# Monetary Policy in the Overlapping Generations Model

David Andolfatto

December 2002

## 1   Introduction

In the previous chapter, we studied the impact of government debt on the operation of the economy. The key insight of that analysis was that by introducing debt (which we thought of as a one-period real bond), the government was able to acquire resources from some agents (the young) and hence (for a given expenditure program) lower the tax burden on other agents (the initial old). Obviously, the initial old are made better off from such a program. But there was also a possibility that young generations might be better off too. This happy result would be possible for the case in which the real rate of interest was lower than the population growth rate (i.e., $zf'(k^*) < 1$ in the context of the model studied in the previous chapter). The optimal government policy in this case is to issue a constant amount of zero-interest debt. While the young transfer output to the old, the zero interest rate earned on their savings (debt and capital) is higher than the negative return that would be earned in the absence of government debt. On the other hand, if the real rate of interest was higher than the population growth rate (i.e., $zf'(k^*) > 1$), then all young generations would be made worse off from such a policy.

The key to generating these results is the redistributive aspect of this type of fiscal policy, which in the present context, takes the form of an intergenerational redistribution. This redistributive aspect is obviously absent in a representative agent model. But keep in mind that the introduction of government debt need not induce an intergenerational transfer even in the OLG model. Whether it does or not depends on the specifics of the fiscal policy. For example, if the government debt was not used to reduce the tax burden of the initial old, but was instead used to reduce the tax burden of the young at the expense of higher taxes in their old age, then Ricardian equivalence holds.

In this chapter, we will introduce a special type of government debt called fiat money. Fiat money can be thought of a zero-interest perpetual bond (a perpetual bond has no fixed maturity date). Unlike the real bond considered in the previous chapter, fiat money is a nominal bond; i.e., it represents a claim against fiat money (instead of output). But why should people want to hold a financial instrument that simply represents a claim against itself? There are two potential reasons. The first reason is that while fiat money has no maturity date, it may nevertheless be

marketable (exchangable for output) and therefore earn a real rate of return. The second reason is that there may be legal restrictions that compel individuals to hold (purchase) fiat money.

## 2  Physical Environment

The physical environment is similar to the one considered in the last chapter. There is a constant population $2N$. Each young person has preferences given by $U = u(c_{1t}) + \beta c_{2t}$. In addition, there is an 'initial' old generation that, from the perspective of $t = 1$, cares only for second-period consumption $c_{20}$. The initial old have an endowment of $k_0$ units of capital. The young have an endowment $y > 0$ and have access to a storage technology where $k$ units of investment yields $zf(k)$ units of future output.

## 3  Government Money

The government has a fixed expenditure program in place, which requires $Ng$ units of output per period. There is no interest-bearing government debt. There is a fixed nominal stock of fiat money $M_0$ that is held by the initial old. The government levies lump-sum taxes $(\tau_{1t}, \tau_{2t})$. Suppose that people expect the government to keep the supply of money fixed over time.

Now while fiat money is not redeemable for output, the old may nevertheless potentially use their money to buy output from the young. Let $v_t$ denote the purchasing power of money (the inverse of the price-level) and let $q_t$ denote the real value of the money purchased by the young. A representative young person faces the following sequence of period budget constraints:

$$
\begin{aligned}
c_{1t} &= y - \tau_{1t} - q_t - k_t; \\
c_{2t} &= \Pi^{-1} q_t + zf(k_t) - \tau_{2t};
\end{aligned}
$$

where $\Pi^{-1} \equiv v_{t+1}/v_t$ represents the (gross) real rate of return on noninterest-bearing fiat money (i.e., the inverse of the gross inflation rate). As well, individuals face the constraint $q_t \geq 0$; i.e., individuals are not allowed to issue their own fiat money; let $\lambda$ denote the Lagrange multiplier associated with this constraint. Then the following conditions characterize a young person's desired money and capital holdings:

$$
\begin{aligned}
u'(y - \tau_{1t} - q_t^d - k_t^d) &= \Pi^{-1}\beta + \lambda; \tag{1} \\
u'(y - \tau_{1t} - q_t^d - k_t^d) &= zf'(k_t^d)\beta.
\end{aligned}
$$

What these conditions tell us is that if both money and capital are to be willingly held in the savings portfolio of the young, then both assets must yield the same rate of return: i.e.,

$$
zf'(k_t^d) = \Pi^{-1}. \tag{2}
$$

That is, since $q_t^d > 0$ in this case, then $\lambda = 0$.

Suppose that the government sets $\tau_{1t} = 0$ for all $t$. The government budget constraint is therefore given by:

$$Ng = N\tau_{2t-1},$$

which implies a constant second-period tax $\tau_2 = g$ on every generation.

The money-market clearing condition in this economy is given by:

$$v_t M_0 = N q_t^d, \tag{3}$$

for all $t = 0, 1, ..., \infty$. Since we are considering stationary equilibria, $q_t^d = q^d$. Consequently, the equilibrium value of money will be given by:

$$v_0 = \frac{N q^d}{M_0}.$$

Note that this condition implies that $\Pi^{-1} = 1.0$ (zero inflation). As well, note money will be valued only in the case for which it is demanded; i.e., $q^d > 0$. Under what conditions will money be in demand?

## 3.1 When Capital Yields a Low Return

Condition (2) tells us that if money is to be valued, then it must earn the same rate of return as capital. Since $\Pi^{-1} = 1.0$, it makes sense to suppose that the young would be willing to cut back on capital expenditure in order to purchase money if $zf'(k^*) < 1$. Let us suppose that this condition holds (this is what is meant by capital earning a low rate of return). Then the equilibrium is characterized by the following conditions:

$$
\begin{aligned}
zf'(k) &= 1; \tag{4} \\
u'(y - q - k) &= \beta;
\end{aligned}
$$

where $k < k^*$. In addition, the intial old consume $c_{20} = q + zf(k_0) - g$. The second-period consumption for all future generations is given by $c_2 = q + zf(k) - g$.

**Exercise 1** *Assume that $zf'(k^*) < 1$. Demonstrate that the allocation characterized in (4) is Pareto optimal. Hint: how would a planner maximize the utility of a repre- sentative young person subject to the constraint that the initial old consume no less than what is implied in the above allocation?*

The careful reader should notice a close correspondence between what we have just done here with the optimal fiscal policy considered in the previous chapter under the condition $zf'(k^*) < 1$. Go back to that chapter now and look up the relevant

exercise (exercise 4, I believe). The condition characterizing the optimal tax $\tau_1$ in that problem is given by:

$$u'(y - \tau_1 - k) = \beta,$$

which exactly corresponds to the second equation in (4) when $\tau_1 = q$. In that problem, the initial old consumed $c_{20} = zf(k_0) - \tau_2 = zf(k_0) + \tau_1 - g$ with second-period consumption for all future generations given by $c_2 = zf(k) + \tau_1 - g$.

In other words, when $zf'(k^*) < 1$, there appear to be two equivalent ways for a government to implement a Pareto optimal allocation. First, it can implement a fiscal policy in which it taxes the young $\tau_1$ and redistributes the proceeds to the old. The young are made better off because they anticipate receiving a similar transfer in the future (and their sacrifice of $\tau_1$ today yields a net return of zero, which is higher than the net return they would have enjoyed by investing in capital). Alternatively, the government might try to accomplish the same thing via a 'monetary' policy instead of a fiscal policy. That is, the government could transfer some arbitrary (but finite) nominal quantity of intrinsically useless money $M_0$ to the initial old, promising to keep this quantity constant over time. If the young believe that this money will retain its value, then they willingly give up $q$ units of output to acquire this money (in contrast to having $\tau_1$ units of output taken from them by the government). As is well-known in the literature, there are two self-fulling equilibria associated with this monetary policy; one in which money is valued and one in which it is not (only the equilibrium with valued fiat money achieves the Pareto optimal allocation). What is less well-known in the literature (actually, I've never seen it mentioned anywhere), is if the government is willing and able to commit to reverting to the optimal fiscal policy in the event that money is not valued, then the equilibrium with valued fiat money is the only equilibrium. Furthermore, it would be wrong to interpret the money in this equilibrium as being 'unbacked' since it is being backed by the government's credible promise to enforce the value of money via taxation.

**Exercise 2** *Suppose that news suddenly (and unexpectedly) arrives that causes agents to revise upward their forecast of the future productivity of their capital projects $z$. Use condition (4) to deduce what effect such a shock will have on capital spending, the quantity of real money balances, and the value of money (or the price level). Will such a shock have any impact on inflation expectations? Explain.*

## 3.2  When Capital Earns a High Rate of Return

Let us now consider the situation in which $zf'(k^*) > 1$ (this is arguably a more empirically relevant case for most economies). In this case, there is no fiscal policy that can effect a Pareto improvement (since the allocation is already Pareto optimal). The analogue to this for monetary policy is the fact that fiat money will have no value (the young are unwilling to purchase a zero interest security when they can earn a postive rate of return by saving in the form of capital).

Nevertheless, the government may want (for redistributive purposes) to impose a tax on the young. Another way of doing this would be to force the young to accept government money in at least some of their transactions. In fact, this is what governments typically do in one way or another. The simplest way to model such a legal restriction is to assume that the young face the following constraint:

$$q_t \geq \sigma, \tag{5}$$

where $\sigma > 0.$[1] If $zf'(k^*) > 1$, then the constraint (5) will always be binding in equilibrium.

Since $q = \sigma$, the money-market clearing condition (3) implies that the equilibrium value of money must satisfy:

$$v_0 = \frac{N\sigma}{M_0},$$

which also implies $\Pi = 1$. As before, assume that the government sets $\tau_1 = 0$ and $\tau_2 = g$. From the first-order conditions in (1), we see that the equilibrium value $k$ and $\lambda$ must satisfy:

$$
\begin{aligned}
u'(y - \sigma - k) &= zf'(k)\beta; \tag{6}\\
[zf'(k) - 1]\beta &= \lambda.
\end{aligned}
$$

The initial old consume $c_{20} = zf(k_0) + \sigma - g$, while future generations consume $c_2 = zf(k) + \sigma - g$ in their old age.

**Exercise 3** *Use (6) to show what happens to equilibrium capital expenditure when the government increases $\sigma$. How does such a policy change affect the welfare of different generations?*

## 4 Monetary Policy Shock

Imagine that the economy described above (for the case in which $zf'(k^*) > 1$) has been in operation for some time when suddenly, and unexpectedly, the government increases the money supply by $M - M_0$ dollars in period $t = 1$, afterward keeping the money supply constant at the new higher level $M$ (individuals in the economy are assumed to understand this). It turns out that the economic impact of such a shock depends crucially on how the government goes about injecting the new money into the economy. There are several options available to the government. It could use the new money issue to finance: (i) a one-time increase in $g_1$; (ii) a one-time tax cut $\tau_{11}$ (accruing to the initial young); or a one-time tax cut $\tau_{20}$ (accruing to the initial old). In the latter two cases, the new money issue is essentially transferred to various agents in the economy.

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[1] Alternatively, one might consider a constraint that looks something like a reserve requirement; i.e., $q \geq \sigma k$. Another possibility is to have the government demand payment of taxes in the form of its own money; i.e., $q \geq \sigma\tau_2$.

Suppose that the government keeps $g$ constant. Let us first consider the case in which the new money is distributed to the initial old in a lump-sum payment:

$$a = v\left(\frac{M - M_0}{N}\right). \tag{7}$$

In this case, the budget constraint of the initial old is given by: $c_{20} = zf(k_0) + (vM_0/N) - g + a$. The budget constraints facing the young remain unchanged. Since $q = \sigma$, the money-market clearing condition tells us that:

$$v = \frac{N\sigma}{M} < v_0;$$

i.e., the effect of this policy will be to reduce the value of money (increase the price-level). Consequently, the initial old will find that their initial money holdings $(M_0/N)$ will have less purchasing power. On the other hand, the old receive the transfer $a$. Is this transfer enough to offset the loss in purchasing power of the initial old? In order to find out, we need to compare the loss in purchasing power $(v_0 - v)M_0$ to the value of the transfer $Na = v(M - M_0)$. As it turns out, the value of the transfer exactly offsets the loss in purchasing power (show this). As a consequence, this surprise monetary policy is *neutral*; i.e., it only affects the price-level. Note that while the realized inflation rate jumps up for one period (between period $t = 0$ and $t = 1$), the expected (and actual) inflation rate from period $t = 1$ onward remains unchanged.

As you may have already guessed, in order for this type of monetary policy shock to have a real effect, it must somehow effect an intergenerational transfer of resources. One way in which this can be accomplished is to have the transfer in (7) distributed to the initial young instead of the intial old. In this case, the old suffer a capital loss on their money holdings that is not compensated by the government (the government has, in effect, partially defaulted on its outstanding stock of debt). The beneficiaries of this policy are the initial young, who sell their goods at higher prices to the old (i.e., they give up less output in order to acquire the minimum amount of money they require to satisfy the legal restriction).

Since the initial young receive a monetary transfer equal to $a$, their first-period budget constraint is given by $c_{11} = y - q_1 - k_1 + a$. Of course, the young will want to hold the minimum amount of money possible, so that $q_1 = \sigma$. Consequently, the condition characterizing the capital expenditure for the initial young is given by:

$$u'\left(y - \sigma\frac{M_0}{M} - k_1\right) = zf'(k_1)\beta, \tag{8}$$

where use has been made of the fact that $a = \sigma(1 - M_0/M)$ in equilibrium. Clearly, $k_1$ is an increasing function of $M$. Likewise, the one-period (short term) real rate of interest $R_1 = zf'(k_1)$ is a decreasing function of $M$. Also note that these real effects are transitory; i.e., equilibrium capital investment returns to its original level in subsequent periods.

**Exercise 4** *Explain how the monetary authority would go about engineering a short-term increase in the real interest rate. Make sure to explain the economic mechanism that causes the interest rate to rise.*