Pre-Match Investment with Frictions*

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Abstract

The paper explores an environment in which agents are motivated to make unproductive investments with the sole aim of improving their matching opportunities. In contrast to existing work, I add frictions by allowing the investment to be imperfectly observed. The analysis allows for a deeper understanding of the trade-off inherent in related models: investments waste resources but facilitates more efficient matching patterns. I show that greater frictions i) do not always lead to inferior matching patterns, and ii) can force the economy into to a Pareto preferred equilibrium.

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1 Introduction

Our productivity is shaped, in large part, by our particular work environment. In many instances, especially in 'high skill' jobs, the relevant difference between work environments lies in differences in the quality of coworkers that one is exposed to.[1] While many interesting insights and important implications of such coworker interdependence have been derived, the analysis has typically unfolded in an 'idealized' competitive setting in which the price of skills directs workers to firms (e.g. Kremer (1993), Kremer and Maskin (1996), and Costrell and Loury (2004))).

Recognizing that the feasibility of this allocation mechanism is greatly diminished when the relevant skills are unobservable, some recent work proposes instead that workers take costly actions in order to 'compete' for the more desirable coworkers (e.g. Hoppe, *et al*. (2009), Rege (2007), and Bidner (2008)). These papers share the feature that ex-ante heterogeneous workers make a costly investment, then enter a matching market in which they are assigned a partner on the basis of the investment.[2] When the investment is unproductive, the essential welfare trade-off lies in the fact that investing consumes resources, but facilitates a more efficient matching pattern.[3] Basically, the fact that workers are complements means that output is maximized when matching is positive assortative on type (Becker (1973)). There is a 'no-investment' equilibrium in which no resources are wasted, but matching is inefficient (random). There is also an 'investment equilibrium' that achieves the efficient matching pattern, but comes at a resource cost since higher types make higher investments in order to differentiate themselves in the matching market.

The present paper seeks to extend this type of analysis by developing a model in which the relationship between investment and matching opportunities is hindered by frictions. Extensions along this dimension are useful because one limitation of fric-

[1]This is as opposed to differences in capital - a feature stressed in standard assignment models (e.g. Sattinger (1993)). Hopkins (2005) studies a model that incorporates many of the features highlighted here, but assumes that firms are (exogenously) differentiated by their capital.

[2]In related work, Pesendorfer (1995) studies a model in which an agent's consumption behaviour influences their matching opportunities, but focuses on the resulting incentives for a monopolist to induce 'fashion cycles'. Although not necessarily grounded in an explicit matching context, similar themes are present in the literature on status and conspicuous consumption, e.g. Frank (1985) and Bagwell and Bernheim (1996).

[3]The investment is unproductive in Hoppe, *et al*. (2009) and Rege (2007). A welfare trade-off of a different nature arises when investment is productive (see Bidner (2008)).

tionless environments is that workers are always either perfectly segregated or randomly matched in equilibrium, making it difficult to rationalize intermediate degrees of segregation. One can not simply append 'noise', because this will change the *nature* of equilibrium since noise affects the relative return to making investments. That is, randomness must be embedded within the structure of the economy, not simply added as an afterthought.

I embed frictions by modeling a situation in which each agent is characterized by an informative (but noisy) signal of their investment, and matches are then formed on the basis of the signal. I adopt a specification that, although stylized, is both tractable and transparent. In particular, I assume that there are two types, two investment levels, and two signals. A single parameter determines the probability with which the signal correctly identifies whether the investment was made. One is then able to analyze how equilibrium segregation (and therefore output and inequality) is affected by this parameter. In addition, we can examine how welfare is affected both within and across equilibria.

The first set of results are quite general in that they take a general class of functions that describe coworker spillovers and i) derive the relevant measure of segregation, and ii) describe how this measure affects output and inequality. The relevant measure is shown to be a correlation coefficient, and greater segregation raises both average output and inequality.

From here I impose a particular function from this class and adopt the structure described above in order to analyze i) the existence of 'investment' and 'no-investment' equilibria, and ii) how frictions affect the segregation measure within each class of equilibria. I show that a no-investment equilibrium always exists, and that an investment equilibrium exists when frictions and investment costs are sufficiently low. Frictions never affect segregation in the no-investment equilibrium, and do not always lower segregation in the investment equilibrium. This implies that output and inequality are not always monotonically decreasing in frictions in the investment equilibrium.

I then analyze how frictions affect welfare. I show that welfare is not always monotonically decreasing in frictions in the investment equilibrium and show that a marginal increase in frictions past the point at which an investment equilibrium fails to exist always generates a Pareto improvement.

The paper is related to models of informational frictions in the labor market. The seminal papers were concerned with wage contracting when workers' (education) investments are perfectly observed, but their productivity is unobserved (Arrow (1973) and Spence (1973)), and this has been extended to settings in which productivity is imperfectly observed (Altonji and Pierret (2001), Farber and Gibbons (1996), Lang and Manove (2006) and Blankenau and Camera (2008)). A related set of papers focuses instead on consequences of agents having imperfect information about their own type (e.g. Eckwert and Zilcha (2007) and (2008)). Interaction with coworkers - and therefore the issue of matching - does not play any role in this strand of the literature.

The quality of information may be relevant in a matching set-up because it influences whether a matched pair produces with the appropriate technology. Blankenau and Camera (2006) explore this possibility in a model where productivity-enhancing investment is imperfectly observed within a match. In contrast to the current paper, agents are not motivated to invest by the anticipation of better matches: there, agents are assumed to be randomly re-matched each period, making it uninteresting to study equilibrium segregation (since it is determined exogenously).

Costly search is another common departure from an idealized setting. Equilibrium matching patterns are of central interest both in models where workers match with firms (e.g. Acemoglu (1999), and Albrecht and Vroman (2002)) and in models where workers match with other workers (e.g. Shimer and Smith (2000), Burdett and Coles (1997), Smith (2006)). Apart from the fact that types are observable in these models (although, see Chade (2006)), the important difference between these models and the present model is that agents do not undertake costly actions in order to overcome the inherent search problem (the matching function is unaffected by agents' actions) in these models, whereas this is a central element to the welfare trade-off stressed in this model.[4]

The remainder of the paper is organized as follows. The model is laid out in Section 2, and analyzed in Section 3. Conclusions are drawn in Section 4. Proofs not in the text are contained in the Appendix.

[4]Burdett and Coles (2001) present a search and matching model in which agents invest to make themselves more attractive as marriage partners. Although the investent improves marriage prospects in the sense that it expands the set of agents that are willing to agree to marriage, the model does not have the feature that the rate at which any given type is *encountered* is influenced by the investment decision. There are models in which this feature is central (e.g. Peters and Siow (2002) and Cole, Mailath, and Postelwaite (2001)), but there is no relevant private information: agents preferences are defined over the (observed) investment made by their partner, and not their partner's type *per se*.

2 A Model

2.1 Spillovers and General Results

The results in this section are 'general' in the sense that they apply to any distribution of types. Although the results are expressed in terms of a specific functional form for simplicity, this is generalized in the Appendix.

Consider some worker i, of type θ_i, that works with a worker of type θ_{-i}. There are spillovers in the sense that worker i's productivity, y_i, is influenced by their partner according to:

$$y_i = \theta_i^{1-\phi}\theta_{-i}^{\phi}, \tag{1}$$

where $\phi \in (0, 0.5)$ is a parameter that measures spillovers. Consider four workers, two of type θ and two of type θ'. Since the total output within a match, $Y(\theta_i, \theta_{-i}) = y_i + y_{-i}$, is a symmetric function with a positive cross-partial derivative, it is well-known that an output-maximizing planner will prefer the segregated outcome whereby agents are matched with their own type (Becker (1973)). The purpose of this section is to derive a summary measure, incorporating spillovers and imperfect segregation, so that we are better able to gauge the extent to which frictions force us away from this first-best outcome.

To begin, observe that:

$$\ln y_i = (1-\phi) \cdot \ln \theta_i + \phi \cdot \ln \theta_{-i}. \tag{2}$$

Spillovers are neutral in the sense that they do not affect expected log-output. More generally, expected log-output depends only on the distribution of types (i.e. not on the particular matching pattern):

Result 1. *Expected log-output equals expected log-type:* $\mathbb{E}[\ln y_i] = \mathbb{E}[\ln \theta_i]$.

The variance of log-output, var $[\ln y]$, *does* in general depend on both spillovers and the matching pattern. To describe the latter, define ρ as the correlation coefficient between the random variables $(\ln \theta_i, \ln \theta_{-i})$:

$$\rho \equiv \mathbf{corr}[\ln \theta_i, \ln \theta_{-i}]. \tag{3}$$

Positive (negative) values of ρ imply positive (negative) assortative matching, and the magnitude of the absolute value of ρ provides a measure of the strength of 'assortativeness'. A value of -1 corresponds to perfect negative assortative matching, a value of 0 corresponds to random matching, and a value of 1 corresponds to perfect positive assortative matching.[5]

The value of ρ serves as a relevant measure of segregation in the following sense.

Result 2. *Let* $M \equiv [1 - 2\phi(1-\phi) \cdot (1-\rho)]$*. The variance of log-output is given by:*

$$\textit{var}\left[\ln y_i\right] = M \cdot \textit{var}\left[\ln \theta_i\right], \tag{4}$$

and therefore depends on both spillovers and the matching pattern (as fully captured by ρ*).*

Note that M is decreasing in ϕ and increasing in ρ. An increase in M raises inequality (as captured by the variance of log-output). However, an increase in M also raises average output, $\mathbb{E}[y_i]$. An explicit expression for $\mathbb{E}[y_i]$ can be calculated with certain distributions of types,[6] but even when this is not possible, the following Result shows that we can be sure about the effect of M.

Result 3. *The expected value of output,* $\mathbb{E}[y]$*, is increasing in* M*.*

To the extent that one is interested in i) the variance of log output, and ii) the level of mean output, the results presented here indicate that M is a convenient summary variable to analyze when studying models of imperfect matching. One such model is now developed.

2.2 A Simple Model of Imperfect Matching

There is a continuum of agents, that each have a type $\theta \in \Theta \equiv \{L, H\}$. A proportion ψ of agents have a type of H. An agent's consumption equals their output, which in turns depends on who they are matched with. As discussed above, an agent of type θ_i that is matched with an agent of type θ_{-i} produces and consumes:

$$y_i = \theta_i^{1-\phi} \cdot \theta_{-i}^{\phi}, \tag{5}$$

where $\phi \in [0, 1/2]$ is a parameterization of the degree to which a partner's type matters. I model the process through which matches are formed by analyzing the Bayesian Nash equilibria of a static game with the following timing:

[5] In (perhaps) more familiar terms, ρ^2 coincides with the R^2 that would arise if one were to regress $\ln \theta_{-i}$ on $\ln \theta_i$.

[6] For instance, if θ were log-normally distributed with mean and variance parameters (μ, σ^2), then $\mathbb{E}[y] = \exp(\mu + (1/2)M \cdot \sigma^2)$.

1. Agents privately observe their type, and choose an investment level.

2. Agents are allocated a publicly-observed signal, which is informative of the investment decision.

3. Agents enter the matching phase, which assigns them exactly one partner on the basis of their signal. Payoffs are realized.

In the first stage, agents have the option to make an investment, $x \in \{0, 1\}$. There is no cost associated with $x = 0$, but $x = 1$ costs $c > 0$.

In the second stage, agents are allocated a noisy signal that partially reveals whether or not the investment was made. In particular, each agent has a signal $s_i \in \{0, 1\}$. The signal "0" is indicative of having not made the investment and the signal "1" is indicative of having made the investment in the sense that:

$$\mathbf{Pr}\left[s_i = 0 \mid x_i = 0\right] = \mathbf{Pr}\left[s_i = 1 \mid x_i = 1\right] = \lambda \geq 1/2. \tag{6}$$

In the third stage, matches are formed according to the following (exogenous) procedure. Agents are first divided into two pools; those with $s = 0$ and those with $s = 1$. Each agent is then randomly matched to exactly one other agent from their pool.

A strategy for player i is a probability with which they choose $x = 1$ conditional on each possible type they may be, $\sigma_i(\theta_i)$. The expected payoff received by an agent of type θ_i that plays σ_i when others are playing σ_{-i} is:

$$U(\sigma_i, \sigma_{-i}, \theta_i) \equiv \theta_i^{1-\phi} \cdot V(\sigma_i, \sigma_{-i}) - \sigma_i \cdot c, \tag{7}$$

where

$$V(\sigma_i, \sigma_{-i}) \equiv (1 - \pi(\sigma_i)) \cdot V_0(\sigma_{-i}) + \pi(\sigma_i) \cdot V_1(\sigma_{-i}), \tag{8}$$

where $\pi(\sigma_i) \equiv \sigma_i \cdot \lambda + (1 - \sigma_i) \cdot (1 - \lambda)$ is the probability that an agent obtains the $s = 1$ signal, conditional on investing with probability σ_i, and $V_s(\sigma_{-i})$ is the expected quality of the match conditional on obtaining a signal of s. Since an agent with signal s is matched with a randomly selected agent that also has a signal s, this is:

$$V_s(\sigma_{-i}) \equiv (1 - \xi_s(\sigma_{-i})) \cdot L^{\phi} + \xi_s(\sigma_{-i}) \cdot H^{\phi}, \tag{9}$$

where $\xi_s(\sigma_{-i})$ is the probability with which an agent is a H type conditional on having a signal of s. These conditional probabilities are calculated using Bayes' rule, given the distribution of types and the strategies of others. By focusing on symmetric play whereby agents of type T invest with probability σ_T, this requires that:

$$\xi_0(\sigma_{-i}) \equiv \frac{[1-\pi(\sigma_H)]\cdot\psi}{[1-\pi(\sigma_H)]\cdot\psi+[1-\pi(\sigma_L)]\cdot(1-\psi)}, \text{ and} \tag{10}$$

$$\xi_1(\sigma_{-i}) \equiv \frac{\pi(\sigma_H)\cdot\psi}{\pi(\sigma_H)\cdot\psi+\pi(\sigma_L)\cdot(1-\psi)}. \tag{11}$$

An equilibrium (Bayes-Nash) is an investment function, $\sigma(\theta)$, and beliefs, $\{\xi_0, \xi_1\}$, such that investments are optimal given beliefs, and beliefs are consistent with investments. Some initial observations will help narrow the search for equilibria.

First, notice that $V(\sigma_i, \sigma_{-i})$ is linear in σ_i - implying that V_{σ_i} is a constant. If some type finds it optimal to invest with positive probability, then $V_{\sigma_i} > 0$ (otherwise it would never be worthwhile incurring the positive marginal cost). Furthermore, if $V_{\sigma_i} > 0$, then the net marginal return to investment, $U_{\sigma_i} = \theta_i^{1-\phi} \cdot V_{\sigma_i} - c$, is strictly increasing in type. This is due to complementary nature of interaction (i.e. $y_{\theta_i,\theta_{-i}} > 0$), and not due to any assumption regarding type-dependent investment costs (as in the standard signaling framework). This 'single-crossing' property ensures that the probability of investment is strictly increasing in type in any equilibrium in which some type invests with positive probability. Thus, there is never an equilibrium in which both types invest with probability one.

Second, if type θ finds it optimal to use a strictly mixed strategy, then the indifference condition: $U_{\sigma_i} = 0$ for all $\sigma' \in [0, 1]$, must hold. The fact that U_{σ_i} is strictly increasing in type means that this indifference condition can hold for at most one type. Thus, at most one type can be using a strictly mixed strategy in equilibrium.

Third, if high types are using a mixed strategy (and therefore low types are not investing), then such a situation is 'unstable' in the sense that if high types invested with a slightly greater probability, then all high types would find it optimal to invest with probability one. Equilibria of this nature will sometimes exist, but are disregarded because of the stability issue.

Together, these observations allow us to focus on two classes of equilibria. First, a *no-investment equilibrium* in which no agent invests. Second, an *investment equilibrium* in which high types invest with probability one, and low types invest with some probability strictly less than one (possibly zero).

3 Analysis

In this section I first analyze the existence of both no-investment equilibria and investment equilibria. Then, I derive an expression connecting the model's parameters to the equilibrium degree of segregation. I then analyze how frictions affect equilibrium segregation. Finally, I make welfare comparisons both within- and across equilibria.

3.1 Existence

3.1.1 No-Investment Equilibrium

Proposition 1. *A no-investment equilibrium always exists.*

Since the signal does not provide any information about an agent's type, beliefs in such an equilibrium beliefs are $\xi_0 = \xi_1 = \psi$. Since the signal is effectively noise, there is no benefit to investing as this does not change the distribution of types that one will match with. Although there are no resources wasted on investment, matching in this equilibrium is completely random (and therefore average output is lower than it would be under some degree of positive assortative matching).

3.1.2 Investment Equilibrium

An investment equilibrium arises when all high types invest, and some proportion, $\sigma \in [0, 1)$, of low types invest. Consistency of beliefs gives us:

$$\xi_0(\lambda, \sigma) = \frac{(1-\lambda) \cdot \psi}{(1-\lambda) \cdot [\psi + \sigma(1-\psi)] + \lambda \cdot (1-\sigma)(1-\psi)} \tag{12}$$

$$\xi_1(\lambda, \sigma) = \frac{\lambda \cdot \psi}{\lambda \cdot [\psi + \sigma(1-\psi)] + (1-\lambda) \cdot (1-\sigma)(1-\psi)} \tag{13}$$

Note that ξ_0 is decreasing in λ and increasing in σ, while ξ_1 is increasing in λ and decreasing in σ.

In an investment equilibrium, making the investment raises the probability with which one is matched with a high type (relative to the probability without investing). In particular, if we let Λ_1 be the probability with which an investor is matched with a high type, and Λ_0 be the probability that a non-investor is matched with a high type, then:

$$\Lambda_0 \equiv \lambda \cdot \xi_0 + (1-\lambda) \cdot \xi_1 \tag{14}$$

$$\Lambda_1 \equiv \lambda \cdot \xi_1 + (1-\lambda) \cdot \xi_0. \tag{15}$$

To be sure, Λ_k (for $k \in \{0,1\}$) is simply a weighted average of two conditional probabilities - the probability of being matched with a high type i) conditional on having the high signal, and ii) conditional on having the low signal - where the weight placed on the former is the probability an investor (if $k = 1$) or non-investor (if $k = 0$) obtains the high signal.

Define $G(\lambda, \sigma) \equiv \Lambda_1 - \Lambda_0$ to be the additional probability associated with making the investment, given that all L types invest with probability σ. Note that by subtracting (14) from (15), we get:

$$G(\lambda, \sigma) = (2\lambda - 1) \cdot [\xi_1(\lambda, \sigma) - \xi_0(\lambda, \sigma)]. \tag{16}$$

Since the net benefit associated with investing for an agent of type $T \in \{H, L\}$ is $T^{1-\phi} \cdot G(\lambda, \sigma) \cdot [H^\phi - L^\phi]$, an agent invests if and only if this is greater than or equal to the net cost, c. That is, agents of type T invest if and only if:

$$G(\lambda, \sigma) \geq \tilde{c}_T \equiv \frac{c}{T^{1-\phi} \cdot [H^\phi - L^\phi]}.$$

The properties of G are displayed in the top panel of Figure 1. In particular, G is increasing and convex in λ, and is decreasing in σ (see Appendix for a formal proof).

The existence and nature of equilibrium depends on the value of λ. Three ranges are of interest. First, if $G(\lambda, 0) < \tilde{c}_H$, then no separating equilibrium exists, since high types will not invest even if no low types invest (the inequality is preserved for higher values of σ).

Second, if $\tilde{c}_H \leq G(\lambda, 0) \leq \tilde{c}_L$, then high types find it optimal to invest but low types do not. Higher values of σ do not reverse the latter inequality, implying that the separating equilibrium in which low types invest with probability $\sigma^* = 0$ is unique.

Finally, if $\tilde{c}_L < G(\lambda, 0)$ then $\sigma^* = 0$ is no longer an equilibrium since L types now find it optimal to invest. In any investment equilibrium in which L types invest with positive probability strictly less than one, it must be the case that they are indifferent to doing so: the value of σ^* must satisfy $\tilde{c}_L = G(\lambda, \sigma^*)$. In this region of frictions, there exists a unique value of $\sigma^* \in (0, 1)$, that satisfies this (uniqueness follows from G being continuous and strictly decreasing in σ, with $G(\lambda, 1) = 0$). When low types invest with this probability, they are indifferent between investing and not investing, whereas high types strictly prefer to invest. Thus, there exists an equilibrium in which low types use a strictly mixed strategy. This equilibrium is stable, since if $\sigma < \sigma^*$ then all low types find it optimal to invest (raising σ), whereas if $\sigma > \sigma^*$ then no low types find it optimal to invest (lowering σ). The equilibrium value of σ is depicted in the second panel of Figure 1, and the above results are summarized in the following proposition.

Proposition 2. *A unique stable investment equilibrium exists if*

$$\tilde{c}_H \leq G(\lambda, 0). \tag{17}$$

All H types invest with probability one, and all L types invest with probability $\sigma^* \in [0, 1)$, where σ^* is zero if $\tilde{c}_H \leq G(\lambda, 0) \leq \tilde{c}_L$, and satisfies $G(\lambda, \sigma^*) = \tilde{c}_L$ otherwise.

The fact that $\sigma^* < 1$ in any investment equilibrium reflects the intuition that if all low types invested then all agents behave the same and the signal then contains no useful information, implying that there would be no value in investing. Figure 1 illustrates how the 'existence condition' (17) is equivalent to requiring that frictions be sufficiently small. In particular, a separating equilibrium exists for all $\lambda \geq \underline{\lambda}$.

3.2 Equilibrium Segregation

As with the frictionless models, there is a trade-off between the two equilibrium classes: a no-investment equilibrium wastes fewer resources on investment, but average output is greater in an investment equilibrium due to the higher degree of positive assortative matching. As discussed in the previous section, the key variable in this type of analysis is M, which in turn requires a calculation of the segregation measure, ρ. A general statement characterizing ρ in this model is now given.

Proposition 3. *If the equilibrium probability that a high type is matched with another high type is Z, then ρ is given by:*

$$\rho \equiv \frac{Z - \psi}{1 - \psi}. \tag{18}$$

This result is useful because the value of Z is readily calculated. In the no-investment equilibrium $Z = \psi$ and the correlation coefficient is zero (i.e. random matching). In the investment equilibrium, the probability that a high type matches with a high type, Z, is the same as the probability that an investor matches with a high type, Λ_1 (since all high types are investors). Since $\Lambda_1 > \psi$, we have that matching exhibits some segregation ($\rho > 0$) in the investment equilibrium.[7] In fact, the expression for ρ turns out to be particularly simple in the investment equilibrium.

[7] The fact that $\Lambda_1 > \psi$ in the investment equilibrium follows from the following observations: i) $\xi_1 > \xi_0$ (otherwise there would be no incentive to invest), ii) $\psi = q\xi_1 + (1-q)\xi_0$, where $q \in (0, 1)$ is

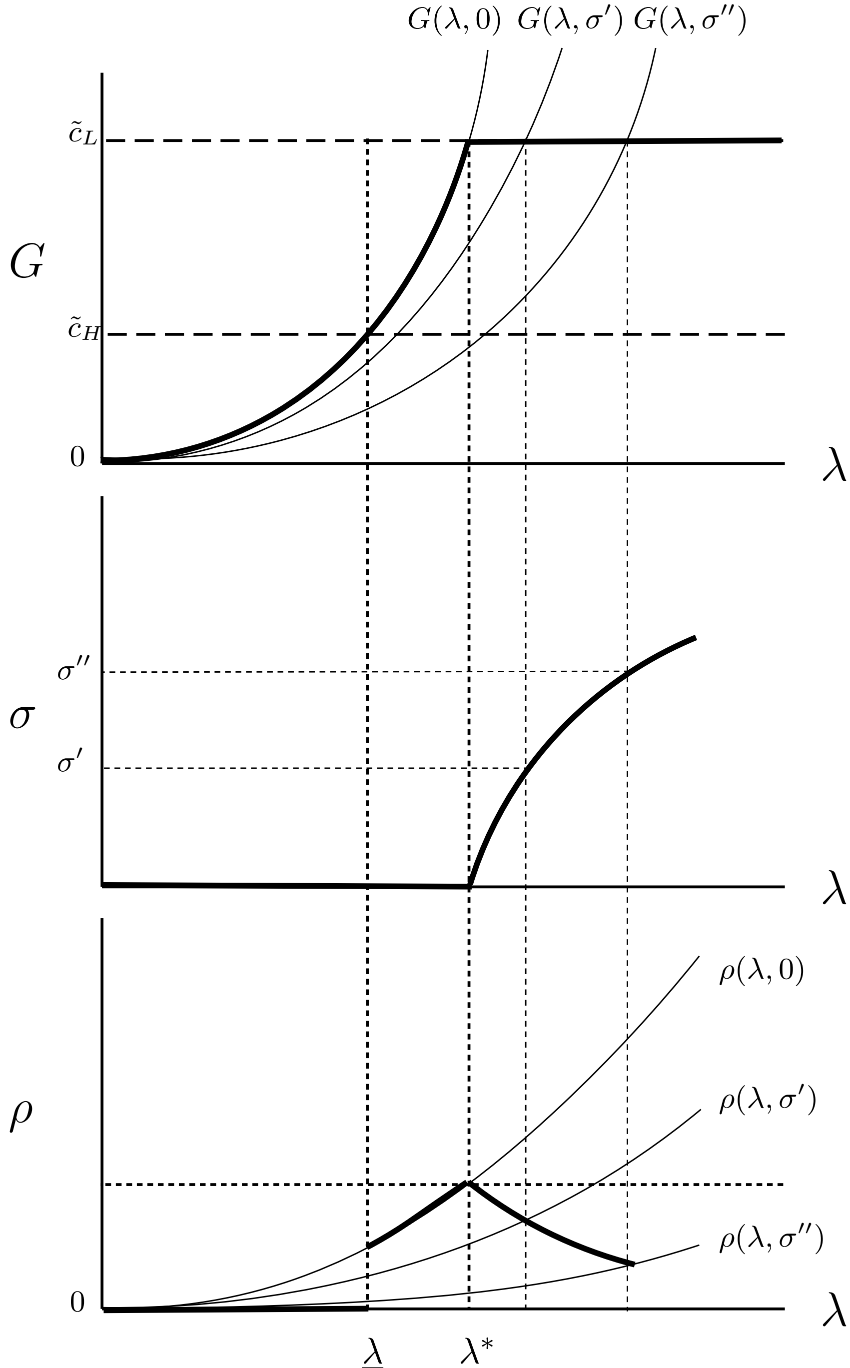

Figure 1: A Geometric Approach to the Investment Equilibrium

Proposition 4. *In the investment equilibrium, where L types invest with probability σ^*, we have*

$$\rho = (1 - \sigma^*) \cdot G(\lambda, \sigma^*). \tag{19}$$

Proof. By expressing the probability of observing a 'cross' match in two different ways, we have:

$$\psi(1 - \Lambda_1) = (1 - \psi)[\sigma^* \Lambda_1 + (1 - \sigma^*)\Lambda_0].$$

Using the fact that $G \equiv \Lambda_1 - \Lambda_0$, we get that

$$\Lambda_1 = \psi + (1 - \psi)(1 - \sigma^*) \cdot G(\lambda, \sigma^*). \tag{20}$$

Using this, along with the observation that $Z = \Lambda_1$ in the investment equilibrium, in (18) produces the result. □

For a given value of σ, the value of ρ is proportional to the value of G. As such, the function ρ inherits essentially the same properties as the function G, as indicated by the three lighter lines in the bottom panel of Figure 1.

3.3 The Impact of Frictions

Having characterized ρ, I now analyze how this variable is affected by frictions.

Proposition 5. *A reduction in frictions (i.e. an increase in λ):*

- *Has no effect on ρ in the no-investment equilibrium,*
- *Raises ρ in the investment equilibrium if $\sigma^* = 0$, and*
- *Lowers ρ in the investment equilibrium if $\sigma^* > 0$.*

Proof. In the no-investment equilibrium, $Z = \psi$ (implying $\rho = 0$) for all λ.

In the investment equilibrium, the results follow from (19). If $\sigma^* = 0$ (or any constant for that matter), then ρ is increasing in λ since G is increasing in λ. If $\sigma^* > 0$, then L types are indifferent to investing and G is a constant ($\tilde{c}_L$). Increases in λ lead to increases in σ^*, which lowers ρ. □

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the proportion of agents that receive the high signal (since both sides of the equation represent the probability that a random agent is of a high type), and iii) $q < \lambda$ (since the non-investors get the high signal with probability $1 - \lambda$, which is less than λ).

The non-monotonicity in the effect of frictions is illustrated in the bottom panel of Figure 1. Low types do not invest in the investment equilibrium when $\lambda \in [\underline{\lambda}, \lambda^*]$, and as such the degree of segregation is decreasing in frictions in this region. However, for $\lambda > \lambda^*$, a positive proportion of low types invest inducing a situation in which segregation is actually increasing in frictions.

One intuition for this non-monotonicity is as follows. Start with the investment equilibrium when frictions are great enough such that no low types are investing. Lowering frictions further will increase equilibrium segregation since investors are more likely to match with other investors, and there is no change in the composition of the investors pool. However, as frictions are further decreased, there may come a point at which some low types find it optimal to invest. Reducing frictions any further then only encourages more non-investors (low types) to invest. Lower frictions tends to increase segregation to the extent that that investors are more likely to match with other investors (the *direct effect*), and tends to decrease segregation to the extent that high types represent a smaller proportion of all investors due to the influx of low types (the *composition effect*).

To see that the composition effect must dominate, notice that low types must be made worse off following a reduction in frictions. This follows from the observation that those low types that remain non-investors are less likely to encounter a high type. But, since low types are indifferent to investing, this means that those low types that remain investors are also worse off. But this can only be true if their probability of encountering a high type decreases. Since this is also the probability that a high type encounters a high type (since all high types are also investors), it must be that segregation also falls.

The non-monotonicity may not arise in equilibrium in some cases because there is nothing that ensures the value of λ^* in Figure 1 lies below one. The following provides a sufficient condition for non-monotonicity.

Corollary 1. *If $\tilde{c}_L < G(1, 0)$, then ρ is inverse-U shaped in λ.*

3.4 Welfare

Although there are many plausible alternatives, define welfare to be the average payoff:

$$W \equiv \int_I u_i di = \mu_y - c \cdot \int_I \sigma_i di \tag{21}$$

I now discuss how welfare changes both *within* and *across* equilibria.

3.4.1 Within Equilibria

We know from the above discussion that frictions do not affect welfare in the no-investment equilibrium, since neither segregation nor the propensity to invest are affected by frictions. The remainder of the section focuses on an investment equilibrium.

When it is only the high types that invest, lower frictions raise welfare because matching displays greater segregation, raising average output (without any change in investment costs). However, when some low types are investing, welfare is reduced on two fronts: i) matching becomes less segregated, lowering average output, and ii) more agents are incurring investment costs. Thus, total welfare reacts the same way to frictions as does segregation. This leads to the following Corollary.

Corollary 2. *If $\tilde{c}_L < G(1,0)$, then welfare in the investment equilibrium is inverse-U shaped in λ.*

The welfare of high types also reacts to frictions in the same way as does segregation (lower frictions always reduces the welfare of low types). It then follows that total welfare tracks the welfare of high types (as a function of frictions). This implies that the welfare-reducing effects of lower frictions is robust in the following sense.

Proposition 6. *If a decrease in frictions produces a decrease in average payoffs, then it also produces a decrease in all agents' payoffs (i.e. represents a Pareto deterioration).*

In other words, the particular choice of welfare weights is immaterial in reaching the conclusion that lower frictions can lower welfare.

3.4.2 Across Equilibria

Let $\underline{\lambda}$ be the smallest value of λ such that an investment equilibrium exists. To make this section relevant, I assume throughout that $\underline{\lambda} < 1$.

Welfare in the no-investment equilibrium, W^N, is unaffected by frictions. This forms a natural benchmark against which to compare welfare in the investment equilibrium. Let welfare in the investment equilibrium be denoted $W^I(\lambda)$.

To begin, note that the investment equilibrium can never Pareto dominate the no-investment equilibrium (since the low types are always worse off in the investment equilibrium). The reverse is not true.

Proposition 7. *There always exists a region of frictions, $[\underline{\lambda}, \lambda']$, where $\underline{\lambda} < \lambda'$, for which the no-investment equilibrium Pareto dominates the investment equilibrium for all $\lambda \in [\underline{\lambda}, \lambda']$.*

Figure 2 provides a geometric proof of this proposition. The figure plots the payoffs obtained by a high type as a function of frictions, in various scenarios. The horizontal line is the payoff in the no-investment equilibrium, and is simply the expected output produced by a high type in that equilibrium. The two dark lines represent the expected output of a high type given that all other high types, and no low types, are investing. The upper line corresponds to the expected output obtained if the high type invests, and the lower line corresponds to the expected output if the high type does not invest. The two lines meet when $\lambda = 1/2$ since the signal is uninformative in this case. The lower of these two lines also represents the payoff of a high type when they do not invest, since investment costs are zero. To get the payoff from investing, the upper line needs to be shifted down by the investment cost, as indicated. The value of $\underline{\lambda}$ is obtained by finding the point at which the payoff lines cross: for values of λ below this point, the payoff to not investing is higher. The figure shows that the payoff to a high type when frictions are at $\underline{\lambda}$ is less than the payoff associated with the no-investment equilibrium (as indicated by the leftmost set of arrows). By continuity, the welfare of high types remains lower in the investment equilibrium for values of λ marginally above $\underline{\lambda}$. For low types, it is always the case that the investment equilibrium delivers a lower payoff than the no-investment equilibrium. Thus, there is a range of λ, just above $\underline{\lambda}$, for which all agents get a lower payoff in the investment equilibrium than in the no-investment equilibrium.

As frictions decrease (i.e. as λ increases from $\underline{\lambda}$), the payoff to high types increases because we know that $\sigma^* = 0$ in the investment equilibrium at $\underline{\lambda}$ (since high types are indifferent, and low types get a lower payoff from investment). If low types have not started to invest by the time λ reaches the point indicated by $\tilde{\lambda}$, then $\lambda' = \tilde{\lambda}$. This is because this is the point at which high types get the same payoff across equilibria. If, on the other hand, low types do start to invest prior to $\tilde{\lambda}$, then $\lambda' = 1$. This is because the payoff to high types falls with λ once low types start to invest.

The case of $\lambda' = 1$ is not very interesting since it implies that the investment equilibrium is Pareto dominated whenever one exists. The following result, which discusses the weaker notion of welfare dominance[8], applies to the more interesting

[8] By this I mean that welfare dominance is weaker than Pareto dominance (since the former is

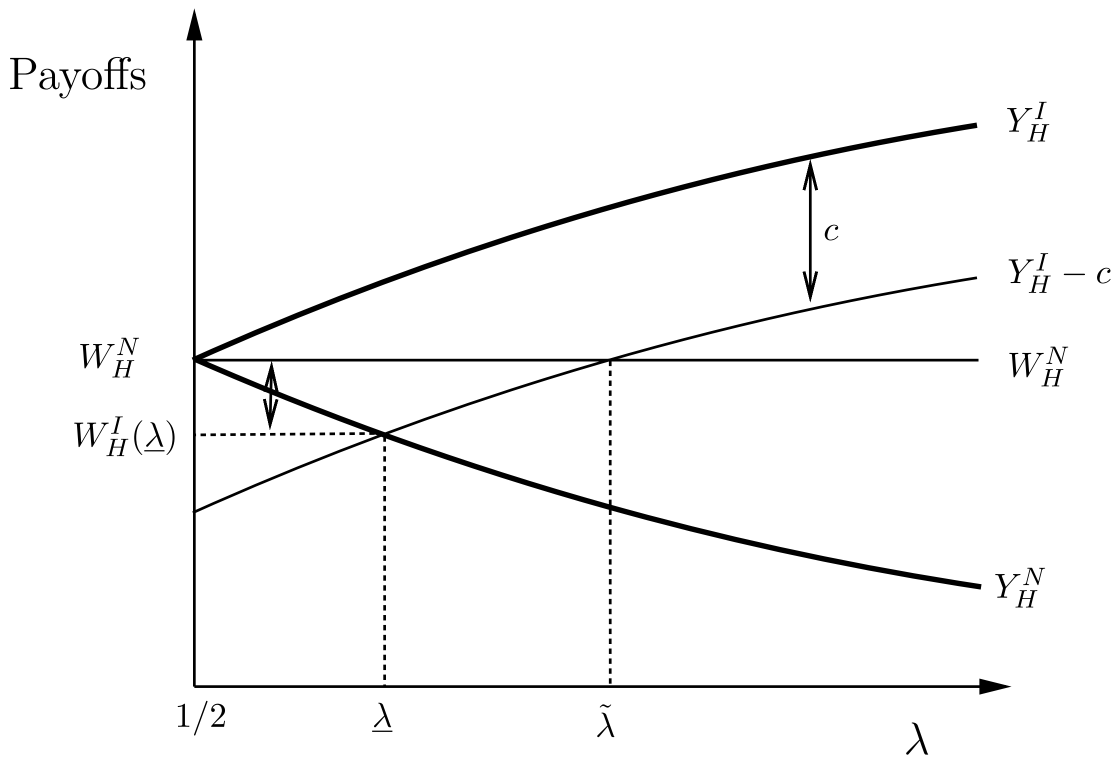

Figure 2: A Geometric Proof of Proposition 7

case of $\lambda' < 1$.

Proposition 8. *If $\lambda' < 1$, then there exists a λ'', where $\lambda' < \lambda''$, such that the no-investment equilibrium welfare dominates the investment equilibrium for all $\lambda \in [\underline{\lambda}, \lambda'']$.*

The proof of this follows from noting that the no-investment equilibrium welfare dominates the investment equilibrium at $\lambda = \lambda'$. The logic is that the high types get the same payoff across the equilibria whereas the low types are strictly worse off in the investment equilibrium. Thus, $W^N > W^I(\lambda')$, and continuity ensures that $W^N > W^I(\lambda' + \varepsilon)$ for small enough $\varepsilon > 0$.

It may very well be the case that the no-investment equilibrium welfare dominates the investment equilibrium for all levels of frictions (i.e. $\lambda'' = 1$). Thus, if the investment equilibrium is to welfare dominate the no-investment equilibrium, then it must be that $\lambda'' < 1$. But even then, the fact that welfare in the investment equilibrium ($W^I(\lambda)$) can be inverse U-shaped means that there is no guarantee that this welfare dominance does not reverse itself at even lower frictions (higher λ).

4 Conclusions

I have developed a model of matching with hidden types in which the capacity to secure a better match via unproductive investment is diminished by the existence of frictions. A central result is that equilibrium segregation (and therefore output and inequality) and welfare are generally not monotonic in the degree of frictions. Although lower frictions allow investors to be more readily matched with each other, lower frictions raise the return to investing which can change the composition of the pool of investors, reducing the degree of segregation.

The model broadens our understanding of the central welfare trade-off inherent in models of premarital investment with hidden types, and represents a first step toward a fuller analysis. Specifically, extending this model to a dynamic setting would allow for a richer understanding of the interaction between the informational frictions stressed here and search frictions, thereby providing additional insight into the relationship between skill spillovers, output and inequality.

implied by, but does not imply, the latter).

References

ACEMOGLU, D. (1999): "Changes in Unemployment and Wage Inequality: An Alternative Theory and Some Evidence," *American Economic Review*, 89(5), 1259–1278.

ALBRECHT, J., AND S. VROMAN (2002): "A Matching Model with Endogenous Skill Requirements," *International Economic Review*, 43(1), 283–305.

ALTONJI, J. G., AND C. R. PIERRET (2001): "Employer Learning And Statistical Discrimination," *The Quarterly Journal of Economics*, 116(1), 313–350.

ARROW, K. J. (1973): "Higher education as a filter," *Journal of Public Economics*, 2(3), 193–216.

BAGWELL, L. S., AND B. D. BERNHEIM (1996): "Veblen Effects in a Theory of Conspicuous Consumption," *American Economic Review*, 86(3), 349–73.

BECKER, G. S. (1973): "A Theory of Marriage: Part I," *Journal of Political Economy*, 81(4), 813–46.

BIDNER, C. (2008): "A Spillover-Based Theory of Credentialism," Working paper, University of British Columbia.

BLANKENAU, W., AND G. CAMERA (2006): "A Simple Economic Theory of Skill Accumulation and Schooling Decisions," *Review of Economic Dynamics*, 9(1), 93–115.

——— (2008): "Public Spending on Education and the Incentives for Student Achievement," *Economica*, Forthcoming.

BURDETT, K., AND M. G. COLES (1997): "Marriage and Class," *The Quarterly Journal of Economics*, 112(1), 141–68.

——— (2001): "Transplants and Implants: The Economics of Self-Improvement," *International Economic Review*, 42(3), 597–616.

CHADE, H. (2006): "Matching with noise and the acceptance curse," *Journal of Economic Theory*, 127(1), 81–113.

COLE, H. L., G. J. MAILATH, AND A. POSTLEWAITE (2001): "Efficient Non-Contractible Investments in Large Economies," *Journal of Economic Theory*, 101(2), 333–373.

COSTRELL, R. M., AND G. C. LOURY (2004): "Distribution of Ability and Earnings in a Hierarchical Job Assignment Model," *Journal of Political Economy*, 112(6), 1322–1363.

ECKWERT, B., AND I. ZILCHA (2007): "The Effect of Better Information on Income Inequality," *Economic Theory*, 32(2), 287–307.

——— (2008): "Efficiency of Screening and Labor Income Inequality," *Journal of Public Economic Theory*, 10(1), 77–98.

FARBER, H. S., AND R. GIBBONS (1996): "Learning and Wage Dynamics," *The Quarterly Journal of Economics*, 111(4), 1007–47.

FRANK, R. H. (1985): "The Demand for Unobservable and Other Nonpositional Goods," *American Economic Review*, 75(1), 101–16.

HOPKINS, E. (2005): "Job Market Signalling of Relative Position, or Becker Married to Spence," ESE Discussion Papers 134, Edinburgh School of Economics, University of Edinburgh.

HOPPE, H. C., B. MOLDOVANU, AND A. SELA (2009): "The Theory of Assortative Matching Based on Costly Signals," *The Review of Economic Studies*, 76(1), 253–281.

KREMER, M. (1993): "The O-Ring Theory of Economic Development," *The Quarterly Journal of Economics*, 108(3), 551–75.

KREMER, M., AND E. MASKIN (1996): "Wage Inequality and Segregation," Harvard Institute of Economic Research Working Papers 1777, Harvard - Institute of Economic Research.

LANG, K., AND M. MANOVE (2006): "Education and Labor-Market Discrimination," NBER Working Papers 12257, National Bureau of Economic Research, Inc.

PESENDORFER, W. (1995): "Design Innovation and Fashion Cycles," *American Economic Review*, 85(4), 771–92.

PETERS, M., AND A. SIOW (2002): "Competing Premarital Investments," *Journal of Political Economy*, 110(3), 592–608.

REGE, M. (2007): "Why do people care about social status?," *Journal of Economic Behavior and Organization*, Forthcoming.

SATTINGER, M. (1993): "Assignment Models of the Distribution of Earnings," *Journal of Economic Literature*, 31(2), 831–80.

SHIMER, R., AND L. SMITH (2000): "Assortative Matching and Search," *Econometrica*, 68(2), 343–370.

SMITH, L. (2006): "The Marriage Model with Search Frictions," *Journal of Political Economy*, 114(6), 1124–1146.

SPENCE, A. M. (1973): "Job Market Signaling," *The Quarterly Journal of Economics*, 87(3), 355–74.

Appendix

A A Generalization of the Productivity Function

Let the productivity of agent i be given by a function, $y : \mathbb{R}^2_+ \to \mathbb{R}_+$, that can be expressed as:

$$y(\theta_i, \theta_{-i}) = g^{-1}\left((1-\phi) \cdot g(\theta_i) + \phi \cdot g(\theta_{-i})\right), \tag{22}$$

where $g : \mathbb{R}_+ \to \mathbb{R}$ is a strictly monotone function with the property that $-g''(x)/g'(x) > 0$ for all $x \geq 0$ (i.e. g is any strictly increasing and concave function, or any strictly decreasing and convex function). Monotonicity ensures the inverse is well-defined, and the latter condition ensures that types are complements (i.e. $y_{\theta_i \theta_{-i}} > 0$). It is straightforward to verify that y is increasing in both arguments.

This specification is general enough to encompass the class of CES functions: simply let $g(x) = x^\gamma$ for $\gamma \in \{\gamma \mid \gamma \neq 0, \gamma < 1\}$. The case of $\gamma = 0$ corresponds to Cobb-Douglas case used in the text (and g is the natural log). Types become perfect substitutes as $\gamma \to 1$, and become perfect complements as $\gamma \to -\infty$. The specification is not restricted to the CES functions however, since g can by *any* strictly increasing and concave or strictly decreasing and convex function.

Since we have:

$$g(y_i) = (1-\phi) \cdot g(\theta_i) + \phi \cdot g(\theta_{-i}), \tag{23}$$

arguments virtually identical to those used in the proof of Results 1 and 2 can be used to show i) that the expected value of $g(y_i)$ equals the expected value of $g(\theta_i)$, and therefore depends only on the distribution of types, and ii) that the variance of $g(y)$ is:

$$\text{var}\left[g(y_i)\right] = M \cdot \text{var}\left[g(\theta_i)\right], \tag{24}$$

where M is defined as in the text. The proof of Result 3 below establishes that expected output is also increasing in M. To summarize, the analysis in the text generalizes quite naturally to this wider class of functions, and does not rely on specific features of the Cobb-Douglas form, such as separability.

B Proofs

B.1 Proof of Results 1 and 2

Proof. Follows from:

$$\begin{aligned}\mathbb{E}[\ln y_i] &= (1-\phi)\cdot\mathbb{E}[\ln\theta_i] + \phi\cdot\mathbb{E}[\ln\theta_{-i}] \\ &= (1-\phi)\cdot\mathbb{E}[\ln\theta_i] + \phi\cdot\mathbb{E}[\ln\theta_i] = \mathbb{E}[\ln\theta_i].\end{aligned}$$

For the variance expression,

$$\begin{aligned}\textbf{var}\,[\ln y_i] &= (1-\phi)^2\cdot\textbf{var}\,[\ln\theta_i] + \phi^2\cdot\textbf{var}\,[\ln\theta_{-i}] - 2\phi(1-\phi)\cdot\textbf{cov}[\ln\theta_i,\ln\theta_{-i}] \\ &= [(1-\phi)^2+\phi^2]\cdot\textbf{var}\,[\ln\theta_i] - 2\phi(1-\phi)\cdot\textbf{var}\,[\ln\theta_i]\cdot\rho\end{aligned}$$

Simplifying this gives the result. □

B.2 Proof of Result 3

The proof covers the general case discussed above. The case presented in the text corresponds to g being the natural log.

Proof. Let x_i represent $g(y_i)$. From the first two results, we know that x is distributed with a mean of $\mu \equiv \mathbb{E}[g(\theta)]$, and a variance of $\sigma^2 \equiv M\cdot\textbf{var}[g(\theta)]$. Let $q(x) \equiv g^{-1}(x)$, so that $y_i = q(x_i)$. Since we are interested in the expected value of y, we are interested in the expected value of $q(x)$. The assumptions on g ensure that q is convex (twice differentiate both sides of the identity $g(q(x)) = x$). Given μ and σ, we seek to determine how $E[q(x)]$ changes with σ.

Define $\varepsilon_i \equiv (x_i-\mu)/\sigma$. By construction, ε has a mean of zero and a variance of one. If we let the distribution of ε be denoted by H, then we can write:

$$E[q(x)\mid\sigma] = \int q\left(\mu+\sigma\cdot\varepsilon\right)dH(\varepsilon). \tag{25}$$

The derivative of this with respect to σ is:

$$\frac{d}{d\sigma}\{E[q(x)\mid\sigma]\} = \int q'\left(\mu+\sigma\cdot\varepsilon\right)\varepsilon dH(\varepsilon). \tag{26}$$

We are interested in the sign of this expression. The fact that q is convex implies i) if $q'(\cdot) > 0$, then $q'(\mu+\sigma\cdot\varepsilon)$ is an increasing function of ε, and ii) if $q'(\cdot) < 0$, then

$q'(\mu+\sigma\cdot\varepsilon)$ is a decreasing function of ε. In either case, $[q'(\mu+\sigma\cdot\varepsilon)-q'(\mu)]\cdot\varepsilon$ is strictly positive (for all $\varepsilon\neq 0$). Therefore:

$$\int [q'(\mu+\sigma\cdot\varepsilon)-q'(\mu)]\cdot\varepsilon dH(\varepsilon)>0. \tag{27}$$

Splitting this apart, and noting that

$$\int q'(\mu)\,\varepsilon dH(\varepsilon)=q'(\mu)\cdot\int \varepsilon dH(\varepsilon)=0, \tag{28}$$

gives us the desired result:

$$\int q'(\mu+\sigma\cdot\varepsilon)\,\varepsilon dH(\varepsilon)>0. \tag{29}$$

Thus, expected output is increasing in M, since this variable increases σ without affecting μ. □

B.3 Properties of G (underlying Proposition 2)

Claim 1. *The function $G(\lambda,\sigma)$ is i) increasing in λ, ii) decreasing in σ, and iii) convex in λ.*

Proof. The fact that ξ_1 and $-\xi_0$ are both increasing in λ and decreasing in σ, along with (16), proves i) and ii).

For some differentiable function, f, let:

$$\xi(f(\lambda))\equiv\frac{f(\lambda)\cdot\psi}{f(\lambda)\cdot[\psi+\sigma(1-\psi)]+(1-f(\lambda))\cdot(1-\sigma)(1-\psi)}$$

Notice that:

$$\frac{\partial}{\partial\lambda}\{\xi\}=f'(\cdot)\cdot\xi'(\cdot)\ \text{ and }\ \frac{\partial^2}{\partial\lambda^2}\{\xi\}=f''(\cdot)\xi'(\cdot)+f'(\cdot)\xi''(\cdot).$$

Since ξ_1 equals $\xi(\cdot)$ when $f(\lambda)=\lambda$ and ξ_0 equals $\xi(\cdot)$ when $f(\lambda)=1-\lambda$, it follows that:

$$\frac{\partial}{\partial\lambda}\{\xi_1\}=-\frac{\partial}{\partial\lambda}\{\xi_0\}\ \text{ and }\ \frac{\partial^2}{\partial\lambda^2}\{\xi_1\}=-\frac{\partial^2}{\partial\lambda^2}\{\xi_0\}.$$

From (16), we have:

$$\frac{\partial}{\partial\lambda}\{G\}=(2\lambda-1)\cdot\frac{\partial}{\partial\lambda}\{[\xi_1-\xi_0]\}+2\cdot[\xi_1-\xi_0],$$

which implies that

$$\frac{\partial^2}{\partial\lambda^2}\{G\}=(2\lambda-1)\cdot\frac{\partial^2}{\partial\lambda^2}\{[\xi_1-\xi_0]\}+2\cdot\frac{\partial}{\partial\lambda}\{[\xi_1-\xi_0]\}+2\cdot\frac{\partial}{\partial\lambda}\{[\xi_1-\xi_0]\},$$

which, using the above results gives

$$\frac{\partial^2}{\partial\lambda^2}\{G\} = 2(2\lambda-1)\cdot\frac{\partial^2}{\partial\lambda^2}\{\xi_1\} + 8\cdot\frac{\partial}{\partial\lambda}\{\xi_1\}.$$

Since

$$\frac{\partial^2}{\partial\lambda^2}\{\xi_1\} = \frac{\partial}{\partial\lambda}\{\xi_1\}\cdot 2\frac{(1-2\sigma\psi)}{\psi}\xi_1 \geq -2\frac{\partial}{\partial\lambda}\{\xi_1\},$$

we have

$$\frac{\partial^2}{\partial\lambda^2}\{G\} \geq [8-4(2\lambda-1)]\frac{\partial}{\partial\lambda}\{\xi_1\} > 0,$$

which proves iii). □

B.4 Proof of Proposition 3

Proof. There are ψZ high-high matches, and $\psi(1-Z)$ high-low matches. This last observation implies that there are also $\psi(1-Z)$ low-high matches, leaving $(1-\psi Z - 2\psi(1-Z))$ low-low matches. The expected value of log output is therefore:

$$\begin{aligned}\mu_{\ln y} = Z\psi\cdot\ln H + \psi(1-Z)\cdot[\ln H + \ln L] \\ + [1 - Z\psi - 2\psi(1-Z)]\cdot\ln L,\end{aligned} \tag{30}$$

which equals $\psi\cdot\ln H + (1-\psi)\cdot\ln L$. This could have been anticipated by the application of the more general formula, noting that $\mu_{\ln\theta} = \psi\cdot\ln H + (1-\psi)\cdot\ln L$.

After (considerable) manipulation, the variance of log-output can be written as:

$$\sigma^2_{\ln y} = \left[1 - 2\phi(1-\phi)\cdot\left(\frac{1-Z}{1-\psi}\right)\right]\cdot\psi(1-\psi)\cdot[\ln H - \ln L]^2. \tag{31}$$

Since $\sigma^2_{\ln\theta} = \psi(1-\psi)\cdot[\ln H - \ln L]^2$, a comparison with the general formula reveals that $(1-Z)/(1-\psi) = 1-\rho$, producing the result. □

B.5 Proof of Proposition 5

Proof. The first point follows immediately from the observation that $Z = \psi$ in a no-investment equilibrium.

In the investment equilibrium, let Λ_1 be the probability with which an investor is matched with a high type, and Λ_0 be the probability with which a non-investor is matched with a high type. As argued in the text, $Z = \Lambda_1$. Thus, the proof establishes that Λ_1 is increasing in λ when $\sigma^* = 0$ and is decreasing when $\sigma^* > 0$.

By the definition of G, we have

$$G = \Lambda_1 - \Lambda_0. \tag{32}$$

By expressing the probability of observing a 'cross' match in two different ways, it must also be that:

$$\psi(1-\Lambda_1) = (1-\psi)[\sigma\Lambda_1 + (1-\sigma)\Lambda_0].$$

Together, these imply that

$$\Lambda_1 = \psi + (1-\psi)(1-\sigma)\cdot G(\lambda,\sigma). \tag{33}$$

If $\sigma^* = 0$, then Λ_1 is increasing in λ since $G(\lambda, 0)$ is increasing in λ. If $\sigma^* > 0$, then G is a constant ($c \cdot [L^{1-\phi} \cdot [H^\phi - L^\phi]]^{-1}$), and Λ_1 is decreasing in σ^*. The result follows since σ^* is increasing in λ (since $G(\lambda,\sigma)$ is increasing in λ and decreasing in σ, but equal to a constant when $\sigma^* > 0$). □