied inelastically, altering the way in which it is allocated does not affect the total output produced by the group.³² Given that workers are segregated in equilibrium, we need only verify that forming a non-segregated group of N workers produces a total output no greater than the sum of what the workers produce when segregated. This is ensured by the fact that skills are complementary. Specifically, skills are *complementary* if a group's total output, $Y(\mathbf{s})$, displays increasing differences. That is, if $s_A \geq s_a$, $s_B \geq s_b$, and $\mathbf{s}_{N-2} \in \mathbb{R}_+^{N-2}$, then

$$Y(s_A, s_B, \mathbf{s}_{N-2}) - Y(s_a, s_B, \mathbf{s}_{N-2}) \geq Y(s_A, s_b, \mathbf{s}_{N-2}) - Y(s_a, s_b, \mathbf{s}_{N-2}). \quad (29)$$

That is, the additional output arising from increasing the skill of a worker (replacing s_a with s_A) is larger when coworkers have higher skills (when coworker skills are (s_B, \mathbf{s}_{N-2}) as opposed to (s_b, \mathbf{s}_{N-2})). Skills are *strictly complementary* if (29) holds with strict inequality. When Y is twice continuously differentiable, (29) is implied by $d^2Y/ds_i ds_j \geq 0$ (where i and j are distinct coworkers), and strict complementarity is implied by $d^2Y/ds_i ds_j > 0$. Using the CES specification, skills are complementary for all $\rho \leq 1$, and are strictly complementary for all $\rho < 1$.

The key to establishing that there are no incentives to form a non-segregated firm is to invoke the well-known result that segregation maximizes total output when skills are complementary (see [Becker \(1973\)](#) for the case of $N = 2$, and [Topkis \(1998\)](#), [Lin \(1992\)](#), or [Durlauf and Seshadri \(2003\)](#) for generalizations to the $N > 2$ case). For one way to see this, consider an arbitrary N workers with an associated skill vector $\tilde{\mathbf{s}} = (s_1, \dots, s_N)$. For each $s_n \in \tilde{\mathbf{s}}$, let $y_n = y(s_n, s_n)$ be the segregated output produced by a worker with skill s_n .

Consider a population of N^2 workers formed by taking N workers of each skill type $s_n \in \tilde{\mathbf{s}}$. Since skills display complementarity, the aggregate output produced when these N^2 workers are sorted into N segregated firms is at least as great as the aggregate output when the workers are matched in any other manner - in particular, when N identical firms with

³¹For example, they could adopt a contract that conditions each worker's payment on the output of their coworkers: e.g. a team performance contract that divides the group's total output evenly, or the reverse in which those producing relatively low output make transfers to those that produce relatively high output.

³²If the model were slightly modified to include costly effort, then the group's available surplus can only be reduced when wages depend on the output of coworkers: since there are no effort externalities across workers, such possibilities can only distort effort decisions away from the first-best.

a vector of worker skills $\bar{\mathbf{s}}$ are formed. That is, $\sum_{n=1}^N Y(\mathbf{s}_n) \geq \sum_{n=1}^N Y(\bar{\mathbf{s}})$, where $\mathbf{s}_n \equiv \{s_n, \dots, s_n\}$ is a vector of N identical skill levels. Since $Y(\mathbf{s}_n) = N \cdot y_n$, the left side of this is $\sum_{n=1}^N N \cdot y_n$. Since the summand on the right side is independent of n , the right side equals $N \cdot Y(\bar{\mathbf{s}})$. Therefore, by dividing both sides by N we see that complementarity implies $\sum_{n=1}^N y_n \geq Y(\bar{\mathbf{s}})$. But this implies that matching an arbitrary N workers together can not produce a total output greater than the sum of their segregated output levels. As such, there are no incentives for workers to form non-segregated groups even if workers had freedom in determining how the total output from such a group is allocated.³³

While skill complementarity permits various modeling simplifications (such as the description of the matching market) and is adopted almost exclusively in the related literature, there are potentially situations of interest in which skills are *not* complementary. One example (suggested by an anonymous referee) is $Y_i(\mathbf{s}) = (1 - \phi) \cdot s_i + \phi \cdot \max\{s_j \mid s_j \in \mathbf{s}\}$. This reflects a situation in which lower skilled workers benefit from the presence of higher skilled coworkers, but higher skilled workers are not harmed by the presence of lower skilled coworkers. Imposing positive assortative matching will not be appropriate in this setting since it will produce unstable matches. Understanding credentialism in such settings is certainly of interest, but a more general analysis encompassing such possibilities is left for future research.

A.2 Partial Pooling Equilibria

Partial pooling equilibria are those in which more than one type, but not all types, make the same investment. To begin exploring such equilibria, we begin with a slight strengthening of proposition 1 in order to rule out some possibilities.

Proposition 7. *There does not exist an equilibrium in which an interval of types that include the lowest type all make the same investment.*

Proof. Suppose to the contrary that agents with types in $[\theta_0, \theta')$ all invest x_p (there may be higher types that also invest this amount). Since investment is weakly increasing in type in any equilibrium (corollary 2), m is weakly increasing and has the property that $m(x) \in \sigma(\Theta)$, it must be that $\mu(x) = \mu(x_p)$ for $x \leq x_p$ and $\mu(x) \geq \mu(x_p)$ for $x > x_p$. Since types in $[\theta_0, \theta')$ do not want to deviate from x_p , it must be that $u(x_p, \mu(x_p), \theta) \geq u(x, \mu(x), \theta) \geq u(x, \mu(x_p), \theta)$. That is, $u(x, \mu(x_p), \theta)$ is maximized at $x = x_p$ for all $\theta \in (\theta_0, \theta')$. But, by definition, $u(x, \mu(x_p), \theta)$ is maximized at the match insensitive investment, $x = \bar{\sigma}(\theta)$, which varies by type, contradicting the supposition that all $\theta \in [\theta_0, \theta')$ invest the same amount. \square

³³If skills are strict complements, as in the CES case with $\rho < 1$, then the total output of an arbitrary non-segregated firm will be strictly lower than the sum of the segregated outputs.

The only significance of the lowest type is that it ensures that no worker invests less than x_p , and, as a result, we have $\mu(x) = \mu(x_p)$ for $x \leq x_p$. Proposition 7 leads us to the question of whether *any* partial pooling equilibria exist. The answer is generally yes, since positive assortative matching does not ensure that μ is right-continuous when intervals not containing θ_0 pool.

To see that partial pooling equilibria may exist, start with the most extreme case in which all but the lowest types pool. Specifically, workers of the lowest type invest $\bar{\sigma}(\theta_0)$ and all others invest some $x_p > \bar{\sigma}(\theta_0)$. Recalling that V_0 is the equilibrium payoff of the lowest type workers, choose x_p so that the lowest type is indifferent between their equilibrium outcome and pooling with the others: i.e. x_p is the unique value of $x > \bar{\sigma}(\theta_0)$ that satisfies $u(x, \hat{\mu}(x), \theta_0) = V_0$ where $\hat{\mu}(x) \equiv E[\theta] \cdot g(x)$. By positive assortative matching, we must have $\mu(x) = \mu_0 \equiv s(\bar{\sigma}(\theta_0), \theta_0)$ for $x \leq \bar{\sigma}(\theta_0)$ and $\mu(x) = \mu_p \equiv E[\theta] \cdot g(x_p)$ for $x \geq x_p$. We can set $\mu(x) = \mu_0$ for $x \in (\bar{\sigma}(\theta_0), x_p)$. This ensures that θ_0 types are optimizing, and, as long as $x_p \geq \bar{\sigma}(\theta_1)$ (nothing guarantees this), so too are all higher types, so that this constitutes an equilibrium.

Such an equilibrium is not fully convincing in the sense that there are multiple matching functions that are positive assortative relative to σ . That is, one could also have chosen $\mu(x) = \mu_p$ for $x \in (\bar{\sigma}(\theta_0), x_p)$. In this case, the proposed strategies are not part of an equilibrium since workers have a profitable deviation; e.g. the lowest types would want to marginally increase their investment (at slight increase in cost but a discrete jump in expected coworker skill). The equilibrium is therefore non-robust in the sense that the strategies are not optimal relative to *all* μ that are consistent with σ and some m that is positive assortative relative to σ . This type of argument regarding existence but non-robustness of partial pooling equilibria extends easily to other partial pooling possibilities, such as types in $[\theta_0, \theta']$ separating and types $(\theta', \theta_1]$ pooling.

B Proofs

Proposition 8. *If $y_{s\bar{s}} = 0$, then for all $x' > x$ and $\theta' > \theta$,*

$$u(x', \bar{s}', \theta) \geq u(x, \bar{s}, \theta) \Rightarrow u(x', \bar{s}', \theta') > u(x, \bar{s}, \theta'). \quad (30)$$

If $y_{s\bar{s}} > 0$, then (30) holds under the additional requirement that $\bar{s}' > \bar{s}$.

Proof. First note that $u_{s\theta} = \frac{d}{d\theta} y_s = \frac{ds}{d\theta} \cdot \frac{dy_s}{ds} = g(x) \cdot \frac{dy_s}{ds}$. Therefore $y_{s\bar{s}} = 0$ implies $u_{s\theta} = 0$. This implies

$$u(x', \bar{s}', \theta') - u(x', \bar{s}', \theta) = u(x', \bar{s}, \theta') - u(x', \bar{s}, \theta). \quad (31)$$

Second, note that $u_{x\theta} = \frac{d}{d\theta} y_s \cdot s_x = g'(x) \cdot \frac{d}{d\theta} y_s \cdot \theta = \frac{g'(x)}{g(x)} \cdot \frac{d}{d\theta} y_s \cdot s = \frac{g'(x)}{g(x)} \cdot \frac{ds}{d\theta} \cdot \frac{d}{ds} y_s \cdot s = g'(x) \cdot \frac{d}{ds} y_s \cdot s$. Therefore $d/ds\{y_s \cdot s\} > 0$ implies $u_{x\theta} > 0$. Using this, we have that $x' > x$ and $\theta' > \theta$ implies

$$u(x', \bar{s}, \theta') - u(x', \bar{s}, \theta) > u(x, \bar{s}, \theta') - u(x, \bar{s}, \theta). \quad (32)$$

Using (31) and (32) gives

$$u(x', \bar{s}', \theta') - u(x', \bar{s}', \theta) > u(x, \bar{s}, \theta') - u(x, \bar{s}, \theta). \quad (33)$$

Rearrange this to get

$$u(x', \bar{s}', \theta') - u(x, \bar{s}, \theta') > u(x', \bar{s}', \theta) - u(x, \bar{s}, \theta). \quad (34)$$

Therefore $u(x', \bar{s}', \theta) - u(x, \bar{s}, \theta) \geq 0$ implies $u(x', \bar{s}', \theta') - u(x, \bar{s}, \theta') > 0$ and the result follows.

From the derivation of $u_{\bar{s}\theta}$, we have that $y_{\bar{s}\bar{s}} > 0$ implies $u_{\bar{s}\theta} > 0$. In this case, if the additional requirement $\bar{s}' > \bar{s}$ holds, then we have

$$u(x', \bar{s}', \theta') - u(x', \bar{s}', \theta) > u(x', \bar{s}, \theta') - u(x', \bar{s}, \theta). \quad (35)$$

Since (32) still holds, it can be used with (35) to imply (33), and the proof continues unchanged from there. \square

Corollary 2. *If $y_{\bar{s}\bar{s}} = 0$, then investment is a weakly increasing function of type in any equilibrium. If $y_{\bar{s}\bar{s}} > 0$, then investment is a weakly increasing function of type in any equilibrium in which μ is a weakly increasing function.*

Proof. Suppose not. Take x' to be the investment of θ types, $x < x'$ to be the investment of $\theta' > \theta$ types, and take $\bar{s}' = \mu(x')$ and $\bar{s} = \mu(x)$. If $y_{\bar{s}\bar{s}} = 0$, then from proposition 8 we see that if the θ types are optimizing, then θ' types cannot be. If $y_{\bar{s}\bar{s}} > 0$ and μ is increasing, then the application of proposition 8 leads to the same conclusion. \square

Note that μ is weakly increasing in (i) any pooling equilibrium (μ is flat), and (ii) any separating equilibrium if $\theta_0 = 0$ (since such types invest zero, the lowest possible investment). Furthermore, if workers are able to “hide” part of their investment from the matching market, then μ will always be weakly increasing.

Proposition 9. *A unique solution exists for the initial values problem defined by (16), and it is given by (17).*

Proof. Let $a(x) \equiv \frac{-g_x(x)}{\phi g(x)}$ and $b(x) \equiv \frac{c}{\phi g(x)}$, so that we can write:

$$t'(x) = a(x) \cdot t(x) + b(x).$$

This is a linear ordinary differential equation, and therefore the initial values problem with an arbitrary initial condition (x_0, t_0) has a unique solution as long as a and b are continuous at $x = x_0$: i.e. if $x_0 > 0$. In this case, the well-known solution is:

$$t(x) = \left[K + \int^x b(z) \cdot \exp\left(-\int^z a(z') dz'\right) dz \right] \cdot \exp\left(\int^x a(z) dz\right),$$

where K is a constant that adjusts so that the initial condition is satisfied and the notation $\int^x f(z) dz$ represents the indefinite integral of $f(x)$. Note that $\int^x a(z) dz = -(1/\phi) \ln(g(x))$, so that $\exp(\int^x a(z) dz) = g(x)^{-1/\phi}$, which lets us write:

$$t(x) = \left[K + \int^x b(z) \cdot g(z)^{\frac{1}{\phi}} dz \right] \cdot g(x)^{-\frac{1}{\phi}} = \left[K + \frac{1}{\phi} \int^x c \cdot g(z)^{\frac{1-\phi}{\phi}} dz \right] \cdot g(x)^{-\frac{1}{\phi}}$$

For any given (x_0, t_0) , we have:

$$K = t_0 \cdot g(x_0)^{\frac{1}{\phi}} - \frac{c}{\phi} \int^{x_0} g(z)^{\frac{1-\phi}{\phi}} dz,$$

and as a result, we get

$$t(x) = \left[t_0 \cdot g(x_0)^{\frac{1}{\phi}} + \frac{c}{\phi} \int_{x_0}^x g(z)^{\frac{1-\phi}{\phi}} dz \right] \cdot g(x)^{-\frac{1}{\phi}}.$$

Since $x_0 = \sigma^+(\theta_0)$, and $\sigma^+(\theta_0) > 0$ for $\theta_0 > 0$, this is the unique solution to the initial values problem (where $t_0 = \theta_0$ and $x_0 = \sigma^+(\theta_0)$) when $\theta_0 > 0$.

This also serves as a solution when $\theta_0 = 0$ by setting $t_0 = x_0 = 0$. To show that this solution is unique, suppose that there are two distinct solutions, t_1 and t_2 , that emanate from the origin. Then, for some $\hat{x} > 0$ we have $t_1(\hat{x}) \neq t_2(\hat{x})$. Since $\hat{x} > 0$, we use the above results to show that for solution k is

$$t_k(x) = \left[t_k(\hat{x}) \cdot g(\hat{x})^{\frac{1}{\phi}} + \frac{c}{\phi} \int_{\hat{x}}^x g(z)^{\frac{1-\phi}{\phi}} dz \right] \cdot g(x)^{-\frac{1}{\phi}}.$$

The only way that $t_k(0) = 0$ is if the bracketed term is zero at $x = 0$. But, there is a unique value of $t_k(\hat{x})$ for which this holds:

$$t_k(\hat{x}) = \frac{c}{\phi} \cdot \frac{\int_0^{\hat{x}} g(z)^{\frac{1-\phi}{\phi}} dz}{g(\hat{x})^{\frac{1}{\phi}}}. \quad (36)$$

Therefore, all solutions emanating from the origin must coincide at \hat{x} . But \hat{x} is arbitrary, proving the uniqueness of the solution. It then follows that a unique solution exists for all $\theta_0 \geq 0$. \square

Proposition 10. *Let $t(x)$ given by (17). Then (i) $t'(x) > 0$ for all $x > x_0$, and (ii) $\lim_{t \rightarrow \infty} t(x) = \infty$.*

Proof. For part (i), after applying the chain rule to (17) we have

$$\begin{aligned}
t'(x) &= \frac{1}{\phi \cdot g(x)} \left[c \cdot \left[1 - g(x)^{-\frac{1}{\phi}} \int_{x_0}^x \frac{1}{\phi} \cdot g'(z) \cdot g(z)^{\frac{1-\phi}{\phi}} dz \right] - \theta_0 \cdot g'(x) \cdot \left[\frac{g(x_0)}{g(x)} \right]^{\frac{1}{\phi}} \right] \\
&> \frac{1}{\phi \cdot g(x)} \left[c \cdot \left[1 - g(x)^{-\frac{1}{\phi}} \int_{x_0}^x \frac{1}{\phi} \cdot g'(z) \cdot g(z)^{\frac{1-\phi}{\phi}} dz \right] - \theta_0 \cdot g'(x) \cdot \left[\frac{g(x_0)}{g(x)} \right]^{\frac{1}{\phi}} \right] \\
&= \frac{c - \theta_0 \cdot g'(x)}{\phi \cdot g(x)} \cdot \left[\frac{g(x_0)}{g(x)} \right]^{\frac{1}{\phi}} > \frac{c - \theta_0 \cdot g'(x_0)}{\phi \cdot g(x)} \cdot \left[\frac{g(x_0)}{g(x)} \right]^{\frac{1}{\phi}} \geq 0,
\end{aligned}$$

where the first and second inequalities follow from $g'(x)$ being a decreasing function and $x > x_0$, and the final inequality follows from $x_0 = \sigma^+(\theta_0)$ and $-(c - \theta_0 \cdot g'(x_0)) = V_x(\sigma^+(\theta_0), \theta_0) \leq 0$.

For part (ii), we have

$$\lim_{x \rightarrow \infty} t(x) = \lim_{x \rightarrow \infty} \left\{ \frac{\theta_0 \cdot g(x_0)^{\frac{1}{\phi}} + \frac{c}{\phi} \int_{x_0}^x g(z)^{\frac{1-\phi}{\phi}} dz}{g(x)^{\frac{1}{\phi}}} \right\}. \quad (37)$$

If g is bounded, then the result follows immediately since the numerator is unbounded whereas the denominator is bounded. If g is unbounded, then by L'Hospital's rule

$$\lim_{x \rightarrow \infty} t(x) = \lim_{x \rightarrow \infty} \frac{\frac{c}{\phi} \cdot g(x)^{\frac{1-\phi}{\phi}}}{\frac{1}{\phi} \cdot g(x)^{\frac{1-\phi}{\phi}} \cdot g'(x)} = c \lim_{x \rightarrow \infty} \frac{1}{g'(x)} = \infty \quad (38)$$

as required. \square

Lemma 3. *In any separating equilibrium, $\sigma(\theta_1) = t^{-1}(\theta_1)$ where t is the solution to (16).*

Proof. Let $x_1 \equiv t^{-1}(\theta_1)$ and note that $x_1 = \lim_{\theta \nearrow \theta_1} \sigma(\theta)$. Since σ is strictly increasing, we must have $\sigma(\theta_1) \geq x_1$. Since type θ_1 agents do not find it profitable to masquerade as any lower type, for all $\theta < \theta_1$ we have $V(\sigma(\theta_1), \theta_1) \geq y(s(\sigma(\theta), \theta_1), s(\sigma(\theta), \theta)) - c \cdot \sigma(\theta))$. The right side is greater than $V(\sigma(\theta), \theta)$, so we have that $V(\sigma(\theta_1), \theta_1) > V(\sigma(\theta), \theta)$ for all $\theta < \theta_1$. Taking θ to θ_1 , noting that V is continuous in θ at each $\theta \in \text{int}(\Theta)$, we see that it must be that $V(\sigma(\theta_1), \theta_1) \geq V(x_1, \theta_1)$. But $V_x(x_1, \theta_1) = s_x(x_1, \theta_1) - c = -\phi \cdot s_\theta(x_1, \theta_1) \cdot t'(x_1) < 0$. Since V is strictly concave in x with $V_x(x_1, \theta_1) < 0$, it must be that $V(\sigma(\theta_1), \theta_1) \geq V(x_1, \theta_1) \Rightarrow \sigma(\theta_1) \leq x_1$. Therefore we have $x_1 \leq \sigma(\theta_1) \leq x_1$, and consequently, $\sigma(\theta_1) = x_1$ as desired. \square

Proposition 11. *No worker has a profitable deviation from the proposed equilibrium investment function given in proposition 2.*

Proof. The proof first establishes that no type has a profitable deviation to any equilibrium investment and then establishes that no type has a profitable deviation to any off-equilibrium investment by showing at each type prefers some equilibrium investment to any off-equilibrium investment. To establish some preliminaries, recall that $v(x, \theta) = u(x, s(m(x), t(m(x))), \theta)$ and note that v is continuous in x at all points except x_0 (it is also continuous at this point if $\theta_0 = 0$). Furthermore v is differentiable at all points except x_0 (a jump discontinuity) and $\sigma(\theta_1)$ (a kink). Whenever differentiable, we have $v_x = u_x + u_s \cdot d/dx\{s(m(x), t(m(x)))\}$ so that $v_{x\theta} = u_{x\theta} + u_{s\theta} \cdot d/dx\{s(m(x), t(m(x)))\}$. Since s is increasing in both arguments and both m and t are weakly increasing, we have $d/dx\{s(m(x), t(m(x)))\} \geq 0$. Therefore $v_{x\theta} > 0$ since $u_{x\theta} > 0$ and $u_{s\theta} \geq 0$ (see proof of proposition 8).

No type $\theta \in \Theta$ has a profitable deviation to any $x \in \sigma((\theta_0, \theta_1])$. At each $x \in \sigma((\theta_0, \theta_1])$, $v(x, \theta) = u(x, s(x, t(x)), \theta)$ is differentiable. Suppose a type θ makes a deviation to a lower investment in $\sigma((\theta_0, \theta_1])$. This corresponds to $\sigma(\theta')$ for some $\theta' < \theta$. But $v_x(\sigma(\theta'), \theta) > v_x(\sigma(\theta'), \theta') = 0$, where the first inequality follows from $v_{x\theta} > 0$ and the equality follows from the first order necessary condition of type θ' . Therefore $v_x(x, \theta) > 0$ at all $x \in \sigma((\theta_0, \theta_1])$, and thus for all $\theta \in \Theta$ we have $v(\sigma(\theta), \theta) \geq v(x, \theta)$ for all $x \in \sigma((\theta_0, \theta_1])$.

Now suppose a type θ makes a deviation to a higher investment in $\sigma((\theta_0, \theta_1])$. This corresponds to $\sigma(\theta')$ for some $\theta' > \theta$. But $v_x(\sigma(\theta'), \theta) < v_x(\sigma(\theta'), \theta') = 0$, using the same reasoning as above (adding that the left derivative is used if $\theta' = \theta_1$). Therefore $v_x(x, \theta) < 0$ at all $x \in \sigma((\theta_0, \theta_1])$. For $\theta > \theta_0$, this implies that $v(\sigma(\theta), \theta) > v(x, \theta)$ for any $x \in \sigma((\theta_0, \theta_1])$. For $\theta = \theta_0$, this implies that $v(x, \theta_0) < \lim_{\theta \searrow \theta_0} v(\sigma(\theta), \theta_0) = V_0 = v(\sigma(\theta_0), \theta_0)$. Therefore $v(\sigma(\theta_0), \theta_0) > v(x, \theta_0)$ for all $x \in \sigma((\theta_0, \theta_1])$. Therefore, for all $\theta \in \Theta$ we have $v(\sigma(\theta), \theta) > v(x, \theta)$ for all $x \in \sigma((\theta_0, \theta_1])$.

Therefore, for all $\theta \in \Theta$ we have $v(\sigma(\theta), \theta) \geq v(x, \theta)$ for all $x \in \sigma((\theta_0, \theta_1])$.

No type $\theta \in (\theta_0, \theta_1]$ has a profitable deviation to $\sigma(\theta_0)$. If $\theta_0 = 0$, then $\sigma(\theta_0) = 0$ and $v_x(0, \theta) > v_x(0, \theta_0) = 0$ for all $\theta > \theta_0$. Since σ is continuous at θ_0 in this case, it follows that all types $\theta > \theta_0$ prefer to make some equilibrium investment (marginally above zero). If $\theta_0 > 0$, then by construction we have $u(x_0, s(x_0, \theta_0), \theta_0) = u(\sigma(\theta_0), s(\sigma(\theta_0), \theta_0), \theta_0)$. Since $x_0 > \sigma(\theta_0)$ and $s(x_0, \theta_0) > s(\sigma(\theta_0), \theta_0)$, by proposition 8 we have that $u(x_0, s(x_0, \theta_0), \theta) > u(\sigma(\theta_0), s(\sigma(\theta_0), \theta_0), \theta)$ for all $\theta > \theta_0$. Since the left side is continuous in x_0 , there exists investments marginally above x_0 such that the inequality remains. But such investments are equilibrium investments. Therefore for any $\theta > \theta_0$ there exists an equilibrium investment in $\sigma((\theta_0, \theta_1])$ that they prefer to $\sigma(\theta_0)$.

No type has a profitable deviation to any $x \notin \sigma(\Theta)$. First note that the set of off-equilibrium investments consists of three intervals: $[0, \sigma(\theta_0))$, $(\sigma(\theta_0), x_0]$, and $(\sigma(\theta_1), \infty)$ - only the last of these is relevant (i.e. non-empty) if θ_0 . For each of the three regions we show that each type prefers some equilibrium investment to each investment within the region. We have shown

that all types prefer their equilibrium investment to any other equilibrium investment, so this implies that there are no profitable off-equilibrium deviations.

Consider $x \in [0, \sigma(\theta_0))$. For such values of x we have $v_x(x, \theta) > v_x(x, \theta_0) > 0$, where the first inequality follows from $v_{x\theta} > 0$ and the second follows from $x < \bar{\sigma}(\theta_0)$. Therefore all types prefer $\bar{\sigma}(\theta_0)$ (which equals $\sigma(\theta_0)$) to all $x \in [0, \sigma(\theta_0))$.

Consider $x \in (\sigma(\theta_1), \infty)$. For such values of x we have $v_x(x, \theta) < v_x(x, \theta_1) < 0$, where the first inequality follows from $v_{x\theta} > 0$ and the second follows from $v_x(x, \theta_1) = y_s \cdot s_x(x, \theta_1) - c < s_x(x, \theta_1) - c < s_x(\sigma(\theta_1), \theta_1) - c = -y_{\bar{s}} \cdot s_{\theta}(\sigma(\theta_1), \theta) \cdot t'(\sigma(\theta_1)) < 0$. Therefore all types prefer $\sigma(\theta_1)$ to all $x \in (\sigma(\theta_1), \infty)$.

Consider $x \in (\sigma(\theta_0), x_0]$. Since $v(x, \theta_0) = u(x, s(\bar{\sigma}(\theta_0), \bar{\sigma}(\theta_0)), \theta_0)$ for any such x , we have that $v(x, \theta_0) < V_0$ (by definition of $\bar{\sigma}(\theta)$). Since $V_0 = \lim_{\theta \searrow \theta_0} v(\sigma(\theta), \theta_0)$, there exists a type $\theta' > \theta_0$ for which $v(\sigma(\theta'), \theta_0) > v(x, \theta_0)$. That is, $u(\sigma(\theta'), s(\sigma(\theta'), \theta'), \theta_0) > u(x, s(x, \theta_0), \theta_0)$. Since $\sigma(\theta') > x$ and $s(\sigma(\theta'), \theta') > s(\sigma(\theta_0), \theta_0)$, by proposition 8 we have that $u(\sigma(\theta'), s(\sigma(\theta'), \theta'), \theta) > u(x, s(\sigma(\theta_0), \theta_0), \theta)$ for all $\theta > \theta_0$. Therefore for all $x \in (\sigma(\theta_0), x_0]$ we can find a θ' for which all types prefer $\sigma(\theta')$ to x .

Therefore for all off-equilibrium investments, we can find an equilibrium investment that all types prefer. We have already established that each type prefers their proposed equilibrium investment to any other equilibrium investment, and therefore we conclude that each type prefers their proposed equilibrium investment to any other investment. \square

Proof of Proposition 2

Proof. Theorem 2.1 from [Mailath and von Thadden \(2012\)](#) establishes that any separating equilibrium in this environment must be differentiable on (θ_0, θ_1) with a positive derivative. As a result (as established in the text), for $\theta \in (\theta_0, \theta_1)$, $\sigma^{-1}(\theta)$ must coincide with the solution to the initial values problem (16). Proposition 9 establishes the existence and uniqueness of this solution, and proposition 10 establishes that the solution possesses the required properties. Lemma 1 establishes that any separating equilibrium must have $\sigma(\theta_0) = \bar{\sigma}(\theta_0)$, and Lemma 3 establishes that $\sigma(\theta_1) = t^{-1}(\theta_1)$. The fact that no type has a profitable deviation from this proposed strategy is established in proposition 11. \square

Proof of Proposition 3

Proof. Since $t'(x) = T(t, x)$, a direct consequence of $t'(x) > 0$ for all $x > x_0$ (see proposition 10) is $c - \theta \cdot g'(\sigma(\theta)) > 0$ for all $\theta > 0$. But this implies over-investment since the first-order condition for the efficient investment is $c - \theta \cdot g'(\sigma^*(\theta)) = 0$. Since $g'(x)$ is decreasing, it follows that $\sigma(\theta) > \sigma^*(\theta)$. \square

Proof of Proposition 4

Proof. An increase in ϕ lowers the match-insensitive investment of the lowest type (strictly, if $\theta_0 > 0$). This lowers their equilibrium payoff, V_0 , thereby increasing x_0 . Furthermore, T is strictly decreasing in ϕ at any (t, x) pair at which $t'(x) > 0$ (which holds at any $x \in \sigma((\theta_0, \theta_1))$). Together these facts imply that the solution to the initial values problem, $t(x)$, is shifted to the right when ϕ increases (i.e. it starts at a point to the right, and never crosses the original solution since the slope is everywhere flatter). Since $t(x)$ is the inverse investment function for $\theta \in \Theta \setminus \{\theta_0\}$, we have that such types invest strictly more when spillovers are higher. \square

Proof of Proposition 5

Proof. Follows from $\frac{d\Delta_o(p, p')}{d\phi} = \frac{d\Delta_o(p, p')}{dz} \cdot \frac{dz}{d\phi}$ (for $o \in \{y, u\}$), $\frac{d\Delta_y(p, p')}{dz} = \frac{1}{1-\eta} \cdot \frac{\Delta_y(p, p')}{z} > 0$, $\frac{d\Delta_u(p, p')}{dz} = z^{\frac{\eta}{1-\eta}} \cdot \frac{c}{1-\eta} \cdot \frac{\eta - z \cdot c}{z \cdot c} < 0$, and $\frac{d}{d\phi} z = \frac{1-\eta}{c} > 0$.

To see that greater spillovers decrease both income and payoff inequality in the benchmark case, we begin by noting that the inequality measures in this case, denoted $\tilde{\Delta}_y$ and $\tilde{\Delta}_u$, are the same as Δ_y and Δ_u with z being replaced by \tilde{z} . The claimed result follows from $\frac{d\tilde{\Delta}_y(p, p')}{d\tilde{z}} = \frac{1}{1-\eta} \cdot \frac{\Delta_y(p, p')}{\tilde{z}} > 0$, $\frac{d\tilde{\Delta}_u(p, p')}{d\tilde{z}} = \tilde{z}^{\frac{\eta}{1-\eta}} \cdot \frac{c}{1-\eta} \cdot \frac{\eta - \tilde{z} \cdot c}{\tilde{z} \cdot c} > 0$, and $\frac{d}{d\phi} \tilde{z} = -\frac{\eta}{c} < 0$. \square

Proof of Proposition 6

Proof. Write $u(\theta) = \theta^{\frac{1}{1-\eta}} \cdot z^{\frac{1}{1-\eta}} \cdot [z^{-1} - c] = \theta^{\frac{1}{1-\eta}} \cdot c^{-\frac{\eta}{1-\eta}} \cdot [\phi + (1-\phi) \cdot \eta]^{\frac{1}{1-\eta}} \cdot [\frac{1}{\phi + (1-\phi) \cdot \eta} - 1]$ and $\tilde{u}(\theta) = \theta^{\frac{1}{1-\eta}} \cdot c^{-\frac{\eta}{1-\eta}} \cdot [(1-\phi) \cdot \eta]^{\frac{1}{1-\eta}} \cdot [\frac{1}{(1-\phi) \cdot \eta} - 1]$. Some simple re-arranging gives

$$\frac{\tilde{u}(\theta)}{u(\theta)} \equiv R(\eta) = \left[\frac{(1-\phi) \cdot \eta}{\phi + (1-\phi) \cdot \eta} \right]^{\frac{\eta}{1-\eta}} \cdot \frac{1 - (1-\phi) \cdot \eta}{(1-\phi) \cdot (1-\eta)}.$$

The result follows from $R(\eta)$ being a strictly decreasing continuous function with $R(0) = \frac{1}{1-\phi} > 1$, $R(1/2) = 1$, and $\lim_{\eta \rightarrow 1} R(\eta) = 0$. \square

C Illustration Details

The equilibrium investments are derived by first deriving the inverse investment function using (17):

$$t(x) = \frac{c}{\phi} \cdot \frac{\int_0^x z^{\frac{\eta}{\phi}(1-\phi)} dz}{x^{\frac{\eta}{\phi}}} = \frac{c}{\phi} \cdot \frac{\frac{1}{1+\frac{\eta}{\phi}(1-\phi)} \cdot x^{1+\frac{\eta}{\phi}(1-\phi)}}{x^{\frac{\eta}{\phi}}} = \frac{c}{\phi + \eta \cdot (1-\phi)} \cdot x^{1-\eta}.$$

Therefore

$$t^{-1}(\theta) = \sigma(\theta) = \left[\frac{\phi + \eta \cdot (1-\phi)}{c} \right]^{\frac{1}{1-\eta}} \cdot \theta^{\frac{1}{1-\eta}}.$$

Then we have $y(\theta) = \theta \cdot \sigma(\theta)^\eta$ and $u(\theta) = y(\theta) - c \cdot \sigma(\theta)$. The exogenous segregation benchmark investments are derived from the first-order condition:

$$(1 - \phi) \cdot \eta \cdot \theta \cdot \bar{\sigma}(\theta)^{\eta-1} = c.$$

Simple re-arranging gives

$$\bar{\sigma}(\theta) = \left[\frac{(1 - \phi) \cdot \eta}{c} \right]^{\frac{1}{1-\eta}} \cdot \theta^{\frac{1}{1-\eta}}.$$

Then we have $\bar{y}(\theta) = \theta \cdot \bar{\sigma}(\theta)^\eta$ and $\bar{u}(\theta) = \bar{y}(\theta) - c \cdot \bar{\sigma}(\theta)$. The efficient outcomes are derived in the same way, except setting $\phi = 0$.

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