Econ 807: Macroeconomic Theory and Policy
Assignment 7: Labor Market Search Model

Consider the following simple labor market search model. The economy is populated by a fixed number of individuals (normalized to unity). Individuals have a simple choice problem: they either work if they have a job; otherwise they search for a job. A job consists of a match between a worker and a firm. A productive job match produces $y_t = \exp(z)$ units of output, where z is an AR(1) productivity shock. Assume that workers and firms split output according to a fixed sharing rule $0 < \xi < 1$; so that profit is given by $(1-\xi)\exp(z)$.
Firms exist in three states: they are either productive, vacant, or inactive. Let $J(z)$ denote the capital value of a productive firm. Let $Q(z)$ denote the capital value of a vacant firm; and normalize the value of an inactive firm to zero. Assume that productive matches break down with exogenous probability δ. Let $0 < \beta < 1$ denote the discount factor. Then $J(z)$ must satisfy the following recursive relationship:

$$J(z) = (1-\xi)\exp(z) + \beta E_{z'}\left[(1-\delta)J(z') + \delta \max\left\{Q(z'), 0\right\} \mid z\right].$$

Vacant firms and unemployed workers are brought together by a matching technology $M(v,u) = \chi v^{\alpha} u^{1-\alpha}$. Let $q(\theta) = \chi\theta^{\alpha-1}$ denote the probability of a vacant firm recruiting successfully, where $\theta \equiv v/u$. Recruiting activity entails a cost $\kappa > 0$. If the firm is successful in hiring, the job only begins to produce output in the subsequent period. Consequently, $Q(z)$ must satisfy the following recursive relationship:

$$Q(z) = -\kappa + \beta E_{z'}\left[q(\theta(z))J(z') + \left[1 - q(\theta(z))\right]\max\left\{Q(z'), 0\right\} \mid z\right],$$

where here we have anticipated that θ will depend on z.
Finally, assume that there is free-entry in the creation of job vacancies. In this case, $Q(z) = 0$ for all z. The capital value of a productive firm must then satisfy:

$$J(z) = (1-\xi)\exp(z) + \beta(1-\delta)E_{z'}\left[J(z') \mid z\right].$$

[1] Use the Coleman algorithm to solve for $J(z)$. From the data, we can set $\xi = 0.8$, $\delta = 0.15$, and $\beta = 0.99$. Assume that z follows an AR(1); i.e., $z' = \rho z + \varepsilon$, where $\rho = 0.96$ and where $\varepsilon \in \{-\sigma, \sigma\}$. Assume that $\sigma = 0.007$ and that the high and low shock occur with equal probability.

Having computed $J(z)$, we can now easily calculate $\theta(z)$ from the second equation above. In particular;

$$\theta(z) = \left(\frac{\beta\chi E_{z'}\left[J(z') \mid z\right]}{\kappa}\right)^{\frac{1}{1-\alpha}}.$$

In order to calculate θ, we will need numbers for χ, α and κ. Here is how I went about calibrating these parameter values. First, we know that the steady-state stock of employment must satisfy:

$$n^* = \frac{p^*}{p^* + \delta}.$$

From the data, we can set $n^* = 0.65$ so that $p^* = \delta n^*/(1-n^*)$. We also know that $p^* = \theta^* q^*$. From the data, choose $q^* = 0.90$, so that $\theta^* = p^*/q^*$ is now determined. Assume that $\alpha = 0.90$ (this is a bit higher than what is usually estimated in the data). Now, since $p^* = \chi(\theta^*)^\alpha$, use this relationship to calibrate $\chi = p^*(\theta^*)^{-\alpha}$. Finally, calibrate the parameter κ such that:

$$\kappa = \beta q^* J^*.$$

[2] With the parameterization now complete, compute $\theta(z)$.

Note that you are now in a position to simulate the model. In particular, the equilibrium level of employment will obey:

$$n' = (1 - \delta)n + p(\theta(z))(1 - n),$$

so that by generating a random sequence of technology shocks z, one can use the equation above (together with knowledge of $\theta(z)$) to simulate the implied path for employment. But instead of simulating the model in this manner, let us instead undertake what is called an 'impulse-response' experiment.

[3] Beginning in a steady state (i.e., $n_0 = n^*$ and $z_0 = 0$), consider the effect of a *one-time* one-standard deviation shock to z. The time-path for z is then given by $z_t = \rho^{t-1}\sigma$. Use your INTERP routine to trace out the effect on θ_t and hence n_t. As well, on a single diagram, trace out the time-path for vacancies and unemployment (this is known as the *Beveridge* curve).