Matching for Social Mobility with Unobserved Heritable Characteristics*

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Abstract

We analyse the intergenerational transmission of 'innate ability', focusing on the role of marital sorting. The heritability of ability induces a concern for the ability of potential spouses *independently* of a concern for their earning capacity. Marriages form on the basis of beliefs about ability (since it is not observed) as well earning capacity (which is observed). Beliefs are informed by earning capacity, but crucially also by family background. We show how the intergenerational transmission of ability becomes sensitive to elements of the economic environment once marital sorting is endogenously determined, but also that policy variables (e.g. income redistribution) generally have no impact. The analysis also reveals a novel 'status motive' for parental investment and channel through which the fortunes of grandparents and prior generations persist.

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1 Introduction

Even in the most meritocratic of societies, individuals' economic prospects are circumscribed by their family background. Parents actively shape the economic prospects of offspring in various ways, for instance by investing in human capital. However, parents also *passively* transmit a range of productive capabilities–"innate ability" for short–to their offspring.[1] Our goal in this paper is to understand the economic forces behind the intergenerational transmission of innate ability.

The process underlying the intergenerational transmission of innate ability has implications for inequality and social mobility. In particular, it is central in understanding the 'unusually' strong persistence documented over multiple generations (Clark [2014]). But beyond this, the process is important to understand because, by its very nature, it is resistant to standard policy tools–innate ability cannot readily be redistributed, subsidized or publicly provided and efforts to provide information, training or nudges will be futile.[2] This resistance to policy intervention lies at the heart of fears surrounding the formation of a 'cognitive elite' and the associated detrimental social consequences.[3]

Economic forces are relevant for the transmission of ability, despite the passive nature of transmission, because of spouse choice. When the ability of both parents contribute to the formation of offspring ability, the intergenerational persistence of ability hinges on the strength of marital sorting on ability.

How does marital sorting on innate ability unfold? Naturally, potential spouses are evaluated on the basis of their earnings potential, as indicated by their human capital and parental income. However, the heritable nature of ability (along with altruism toward descendants) induces a concern for the innate ability of a potential spouse, *independent* of the potential spouse's earnings potential. That is, a spouse's innate ability influences the earnings capacity of one's descendants independently of the spouse's earning capacity.

Yet, identifying the innate ability of a potential spouse is difficult. Their innate

[1]The term "innate ability" is chosen because of the emphasis it places on the passive nature of transmission. That is, it refers to the aggregate of those productive capabilities which resist the active interventions of parents (or anyone else). We are primarily motivated by the genetic component of cognitive ability, but we are by no means restricted to this interpretation: the term is completely analogous to the "endowments" of Becker and Tomes [1979], which include cognitive abilities but also "family reputation and "connections", and knowledge, skills, and goals provided by their family environment" (p.1153). The term is also completely analogous to the "social competence" of Clark [2014].

[2]Even beyond standard policy tools, evidence from Clark [2014] suggests that this passive transmission process is unaffected by wildly different institutional settings.

[3]See Murray [2012], Arrow et al. [2000], Herrnstein and Murray [1994], Hernnstein (1973) and Young [1958].

ability is not revealed by their human capital since the latter is also influenced by investments made by their parents and by luck. Together then, potential spouses are evaluated on the basis of their human capital and on beliefs about their innate ability. Clearly, a potential spouse's human capital will inform such beliefs. However, the heritability of innate ability means that beliefs will also be informed by the family history of a potential spouse.

The model we develop in section 2 is intended to capture these ideas in a tractable manner. In each period each family starts with a couple. The couple uses their human capital to earn income, and then produces offspring. Income is divided between consumption and investment. The human capital of offspring depends on parental investment, their innate ability, and luck. The innate ability of offspring is sensitive to the innate ability of each parent, and is not observed. Beliefs about the ability of offspring are rationally formed on the basis of their human capital, their family income, and the beliefs associated with their parents' abilities. The offspring enter a marriage market, characterised by their human capital, their parents' income, and the commonly-held beliefs about their ability. Parents die once their offspring are married. The new couples become the families of the following period, and the process starts over again.

The analysis emphasizes the role of the family as a repository of information. The experiences of a generation are incorporated into their family history, which then in turns shapes the experiences of future generations. Our model formalizes this abstract description in a tractable manner. We show how the beliefs associated with each offspring can be summarized by a scalar, which we call their *status*. Similarly, we show how the relevant family history is summarized by a scalar, which we call *family status*. An offspring's status depends on family history only via family status, and family status is derived only from the status of each parent. This dynamic interaction produces a simple law of motion associated with family status. In addition, the model delivers an endogenous measure of the importance of family background. This measure is the weight that agents optimally place on an individual's family status relative to their human capital during the process of belief updating.

In order to focus on the central issue of marital sorting on unobserved innate ability, we abstract from all other frictions in the marriage market. This will mean that marriages are perfectly (positively) sorted on the human capital dimension. Marriages will similarly be perfectly (positively) sorted on individual status; i.e. there is perfect sorting on *expected* ability. Of course, there will be imperfect sorting on actual ability. The strength of sorting on ability depends on the precision of beliefs–the *quality of information*. We show how the quality of information evolves over time, and globally converges to a unique steady state value. The quality of information will, via its effect on sorting, affect the intergenerational persistence of ability and the dispersion of ability in society. Our emphasis on unobservability distinguishes this paper from the existing literature concerned with intergenerational implications of marital matching (e.g. Aiyagari et al. [2000], Fernández et al. [2005] Cole et al. [1992], Anderson [2015], Zak and Park [2002]) Comerford et al. [2017] also consider unobserved ability in a dynastic setting, but do not consider marital matching. Rather, they are concerned with statistical discrimination in the labour market.

When the strength of marital sorting on ability is treated as fixed, the economic environment plays no role in the intergenerational transmission of ability.[4] In contrast, technological conditions governing the sensitivity of human capital to ability and to luck will have an effect on transmission in our setting. This is purely because of the effect they exert on the quality of information and thus sorting. On the other hand, institutional conditions–such as those governing the equality of outcomes (redistribution of income) and opportunity (redistribution of parental inputs) generally have no effect. We explore extensions in which institutional conditions can have an impact. This endogeneity of marital sorting, along with focus on the passive role of parents, distinguishes this paper from a literature on intergenerational transmission of income (e.g. Becker and Tomes [1979, 1986] Solon [2004], Clark [2014], Benabou [2002], Kremer [1998]).[5]

The central role of family history in our model introduces two main features. The first feature has to do with the impact of prior generations. We show how the economic luck experienced by an individual will percolate to descendants via a novel channel. Luck raises the individual's status, which allows them to marry a higher-ability spouse on average. The higher-ability spouse in turn contributes to the ability of descendants. But the impact is persistent since the higher individual status becomes embedded into family status, which has an independent effect on the marital prospects of all descendants. The second feature has to do with investment incentives facing parents. If investments are not observed, then parents will be motivated to invest, in part, by an attempt to raise the status of offspring. This will not affect the ability of offspring (as it is determined by parental abilities, not investment), but it will affect the expected

[4]This is a direct consequence of passive transmission. The economic environment could shape the distribution of ability (but not its transmission conditional on sorting) via standard evolutionary dynamics if there were differential fertility. In any case, we abstract from differential fertility to focus in on the sorting issue.

[5]In addition to making human capital investments, the active role of parents includes decisions about where to live (Benabou [1996], Durlauf [1996], Fernandez and Rogerson [1996]), which cultural traits to inculcate (Bisin and Verdier [2001], Francois and Zabojnik [2005], Tabellini [2008]), and which parenting style to adopt (Doepke and Zilibotti [2017]).

ability of further generations. A greater status allows offspring to attract a spouse of higher ability on average, and this will raise the expected ability of grandchildren. Furthermore, the higher status of offspring will be preserved in higher family status which will help all future generations attract higher-ability spouses. This emphasis on intergenerational aspects distinguishes this paper from a literature on marital matching with unobserved characteristics (Hoppe et al. [2009], Cole et al. [1995], Hopkins [2012], Bidner et al. [2016], Bidner [2010], Rege [2008], Anderson and Smith [2010], Bergstrom and Bagnoli [1993], Chade [2006]). Indeed, the intergenerational connection introduced by heritable ability is the *only* reason that agents care about the status of potential partners in the model.

1.1 Discussion of 'Innate Ability'

As noted, our notion of "innate ability" permits a variety of interpretations. As such, transmission need not be genetic in nature. Indeed, our model accommodates cultural transmission, whereby ability is encoded in "memes" rather than genes (Dawkins [2006]). However, we put much weight on the genetic channel for three main reasons. First, it fits more naturally with our maintained assumptions that transmission (i) is passive, and (ii) necessarily involves the contribution of both parents. Second, it accords well with available evidence. The evidence from behavioural genetics, typically grounded in a comparison of identical and fraternal twins or of biological and adopted siblings, has long established a genetic basis for a variety of relevant traits from IQ to financial literacy (see Polderman et al. [2015] and Sacerdote [2011] for a review of this vast literature). These findings have been supported in recent years by evidence from molecular genetics, whereby an individual's relevant traits are associated directly with their genome (e.g. see Okbay et al. [2016]). In addition, recent evidence on persistence over the long run suggests that the transmission process is remarkably stable across wildly different economic environments, and is also consistent with a variety of other signatures of genetic transmission (see Clark [2014], especially Chapter 7). Third, if sorting is to be of quantitative relevance, then the transmission of ability between parents and offspring must be of sufficiently high fidelity (Kremer [1998]). The genetic channel offers a far higher parent-child fidelity because, unlike genes, the transmission of memes is not restricted to parent-child pairs and can operate with equal force within groups of peers, colleagues and extended family. This is to say nothing of memes' high susceptibility to mutation. Of course, none of this is to suggest that cultural transmission is unimportant (only that it will have little quantitative impact on our objects of study–parent-child correlations–unless it operates in a manner that mimics genetic transmission), or that human capital is unresponsive to environmental influences (we explicitly include environmental contributions in the model).

2 Model

2.1 Fundamentals

2.1.1 Population

We consider an infinite horizon discrete time economy with time indexed by $t = 0, 1, 2, ...$ Each date t starts with a continuum of households, indexed by i, each consisting of one male and one female parent. Household (i, t) has two offspring, one male and one female. The offspring from each household then are paired in a marriage market with an individual of the opposite sex. The parents of each household die at the end of the period, and the newly-paired offspring form the households for the following period. By convention, the household index is retained by the male offspring,[6] and we denote the household index of his wife by i'. That is, household $(i, t + 1)$ is formed by the marriage of the male offspring from household (i, t) with his spouse, the female from household (i', t). Thus, i' is the 'maiden index' of the mother in household i. By construction the measure of males, females and households is the same in each period, which we normalize to unity.

2.1.2 Timing

Household (i, t) consists of the male offspring from household $(i, t - 1)$ and the female offspring from household $(i', t - 1)$. The parents each have an *ability* which was determined in the previous period, denoted by the scalars $(\theta_{i,t-1}, \theta_{i',t-1})$. Ability is unobserved, yet beliefs about the ability of each parent were formed in the previous period, and are described by the probability density functions $(\psi_{i,t-1}, \psi_{i',t-1})$. Parents also have (log) human capital levels that were determined in the previous period, denoted by the scalars $(x_{i,t-1}, x_{i',t-1})$.

The human capital of parents in household (i, t) is used to produce family output, and this output, along with redistributive factors, determine household income. Redistribution arises from two sources–a progressive taxation system, as well as deviations

[6] Since there is no asymmetry by sex, the female's index would work just as well. The emphasis on the male lineage is motivated by the empirical literature which focuses on traits that are easier to measure for the male lineage, such as last names and earnings.

from meritocracy whereby rents are acquired by virtue of parental incomes. The structural equations are fully described in the Appendix, but here we give the reduced-form relationship. In particular, the log income of household (i, t), denoted y_{it}, depends on the average household log human capital and average household log parental income:

$$y_{i,t} = \beta_0 + \beta_1 \cdot [x_{i,t-1} + x_{i',t-1}]/2 + \beta_2 \cdot [y_{i,t-1} + y_{i',t-1}]/2. \tag{1}$$

The inverse of the parameter β_1 reflects the extent of redistributive taxation in the model, while the inverse of the parameter β_2 also reflects the extent of meritocracy. The constant β_0 simply ensures that the resource constraint holds (total output equals total income).

Offspring are born after income is generated. The offspring in household (i, t) are endowed with an *ability*, denoted θ_{it}.[7] This ability is partly inherited from their parents, according to:

$$\theta_{it} = b \cdot [\theta_{i,t-1} + \theta_{i',t-1}]/2 + \upsilon_{it} \tag{2}$$

where $b \in (0, 1)$ and $\upsilon_{it} \sim N(0, \sigma_\upsilon^2)$ is an idiosyncratic component. Ability is unobserved.

The *human capital* of offspring depends on ability, but also on parental investment and luck. Again, the underlying structural model is fully described in the Appendix, but here we present the reduced form equations. In particular, the log human capital of offspring in household (i, t), denoted x_{it}, is given by

$$x_{i,t} = \alpha_0' + \alpha_1 \cdot \theta_{i,t} + \alpha_2 \cdot h_{i,t} + \varepsilon_{i,t}, \tag{3}$$

where h_{it} is log parental investment and $\varepsilon_{i,t}$ is an idiosyncratic component capturing 'luck', where $\varepsilon_{it} \sim N(0, \sigma_\varepsilon^2)$. The value of σ_ε^2 parameterizes the extent of luck. The parameter α_1 captures the extent to which ability (nature) matters for human capital production. The parameter α_2 captures the extent to which parental investment (nurture) matters for human capital production, but also inversely captures the extent of parental investment redistribution. For instance, "equality of opportunity" arises when $\alpha_2 = 0$. The constant, α_0', simply accounts for redistribution (i.e. ensures that the resource constraint binds).

Parental investment must be financed from family income. It is useful to consider

[7] Our assumption that siblings have the same ability is for simplicity–it is not essential but allows us to abstract from the possibility that sibling outcomes are informative about one's ability.

a special case whereby all families invest the same proportion of their income, z. We will go on to show that this is optimal and derive the optimal z. But holding it fixed for now helps clarify the main arguments to follow. Fixing z allows us to write

$$x_{i,t} = \alpha_0 + \alpha_1 \cdot \theta_{i,t} + \alpha_2 \cdot y_{i,t} + \varepsilon_{i,t}, \tag{4}$$

where $\alpha_0 = \alpha_0' + \alpha_2 \cdot \ln z$.

Beliefs about the ability of offspring are updated on the basis of family income and the offspring's realized human capital. These (posterior) beliefs, denoted ψ_{it}, are given by Bayes rule:

$$\psi_{it}(\theta) = \frac{\bar{\psi}_{it}(\theta \mid y_{it}) \cdot f(x_{it} \mid \theta, y_{it})}{\int \bar{\psi}_{it}(\tilde{\theta} \mid y_{it}) \cdot f(x_{it} \mid \tilde{\theta}, y_{it}) d\tilde{\theta}}, \tag{5}$$

where f is the conditional distribution of log human capital implied by (4), and $\bar{\psi}_{it}(\theta \mid y_{it})$ are (prior) beliefs conditioned on family income implied by $(\psi_{i,t-1}, \psi_{i',t-1})$, (2) and (1).

Offspring then enter the marriage market, characterized by their human capital, parental income, and updated beliefs about their ability: $\omega_{it} \equiv \{x_{it}, y_{it}, \psi_{it}\}$. Offspring are then each allocated a partner in the marriage market (as described below), forming the households of the next period.

2.1.3 Preferences

Parents have preferences over their own consumption and the infinite sequence of consumption of their descendants. Specifically, agents care about the present discounted value of utilities:

$$U_{i,t} = u(C_{i,t}) + \mathbb{E}\left[\sum_{\tau=1}^{\infty} \delta^{\tau} \cdot u(C_{i,t+\tau})\right] \tag{6}$$

where $\delta \in (0, 1)$ is the discount factor and $C_{i,t}$ is the consumption of household (i, t).[8] We impose log utility, which, given that all households invest a fraction z of their income

[8] Strictly speaking, this says that parents care about the consumption of descendants down the male line. Given symmetry across the sexes, this is equivalent to caring about an average across all descendants within each subsequent generation. We adopt this specification purely for notational convenience.

in the human capital of offspring, implies that:

$$U_{i,t} = y_{it} + \mathbb{E}\left[\sum_{\tau=1}^{\infty} \delta^{\tau} \cdot y_{i,t+\tau}\right] + U_0, \tag{7}$$

where $U_0 \equiv \ln(1-z) + [\delta/(1-\delta)] \cdot \ln(1-z)$ is a constant. Thus, when families invest a fixed share of their income, log utility implies agents act as if they care about the present discounted value of (log) family income.

2.1.4 Marriage Market

The offspring from households at date t are characterized by their *marriage type* $\omega_{it} \equiv \{x_{it}, y_{it}, \psi_{it}\}$. The marriage market is described by a matching set, which we require to be feasible and stable. Formally, let Ω denote the set of possible marriage types. Define a *matching set* M as a subset of Ω^2 such that $M_m(A) \equiv \{\omega_f | \omega_m \in A \text{ and } (\omega_m, \omega_f) \in M\}$ and $M_f(A) \equiv \{\omega_m | \omega_f \in A \text{ and } (\omega_m, \omega_f) \in M\}$ are non-empty for each $A \subseteq \Omega$. A matching set describes how males and females are to be paired on the basis of their marriage types. Males with marriage types in A are to be paired with females who have marriage types in $M_m(A)$, and similarly females with marriage types in A are to be paired with males who have marriage types in $M_f(A)$. A matching set is *feasible* if the measure of males with marriage types in A equals the measure of females with marriage types in $M_m(A)$ and the measure of females with marriage types in A equals the measure of males with marriage types in $M_f(A)$ for each measurable $A \subseteq \Omega$. A matching set is *stable* if no unmatched pair prefer to marry than remain with their assigned partner.

2.1.5 Initial Conditions

At $t = 0$ each household is endowed with a log income $y^0 \equiv \ln Y^0$, where $Y^0 > 0$. Nature endows the offspring in household i with an ability, $\theta_{i,0} \sim N(0, \bar{\gamma}_0)$ where $\bar{\gamma}_0 > 0$. Prior beliefs about offspring ability at $t = 0$ thus coincide with this normal distribution.

2.1.6 Equilibrium

An equilibrium is a sequence of matching sets and parental investment rules (each mapping disposable income into parental human capital investment) such that each matching set is feasible and stable given the investment rules and the investment rules are optimal given the matching sets.

3 Analysis and Results

3.1 Segregation Matching Equilibrium

Our assumptions of a large population and gender symmetry are immensely valuable here since they allows us to consider a simple candidate for matching sets: segregation. That is, agents marry someone with identical characteristics: $M_m(A) = A$ and $M_f(A) = A$ for all $A \subseteq \Omega$.[9] This allows us to focus attention on equilibrium ability sorting holding fixed the sorting patterns along other dimensions (such as human capital and parental income). In the appendix (section C.1.1) we verify that segregation is indeed a feasible and stable matching set.

Under segregation, equation (1) simplifies to:

$$y_{i,t} = \beta_0 + \beta_1 \cdot x_{i,t-1} + \beta_2 \cdot y_{i,t-1}.$$

Using (4) to substitute for $x_{i,t-1}$ gives the reduced-form relationship:

$$y_{i,t+1} = \pi_0 + \pi_1 \cdot \theta_{i,t} + \pi_2 \cdot y_{i,t} + \varepsilon^y_{i,t}$$

where $\pi_0 \equiv \beta_0 + \beta_1\alpha_0$, $\pi_1 \equiv \beta_1\alpha_1$, $\pi_2 \equiv \beta_1\alpha_2 + \beta_2$, and $\varepsilon^y_{it} \equiv \beta_1 \cdot \varepsilon_{it}$. Thus, the reduced-form luck component, $\varepsilon^y_{i,t}$, is normally distributed with mean zero and a variance $\sigma^2_{\varepsilon^y} \equiv \beta_1^2 \cdot \sigma^2_\varepsilon$.

3.2 Steady State

A steady state arises when the variance of ability σ^2_θ (dispersion), the husband-wife ability correlation ρ^{HW}_θ (sorting), and the parent-child ability correlation ρ^{PC}_θ (persistence) are all independent of time. In a steady state, the type transmission equation alone allows to derive the variance of types and the parent-child type correlation as functions of the husband-wife correlation:[10]

$$\sigma^2_\theta = \frac{\sigma^2_\upsilon}{1 - \frac{b^2}{2} \cdot (1 + \rho^{HW}_\theta)} \tag{8}$$

$$\rho^{PC}_\theta = \frac{b}{2} \cdot (1 + \rho^{HW}_\theta) \tag{9}$$

[9]This should not be interpreted as siblings marrying. Rather, it is a large-population idealization of marrying someone (from a different family) with 'similar' characteristics.

[10]These are the equations underlying Kremer (1997), where the husband-wife correlation is taken as exogenous. For instance, if we had instead assumed that type were observed then segregation would imply $\rho^{HW}_\theta = 1$ and therefore $\sigma^2_\theta = \sigma^2_\upsilon/[1 - (b^2/2)]$ and $\rho^{PC}_\theta = b$.

Our interest lies in deriving the steady state husband-wife ability correlation, ρ_θ^{HW}. To this end, it is useful to make some definitions as follows. Given beliefs, ψ_{it}, define the *status* of offspring from household (i,t) as $\phi_{it} \equiv \int \theta \psi_{it}(\theta) d\theta$ (i.e. their expected ability). Furthermore, define the *quality of information* associated with offspring of household (i,t) as γ_{it}^{-1} where $\gamma_{it} \equiv \int (\theta - \phi_{it})^2 \psi_{it}(\theta) d\theta$ is the variance of beliefs.

We now first derive a relationship between ρ_θ^{HW} and σ_θ^2 under the assumption that the quality of information is exogenous and constant across families and time, denoted γ^{-1}. We then go on to show that the quality of information does indeed have this property in the steady state, and then derive the steady state value.

Lemma 1 *If the quality of information is constant across families and time, denoted* γ^{-1}*, then*

$$\rho_\theta^{HW} = 1 - \frac{\gamma}{\sigma_\theta^2}. \tag{10}$$

To see why this is so, note that we can express an agent's ability as the sum of their status and a belief error:

$$\theta_{it} = \phi_{it} + \varepsilon_{it}^\gamma,$$

where ε_{it}^γ is mean zero, uncorrelated with ϕ_{it}, and has a variance of $\gamma \in [0, \sigma_\theta^2]$. This implies that the variance of ability equals the variance of status plus γ.[11] Segregation implies $\phi_{it} = \phi_{i't}$ and that ε_{it}^γ and $\varepsilon_{i't}^\gamma$ are uncorrelated, which further implies that the covariance of spouse abilities is simply the variance of status. Simple algebra then establishes the result.[12]

Given a constant quality of information, γ^{-1}, the steady-state values of $(\rho_\theta^{HW}, \sigma_\theta^2, \rho_\theta^{PC})$ are therefore given by the solution to equations (8), (9) and (10), as we now report.[13]

[11] Notice that this implies that γ cannot be set arbitrarily: since the variance of status is non-negative, it must be that $\gamma \in [0, \sigma_\theta^2]$. Intuitively, the variance of beliefs goes from zero, when beliefs are perfectly accurate, to σ_θ^2, when beliefs are completely uninformative.

[12] The intuition for the dependence on σ_θ^2 is as follows. A husband and wife will share the same status. A given variance of belief errors tells us the expected difference in the pair's abilities. A given expected difference will have a large impact on the correlation if the distribution of ability is tight (e.g. there is a higher chance that the 'top' abilities will end up with the 'bottom' abilities). The impact on the correlation is small if the distribution of ability is disperse (e.g. matches will tend to be more 'local' relative to the range of abilities in the population).

[13] The fact that $\gamma \in [0, \sigma_\theta^2]$ ensures that each of the quantities in the proposition fall within the appropriate ranges (i.e. variance is non-negative and correlations are between -1 and 1. Specifically, $\gamma \in [0, \sigma_\theta^2]$ ensures that $\rho_\theta^{HW} \in [0, 1]$, that $\rho_\theta^{PC} \in (b/2, b)$, and that $\sigma_\theta^2 \in [\sigma_\upsilon^2/(1 - b^2/2), \sigma_\upsilon^2/(1 - b^2)]$.

Proposition 1 *Given a constant quality of information, γ^{-1}, the steady state dispersion, persistence and sorting properties of ability are given by:*

$$\sigma_\theta^2 = \frac{\sigma_\upsilon^2 - \gamma \cdot \frac{b^2}{2}}{1 - b^2} \tag{11}$$

$$\rho_\theta^{PC} = b \cdot \frac{\sigma_\upsilon^2 - \frac{\gamma}{2}}{\sigma_\upsilon^2 - b^2 \cdot \frac{\gamma}{2}} \tag{12}$$

$$\rho_\theta^{HW} = \frac{\sigma_\upsilon^2 - \gamma \cdot (1 - \frac{b^2}{2})}{\sigma_\upsilon^2 - b^2 \cdot \frac{\gamma}{2}}. \tag{13}$$

Each of these outcomes is increasing in the quality of information.

The proof is contained in the derivations in appendix section B. Figure 1 illustrates how ρ_θ^{HW}, ρ_θ^{PC} and σ_θ^2 are jointly determined by γ (using (8), (9) and (10)).

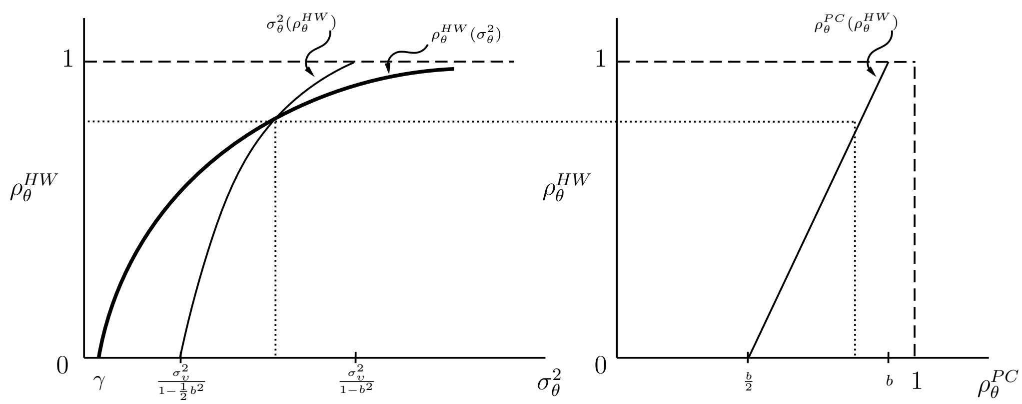

Figure 1: Long-Run Ability Sorting, Persistence and Dispersion

While understanding the factors driving the sorting, persistence, and dispersion of ability is important in its own right, we are also interested in the consequences for social mobility. To this end, we have the following.

Proposition 2 *Given a constant quality of information, γ^{-1}, the steady state dispersion and persistence properties of income are given by:*

$$\sigma_y^2 = \frac{\left[\frac{1+b\pi_2}{1-b\pi_2}\right] \pi_1^2 \cdot \sigma_\theta^2(\gamma) + \sigma_{\varepsilon^y}^2}{1 - \pi_2^2} \tag{14}$$

$$\rho_y^{PC} = \pi_2 + b \cdot \frac{(1-\pi_2^2) \cdot [\pi_1^2 \cdot \sigma_\theta^2(\gamma)]}{(1+b\pi_2) \cdot [\pi_1^2 \cdot \sigma_\theta^2(\gamma)] + (1-b\pi_2) \cdot [\sigma_{\varepsilon^y}^2]}, \tag{15}$$

where $\sigma_\theta^2(\gamma)$ is given by (11). Both of these outcomes are increasing in the quality of information, γ^{-1}.

The proof is contained in the derivations in appendix section B.

The expression for the persistence of income, ρ_y^{PC}, makes clear the role of heritable ability in social mobility: a positive intergeneration income correlation would be observed *even if* parental income had no causal impact on income (i.e. even if $\pi_2 = 0$).

We stress that the steady state quality of information, γ^{-1}, is endogenously determined. Fully analysing the properties of a steady state therefore requires that the information structure be sufficiently tractable that the steady state precision of beliefs can be characterized. We now turn to this.

3.3 Steady State Quality of Information

3.3.1 Prior Beliefs

Prior beliefs about offspring ability follow from the ability transmission equation, (2). *Family status*, denoted $\bar{\phi}_{it}$, is defined as the expected ability of offspring conditional on the status profile of parents. From (2), this is $\bar{\phi}_{it} \equiv b \cdot [\phi_{i,t-1} + \phi_{i',t-1}]/2$, which under segregation is

$$\bar{\phi}_{it} = b \cdot \phi_{i,t-1}. \tag{16}$$

Note that household income does not affect beliefs about parental types since such beliefs were already conditioned on $x_{i,t-1}$ and $y_{i,t-1}$. Thus family status is also the expected ability of offspring conditional on parental status profile and family income.

The variance of prior beliefs, under segregation, is $\bar{\gamma}_{it} = (b^2/2)\gamma_{i,t-1} + \sigma_v^2$. If the variance of beliefs is the same across households in the previous period, denoted γ_{t-1}, then the variance of prior beliefs is the same across households, equal to:

$$\bar{\gamma}_t = (b^2/2)\gamma_{t-1} + \sigma_v^2. \tag{17}$$

If beliefs about parental types are normally distributed with a constant variance across households, then prior beliefs are also normally distributed with a constant variance across households. That is, prior beliefs are described by

$$\theta_{it} \sim N(\bar{\phi}_{it}, \bar{\gamma}_t). \tag{18}$$

3.3.2 Signal

These prior beliefs are then updated on the basis of an agent's human capital, x_{it}, and parental income, $y_{i,t}$. Whilst this sort of Bayesian updating can quickly become complicated, our structure allows us to conduct this updating in a very tractable manner. By re-arranging the human capital equation we get

$$s_{i,t} \equiv \frac{x_{it} - \alpha_0 - \alpha_2 \cdot y_{i,t}}{\alpha_1} = \theta_{it} + \xi_{i,t}, \tag{19}$$

where $\xi_{i,t} \equiv \varepsilon_{i,t}/\alpha_1$. That is, s_{it} is a signal of ability since

$$s_{it} \sim N(\theta_{it}, \sigma_\xi^2) \tag{20}$$

where $\sigma_\xi^2 = \sigma_\varepsilon^2/\alpha_1^2$.

3.3.3 Posterior Beliefs

If prior beliefs are normal, then from (18) and (20) standard results tell us that the posterior is also normal:

$$\theta_{it} \mid s_{it} \sim N(\phi_{it}, \gamma_t), \tag{21}$$

where

$$\gamma_t \equiv \frac{\sigma_\xi^2 \cdot \bar{\gamma}_t}{\sigma_\xi^2 + \bar{\gamma}_t}, \tag{22}$$

and

$$\phi_{it} \equiv \lambda_t \cdot \bar{\phi}_{it} + (1 - \lambda_t) \cdot s_{it}, \tag{23}$$

where

$$\lambda_t \equiv \frac{\sigma_\xi^2}{\sigma_\xi^2 + \bar{\gamma}_t} \tag{24}$$

is the weight that agents place on the prior.

3.3.4 Implications

We start by noting a convenient implication of our maintained assumptions.

Lemma 2 *At each date, the quality of information is the same for all families: for all t, we have $\gamma_{it}^{-1} = \gamma_t$ for all $i \in [0, 1]$.*

To see this, note that at $t = 0$ prior beliefs are indeed normal with a variance that is independent of household. The above analysis then implies that posterior beliefs at $t = 0$ will also have this property, and therefore that so too will prior beliefs at $t = 1$. This logic iteratively repeats for all future periods.

The analysis above not only establishes that the quality of information only depends on the date (and not the family), it also establishes how this quality evolves over time. In particular, from (17) and (22) we have that the variance of beliefs evolves according to the following difference equation:

$$\gamma_t = \frac{\sigma_\xi^2 \cdot [(b^2/2) \cdot \gamma_{t-1} + \sigma_v^2]}{\sigma_\xi^2 + (b^2/2) \cdot \gamma_{t-1} + \sigma_v^2}. \tag{25}$$

It is straightforward to see that this implies global convergence. In particular, the steady state belief variance, denoted γ, is the unique positive solution to

$$\gamma = \frac{\sigma_\xi^2 \cdot [(b^2/2) \cdot \gamma + \sigma_v^2]}{\sigma_\xi^2 + (b^2/2) \cdot \gamma + \sigma_v^2}. \tag{26}$$

Proposition 3 *The variance of beliefs converges to the steady state value, γ, which satisfies (26). The steady state quality of infomation, γ^{-1}, is (i) decreasing in $\sigma_\xi^2 \equiv \sigma_\varepsilon^2/\alpha_1^2$, (ii) decreasing in b and σ_v^2, and is (iii) independent of α_2, β_1, and β_2.*

The proof follows from the above discussion with the comparative statics being easily derived from (26).[14] Intuitively, the steady state quality of information is reduced as the signal from human capital becomes noisier (i.e. as σ_ξ^2 increases) and as the signal from parental status becomes noisier (i.e. as b and σ_v^2 increase).

Given γ we can derive the steady state weight that agents place on their prior, λ. This value indicates the importance that agents place on family background relative to individual performance. We have that $\lambda_t \to \lambda$ where

$$\lambda \equiv \frac{\sigma_\xi^2}{\sigma_\xi^2 + (b^2/2) \cdot \gamma + \sigma_v^2}. \tag{27}$$

Proposition 4 *The long-run relative importance of family background, λ, is (i) increasing in $\sigma_\xi^2 \equiv \sigma_\varepsilon^2/\alpha_1^2$, (ii) decreasing in b and σ_v^2, and is (iii) independent of α_2, β_1, and β_2.*

[14] The right side of (26) is increasing in σ_ξ^2 (which is $\sigma_\varepsilon^2/\alpha_1^2$), implying that so too is γ. The right side of (26) is increasing in σ_v^2 and in b, implying that so too is γ.

Notice that an increase in signal noise, σ_ξ^2, in steady state raises both the variance of beliefs and the relative importance of family background. On the other hand, an increase in b or σ_υ^2 in the steady state also raises the variance of beliefs yet reduces the relative importance of family background.

An individual's status and family status depend on each other. Family status depends on parental status according to (16), and an individual's status depends on family status according to (23). As such, family status evolves according to

$$\bar{\phi}_{i,t+1} = b \cdot [\lambda \cdot \bar{\phi}_{i,t} + (1-\lambda) \cdot \theta_{i,t} + (1-\lambda) \cdot \xi_{i,t}] \tag{28}$$

and individual status evolves according to

$$\phi_{it} = \lambda \cdot b \cdot \phi_{i,t-1} + (1-\lambda) \cdot \theta_{i,t} + (1-\lambda) \cdot \xi_{i,t} \tag{29}$$

We are now in a position to discuss the factors that shape the sorting, persistence and dispersion of ability by combining propositions 1 and 3. We are also in a position to discuss the factors that shape the social mobility and inequality by combining propositions 2 and 3. We now turn to such issues.

4 Discussion of Results

4.1 Economic Environment

By construction, an agent's inherited ability is not affected by their environment. Nevertheless, the environment shapes the dispersion and persistence of ability in a society. It does this by changing the availablity of information about ability, thereby influencing sorting. The model is useful for identifying which aspects of the environment can be expected to have an impact and which aspects will not.

Corollary 1 *An increase in α_1 or a decrease in σ_ε^2 will strengthen sorting, persistence and dispersion of ability.*

The return to ability, α_1, has an impact on how similar an individual's ability is to that of their spouse, their parents and children, and a random member of society. To be sure, this is because an increase in the return to ability makes an agent's human capital a more reliable signal of ability and this fact facilitates stronger sorting in the marriage market.

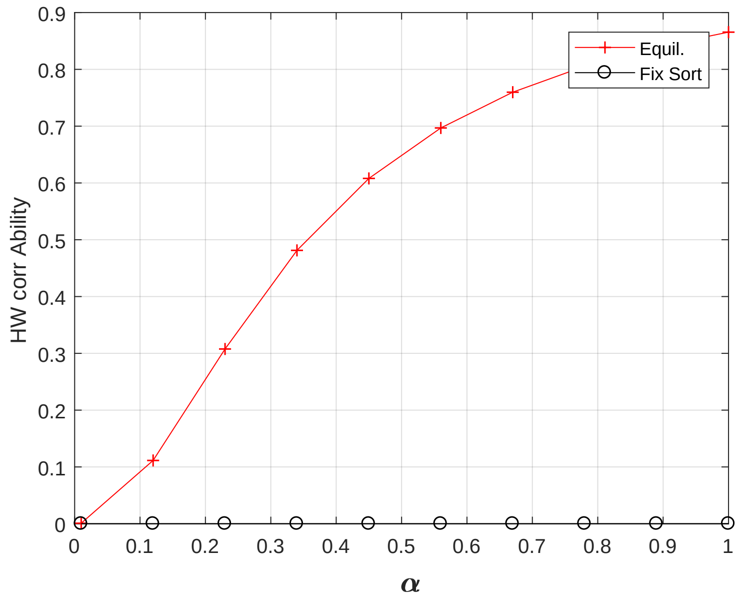

Figure 2: A Higher Return to Ability Strengthens Sorting

Another plausible mechanism that would lead to a similar conclusion has to do with frictional matching: a greater return to ability provides incentives to search more intensely for a high ability partner. To distinguish the two mechanisms, we note that the competing explanation would also predict sorting systematically varies with the policy environment (e.g. lower redistributive taxation should also provide incentives to search more intensely for a high ability partner) whereas this is not true in our case (see the section that follows). The evidence from Clark [2014] is supportive of our mechanism against this alternative.

In terms of income, the variables $(\alpha_1, \sigma_\varepsilon^2)$ will have a direct effect on inequality and social mobility (i.e. holding ability sorting fixed) and an indirect sorting effect.

Corollary 2 *The direct effect of α_1 on social mobility and on inequality is exacerbated by the sorting effect. The direct effect of σ_ε^2 on social mobility is exacerbated by the sorting effect, whereas the direct effect of σ_ε^2 on inequality is mitigated by the sorting effect.*

Intuitively, if we hold ability sorting fixed, a larger return to ability lowers social mobility and raises inequality. But it also facilitates stronger marital sorting on ability, and thereby raises the persistence and dispersion of ability, which in turn lowers social mobility and raises inequality further.

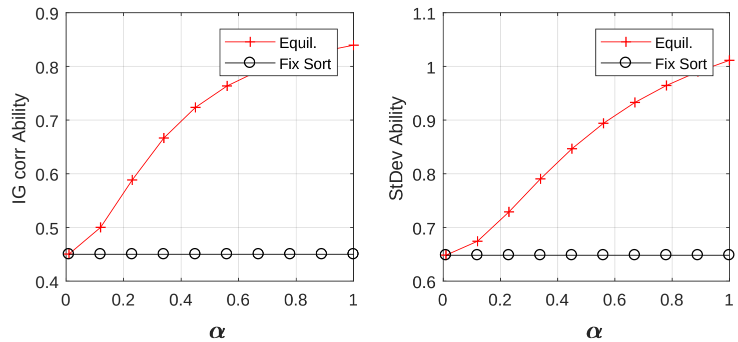

Figure 3: A Higher Return to Ability Strengthens Persistence and Dispersion of Ability

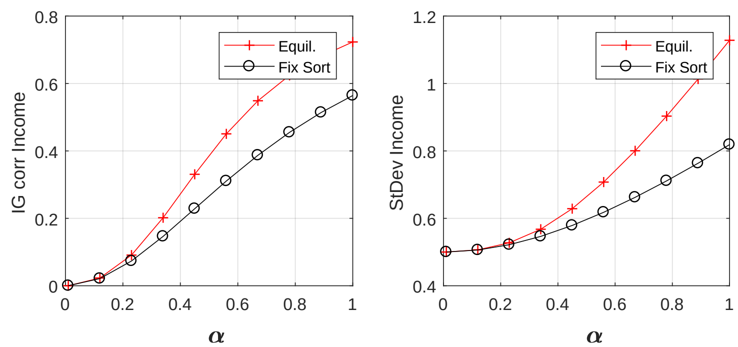

Figure 4: A Higher Return to Ability Strengthens Persistence and Dispersion of Income via Sorting Effect

Similarly, if we hold ability sorting fixed, a larger luck component raises social mobility and raises inequality. But it also weakens marital sorting on ability, and thereby lowers the persistence and dispersion of ability, which in turn raises social mobility further but also lowers inequality. This leads to the possibility that luck will have a non-monotonic effect on income inequality: luck raises inequality holding ability sorting fixed, but lowers inequality by weakening ability sorting. This is illustrated in Figure 5.

4.2 Institutional/Policy Environment

The institutional/policy environment is captured by our parameters α_2, β_1, and β_2. Recall that these describe the extent to which parental human capital inputs are redistributed, as well as the extent to which income is redistributed via taxation and departures from meritocracy. In terms of income, these parameters clearly have an impact on social mobility and inequality (see proposition 2). However, in the base model

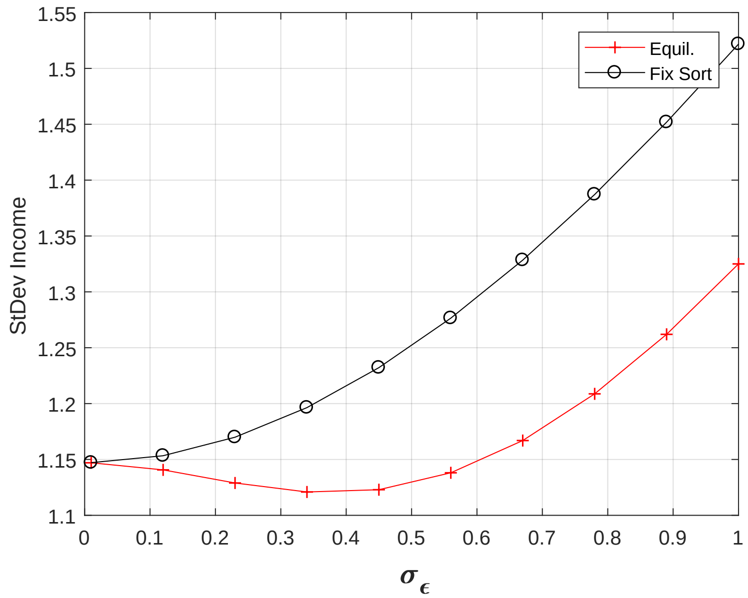

Figure 5: Non-Monotonic Effects of Luck on Income Inequality

at least, they have no impact on the extent of *ability* sorting, persistence or dispersion.

Corollary 3 *The institutional/policy environment variables captured by the reduced-form parameters α_2, β_1, and β_2 have no effect on the sorting, persistence or dispersion of ability.*

This result highlights the difficulty in inferring (unobservable) changes in the sorting, persistence and dispersion of ability from (observable) changes in the sorting, persistence and dispersion of income. The evidence from Clark [2014] is supportive of this implication of our model.

4.3 Heritability Environment

The 'heritability' variables, b and σ_v^2, will clearly have a direct impact on the dispersion and persistence of the heritable characteristic (see proposition 1). But, less obviously, they will also have an effect on sorting and thus an indirect sorting effect on dispersion and persistence of ability via their effect on steady state belief precision.

Corollary 4 *The direct effect of b on the persistence and dispersion of ability is mitigated by the sorting effect. The direct effect of σ_v^2 on the persistence and dispersion of ability is exacerbated by the sorting effect.*

4.4 Summary

We have shown how the quality of information is key to understanding the sorting, persistence and dispersion of ability as well as social mobility and inequality. We then showed which factors shape the quality of information in the long run.

This exercise had little to say about income *levels* are affected by the presence of unobserved heritable characteristics because we took parental investment to be a fixed proportion of income. In the following section we endogenize parental investment.

The exercise also has little to say about sorting in the human capital dimension. Indeed, we have held this fixed throughout in order to isolate the impact on ability sorting. To demonstrate that nothing in the analysis hinges on perfect sorting on the human capital dimension we present an extension where an agent's human capital is not perfectly observed in the marriage market.

Finally, the conclusion that the institutional/policy environment does not matter arises because the relevant parental characteristics are assumed to be perfectly observed in the marriage market. We relax this assumption by supposing that parental investment has an unobserved idiosyncratic component. This feature breaks perfect sorting on parental investment and introduces a role for some policy variables.

5 Analyzing Multiple Generations

5.1 Multi-generational Correlations

5.1.1 Ability

What does the one-generation ability correlation tell us about multi-generation ability correlations? We show that the true multi-generation ability correlation is larger than that implied by a geometric extrapolation of the one-generation correlation. This is due entirely to imperfect sorting on ability. As such, the extent of this bias is endogenously determined in our setting.

The ability correlation between family members $k \in \{2, 3, ..\}$ generations apart is given by:

$$\rho_{\theta,k}^{PC} \equiv \frac{\mathrm{Cov}(\theta_{it}, \theta_{i,t-k})}{\sigma_\theta^2} = b^{k-1} \cdot \rho_\theta^{PC} = b^k \cdot \left[\frac{\sigma_v^2 - \frac{\gamma}{2}}{\sigma_v^2 - b^2 \cdot \frac{\gamma}{2}} \right]. \tag{30}$$

Thus the long correlation implied by extrapolating the short correlation geometrically understates the true long correlation:

$$\frac{(\rho_\theta^{PC})^k}{\rho_{\theta,k}^{PC}} = \left[\frac{\sigma_\upsilon^2 - \frac{\gamma}{2}}{\sigma_\upsilon^2 - b^2 \cdot \frac{\gamma}{2}}\right]^{k-1} \in (0,1). \tag{31}$$

The extent of the bias increases in k as this ratio goes to zero as k increases. Furthermore this bias in endogenous in our setting, as the ratio is decreasing in γ.

5.1.2 Income

We now turn to the same issue applied to incomes. For $k \in \{2, 3, ...\}$, let $\rho_{y,k}^{PC} \equiv \text{Cov}(y_{it}, y_{i,t-k})/\sigma_y^2$ be the k-generation income correlation in the steady state. In the appendix we show that these correlations can be defined recursively by:

$$\rho_{y,k}^{PC} = b^k \cdot \Phi(\gamma) + \pi_2 \cdot \rho_{y,k-1}^{PC}, \tag{32}$$

where

$$\Phi(\gamma) \equiv \frac{(1-\pi_2^2) \cdot [\pi_1^2 \cdot \sigma_\theta^2(\gamma)]}{(1+b\pi_2) \cdot [\pi_1^2 \cdot \sigma_\theta^2(\gamma)] + (1-b\pi_2) \cdot [\sigma_{\varepsilon^y}^2]}. \tag{33}$$

We can compare the k-generation correlation with that implied by the geometric extrapolation of the 1-generation correlation. In particular, the multi-generation correlation can be over- or under-stated by the geometric extrapolation of the 1-generation correlation depending on the relative size of ρ_1 and b.

Proposition 5 *Extrapolating the single-generation income correlation* understates *the true multi-generation income correlation (i.e. $\rho_1^k < \rho_k$) if*

$$\pi_2 < b \cdot (1 - \Phi(\gamma)). \tag{34}$$

The extrapolation overstates *the true correlation (i.e. $\rho_1^k > \rho_k$) if the inequality is reversed. The extrapolation* equals *the true correlation (i.e. $\rho_1^k = \rho_k$) if the inequality is replaced with an equality.*

The proof is in the appendix.

In other words, the extrapolation can either overstate or understate the true persistence of income across multiple generations. The condition on parameters that determines which case arises relates to the relative strength of parental transmission of income and ability. For instance, the extrapolation *overstates* the true persistence in

Becker and Tomes [1979] but *understates* it in Clark [2014]. In the former, there is no luck component, so that $\Phi = 1$ and the condition in the proposition can never hold. In the latter, parental income does not matter ($\pi_2 = 0$), so that the condition is always satisfied (as long as there is a luck component).[15]

5.2 Role of Luck and Prior Generations

In this section we show how our analysis introduces new forces driving persistence of economic fortune. In short, the expected ability of one's offspring, conditional on one's own ability, is increasing in one's economic luck and in inherited family status. It is also increasing in the ability of parents, grandparents and all previous generations. Even conditional on these abilities, it is also increasing in the economic fortunes of parents, grandparents and all previous generations. These results do not violate the maintained assumption that only parental abilities (and not income) matter for offspring ability. The key of course is that the ability of an agent's offspring depends on the ability of the agent's eventual spouse, and the quality of spouse that the agent can attract will be affected by the variables above. This effect cannot arise in models that take husband-wife ability correlation as exogenous. Such models imply that the distribution of spousal abilities depends only on one's own ability, and importantly, not on one's income or human capital.

One measure of dynastic ability is:

$$V_{it}^{\theta} \equiv \sum_{\tau=0}^{\infty} \delta^{\tau} \mathbb{E}_t \left[\theta_{i,t+\tau}\right] \tag{35}$$

In the appendix, we show the following.

Proposition 6 *Holding ability θ_{it} fixed, dynastic ability is increasing in economic luck. In particular,*

$$\frac{d\mathbb{E}_t \left[V_{it}^{\theta} | \theta_{it}\right]}{d\varepsilon_{it}} = \frac{b\delta}{1 - b\delta} \frac{1 - \lambda}{2 - b\delta\lambda} \cdot \frac{1}{\alpha} > 0. \tag{36}$$

This effect is decreasing in λ. There are two effects associated with the return to ability, α_1. First, a higher return to ability lowers λ and thus raises the sensitivity of dynastic

[15] Solon (2014) points out that the long run persistence of measured income can be understated by the short run persistence if there is measurement error. In contrast, the presence of luck does not require any measurement error. The two are not equivalent since the measurement error is not transmitted to offspring whereas luck is.

ability to the signal. However, the final fraction reveals an offsetting direct effect: a higher return to ability makes the signal less sensitive to luck.

Proposition 7 *Holding ability θ_{it} fixed, dynastic ability is increasing in family status. In particular,*

$$\frac{d\mathbb{E}_t\left[V_{it}^{\theta}|\theta_{it}\right]}{d\bar{\phi}_{it}} = \frac{b\delta}{1-b\delta}\frac{\lambda}{2-b\delta\lambda} > 0. \tag{37}$$

This result suggests an novel role for grandparents and prior generations. Consider two households, (i,t) and (j,t), that are identical at the start of period t. During the period, they experience different luck. This necessarily means that households $(i,t+1)$ and $(j,t+1)$ differ in their family status. Suppose it turned out that the offspring from these families go on to earn the same income at date $t+2$, and that they have offspring with identical abilities. In short, families $(i,t+2)$ and $(j,t+2)$ have the same parental income and have offspring with the same ability. The only difference between them is that one has a higher family status owing to the initial luck differential experienced by their grandparents. This luck differential matters *for dynastic ability* even though the households are now otherwise identical.

These results are relevant for understanding some additional incentives for parental investment that our analysis introduces. If offspring ability is affected by parental luck, then grandparents have an incentive to 'manufacture' economic luck by investing in their offspring (i.e. the parents). Although such investment cannot influence the ability of their offspring it will influence the ability of grandchildren (since, by the above logic, the investment will help one's child attract a higher-ability partner). We now turn to this issue.

6 Optimal Parental Investment

We now allow each household to make human capital investments in offspring optimally. We allow a limited form of asymmetric information whereby parents are better informed than the public about their investment. As a result, parents have two motivations for investing: a standard one of raising the income-generating capacity of offspring (Becker and Tomes [1986, 1979]), and a novel one of manipulating the market's assessment of their offspring's ability. By raising the market's assessment, offspring are able to secure partners with higher expected ability. Since matching is assortative on human capital and parental income, this has no impact on offspring income. However, it *will* have an impact on the income of grandchildren (and subsequent generations) because it raises their expected ability.

If family i makes a human capital investment given by a proportion z_{it} of their income, then their log consumption is

$$c_{it} = \ln(1 - z_{it}) + y_{it}. \tag{38}$$

This captures the cost of investing. The first benefit from investing has to do with the fact that investment raises the expected income of offspring. Recalling (3), their offspring will have a human capital of:

$$x_{i,t} = \alpha_0' + \alpha_1 \cdot \theta_{i,t} + \alpha_2 \cdot [\ln z_{it} + y_{it}] + \varepsilon_{i,t},$$

so that the expected income of offspring is therefore:

$$\mathbb{E}_t[y_{i,t+1}] = \pi_0' + \pi_1 \cdot \mathbb{E}_t[\theta_{i,t}] + \pi_2 \cdot y_{i,t} + \beta_1 \alpha_2 \cdot \ln z_{it}. \tag{39}$$

The second benefit of investing is that it will raise the status of offspring. As previously, the market observes human capital and parental income. But the market also has rational expectations about the investment that was made by agents' parents. If the market expects an investment share of z_{it}^*, then the relevant signal given an actual investment of z_{it} is:

$$s_{i,t} \equiv \frac{x_{i,t} - \alpha_0' - \alpha_2 \cdot y_{i,t}}{\alpha_1} = \theta_{i,t} + \xi_{i,t} + \frac{\alpha_2}{\alpha_1} \cdot (\ln z_{it} - \ln z_{it}^*).$$

The signal translates into individual status according to (23). That is:

$$\begin{aligned} \mathbb{E}_t[\phi_{i,t}] &= \lambda \cdot \bar{\phi}_{i,t} + (1 - \lambda) \cdot \mathbb{E}_t[s_{i,t}] \\ &= \lambda \cdot \bar{\phi}_{i,t} + (1 - \lambda) \cdot \mathbb{E}_t[\theta_{i,t}] + (1 - \lambda) \cdot \frac{\alpha_2}{\alpha_1} \cdot (\ln z_{it} - \ln z_{it}^*) \end{aligned}$$

Thus the signal translates into family status according to:

$$\begin{aligned} \mathbb{E}_t[\bar{\phi}_{i,t+1}] &= b \cdot \mathbb{E}_t[\phi_{i,t}] \\ &= b\lambda \cdot \bar{\phi}_{i,t} + b(1 - \lambda) \cdot \mathbb{E}_t[\theta_{i,t}] + b(1 - \lambda) \frac{\alpha_2}{\alpha_1} \cdot (\ln z_{it} - \ln z_{it}^*). \end{aligned} \tag{40}$$

Recall that family i's payoffs are given by

$$U_{it} = c_{it} + \mathbb{E}_t \left[\sum_{\tau=1}^{\infty} \delta^{\tau} c_{i,t+\tau} \right].$$

In the appendix we show that under the optimal investment strategy, the expectation term above is linear in $\mathbb{E}_t[y_{i,t+1}]$ and $\mathbb{E}_t[\bar{\phi}_{i,t+1}]$. From (39) and (40) we see that these expectations are linear in $\ln z_{it}$. This, along with (38), tells us that family i's investment problem boils down to a simple problem of the form:

$$\max_{z_{it} \in [0,1]} \left\{ \ln(1 - z_{it}) + \zeta_1 \cdot \ln z_{it} + \zeta_2 \cdot \ln z_{it} \right\}.$$

This expression allows a clear view of the relevant forces at play. The first term is the cost of investment whereas the second and third term are the two sources of benefit. Specifically, ζ_1 captures the standard motivation to invest based on raising the earning capacity of offspring, whereas ζ_2 captures the novel motivation to invest based on raising offspring status.

Proposition 8 *All families optimally invest the same fraction of their income:*

$$z_{it}^* = z^* = \frac{\zeta_1 + \zeta_2}{1 + \zeta_1 + \zeta_2},$$

where

$$\zeta_1 \equiv \frac{\delta \beta_1 \alpha_2}{1 - \delta[\beta_1 \alpha_2 + \beta_2]}$$
$$\zeta_2 \equiv \zeta_1 \cdot \frac{b\delta}{1 - b\delta} \cdot \frac{1 - \lambda}{2 - b\delta\lambda}.$$

The term ζ_1 captures the sort of incentives analyzed in standard models such as Becker and Tomes [1986, 1979]. The new force that we identify here is the ζ_2 term, and in particular the final component, $(1 - \lambda)/(2 - b\delta\lambda)$. This term is decreasing in the relevance of family background, λ. There are two competing effects: a higher λ lowers investment incentives since status is becomes less sensitive to investment efforts, however this is partly offset by the fact a higher status will persist for longer.

Since higher parental investment is associated theoretically and empirically with economic development, the analysis suggests a new mechanism through which economic development is hindered in societies where family background plays a central concern in the marriage market. Intuitively, in such cases it is difficult to shift the market's beliefs about offspring ability when the market places little weight on offspring performance relative to the prior.

From proposition 4, we have that ζ_2 (and thus investment via the status channel) is increasing in the return to ability α_1, decreasing in the importance of luck σ_ε^2, and increasing in b and σ_υ^2. For instance, a higher return to ability will raise the incentive to invest in offspring (even though such investment does not raise ability directly, nor is complementary to it)–rather, a higher return to ability makes the market's beliefs about ability more sensitive to human capital (and thus parental investment efforts) in steady state. Also of note is the fact that the policy parameters continue to have no impact on the 'status-based' incentive to invest (although they will of course have an effect on the standard 'income-based' incentive to invest–greater redistributive taxation lowers the incentive to invest, etc.).

Note that the 'status' motive for investment can be quantitatively important relative to the standard motive. That is, $\zeta_2 > \zeta_1$ for $b\delta$ large enough and λ small enough.

7 Extensions

7.1 Imperfect Sorting on Human Capital

By allowing perfect sorting on human capital and parental income, we are able to focus on the equilibrium sorting of ability. In this section we consider a version in which there is imperfect sorting on human capital. We show that the main insights are not exclusive to the perfect sorting setting. Further, for empirical purposes, it may also be helpful to evaluate how the parameters map into spousal human capital correlations. In particular, it will allow us to examine the nature of the relationship between (observable) spousal human capital correlations and (unobservable) spousal ability correlations.

We now consider an extension in which the human capital of agents in the marriage market is not perfectly observed (even by the agent themselves). Instead, everyone observes a noisy signal of human capital, generated according to:

$$\hat{x}_{it} \equiv x_{it} + \nu_{it} \tag{41}$$

where $\nu_{it} \sim N(0, \sigma_\nu^2)$. This signal of human capital is then used to update prior beliefs, $\bar{\psi}_{it}$, to form *interim* beliefs, $\hat{\psi}_{it}$. Marriage forms on the basis of interim beliefs. After the formation of marriages, human capital x_{it} is observed and posterior beliefs, ψ_{it} are formed. These posterior beliefs then form the basis of the prior beliefs inherited by the next generation.

Since

$$\hat{x}_{it} = \alpha_0 + \alpha_1\theta_{it} + \alpha_2 \cdot y_{i,t} + \varepsilon_{it} + \nu_{it}, \tag{42}$$

it follows that the relevant signal is:

$$\hat{s}_{it} \equiv \frac{\hat{x}_{it} - \alpha_0 - \alpha_2 \cdot y_{i,t}}{\alpha_1} = \theta_{it} + \frac{\varepsilon_{it} + \nu_{it}}{\alpha_1}. \tag{43}$$

The error component of this signal is

$$\hat{\xi}_{it} \equiv \frac{\varepsilon_{it} + \nu_{it}}{\alpha_1} \tag{44}$$

which has a variance of

$$\sigma^2_{\hat{\xi}} = \frac{\sigma^2_{\varepsilon} + \sigma^2_{\nu}}{\alpha_1^2}. \tag{45}$$

Essentially the same updating procedure as in the base model applies. Indeed, since human capital x_{it} is observed after marriages are formed, the market updates beliefs on the basis of x_{it} since the signal $\hat{x}_{it}$ does not provide any additional information about ability over-and-above that provided by x_{it}. As such, belief updating occurs as in the baseline model without observation noise. That is, the variance of posterior beliefs converges to γ in the steady state. However, it is the variance of *interim* beliefs that matters for the strength of sorting on human capital. Similar arguments to the base case apply so that the steady state variance of interim beliefs is:

$$\hat{\gamma} \equiv \frac{\sigma^2_{\hat{\xi}}[\frac{b^2}{2} \cdot \gamma + \sigma^2_{\upsilon}]}{\sigma^2_{\hat{\xi}} + \frac{b^2}{2} \cdot \gamma + \sigma^2_{\upsilon}}. \tag{46}$$

Since $\sigma^2_{\hat{\xi}} > \sigma^2_{\xi}$ whenever there is income noise ($\sigma^2_{\nu} > 0$), it naturally follows that $\hat{\gamma} > \gamma$ in such cases. The expressions for spousal ability correlation, ability variance and intergenerational persistence of ability are the same as those derived before with γ replaced with $\hat{\gamma}$. Thus the new element introduced by imperfectly observed income is captured by $\sigma^2_{\hat{\xi}}$. Thus nothing of substance changes in terms of ability sorting–it just becomes noisier, although the institutional/policy parameters still exert no influence on the strength of ability sorting or persistence.

7.2 Imperfect Sorting on Parental Investment

In order for policy variables to matter (i.e. to affect the sorting on genes or the intergenerational correlation of ability), the parental contribution must not be observed perfectly. The most transparent way to model this (since it preserves symmetric imperfect information) is to suppose that parental contributions are a stochastic function of parental investment. That is, allocating a proportion z of income to human capital investment translates into an effective (log) contribution of

$$h_{it} = \ln z + y_{it} + \varepsilon^h_{it} \tag{47}$$

where $\varepsilon^h_{it} \sim N(0, \sigma^2_{\varepsilon^h})$. Thus human capital of offspring is therefore:

$$x_{it} = \alpha_0 + \alpha_1 \cdot \theta_{it} + \alpha_2 \cdot y_{i,t} + \alpha_2 \cdot \varepsilon^h_{it} + \varepsilon_{it}, \tag{48}$$

where, as before, $\alpha_0 \equiv \alpha'_0 + \alpha_2 \cdot z$. By defining $\varepsilon^x_{it} \equiv \alpha_2 \cdot \varepsilon^h_{it} + \varepsilon_{it}$ as 'aggregate' luck, we can express human capital in a generalized form:

$$x_{it} = \alpha_0 + \alpha_1 \cdot \theta_{it} + \alpha_2 \cdot y_{it} + \varepsilon^x_{it}. \tag{49}$$

This generalization is useful because our results need only be adjusted by replacing ε_{it} with ε^x_{it}. In particular, when considering $\sigma^2_{\varepsilon^h} > 0$, the variance of human capital luck generalizes beyond σ^2_ε to $\sigma^2_{\varepsilon^x} \equiv \sigma^2_\varepsilon + \alpha_2^2 \cdot \sigma^2_{\varepsilon^h}$.

This generalization has four important implications. First, we see that now a policy parameter, α_2, will matter for ability sorting. Recall that α_2 captures the strength with which parental investment translates into offspring human capital, and reflects technology but also policy aimed at redistributing parental inputs (equality of opportunity arises when $\alpha_2 = 0$). Changes in α_2 will matter for ability sorting because such changes will impact the steady state precision of beliefs.

Second, the impact of this policy parameter is counterintuitive. A greater equality of opportunity (a lower α_2) reduces signal noise, thereby making beliefs more precise in the steady state. This facilitates *stronger* ability sorting and persistence. Intuitively, some component of the signal noise is due to the randomness associated with the return on parental inputs–when parental inputs are less important (due to a greater equality of opportunity) this component of the noise diminishes and underlying ability is better revealed.

Third, the impact of a greater equality of opportunity (lower α_2) on social mobility becomes non-monotonic in general. A greater equality of opportunity will raise social mobility *holding 'ability mobility' fixed*, but will reduce ability mobility. Note too that while an decrease in α_2 will raise the persistence of ability, it will *lower* the persistence of income. Thus, movements toward greater social mobility in incomes may coincide with lower social 'ability' mobility.

Fourth, changes in a society's ability mobility will not be reliably reflected in changes in income mobility. Greater income mobility will in fact coincide with *less* ability mobility, if driven by greater equality of opportunity. On the other hand, greater income mobility will coincide with *greater* ability mobility, if driven by other parameters (e.g. the return to ability, α_1).

8 Conclusions

Understanding the transmission of innate ability is an important component of understanding the perpetuation of economic advantage. The marriage market is central in this understanding when ability is influenced by the ability of both parents. Indeed, the heritable nature of ability, along with a concern for the welfare of descendants, induces a concern for the ability of potential partners independently of the standard concern for their earning capacity. A satisfactory model of such a marriage market must contend with the unobserved nature of ability, and in particular how beliefs are shaped by observed characteristics of participants as well as their family background.

Our attempt at such a model hones in on such information frictions by removing all impediments to perfect sorting on earning capacity. This affords us a clear view of some broad new insights.

The first is that, despite ability being passively transmitted by parents, the economic environment will influence the process. It does so by shaping the steady state quality of information available to marriage market participants. Whilst the return to ability and the prevalence of economic luck will exert such an influence, institutional characteristics (such as the degree of meritocracy) and policy variables (such as income redistribution) will generally have no effect.

The second is that, *holding their ability fixed*, an individual's economic outcomes will have an impact on the expected ability of descendants. This holds, even though transmission is not influenced by economic outcomes by construction, because economic outcomes help determine the expected ability of the other individual that contributes to the ability of offspring (i.e. the spouse). This uncovers a new channel through which the economic fortunes of the current generation are influenced by the economic fortunes of prior generations: today's economic fortunes translate into marital fortunes which translate into the next generation's ability fortunes and thereby their economic fortunes. It also introduces a new motive for parental investment: it affects the market's perception of ability, thereby allowing offspring to marry higher-ability spouses, and therefore to generate higher-ability descendants.

The model rests on various simplifying assumptions in order to gain tractability, yet relaxing some of these offers scope for additional insight in future work. For instance, allowing for asymmetries by gender, embedding fertility decisions, and adding an explicit political economy layer (which influences and is influenced by the distribution of ability) all seem promising avenues.

The model contains implications for existing empirical work–e.g. in making sense of the 'policy invariance' reported by Clark [2014], and in cautioning against making inferences about ability persistence based on income persistence–but we are hopeful that the model will prove useful for more direct empirical work in the future. The model makes clear predictions that are testable in principal. The key barrier is obtaining reliable large scale data on the ability of individuals, their spouse and children. To the extent that ability is interpreted as being genetic in nature, we hold much hope in the increasing availability of genetic markers in large datasets.

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A Structural Model

In this section we develop the structural model that underlies the reduced-form equations describing human capital and income (i.e. equations (3), (4) and (1)).

A.1 Human Capital

Let X_{it} denote the human capital of a single agent of family i of generation t. We assume that X_{it} is determined by the contribution of ability, $G_{it} \equiv \exp(\theta_{it})$, effective parental inputs, $P_{i,t-1}$, and luck, $\varepsilon_{it} \sim N(0, \sigma_\varepsilon^2)$:

$$X_{it} = A_1 \cdot G_{it}^{\alpha_1} \cdot P_{i,t-1}^{\chi} \cdot \exp(\varepsilon_{it}),$$

where A_1, α_1 and χ are parameters with positive values. Effective parental inputs are a function of parental investment, $H_{i,t-1}$:

$$P_{i,t-1} \equiv H_{i,t-1}^{1-\sigma} \hat{P}_{t-1},$$

where $\sigma \in [0,1]$ parameterizes the redistribution of parental investment (Benabou [2002]) and thus captures equality of opportunity,[16] and $\hat{P}_{t-1}$ is a term that adjusts to ensure that the following resource constraint holds:

$$\int P_{i,t-1} di = \int H_{i,t-1} di$$

That is:

$$\hat{P}_{t-1} = \frac{\int H_{i,t-1} di}{\int H_{i,t-1}^{1-\sigma} di}.$$

Finally, parental investment depends on household income, $Y_{i,t-1}$, and an idiosyncratic component, $\varepsilon_{it}^h \sim N(0, \sigma_{\varepsilon^h}^2)$:

$$H_{it} = z \cdot Y_{i,t-1} \cdot \exp(\varepsilon_{it}^h),$$

where $z \in [0,1]$. In the main model, z is exogenous and $\sigma_{\varepsilon^h}^2 = 0$ for all (i,t). We endogenize z and allow $\sigma_{\varepsilon^h}^2 > 0$ in extensions.

By taking logs we get the following reduced form relationship:

$$x_{it} = \alpha_{0t} + \alpha_1 \cdot \theta_{it} + \alpha_2 \cdot y_{i,t-1} + \varepsilon_{it}^x, \tag{50}$$

where $\alpha_{0t} \equiv \ln A_1 + \chi \cdot \ln \hat{P}_{t-1} + \ln z$, $\alpha_2 \equiv \chi \cdot (1-\sigma)$ and $\varepsilon_{it}^x \equiv \varepsilon_{it} + \alpha_2 \cdot \varepsilon_{it}^h$. The value of α_{0t} depends only on the distribution of income, and thus will be a constant, α_0, in the steady state.

[16] An alternative approach would be to have human capital depend on a public input that is stochastically biased toward higher-income families. By reducing the bias, parental inputs are more reliably revealed by parental income.

A.2 Income

Household income, Y_{it}, results from the redistribution of pre-tax income, Y_{it}^{pre}:

$$Y_{it} = [Y_{it}^{\text{pre}}]^{1-\tau}\,\hat{Y}_t$$

where $\tau \in [0,1]$ parameterizes *income redistribution* (Benabou (2002)), and $\hat{Y}_t$ ensures that the resource constraint holds:

$$\int Y_{it}di = \int Y_{it}^{\text{pre}}di.$$

That is,

$$\hat{Y}_t = \frac{\int Y_{it}^{\text{pre}}di}{\int [Y_{it}^{\text{pre}}]^{1-\tau}\,di}.$$

Pre-tax income, Y_{it}^{pre}, depends on household output, Q_{it}, and parental income, $\bar{Y}_{i,t-1}$:

$$Y_{it}^{\text{pre}} = Q_{it}^{\mu}\bar{Y}_{i,t-1}^{1-\mu}\cdot\hat{Y}_t^{\text{pre}}$$

where μ parameterizes *meritocracy* and $\hat{Y}_t^{\text{pre}}$ ensures that the resource constraint holds:

$$\int Y_{it}^{\text{pre}}di = \int Q_{it}di.$$

That is,

$$\hat{Y}_t^{\text{pre}} = \frac{\int Q_{it}di}{\int Q_{it}^{1-\mu}di}.$$

Parental income, $\bar{Y}_{i,t-1}$, depends on the parental income of both household members:

$$\bar{Y}_{i,t-1} = Y_{i,t-1}^{1/2}Y_{i',t-1}^{1/2}.$$

Finally, household output, Q_{it}, is produced by the human capital of both household members:

$$Q_{it} = X_{it}^{1/2}X_{i't}^{1/2}$$

Taking logs gives us the following relationship:

$$y_{it} = \beta_{0t} + \beta_1 \cdot [x_{i,t} + x_{i',t}]/2 + \beta_2 \cdot [y_{i,t-1} + y_{i',t-1}]/2,$$

where $\beta_{0t} \equiv \ln \hat{Y}_t + (1-\tau) \cdot \ln \hat{Y}_t^{\text{pre}}$, $\beta_1 \equiv (1-\tau) \cdot \mu$, and $\beta_2 \equiv (1-\tau) \cdot (1-\mu)$. The value of β_{0t} depends only on the distribution of income and output, and thus will be a constant, β_0, in the steady state.

B Deriving Correlations

B.1 Ability

The relevant equations are reproduced here:

$$\theta_{it} = \frac{b}{2} \cdot \theta_{i,t-1} + \frac{b}{2} \cdot \theta_{i',t-1} + \upsilon_{it} \tag{51}$$

$$\theta_{it} = \phi_{i,t} + \varepsilon_{it}^{\gamma} \tag{52}$$

$$\phi_{i,t} = \phi_{i',t} \tag{53}$$

Notice that in the steady state $\bar{\theta} \equiv E[\theta_{it}] = 0$ and $\bar{\phi} \equiv E[\phi_{i,t}] = 0$. Thus this is a system of mean-zero random variables. Recalling that if r_1 and r_2 are mean-zero random variable then $\text{Cov}(r_1, r_2) = \mathbb{E}[r_1 r_2]$, we can use (51) to get:

$$\text{Cov}(\theta_{it}, r_{it}) = \frac{b}{2} \cdot \text{Cov}(\theta_{i,t-1}, r_{it}) + \frac{b}{2} \cdot \text{Cov}(\theta_{i',t-1}, r_{it}) + \text{Cov}(\upsilon_{i,t}, r_{it}) \tag{54}$$

where r_{it} is any mean-zero random variable. Thus, we have the following system:

$$\text{Cov}(\theta_{it}, \theta_{it}) = \frac{b}{2} \cdot \text{Cov}(\theta_{i,t-1}, \theta_{it}) + \frac{b}{2} \cdot \text{Cov}(\theta_{i',t-1}, \theta_{it}) + \sigma_{\upsilon}^2 \tag{55}$$

$$\text{Cov}(\theta_{it}, \theta_{i,t-1}) = \frac{b}{2} \cdot \text{Cov}(\theta_{i,t-1}, \theta_{i,t-1}) + \frac{b}{2} \cdot \text{Cov}(\theta_{i',t-1}, \theta_{i,t-1}) \tag{56}$$

$$\text{Cov}(\theta_{it}, \theta_{i',t-1}) = \frac{b}{2} \cdot \text{Cov}(\theta_{i,t-1}, \theta_{i',t-1}) + \frac{b}{2} \cdot \text{Cov}(\theta_{i',t-1}, \theta_{i',t-1}) \tag{57}$$

Using the steady state conditions $\text{Cov}(\theta_{it}, \theta_{it}) = \text{Cov}(\theta_{i,t-1}, \theta_{i,t-1}) = \text{Cov}(\theta_{i',t-1}, \theta_{i',t-1}) = \sigma_\theta^2$ and $\text{Cov}(\theta_{i,t-1}, \theta_{i',t-1}) = \text{Cov}(\theta_{i,t}, \theta_{i',t})$, and the symmetry property $\text{Cov}(\theta_{it}, \theta_{i',t-1}) =$

$\text{Cov}(\theta_{it}, \theta_{i,t-1})$, simplifies this to:

$$\sigma_\theta^2 = \frac{b}{2} \cdot \text{Cov}(\theta_{i,t-1}, \theta_{it}) + \frac{b}{2} \cdot \text{Cov}(\theta_{i',t-1}, \theta_{it}) + \sigma_\upsilon^2 \tag{58}$$

$$\text{Cov}(\theta_{it}, \theta_{i,t-1}) = \frac{b}{2} \cdot \sigma_\theta^2 + \frac{b}{2} \cdot \text{Cov}(\theta_{i',t}, \theta_{i,t}) \tag{59}$$

These tell us the ability variance and the intergenerational ability covariance as a function of the spousal covariance. Dividing both by σ_θ^2 and solving yields (8) and (9) in the text.

Now we perform the same exercise using (52):

$$\text{Cov}(\theta_{it}, \theta_{i't}) = \text{Cov}(\phi_{it}, \theta_{i't}) \tag{60}$$

$$\text{Cov}(\theta_{it}, \phi_{i't}) = \text{Cov}(\phi_{it}, \phi_{i't}) \tag{61}$$

$$\text{Cov}(\theta_{it}, \theta_{it}) = \text{Cov}(\phi_{it}, \theta_{it}) + \gamma \tag{62}$$

$$\text{Cov}(\theta_{it}, \phi_{it}) = \text{Cov}(\phi_{it}, \phi_{it}). \tag{63}$$

Using $\text{Cov}(\theta_{i't}, \phi_{it}) = \text{Cov}(\theta_{it}, \phi_{i't})$ gives

$$\text{Cov}(\theta_{it}, \theta_{i't}) = \text{Cov}(\phi_{it}, \phi_{i't}) \tag{64}$$

$$\text{Cov}(\theta_{it}, \theta_{it}) = \text{Cov}(\phi_{it}, \phi_{it}) + \gamma. \tag{65}$$

The segregation-on-status equation (53) gives $\text{Cov}(\phi_{it}, \phi_{i't}) = \text{Cov}(\phi_{it}, \phi_{it})$. Therefore

$$\text{Cov}(\theta_{it}, \theta_{i't}) = \text{Cov}(\theta_{it}, \theta_{it}) - \gamma. \tag{66}$$

Thus the steady state covariances of interest are given as the solution to (58), (59), and (66). The solution is

$$\sigma_\theta^2 = \frac{\sigma_\upsilon^2 - \gamma \cdot \frac{b^2}{2}}{1 - b^2} \tag{67}$$

$$\text{Cov}(\theta_{it}, \theta_{i,t-1}) = \frac{b\sigma_\upsilon^2 - \gamma \cdot \frac{b}{2}}{1 - b^2} \tag{68}$$

$$\text{Cov}(\theta_{it}, \theta_{i't}) = \frac{\sigma_\upsilon^2 - \gamma \cdot (1 - \frac{b^2}{2})}{1 - b^2}. \tag{69}$$

Dividing the latter two equations by the first gives the correlations reported in the proposition.

B.1.1 Multi-Generational Correlations

Using the ability transmission equation and symmetry gives, for $k = 2, 3, ...$:

$$\text{Cov}(\theta_{it}, \theta_{i,t-k}) = b \cdot \text{Cov}(\theta_{it}, \theta_{i,t-(k-1)}). \tag{70}$$

Using the steady state conditions, the ability correlation between family members $k \in \{2, 3, ..\}$ generations apart is thus given by:

$$\rho_{\theta,k}^{PC} \equiv \frac{\text{Cov}(\theta_{it}, \theta_{i,t-k})}{\sigma_\theta^2} = b^{k-1} \cdot \rho_\theta^{PC} = b^k \cdot \left[\frac{\sigma_\upsilon^2 - \frac{\gamma}{2}}{\sigma_\upsilon^2 - b^2 \cdot \frac{\gamma}{2}} \right]. \tag{71}$$

Thus the long correlation implied by extrapolating the short correlation always *understates* the true long correlation:

$$\frac{(\rho_\theta^{PC})^k}{\rho_{\theta,k}^{PC}} = \left[\frac{\sigma_\upsilon^2 - \frac{\gamma}{2}}{\sigma_\upsilon^2 - b^2 \cdot \frac{\gamma}{2}} \right]^{k-1} \in (0, 1). \tag{72}$$

The extent of the bias increases in k as this ratio goes to zero as k increases. Furthermore this bias is endogenous in our setting, as the ratio is decreasing in γ.

The divergence between extrapolated and actual long correlations is in this case due entirely to the omission of the omitted parent's ability. Intuitively, the one-generation correlation does not capture the fact that both the parent and offspring abilities are positively correlated with the other parent's ability.

B.2 Income

Notice that in the steady state $\bar{\theta} \equiv E[\theta] = 0$ and $\bar{y} \equiv E[y] = \frac{\pi_0}{1-\beta_2}$. For what follows, take y to be the de-meaned counterpart (to save on notation). In steady state with segregation we have

$$y_{i,t+1} = \pi_1 \cdot \theta_{i,t} + \pi_2 \cdot y_{i,t} + \varepsilon_{it}^y. \tag{73}$$

Using a method identical to that for ability, we get the following system:

$$\text{Cov}(y_{i,t+1}, y_{i,t+1}) = \pi_1 \cdot \text{Cov}(\theta_{it}, y_{i,t+1}) + \pi_2 \cdot \text{Cov}(y_{i,t}, y_{i,t+1}) + \sigma_{\varepsilon^y}^2 \tag{74}$$

$$\text{Cov}(y_{i,t+1}, \theta_{it}) = \pi_1 \cdot \text{Cov}(\theta_{it}, \theta_{it}) + \pi_2 \cdot \text{Cov}(y_{i,t}, \theta_{it}) \tag{75}$$

$$\text{Cov}(y_{i,t+1}, y_{i,t}) = \pi_1 \cdot \text{Cov}(\theta_{it}, y_{i,t}) + \pi_2 \cdot \text{Cov}(y_{i,t}, y_{i,t}). \tag{76}$$

From (51) and $\text{Cov}(\theta_{i',t-1}, y_{i',t}) = \text{Cov}(\theta_{i,t-1}, y_{i,t})$ we have:

$$\text{Cov}(\theta_{it}, y_{i,t}) = \frac{b}{2} \cdot \text{Cov}(\theta_{i,t-1}, y_{i,t}) + \frac{b}{2} \cdot \text{Cov}(\theta_{i',t-1}, y_{i,t}) \tag{77}$$

$$= b \cdot \text{Cov}(\theta_{i,t-1}, y_{i,t}). \tag{78}$$

Applying the steady state conditions gives the following system:

$$\text{Cov}(y_{i,t}, y_{i,t}) = \pi_1 \cdot \text{Cov}(\theta_{it}, y_{i,t+1}) + \pi_2 \cdot \text{Cov}(y_{i,t}, y_{i,t-1}) + \sigma^2_{\varepsilon^y} \tag{79}$$

$$\text{Cov}(y_{i,t+1}, \theta_{it}) = \pi_1 \cdot \text{Cov}(\theta_{it}, \theta_{it}) + \pi_2 \cdot \text{Cov}(y_{i,t}, \theta_{it}) \tag{80}$$

$$\text{Cov}(y_{i,t}, y_{i,t-1}) = \pi_1 \cdot \text{Cov}(\theta_{it}, y_{i,t}) + \pi_2 \cdot \text{Cov}(y_{i,t}, y_{i,t}) \tag{81}$$

$$\text{Cov}(\theta_{it}, y_{i,t}) = b \cdot \text{Cov}(\theta_{i,t}, y_{i,t+1}). \tag{82}$$

Solving gives us the two covariances of interest:

$$\text{Cov}(y_{it}, y_{it}) \equiv \sigma^2_y = \frac{\left(\frac{1+b\pi_2}{1-b\pi_2}\right) \pi_1^2 \cdot \sigma^2_\theta + \sigma^2_{\varepsilon^y}}{1-\pi_2^2} \tag{83}$$

$$\text{Cov}(y_{it}, y_{i,t-1}) = \frac{\left(\frac{b+\pi_2}{1-b\pi_2}\right) \pi_1^2 \cdot \sigma^2_\theta + \pi_2 \cdot \sigma^2_{\varepsilon^y}}{1-\pi_2^2}, \tag{84}$$

where σ^2_θ was derived above in the ability section. The correlation of interest is:

$$\rho^{PC}_y \equiv \frac{\text{Cov}(y_{it}, y_{i,t-1})}{\text{Cov}(y_{it}, y_{it})} = \frac{\left(\frac{b+\pi_2}{1-b\pi_2}\right) \pi_1^2 \cdot \sigma^2_\theta + \pi_2 \cdot \sigma^2_{\varepsilon^y}}{\left(\frac{1+b\pi_2}{1-b\pi_2}\right) \pi_1^2 \cdot \sigma^2_\theta + \sigma^2_{\varepsilon^y}} \tag{85}$$

$$= \pi_2 + b \cdot \Phi(\gamma), \tag{86}$$

where $\Phi(\gamma)$ is defined in (33). Here we see that income would be persistent *even if income did not depend on parental income* (i.e. if $\pi_2 = 0$).

Since Φ is increasing in γ^{-1}, we have that α_1 raises the persistence of income both directly (via π_1) and indirectly (via sorting, γ). Similarly, σ^2_ε lowers the persistence of income both directly (via ($\sigma^2_{\varepsilon^y}$) and indirectly (via sorting, γ).

We also see that ρ^{PC}_y depends on π_2, whereas ρ^{PC}_θ was independent of π_2. Thus, policy that affects the sensitivity of human capital to parental inputs or meritocracy in general, will have an effect on income mobility but will have no impact on ability mobility. As such, (i) changes in persistence of observed characteristics need not be informative about changes in the persistence of unobserved characteristics, and (ii) the effect of such policy will be limited by the fact that income will persist even if parental income has no direct effect on income.

B.2.1 Ability and Parental Income

The same notation can be used to describe the variance of income:

$$\sigma_y^2 = \frac{1}{1-b\pi_2}\frac{\pi_1^2\sigma_\theta^2}{\Phi(\gamma)}. \tag{87}$$

To what extent do richer parents have higher-ability offspring? Using (80) and (82) we have

$$\text{Cov}(\theta_{it}, y_{i,t-1}) = b \cdot \frac{\pi_1}{1-b\pi_2} \cdot \sigma_\theta^2. \tag{88}$$

As such, the correlation is

$$\text{Cor}(\theta_{it}, y_{i,t-1}) = b \cdot \frac{1}{1-b\pi_2} \cdot \frac{\pi_1\sigma_\theta}{\sigma_y} = b \cdot \sqrt{\frac{\Phi(\gamma)}{1-b\pi_2}}. \tag{89}$$

Similarly, if we were to regress ability on parental income by OLS the coefficient on parental income would be

$$\frac{\text{Cov}(\theta_{it}, y_{i,t-1})}{\sigma_y^2} = b \cdot \frac{\pi_1}{1-b\pi_2} \cdot \frac{\sigma_\theta^2}{\sigma_y^2} = \frac{b}{\pi_1} \cdot \Phi(\gamma). \tag{90}$$

Note too that the R-squared from such a regression would be:

$$[\text{Cor}(\theta_{it}, y_{i,t-1})]^2 = \frac{b^2}{1-b\pi_2} \cdot \Phi(\gamma). \tag{91}$$

Both of these measures of association are increasing in the quality of information, γ^{-1}. That is, better information in the steady state will raise the tendency for those with above average incomes to have children of above average ability. Again, the indirect sorting effect of α_1 and σ_ε^2 will exacerbate the direct effects. For instance, an increase in α_1 will mean that income is more sensitive to ability and therefore we would expect a high parental income to be more strongly associated with a high parental ability and thus with a high child ability. In addition, however, a higher α_1 raises the strength of sorting, implying a stronger association between parental ability and child ability.

B.2.2 Longer-Term Income Mobility

To work out $\text{Cov}(y_{it}, y_{i,t-k})$ for $k = 2, 3, ...$ note:

$$\text{Cov}(y_{it}, y_{i,t-k}) = \pi_1 \cdot \text{Cov}(\theta_{it}, y_{i,t-k}) + \pi_2 \cdot \text{Cov}(y_{i,t-1}, y_{i,t-k}) \tag{92}$$

$$= \pi_1 \cdot \text{Cov}(\theta_{it}, y_{i,t-k}) + \pi_2 \cdot \text{Cov}(y_{i,t}, y_{i,t-(k-1)}) \tag{93}$$

and

$$\text{Cov}(\theta_{it}, y_{i,t-k}) = b \cdot \text{Cov}(\theta_{i,t-1}, y_{i,t-k}) \tag{94}$$

$$= b \cdot \text{Cov}(\theta_{i,t}, y_{i,t-(k-1)}) \tag{95}$$

$$= b^k \cdot \text{Cov}(\theta_{i,t}, y_{i,t}) \tag{96}$$

$$= b^k \cdot \frac{\pi_1}{1 - b\pi_2} \cdot \sigma_\theta^2. \tag{97}$$

Thus, letting $\rho_{y,k}^{PC} \equiv \text{Cov}(y_{it}, y_{i,t-k})/\sigma_y^2$ be the k-generation income correlation, we have

$$\rho_{y,k}^{PC} = b^k \cdot \left[\frac{\pi_1^2}{1 - b\pi_2} \cdot \frac{\sigma_\theta^2}{\sigma_y^2} \right] + \pi_2 \cdot \rho_{y,k-1}^{PC} \tag{98}$$

$$= b^k \cdot \Phi(\gamma) + \pi_2 \cdot \rho_{y,k-1}^{PC}, \tag{99}$$

where $\Phi(\gamma)$ is defined in (33).[17]

This expression can be used to compare the k-generation correlation with that implied by the geometric extrapolation of the 1-generation correlation. In particular, the multi-generation correlation can be over- or under-stated by the geometric extrapolation of the 1-generation correlation depending on the relative size of ρ_1 and b.

Proposition 9 *Extrapolating the single-generation income correlation* understates *the true multi-generation income correlation (i.e. $\rho_1^k < \rho_k$) if*

$$\pi_2 < b \cdot (1 - \Phi(\gamma)). \tag{102}$$

The extrapolation overstates *the true correlation (i.e. $\rho_1^k > \rho_k$) if the inequality is*

[17] Solving explicitly:

$$\rho_{y,k}^{PC} = \Phi(\gamma) \cdot \left[\sum_{s=2}^{k} b^s \cdot \pi_2^{k-s} \right] + \pi_2^{k-1} \cdot \rho_y^{PC} \tag{100}$$

$$= \Phi(\gamma) \cdot \left[\sum_{s=1}^{k} b^s \cdot \pi_2^{k-s} \right] + \pi_2^k \tag{101}$$

reversed. The extrapolation equals *the true correlation (i.e.* $\rho_1^k = \rho_k$*) if the inequality is replaced with an equality.*

Proof. We first establish that $\rho_1 < b$ implies $\rho_1^k < \rho_k$ for all $k \in \{2, 3, ...\}$. Since $\rho_1 = b\Phi(\gamma) + \pi_2$, the condition $\rho_1 < b$ is the same as the one stated in the proposition.

To this end, we first we show that if (i) $\rho_1 < b$ and (ii) $\rho_1^k < \rho_k$ for some $k \in \{2, 3, ...\}$, then $\rho_1^{k+1} < \rho_{k+1}$. To see this note

$$\rho_1^{k+1} = \rho_1^k \cdot [b\Phi + \pi_2] \tag{103}$$

$$< b^{k+1}\Phi + \rho_1^k \cdot \pi_2 \tag{104}$$

$$< b^{k+1}\Phi + \rho_k \cdot \pi_2 = \rho_{k+1}, \tag{105}$$

where the first inequality comes from (i) and the second from (ii).

Second we show that (i) implies that (ii) holds for $k = 2$. This follows since

$$\rho_1^2 = \rho_1 \cdot [b\Phi + \pi_2] \tag{106}$$

$$< b^2\Phi + \rho_1 \cdot \pi_2 = \rho_2, \tag{107}$$

where the inequality comes from (i).

By induction we therefore have that $\rho_1 < b$ implies $\rho_1^k < \rho_k$ for all $k \in \{2, 3, ...\}$. Thus long run persistence is larger than that implied by the short run persistence if $\rho_1 < b$ (equivalently, $\pi_2 < b \cdot (1 - \Phi(\gamma))$. The same logic applies when "<" in (i) is replaced with "=" or ">". □

In other words, the extrapolation can either overstate or understate the true persistence of income across multiple generations. The condition on parameters that determines which case arises relates to the relative strength of parental transmission of income and ability. For instance, the extrapolation *overstates* the true persistence in Becker and Tomes [1979] but *understates* it in Clark [2014]. In the former, there is no luck component, so that $\Phi = 1$ and the condition in the proposition can never hold. In the latter, parental income does not matter ($\pi_2 = 0$), so that the condition is always satisfied (as long as there is a luck component).[18]

[18]Solon (2014) points out that the long run persistence of measured income can be understated by the short run persistence if there is measurement error. In contrast, the presence of luck does not require any measurement error. The two are not equivalent since the measurement error is not transmitted to offspring whereas luck is.

C Values

The purpose of this section is to lay the groundwork that will ultimately allow us to derive expressions for the present value of dynastic income and dynastic ability. In particular these quantities will be expressed in terms of variables under which household (i,t) has control. In particular, the expected income and family status of the offspring in household (i,t). This will be useful for describing preferences in the marriage market, as potential partners will affect both of these expectations. It will also be useful for analysing optimal investment, as parental investment will also influence both of these expectations.

Consider the following system of expectations:

$$\mathbb{E}_t\left[y_{i,t+\tau+1}\right] = \pi_0 + \pi_1 \cdot \mathbb{E}_t\left[\theta_{i,t+\tau}\right] + \pi_2 \cdot \mathbb{E}_t\left[y_{i,t+\tau}\right] \tag{108}$$

$$\mathbb{E}_t\left[\theta_{i,t+\tau}\right] = (b/2) \cdot \mathbb{E}_t\left[\theta_{i,t+(\tau-1)}\right] + (1/2) \cdot \mathbb{E}_t\left[\bar{\phi}_{i,t+\tau}\right] \tag{109}$$

$$\mathbb{E}_t\left[\bar{\phi}_{i,t+\tau+1}\right] = b \cdot \left[\lambda \cdot \mathbb{E}_t\left[\bar{\phi}_{i,t+\tau}\right] + (1-\lambda) \cdot \mathbb{E}_t\left[\theta_{i,t+\tau}\right]\right] \tag{110}$$

The first of these is from the reduced-form income equation. The second is from the ability transmission equation, noting that $\mathbb{E}_t\left[\theta_{i',t+(\tau-1)}\right] = \mathbb{E}_t\left[\phi_{i,t+(\tau-1)}\right]$ and that $\mathbb{E}_t\left[\bar{\phi}_{i,t+\tau}\right] = b \cdot \mathbb{E}_t\left[\phi_{i,t+(\tau-1)}\right]$. The third comes from the relationship between family status and individual status, and the expression for individual status. For $\tau = 1, 2, ...$, define

$$\mathbf{w}_\tau \equiv \begin{bmatrix} \mathbb{E}_t\left[y_{i,t+\tau}\right] - \frac{\pi_0}{1-\pi_2} \\ \mathbb{E}_t\left[\theta_{i,t+\tau-1}\right] \\ \mathbb{E}_t\left[\bar{\phi}_{i,t+\tau}\right] \end{bmatrix} \tag{111}$$

so that the system can be written

$$\mathbf{A}_1 \mathbf{w}_{\tau+1} = \mathbf{A}_2 \mathbf{w}_\tau \tag{112}$$

where

$$\mathbf{A}_1 \equiv \begin{bmatrix} 1 & -\pi_1 & 0 \\ 0 & 1 & 0 \\ 0 & -(1-\lambda) \cdot b & 1 \end{bmatrix}, \ \mathbf{A}_2 \equiv \begin{bmatrix} \pi_2 & 0 & 0 \\ 0 & b/2 & 1/2 \\ 0 & 0 & \lambda \cdot b \end{bmatrix}. \tag{113}$$

This gives

$$\mathbf{w}_{\tau+1} = \mathbf{A}\mathbf{w}_\tau \tag{114}$$

where $\mathbf{A} \equiv \mathbf{A}_1^{-1}\mathbf{A}_2$. Therefore we have

$$\mathbf{w}_\tau = \mathbf{A}^{\tau-1}\mathbf{w}_1 \tag{115}$$

so that

$$\mathbf{V} \equiv \sum_{\tau=1}^{\infty} \delta^{\tau-1}\mathbf{w}_\tau \tag{116}$$

$$= \left[\sum_{\tau=1}^{\infty} \delta^{\tau-1}\mathbf{A}^{\tau-1}\right]\mathbf{w}_1 = \left[\sum_{\tau=0}^{\infty} \delta^{\tau}\mathbf{A}^{\tau}\right]\mathbf{w}_1 = \varphi\mathbf{w}_1, \tag{117}$$

where $\varphi \equiv [\mathbf{I_3} - \delta\mathbf{A}]^{-1}$ (where $\mathbf{I_3}$ is the 3×3 identity matrix).

C.1 Dynastic Income

Define

$$V_{it}^y \equiv \mathbb{E}_t\left[\sum_{\tau=1}^{\infty} \delta^{\tau-1} y_{i,t+\tau}\right] \tag{118}$$

The main result is the following.

Lemma 3 *We have*

$$V_{it}^y = \varphi_0 + \varphi_{11} \cdot \mathbb{E}_t\left[y_{i,t+1}\right] + \varphi_{12} \cdot \mathbb{E}_t\left[\theta_{i,t}\right] + \varphi_{13} \cdot \mathbb{E}_t\left[\bar{\phi}_{i,t+1}\right] \tag{119}$$

where $\{\varphi_0, \varphi_{11}, \varphi_{12}, \varphi_{13}\}$ *are positive constants given by*

$$\varphi_0 = \frac{\delta\pi_0}{1-\delta} \cdot \frac{1}{1-\delta\pi_2} \tag{120}$$

$$\varphi_{11} = \frac{1}{1-\delta\pi_2} \tag{121}$$

$$\varphi_{12} = \frac{1}{1-\delta\pi_2} \cdot \frac{\delta\pi_1}{(1-b\delta)(2-b\delta\lambda)} \cdot b(1-b\delta\lambda) \tag{122}$$

$$\varphi_{13} = \frac{1}{1-\delta\pi_2} \cdot \frac{\delta\pi_1}{(1-b\delta)(2-b\delta\lambda)}. \tag{123}$$

To see this note that, by definition, the first element of $\mathbf{V}$ (defined in (116)) is

$$\mathbf{V}_{11} = \sum_{\tau=1}^{\infty} \delta^{\tau-1} \mathbb{E}_t \left[y_{i,t+\tau} \right] - \frac{\pi_0}{1-\pi_2} \sum_{\tau=1}^{\infty} \delta^{\tau-1} \tag{124}$$

$$= V_{it}^{y} - \frac{\pi_0}{1-\pi_2} \frac{1}{1-\delta}. \tag{125}$$

From (117) we have

$$\mathbf{V}_{11} = \varphi_{11} \cdot (\mathbb{E}_t \left[y_{i,t+1} \right] - \frac{\pi_0}{1-\pi_2}) + \varphi_{12} \cdot \mathbb{E}_t \left[\theta_{i,t} \right] + \varphi_{13} \cdot \mathbb{E}_t \left[\bar{\phi}_{i,t+1} \right] \tag{126}$$

where φ_{rc} is the row r column c element of φ. Therefore

$$V_{it}^{y} = \varphi_0 + \varphi_{11} \cdot \mathbb{E}_t \left[y_{i,t+1} \right] + \varphi_{12} \cdot \mathbb{E}_t \left[\theta_{i,t} \right] + \varphi_{13} \cdot \mathbb{E}_t \left[\bar{\phi}_{i,t+1} \right] \tag{127}$$

where $\varphi_0 \equiv \frac{\pi_0}{1-\pi_2} \left[\frac{1}{1-\delta} - \varphi_{11} \right]$. The values given in the lemma are revealed by direct calculation of φ.

C.1.1 Segregation is Stable and Feasible

Segregation is trivially feasible. For stability we note that the attractiveness of individuals in the marriage market is summarized by an index of their observed characteristics. In particular, potential partners are evaluated according to $\mathbb{E}_t U_{i,t+1}$, which is precisely V_{it}^{y} from the previous section. Partner characteristics matter because

$$\mathbb{E}_t \left[y_{i,t+1} \right] = \beta_0 + \beta_1 \cdot [x_{i,t} + x_{i',t}] + \beta_2 \cdot [y_{it} + y_{i't}]/2 \tag{128}$$

$$\mathbb{E}_t \left[\bar{\phi}_{i,t+1} \right] = b \cdot [\phi_{i,t} + \phi_{i',t}]. \tag{129}$$

It therefore follows that the attractiveness of the spouse from household (j,t) is given by:

$$a_{j,t} \equiv (\varphi_{11}\beta_1/2) \cdot x_{j,t} + (\varphi_{11}\beta_2/2) \cdot y_{j,t} + (\varphi_{13} b) \cdot \phi_{j,t}. \tag{130}$$

Stability and feasibility in the period t marriage market requires segregation on a_{it}, which is indeed achieved by segregation. That is, if an agent from household (i,t) were to strictly prefer to marry an agent from household (j,t) to their assigned partner under segregation, (i.e. an agent from household (i',t), where $a_{i',t} = a_{it}$), then it must be that $a_{i',t} < a_{j,t}$. But then this implies $a_{i,t} < a_{j,t} = a_{j',t}$, so that j would strictly prefer to *not* match with i over their assigned partner under segregation j'. Thus, segregation is indeed stable and feasible.

C.1.2 Optimal Parental Investment

Assuming all other households invest a fraction z^* of their income, the preferences of household (i,t) are given by

$$\ln(1 - z_{it}) + y_{it} + \delta \cdot V_{it}^{y}, \tag{131}$$

where V_{it}^{y} is defined above. Investment affects V_{it}^{y} in two ways, since:

$$\mathbb{E}_t\left[y_{i,t+1}\right] = \text{constants} + (\beta_1\alpha_2) \cdot \ln z_{it} \tag{132}$$

$$\mathbb{E}_t\left[\bar{\phi}_{i,t+1}\right] = b \cdot \mathbb{E}_t[\phi_{i,t}] = \text{constants} + b(1-\lambda)\frac{\alpha_2}{\alpha_1} \cdot \ln z_{it}. \tag{133}$$

Therefore, ignoring constants, preferences over z_{it} are given by

$$\ln(1 - z_{it}) + \zeta_1 \cdot \ln z_{it} + \zeta_2 \cdot \ln z_{it} \tag{134}$$

where

$$\zeta_1 \equiv \delta \cdot \varphi_{11} \cdot (\beta_1\alpha_2) = \frac{\delta\beta_1\alpha_2}{1 - \delta[\beta_1\alpha_2 + \beta_2]} \tag{135}$$

$$\zeta_2 \equiv \delta \cdot \varphi_{13} \cdot b(1-\lambda)\frac{\alpha_2}{\alpha_1} = \zeta_1 \cdot \frac{b\delta}{1 - b\delta} \cdot \frac{1-\lambda}{(2 - b\delta\lambda)} \tag{136}$$

C.2 Dynastic Ability

One measure of ability across a dynasty is:

$$V_{it}^{\theta} \equiv \sum_{\tau=0}^{\infty} \delta^{\tau}\mathbb{E}_t\left[\theta_{i,t+\tau}\right] \tag{137}$$

where δ acts as a discount factor for future ability levels. The vector $\mathbf{V}$ is useful for computing this quantity since, by definition, the second element is:

$$\mathbf{V}_{21} = \sum_{\tau=1}^{\infty} \delta^{\tau-1}\mathbb{E}_t\left[\theta_{i,t+\tau-1}\right] = V_{it}^{\theta}. \tag{138}$$

The above analysis thus tells us that

$$V_{it}^{\theta} = \varphi_{21} \cdot \mathbb{E}_t\left[y_{i,t+1}\right] + \varphi_{22} \cdot \mathbb{E}_t\left[\theta_{i,t}\right] + \varphi_{23} \cdot \mathbb{E}_t\left[\bar{\phi}_{i,t+1}\right] \tag{139}$$

where direct computation of φ yields

$$\varphi_{21} = 0 \tag{140}$$

$$\varphi_{22} = \frac{2 - b\delta(1+\lambda)}{(1-b\delta)(2-b\delta\lambda)} \tag{141}$$

$$\varphi_{23} = \frac{\delta}{(1-b\delta)(2-b\delta\lambda)}. \tag{142}$$

C.2.1 Effect of Luck on Dynastic Ability

Thus economic luck, ε_{it}, raises dynastic ability because (and only because) it raises the family status of the next generation. In particular,

$$V_{it}^{\theta} = \text{constants} + \varphi_{23} b(1-\lambda) \cdot \xi_{it} \tag{143}$$

$$= \text{constants} + \frac{b\delta}{1-b\delta}\frac{1-\lambda}{2-b\delta\lambda} \cdot \xi_{it}. \tag{144}$$

Since $\xi_{it} \equiv \varepsilon_{it}/\alpha_1$, we have the following.

Proposition 10 *Dynastic ability is increasing in economic luck. In particular,*

$$\frac{dV_{it}^{\theta}}{d\varepsilon_{it}} = \frac{b\delta}{1-b\delta}\frac{1-\lambda}{2-b\delta\lambda} \cdot \frac{1}{\alpha} > 0. \tag{145}$$

This effect is decreasing in λ.

The first two fractions indicate that the analysis of the effect of luck on dynastic ability closely resembles the analysis of the 'status' motive for parental investment. However there are two effects associated with the return to ability, α_1. First, a higher return to ability lowers λ and thus raises the sensitivity of dynastic ability to the signal. However, the final fraction reveals an offsetting direct effect: a higher return to ability makes the signal less sensitive to luck.