1. Suppose the consumer has a utility function $U(Q_x, Q_y) = \sqrt{Q_x Q_y}$, where Q_x and Q_y are the quantity of good x and quantity of good y respectively. Assume his income is I and the prices of the two goods are P_x and P_y. Write down the consumer's problem.
   The consumer's problem is to:
   $$\max_{Q_x, Q_y} \sqrt{Q_x Q_y} \quad s.t. \quad P_x Q_x + P_y Q_y = I$$

2. Suppose Jack is deciding on the quantities of pizza and movie to buy. He has \$100. The price of a pizza is \$5, and the price of a movie is \$10.
   a. Write down his budget constraint. Draw the diagram and show his opportunity set. What is the slope of the budget line?
   b. Suppose the government imposes a 20% consumption tax on movie. Write down his budget constraint, and add the new budget constraint line to the above diagram. What is the slope of the new budget line?
   c. Because his income is low, suppose Jack gets \$20 food stamp from the government which can only be used to buy pizza. Draw the new budget constraint line.

   (Note: Please label your diagram clearly.)

   Answer: Denote by Q_p and Q_m the quantity of pizzas and movies respectively.

   a. $5Q_p + 10Q_m = 100$
      Slope = $-\frac{1}{2}$ if Q_p is on the horizontal axis.
   b. $5Q_p + 10(1 + 20\%)Q_m = 100$
      Slope = $\frac{5}{12}$ if Q_p is on the horizontal axis.
   c. Because the food stamp can only be used to buy pizzas, the maximum unit of movies Jack can buy is unchanged. See Figure 1.

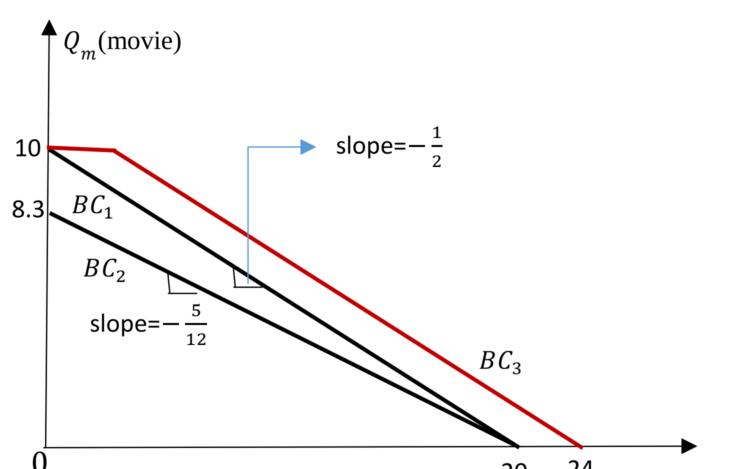


Figure 1

3. Mary doesn't like working but enjoy the leisure time. If she works, she earns a wage of $10/hour and she can uses the income to buy all the goods she need to consume. Assume the price of the goods she consumes is equal to $1. Also assume Mary's utility function is $U(C,N) = \sqrt{CN}$, where C is her consumption (quantity of all the goods she consumes) and N is her leisure hours. Denote by L her labor hours, $L + N = 24$. Derive Mary's demand curve for Leisure and supply curve for Labor.

   (Hint: 1. Her marginal utilities are: $MU_C = \frac{\sqrt{N}}{2\sqrt{C}}$, $MU_N = \frac{\sqrt{C}}{2\sqrt{N}}$; 2. Try different wages and calculate the leisure hours corresponding to each of them.)

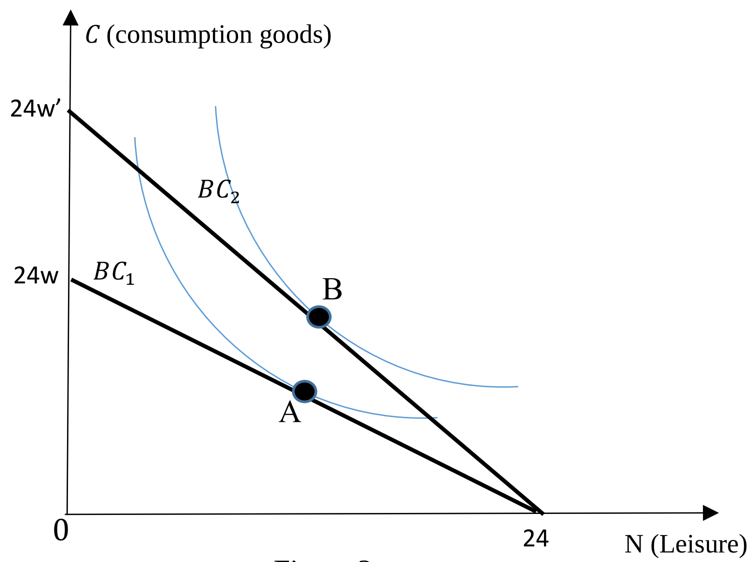


Figure 2

In order to derive the demand curve, we need to calculate the optimal leisure hours (N) corresponding to different wages. For example, what is the optimal N when wage w=10? 5? 20? Here I am making it more general and assuming the wage is w. So the budget line is BC_1. Suppose the optimal point is A. Now let's calculate N at the point A. The point A has two properties: 1) It is on the budget line BC_1, 2) At A the slope of the indifference curve that's tangent to the budget line at A is equal to the slope of the budget line.

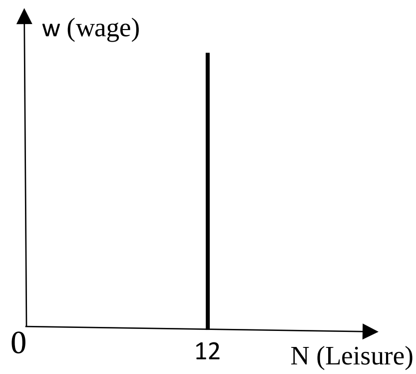
From the first property, we have at A: $P_c C_A + P_N N_A = 24w$, where P_c is the price of the consumption goods, and P_N is the price of one leisure hour. From the question $P_C = 1$. Since the price of leisure is equal to its opportunity cost, $P_N = w$. So $C_A + wN_A = 24w$. This is the budget constraint function.

From the second property, slope of BC_1 = MRS at A $\Leftrightarrow \frac{P_N}{P_C} = \frac{MU_N}{MU_C} \Leftrightarrow w = \frac{\frac{\sqrt{C_A}}{2\sqrt{N_A}}}{\frac{\sqrt{N_A}}{2\sqrt{C_A}}} = \frac{C_A}{N_A} \Leftrightarrow C_A = wN_A$.

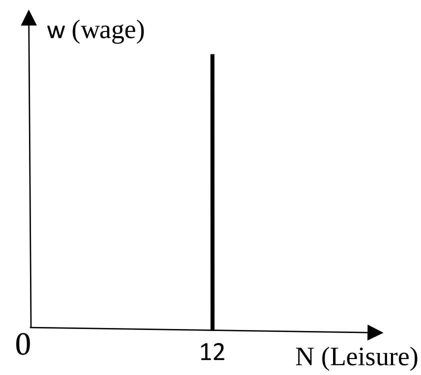
Substitute this result in the budget constraint function:

$C_A + wN_A = 24w \Leftrightarrow wN_A + wN_A = 24w \Leftrightarrow N_A = 12$, then we can get labor hours $L_A = 24 - N_A = 12$.

Then let's change the wage to w' and assume w'>w. Then the budget constraint moves to BC_2 and assume the new optimal point is now B. Following the same steps as shown above, we can get at B: $N_B = 12$ and $L_B = 12$. (Since in this problem the optimal N doesn't depend on the wage, we actually don't need to try different wages. But when N is not a constant, you need to try different wages and get the Ns associated with each wage.)
So when we change the wage, the optimal leisure hour doesn't change and equals 12. So the demand curve of leisure hour is a vertical line at N=12. The supply curve of labor hours is a vertical line at L=12.



Demand curve of Leisure



Supply curve of Leisure

4. *Economist George Stigler once wrote that, according to consumer theory, "if consumers do not buy less of a commodity when their incomes rise, they will surely buy less when the price of the commodity rises." Explain this statement.*
The statement *"if consumers do not buy less of a commodity when their incomes rise,…"* shows that the commodity is a normal good. A normal good cannot be a Giffen good. So when the price rises, the quantity must fall.

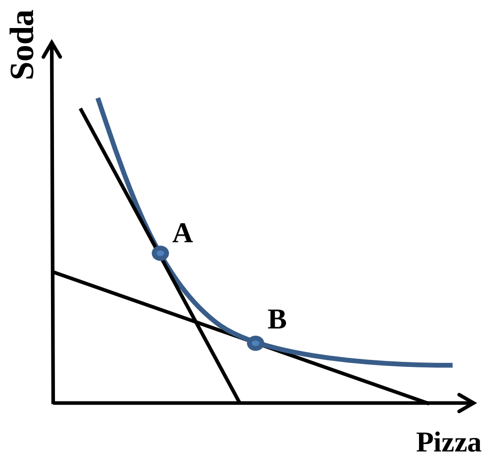
5. What's the economic meaning of MRS?
MRS, the marginal rate of substitution, shows how many units of good2 a consumer must give up in order to consume one more unit of good1 such that his utility level does not change. Or it can be stated as: MRS shows how many units of good2 give a consumer the same amount of utility as one more unit of good1 does.

6. You consume only soda and pizza. One day, the price of soda goes up, the price of pizza goes down, and you are just as happy as you were before the price changes.

   a. Illustrate this situation on a graph.

   b. Can you afford the bundle of soda and pizza you consumed before the price changes?

   a.



   b. No, I cannot afford the old bundle (A) because it is now beyond my budget line.

7. Describe the four properties of indifference curves and explain why they are true. (You may need to refer to the basic assumptions we make on the consumer's preference.)
   The four properties are:
   1) An indifference curve is downward sloping.
   2) A high indifference curve represents a high level of utility
   3) Indifference curves do not cross.
   4) An indifference curve has diminishing MRS.

   Here I am only proving 3). Please see the notes for the others.

   Suppose two indifference curves IC1 and IC2 interact at the point B. Assume A is any point other than B on IC1 and C is any point other than B on IC2. Because both A and B are on IC1, the consumer is indifferent between A and B. Because both C and B are on IC2, the consumer is indifferent between B and C. So the consumer is indifferent between A and C (from the transitivity assumption). So by the definition of an indifference curve, A and C must be on the same indifference curve. This contradict with the fact that A is on IC1 and C is on IC2 and IC1 and IC2 are different indifference curves. So indifference curves do not cross.

8. Compare the following two pairs of goods:
   - Coke and Pepsi
   - Skis and ski bindings

In which case do you expect the indifference curves to be fairly straight, and in which case do you expect the indifference curves to be very bowed? In which case will the consumer respond more to a change in the relative price of the two goods?

Indifference curves between Coke and Pepsi are fairly straight, since there is little to distinguish them, so they are nearly perfect substitutes. Indifference curves between skis and ski bindings are very bowed, since they are complements. A consumer will respond more to a change in the relative price of Coke and Pepsi, possibly switching completely from one to the other if the price changes.

9. Steve's utility function is $U = BC$, where B = veggie burgers per week and C = packs of cigarettes per week. Here, $MU_B = C$ and $MU_C = B$. Steve's income is \$120, the price of a veggie burger is \$2, and that of a pack of cigarettes is \$1.
    a. What is his marginal rate of substitution if veggie burgers are on the vertical axis and cigarettes are on the horizontal axis?
    b. How many burgers and how many packs of cigarettes does Steve consume to maximize his utility?

Now a new tax raises the price of a burger to \$3.

    c. What is his new optimal bundle?
    d. If there is no income effect, what is his optimal bundle as a result of substitution effect?
    e. Is a burger a normal good or an inferior good?
    f. Illustrate your answers above in a graph. (Clearly label the substitution and income effects)

Denote by P_C and P_B the prices of cigarettes and burgers respectively. Assume the optimal point in b. is E, and the optimal point in c. is F. (See Figure 9 below.)

a. MRS = $\frac{MU_C}{MU_B} = \frac{B}{C}$.

b. At the optimal point E, MRS = slope of budget line. So we have $\frac{B}{C} = \frac{P_C}{P_B} = \frac{1}{2} \Leftrightarrow C = 2B$. Also the optimal point must be on the budget line. So $P_C C + P_B B = 120 \Leftrightarrow C + 2B = 120$. Substitute $C = 2B$ into the budget function: $2B + 2B = 120 \Leftrightarrow B = 30$. Then $C = 2B = 60$. So the optimal point is E(C=60, B=30).

c. Now $P_B = 3$. So $\frac{B}{C} = \frac{P_C}{P_B} = \frac{1}{3} \Leftrightarrow C = 3B$. Budget constraint function becomes: $P_C C + P_B B = 120 \Leftrightarrow C + 3B = 120$. Substitute 3B for C, we have $3B + 3B = 120 \Leftrightarrow B = 20$. $C = 3B = 60$. So the optimal point is F(C=60, B=20).

d. As a result of substitution effect only, the original optimal point (in b.) will move to a point S at which 1) the consumer's utility is unchanged (same as that at E), and 2) the slope of the new budget line (BC_2) is equal to the slope of the IC passing through the original optimal point E. From these two properties, we can solve for the B and C at the point S (C_s, B_s). From the first property: utility at the point S = $B_s C_s$. From b. we have that the original optimal point is E(C=60, B=30). His utility at E(C=60, B=30) = BC = 1800. So $B_s C_s = 1800$ (*).

From the second property: MRS at S = slope of new budget line ⇔ $\frac{B_s}{C_s} = \frac{P_C}{P_B} = \frac{1}{3}$ ⇔ $C_s = 3B_s$ (**). Substitute this result into the equation (*), we have $3B_s^2 = 1800 \Rightarrow B_s = 10\sqrt{6}$. Then $C_s = 3B_s = 30\sqrt{6}$.

e. To answer this question, we need to check how quantity changes when ONLY the income changes. Although in the problem what we have is a change in the price, we can study how the quantity changes as a result of income effect only. The move from the point S to the point F is a result of the income effect only. The income is lower (because the price of burger is higher). Now let's see how the quantity of burgers (B) changes. At S, $B_s = 10\sqrt{6}$. At F, $B = 20$. Because $10\sqrt{6} > 10\sqrt{4} = 20$, from the point S to F the quantity of burgers is lower. That is, when the income is lower, the quantity of burgers consumed is lower. So a burger is a normal good.

f.

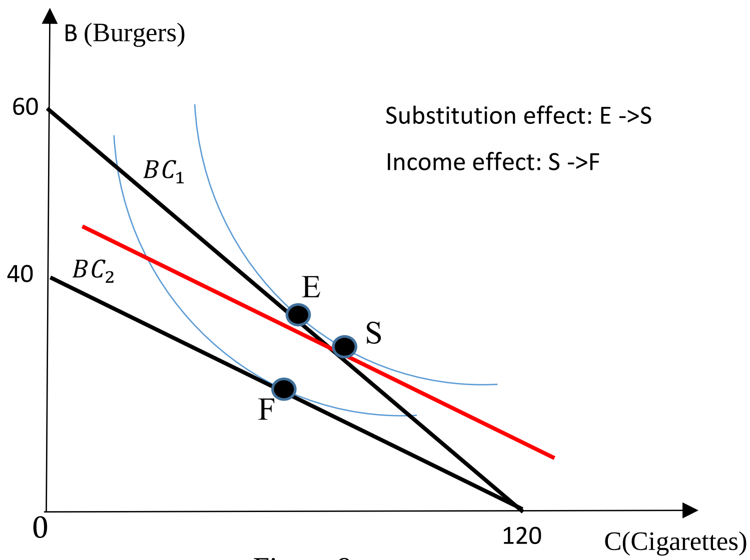


Figure 9

10. Edward's budget on video games and movies is $60/month. The price of a game is $1 and the price of a movie is $2. Edward's utility function is $U = 4G + 3M$, where G is the quantity of games and M is the quantity of movies he buys.
    a. How many games and movies he buys each month.
    b. Now suppose the price of a game increases to $4. How many games and movies he buys each month.

Here indifference curves are straight lines.

a. See Figure 10a. When the IC is steeper than the BC, the optimal point is at G=60, M=0.

b. See Figure 10b. When the BC is steeper than the IC, the optimal point is at G=0, M=30.

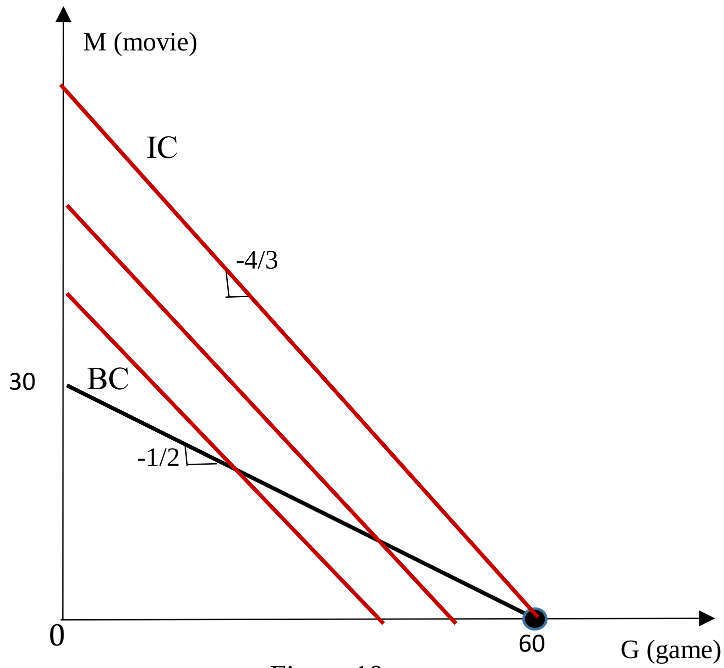


Figure 10a

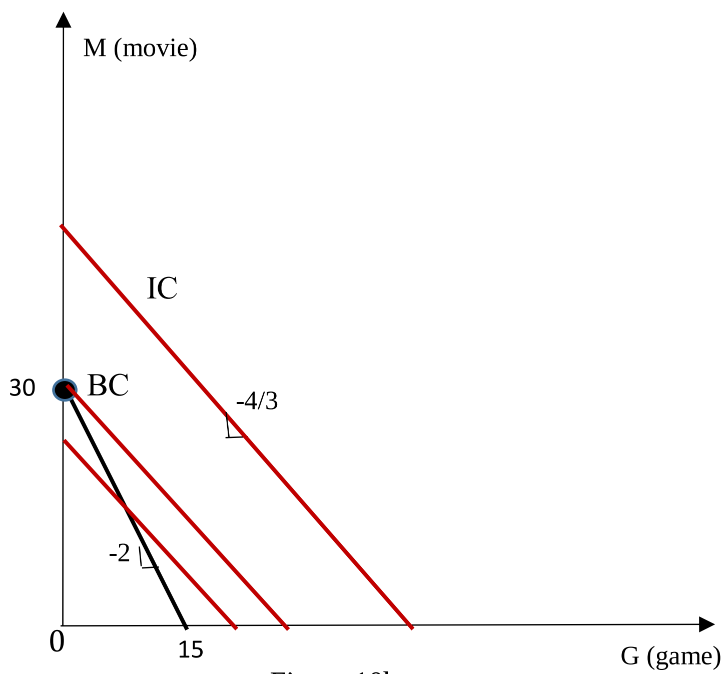


Figure 10b