Econ 807: Macroeconomic Theory and Policy
Assignment 6: Asset Pricing

1. Consider an economy populated by a representative agent. Preferences are given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\alpha} - 1}{1-\alpha} \right],$$

where $0 < \beta < 1$. The economy has a single productive asset, which consists of one unit of durable capital that generates a stochastic stream of nonstorable output y_t. Assume that (the log of) output follows a first-order Markov process; i.e.,

$$\ln y_{t+1} = (1-\rho)\ln \overline{y} + \rho \ln y_t + \varepsilon_t,$$

where $\varepsilon_t \in \{-\sigma, \sigma\}$ with equal probability and $-1 < \rho < 1$.

   (a) Let $\phi_b(y)$ denote the equilibrium price of a one-period risk-free bond that promises one unit of future consumption. Let $y_L(y) = \overline{y}^{1-\rho} y^{\rho} \exp(-\sigma)$ and $y_H(y) = \overline{y}^{1-\rho} y^{\rho} \exp(\sigma)$. Show that:

$$\phi_b(y) = y^{\alpha} \beta \left\{ 0.5 y_L(y)^{-\alpha} + 0.5 y_H(y)^{-\alpha} \right\}.$$

   (b) Let $\phi_e(y)$ denote the equilibrium price of a claim to the (stochastic) stream of output generated by the productive asset. Show that the pricing function $\phi_e(y)$ satisfies: $\phi_e(y) = y^{\alpha}\beta \{0.5 [y_L(y) + \phi_e(y_L(y))] y_L(y)^{-\alpha} + 0.5 [y_H(y) + \phi_e(y_H(y))] y_H(y)^{-\alpha}\}.$

2. Let $\overline{y} = 1$ and assume that $\rho = 0.95$, $\sigma = 0.006$ (so that the standard deviation of log output is approximately 2%). Assume that $\alpha = 2$ (you can play around with this parameter to see how the model behaves for different degrees of risk-aversion). With this parameterization, you can solve for $\phi_b(y)$ directly. Use the Coleman algorithm to solve for $\phi_e(y)$. Conduct a simulation and compare the average rates of return on bonds versus equity (this difference is called the *risk premium*). Also, plot the two functions and explain their properties.

3. What we have called 'equity' here corresponds to a claim on the economy's GDP. In reality, equity represents a claim against the cash flow generated by the corporate sector. Let $y_t = w_t + d_t$, where d_t represents 'cash flow' and w_t represents all other income (wage bill, interest payments to bondholders, etc.). Assume that (w_t, d_t) each follow a Markov process:

$$\begin{aligned} \ln w_{t+1} &= (1-\rho_w)\ln \overline{w} + \rho_w \ln w_t + \varepsilon_t^w; \\ \ln d_{t+1} &= (1-\rho_d)\ln \overline{d} + \rho_d \ln d_t + \varepsilon_t^d; \end{aligned}$$

where $cor(\varepsilon_t^w, \varepsilon_t^d) = 0$. Here, we have $\overline{y} = \overline{w} + \overline{d}$. Again, normalize $\overline{y} = 1$ and calibrate $\overline{d}$ to the data (i.e., set $\overline{d} = 0.05$). Assume that $\rho_w = \rho_d = 0.95$. Since cash flow is much more volatile that other income components, set $std(\varepsilon_t^w) = 0.004$ and $std(\varepsilon_t^d) = 0.016$. As before, assume that both innovation terms lie in a two point set (and occur with equal probability) and have a zero mean. Solve for the bond and equity pricing functions $\phi_b(w,d)$ and $\phi_e(w,d)$, where equity now constitutes a claim against cash flow (and not the GDP). Simulate the model and report the equity premium.

Note: to solve for these asset prices, you will have to modify your INTERP procedure as follows:

```
proc INTERP(rule,xx,yy);
local i,j,w1,w2;
i = maxindc( divid .ge xx );
j = maxindc( wage .ge yy );
i=maxc(i|2);
j=maxc(j|2);
w1 = (1/step1)*( xx - divid[i-1] );
w2 = (1/step2)*( yy - wage[j-1] );
retp (w1*w2*rule[i,j] + (1-w1)*w2*rule[i-1,j] +
w1*(1-w2)*rule[i,j-1] + (1-w1)*(1-w2)*rule[i-1,j-1]);
endp;
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