Fiscal Policy in the Overlapping Generations Model

David Andolfatto

December 2002

1 The Physical Environment

The model developed here is a variant of the frameworks presented in Samuelson (1958) and Diamond (1965). Consider an economy consisting of two-period lived overlapping generations. Let N denote the population of young individuals and assume that the population remains constant at $2N$. A representative young person has preferences defined over time-dated consumption (c_{1t}, c_{2t}) where c_{it} denotes the consumption of output by generation $t = 1, 2, ..., \infty$ in the i^{th} period of life. Let preferences be represented by the by the utility function:

$$U(c_{1t}, c_{2t}) = u(c_{1t}) + \beta c_{2t}, \tag{1}$$

where $u(.)$ has the usual nice properties and $\beta > 0$. There is also an initial old generation that, from the perspective of $t = 1$, cares only about consumption in the remaining period of life, c_{20}.

Each young person is endowed with $y > 0$ units of output when young and nothing when old. There is a storage technology with the following properties: k_t units of output invested in capital yields $zf(k_t)$ units of future output, where $f(.)$ is strictly increasing and concave, and $z > 0$ is a technology parameter. Capital depreciates fully after use in production. The initial old are endowed with k_0 units of capital.

2 Decision Making

In the absence of any government, the equilibrium of this economy is easy to characterize. Each young person faces the following choice problem:

$$\max_{k_t \geq 0} u(y - k_t) + \beta z f(k_t).$$

The optimal (and equilibrium) level of capital investment is characterized by:

$$u'(y - k_t^*) = zf'(k_t^*)\beta. \tag{2}$$

Note that if $zf'(k_t^*) > 1$ (the gross population growth rate), then the equilibrium level of investment k_t^* is also Pareto optimal. Also note that $k_t^* = k^*$ is constant over time $t \geq 1$.

Exercise 1 *How does k_t^* depend on the parameters z and β? Explain (make sure to provide an economic interpretation for these two parameters).*

3 Fiscal Policy

Suppose that the economy described above has been in operation for a number of periods when suddenly, and unexpectedly, a government emerges at some arbitrary date. Let us label this date $t = 0$ and call it the 'initial period.' Assume that the government has an exogenous expenditure requirement: it needs to acquire Ng units of output in each and every period $t \geq 0$ for the foreseeable future. Assume that $0 < g < y$.

3.1 Balanced Budget Policy

Suppose that the fiscal authority decides to finance its expenditure requirements solely by the way of lump-sum taxes; let (τ_{1t}, τ_{2t}) denote the tax imposed on generation $t \geq 1$. Then the period budget constraints for each young person are given by:

$$\begin{aligned} c_{1t} &= y - \tau_{1t} - k_t; \\ c_{2t} &= zf(k_t) - \tau_{2t}. \end{aligned} \tag{3}$$

Note that an initial old person is endowed with k^* units of capital, where k^* satisfies condition (2). Let τ_{20} denote the tax (or transfer) imposed on the initial old. The consumption for the initial is therefore given by: $c_{20} = zf(k^*) - \tau_{20}$. Because preferences are linear in second-period consumption, only the first-period tax influences saving behavior; i.e., optimal capital investment is now characterized by:

$$u'(y - \tau_{1t} - k_t) = zf'(k_t)\beta. \tag{4}$$

The government's budget constraint is given by:

$$Ng = N\tau_{1t} + N\tau_{2t-1}, \tag{5}$$

for all $t \geq 0$. Assume that the government treats all generations 'fairly' in the sense that $(\tau_{1t}, \tau_{2t}) = (\tau_1, \tau_2)$.

Exercise 2 *Suppose that the government finances its purchases solely by taxing the young; i.e., $\tau_1 = g$. Compare the resulting equilibrium level of investment to k^*. Explain any differences.*

Exercise 3 *Suppose that the government finances its purchases solely by taxing the old; i.e., $\tau_2 = g$. Compare the resulting equilibrium level of investment to k^*. Compare the welfare effects of these two tax policies.*

Exercise 4 *Suppose that $zf'(k^*) < 1$. Show that there is a balanced budget fiscal policy with $\tau_1 > 0$ that makes every generation better off relative to the k^* allocation. Hint: set up the individual's choice problem conditional on some τ_1 (noting that $\tau_1 + \tau_2 = g$), find the value function, and then maximize this value function with an appropriate choice of τ_1. Can you explain why the laissez-faire equilibrium not Pareto optimal?*

3.2 Distortionary Taxes

In reality, governments do not typical levy lump-sum taxes. Commonly, taxes are levied in proportion to the value of some economic activity, like an income tax. In the present context, an income tax would leave the typical person with a stream of after-tax earnings equal to $(1-\tau_{1t})y$ and $(1-\tau_{2t})zf(k)$. Note that since y is an endowment, the first-period income tax will not distort behavior. However, since second-period earnings depends on first-period investment, the second-period income tax will in general distort investment behavior. When $\tau_{1t} = 0$ and $\tau_{2t} > 0$, the optimal level of capital investment for a representative young person is now characterized by the following condition:

$$u'(y - k_t) = (1 - \tau_{2t})zf'(k_t)\beta. \tag{6}$$

Exercise 5 *How does an increase in τ_{2t} influence an individual's desired capital spending? Explain. Does the sudden implementation of this tax distort the economic behavior of the initial old? Explain.*

The government's budget constraint is now given by (dividing through by N):

$$\begin{aligned} g &= \tau_{2t}zf(k^*) \text{ for } t = 0; \\ g &= \tau_{2t}zf(k_t) \text{ for } t \geq 1; \end{aligned} \tag{7}$$

where k^* is characterized by condition (4) when $\tau_{1t} = 0$; i.e., this is the capital stock inherited by the initial old from their previous (prior to the implementation of this fiscal policy) investment decision. Assuming that $zf(k^*) > g$, the initial tax rate is easily solved from (7):

$$\tau_{20} = \frac{g}{zf(k^*)}.$$

The second-period tax rate for $t \geq 1$, however, must be solved jointly with condition (6); i.e., combining conditions (6) with (7), we can characterize the equilibrium level of capital investment by:

$$u'(y - k) = \left[1 - \frac{g}{zf(k)}\right] zf'(k)\beta. \tag{8}$$

With k so determined, we can then calculate the equilibrium tax rate for the representative young person as:

$$\tau_2 = \frac{g}{zf(k)}.$$

In order to simplify matters, assume that $f(.)$ is a power function, so that $f'(k)k/f(k) = \alpha$ is a constant (a positive fraction).[1] In this case, condition (8) can be rewritten as:

$$ku'(y-k) = \alpha\beta(zf(k) - g). \tag{9}$$

Exercise 6 *Derive condition (9) and show how equilibrium capital spending reacts to an increase in g. How does this result differ from the case in which τ_2 is a lump-sum tax? Explain.*

Exercise 7 *Show that the distortionary tax τ_2 on the representative young person must be larger than the tax τ_{20} imposed on the initial old. Explain why.*

3.3 Government Debt

Suppose now that the government decides to finance at least a part of its expenditures by issuing debt. The debt contracts considered here are one-period real bonds; i.e., they represent sure claims to output (as opposed to money) in the 'next period.' Let R_t denote the (gross) real rate of interest that is earned on government debt. Let b_t denote the government bond holdings of each young person; their choice problem can now be formulated as follows:

$$\max_{b_t, k_t} u(y - \tau_{1t} - b_t - k_t) + \beta[R_t b_t + zf(k_t) - \tau_{2t}].$$

Desired bond and capital holdings are characterized by the following restrictions:

$$\begin{aligned} zf'(k_t^d) &= R_t; \\ u'(y - \tau_{1t} - b_t^d - k_t^d) &= R_t\beta. \end{aligned} \tag{10}$$

Suppose that the government issues some given amount of debt Nb and endeavors to keep government debt fixed at this level. Then in this case, the 'initial' period budget constraint facing the government is given by (dividing through by N):

$$g = \tau_{11} + \tau_{20} + b.$$

In subsequent periods ($t \geq 1$), the government must pay off maturing debt so that the relevant budget constraint is given by:

$$g + R_{t-1}b = \tau_{1t} + \tau_{2t-1} + b.$$

In what follows, let us restrict our attention to 'stationary' tax policies; i.e., $(\tau_{1t}, \tau_{2t}) = (\tau_1, \tau_2)$ for all generations $t \geq 1$. (Note that the tax imposed on the initial old τ_{20} may differ from τ_2). Given the stationarity that is built into this environment, it will also turn out to be the case that $R_t = R$. Also, keep in mind that we are assuming an environment where $R > 1$.

[1] For example, $f(k) = k^\alpha$.

Now, suppose that the governments sets $\tau_1 = 0$, which leaves it's initial period budget constraint as $g = \tau_{20} + b$. Then, for a given g, we see that the choice of b is essentially a choice τ_{20}. From the perspective of the initial old generation, the lower their tax bill (the larger the government debt), the better. This is because the old are not expecting to be around to pay off (or service) the debt (nor do they care about the future generations who are to be stuck with paying off (or servicing) this debt). Consequently, the bond issue reduces the tax burden that is imposed on the initial old generation and increases the tax burden for all future generations; i.e., from the government budget constraint, $\tau_2 = g + (R-1)b$ for all generations $t \geq 1$.

Recall that the demand for government debt is determined by condition (10). In equilibrium, $b^d = b$ and the conditions in (10) can be combined to yield:

$$u'(y - b - k) = zf'(k)\beta. \tag{11}$$

This condition characterizes the equilibrium level of capital investment for a given level of debt b. Also, for any given b, the equilibrium tax bill imposed on the initial old is $\tau_{20} = g - b$ and $\tau_2 = g + [zf'(k) - 1]b$ for all other generations.

Exercise 8 *Suppose that the government is contemplating two different ways of financing g. The first method entails no debt; instead, the young are taxed fully for the expenditure g; i.e., $\tau_1 = g$. The second method entails $\tau_1 = 0$ and a level of debt $b = g$. Show that the equilibrium level of capital expenditure is identical under these two policies. Is the representative young person indifferent between these two policies? Explain.*

Exercise 9 *In the previous exercise, the two methods of finance implied the same level of equilibrium capital investment. Show that this is no longer the case if τ_2 is a distortionary tax. Explain.*

Exercise 10 *Use condition (11) to show that for a given level of g, a higher levels of government debt increases the real interest rate and 'crowds out' private capital spending. Show that the welfare of the initial old is an increasing function of the level of debt. Explain.*

In the fiscal policy just considered, an increase in b results in a decrease in τ_{20}, resulting in intergenerational transfers of resources. Here, I would like to consider a policy of altering the timing of taxes that does not result in an intergenerational transfer. In order to do so, suppose that $\tau_{20} = 0$ so that the initial period government budget constraint is $g = \tau_1 + b$. Furthermore, assume that the entire amount of maturing debt is paid off by a tax on the (future) old; i.e., $\tau_2 = Rb$.

Exercise 11 *For the fiscal policy described just above, explain why Ricardian equivalence holds.*