Online Appendix: "Investing in Skill and Searching for Coworkers: Endogenous Participation in a Matching Market"

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This online appendix accompanies our paper "Investing in Skill and Searching for Coworkers: Endogenous Participation in a Matching Market" published in the American Economic Journal: Microeconomics. We provide details of the illustrations used in paper as well as the details of an extension beyond the binary investment case.

Online Appendix A: Illustration Details

A1. Details of Illustration Depicted in Figure 1

When F is uniform, we have

$$G(\theta \mid \hat{\theta}) = \begin{cases} 1 & \text{for } \theta > 1 \\ \frac{\theta - \hat{\theta}}{1-\hat{\theta}} & \text{for } \theta \in [\hat{\theta}, 1] \\ 0 & \text{otherwise,} \end{cases} \tag{A1}$$

so that R_k satisfies

$$R_k = \frac{\alpha}{r} \cdot \int_{\max\{R_k, \hat{\theta}\}}^{R_{k-1}} \left[\frac{\theta' - R_k}{1 - \hat{\theta}} \right] \cdot d\theta'. \tag{A2}$$

One can derive R_k as an explicit function of R_{k-1} by considering two cases.

If $R_k \geq \hat{\theta}$ then R_k satisfies

$$R_k = \frac{\alpha}{r} \cdot \int_{R_k}^{R_{k-1}} [\theta' - R_k] \cdot dG(\theta' \mid \hat{\theta}), \tag{A3}$$

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which, by using the uniform functional form, gives

(A4) $$2 \cdot \frac{r}{\alpha} \cdot (1 - \hat{\theta}) \cdot R_k = (R_{k-1} - R_k)^2,$$

which, expressing R_k as an explicit function of R_{k-1}, is:

(A5) $$R_k = \frac{r}{\alpha} \cdot (1 - \hat{\theta}) + R_{k-1} - \sqrt{\frac{r}{\alpha} \cdot (1 - \hat{\theta}) \cdot \left(\frac{r}{\alpha} \cdot (1 - \hat{\theta}) + 2 \cdot R_{k-1}\right)}.$$

Alternatively, if $R_k < \hat{\theta}$ then R_k satisfies

(A6) $$R_k = \frac{\alpha}{r} \cdot \int_{\hat{\theta}}^{R_{k-1}} [\theta' - R_k] \cdot dG(\theta')$$

which is

(A7) $$2 \cdot \frac{r}{\alpha} \cdot (1 - \hat{\theta}) \cdot R_k = R_{k-1}^2 - \hat{\theta}^2 - 2 \cdot R_k \cdot (R_{k-1} - \hat{\theta}),$$

so that

(A8) $$R_k = \frac{1}{2} \cdot \frac{R_{k-1}^2 - \hat{\theta}^2}{\frac{r}{\alpha} \cdot (1 - \hat{\theta}) + R_{k-1} - \hat{\theta}}.$$

In calculating these boundaries, start with $k = 1$ since R_1 is defined on the entire type space $[0, 1]$. Then proceed to $k = 2$, noting that R_2 is defined at the points for which $R_1 > \hat{\theta}$. Successive values of k can be used to compute R_k, noting that R_k is defined at the points for which $R_{k-1} > \hat{\theta}$. The value of K at $\hat{\theta}$ is that value of k for which $R_k \leq \hat{\theta}$. The class boundaries are then $\theta_k = R_k$ for $k = 1, ..., K - 1$ and $\theta_K = \hat{\theta}$.

A2. Details of Illustration Depicted in Figure 5

Using the uniform distribution of type, we have that $\tilde{\theta}_k$ satisfies

(A9) $$\tilde{\theta}_k = \frac{\alpha}{r} \cdot \left[\frac{\tilde{\theta}_{k-1} - \tilde{\theta}_k}{2}\right],$$

which is

(A10) $$\tilde{\theta}_k = \frac{1}{1 + 2 \cdot \frac{r}{\alpha}} \cdot \tilde{\theta}_{k-1}.$$

Using the initial condition that $\tilde{\theta}_0 = 1$, we have an explicit expression for $\tilde{\theta}_k$:

$$\tilde{\theta}_k = \left[\frac{1}{1 + 2 \cdot \frac{r}{\alpha}}\right]^k. \tag{A11}$$

Furthermore,

$$m_k(\hat{\theta}) = \frac{1}{1 + \frac{r}{\alpha}} \cdot \frac{\hat{\theta} + \theta_{k-1}}{2}. \tag{A12}$$

Online Appendix B: More than two investment levels

This section shows that our key results do not hinge on the binary investment assumption. Specifically, we show via a simple example how acceptance constrained equilibria continue to exist when we allow a large, but discrete, investment space. We also show how a richer investment space generates new features too; we leave a more comprehensive treatment to future research.

Suppose that agents can invest in discrete units so that $x_i \in \{0, 1, 2, ...\}$ and $s_i = x_i \cdot \theta_i$. The cost of x units of investment is $c(x)$, which is strictly increasing. Consider an acceptance constrained equilibrium with one class in which all investors invest $x > 0$. Given a cut-off type of $\hat{\theta}^*$, the asset value equation tells us that for $\theta \in [\hat{\theta}^*, 1]$, $U(\theta \mid \hat{\theta}^*)$ satisfies:

$$rU(\theta \mid \hat{\theta}^*) = \frac{\alpha}{r} \cdot [x \cdot \theta \cdot x \cdot \mathbb{E}[\theta' \mid \theta' \in [\hat{\theta}^*, 1] - rU(\theta \mid \hat{\theta}^*)].$$

A type θ' will be accepted by a type $\theta \in [\hat{\theta}^*, 1]$ as long as $x \cdot \theta' \cdot x \cdot \theta / r \geq U(\theta \mid \hat{\theta}^*)$, or

$$\theta' \geq R(\theta) \equiv rU(\theta \mid \hat{\theta}^*)/(x^2 \cdot \theta).$$

Using the asset value equation, this is:

$$R(\theta) = \frac{\alpha}{r} \cdot [\mathbb{E}[\theta' \mid \theta' \in [\hat{\theta}^*, 1] - R(\theta)].$$

Thus, as before, $R(\theta)$ is a constant for those in the first class. Furthermore, since $\hat{\theta}^*$ satisfies $R(\hat{\theta}^*) = \hat{\theta}^*$, it is straightforward to see that $\hat{\theta}^* = \hat{\theta}_1$ (where $\hat{\theta}_1$ is that used in the main analysis).

To produce closed-form expressions, suppose that the distribution of types is uniform on $[0, 1]$. Assuming that a worker is accepted, their payoff when investing x when others are investing x^* is:

$$U(\theta, x \mid \hat{\theta}_1, x^*) = \frac{1}{r} \cdot \frac{\alpha}{r + \alpha} \cdot x^* \cdot \frac{1 + \hat{\theta}_1}{2} \cdot x \cdot \theta = Z \cdot x^* \cdot x \cdot \theta,$$

where $Z \equiv \frac{1}{r} \cdot \frac{\alpha}{r+\alpha} \cdot \frac{1+\hat{\theta}_1}{2}$. For this to constitute an equilibrium a series of conditions must be met. First, all investors prefer investing x to not investing (i.e. $x = 0$). It is sufficient to verify that

$$U(\hat{\theta}_1, x \mid \hat{\theta}_1, x) \geq c(x),$$

which holds by making $c(x)$ small enough. Second, all investors prefer investing x to any other level x_i. This implies two conditions: (i) that $0 < x_i < x$ be deterred, and (ii) that $x_i > x$ be also deterred. For (i) a sufficient condition is that the highest type, when investing $x_i = x - 1$, is not accepted in equilibrium. That is, $1 \cdot (x-1) < x \cdot \hat{\theta}_1$ (the left side is the skill of the highest type when they cut their investment by one step and the right is the minimum skill required for acceptance in equilibrium). This amounts to

$$\hat{\theta}_1 > 1 - (1/x),$$

which holds for x low enough (of course, this always holds when $x = 1$) and can be made to hold for any x by setting α high enough. For (ii), it is sufficient to check that the highest types do not want to raise their investment, which holds when $c(x+1)$ sufficiently high:

$$U(1, x+1 \mid \hat{\theta}_1, x) - c(x+1) < U(1, x \mid \hat{\theta}_1, x) - c(x)$$

Last, all non-investors are acting optimally. As before, the fact that they would never be accepted ensures that non-investors do not want to invest x (or less). The only remaining possibility is that some non-investors would profit by investing in excess of x, say $x+1$ (which necessarily gives those close to the marginal investor an acceptable skill level). To deter this one needs,

$$U(\hat{\theta}_1, x+1 \mid \hat{\theta}_1, x) < c(x+1),$$

which, again, is ensured by setting $c(x+1)$ high enough.

From this we see how even the simplest of acceptance constrained equilibria - one in which all investors invest the same amount and there is a single class - continue to exist in this setting. Richer forms of acceptance constrained equilibria are clearly possible, but here our intention is to highlight that it is the *discreteness* of investment possibilities that is important. We utilize the binary investment decision in our exposition for the simplicity it affords in illuminating the key mechanisms at play. For instance, even analyzing the case with three investment possibilities induces a new form of multiplicity whereby a one-class acceptance constrained equilibrium in which all investors choose $x = 1$ coexists with one in which they invest $x = 2$. To see this, note that the interesting constraints that must be satisfied are, (1) when $x = 1$, a marginal non-investor does not want to invest $x_i = 2$ to be accepted:

$$Z \cdot 1 \cdot 2 \cdot \hat{\theta}_1 - c(2) < 0,$$

and (2) when $x = 2$, a marginal investor wants to invest $x = 2$:

$$Z \cdot 2 \cdot 2 \cdot \hat{\theta}_1 - c(2) > 0.$$

Clearly, a cost function such that

$$c(2) \in (Z \cdot 2 \cdot \hat{\theta}_1, Z \cdot 4 \cdot \hat{\theta}_1)$$

will satisfy both of these. The other constraints can be satisfied by taking $c(\cdot)$ such that $c(1)$ low enough, $c(3)$ large enough, and $\hat{\theta}_1$ such that $\hat{\theta}_1 > 1 - (1/2) = 1/2$ (this requires $\alpha > 2 \cdot r$ with a uniform distribution). The source of this multiplicity is quite distinct from the sources we emphasize in the main model, and therefore leave a comprehensive treatment for future research.