L6 – Theory of the Firm: Cost
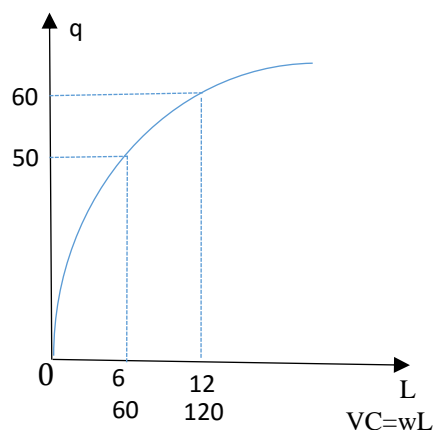
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I.   Short-Run Costs

TC = FC + VC

TC = rK + wL(q)

1. Production Function and Cost

Shape of the Variable Cost Curve

Variable Cost and Total Product of Labor

- The firm's short-run variable cost curve and its total product of labor curve have the same

shape. The total product of labor curve uses the horizontal axis measuring hours of work. The variable cost curve uses the horizontal axis measuring labor cost, which is the only variable cost.

Shape of the Marginal Cost Curve

$$MC = \frac{\Delta VC}{\Delta q} = w\frac{\Delta L}{\Delta q} = \frac{w}{MP_L}$$

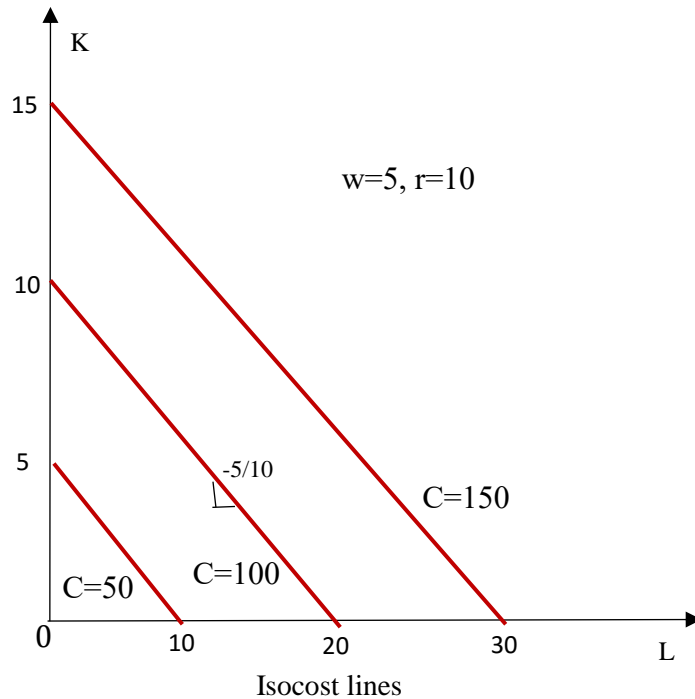- Diminishing marginal product of labor leads to increasing marginal cost
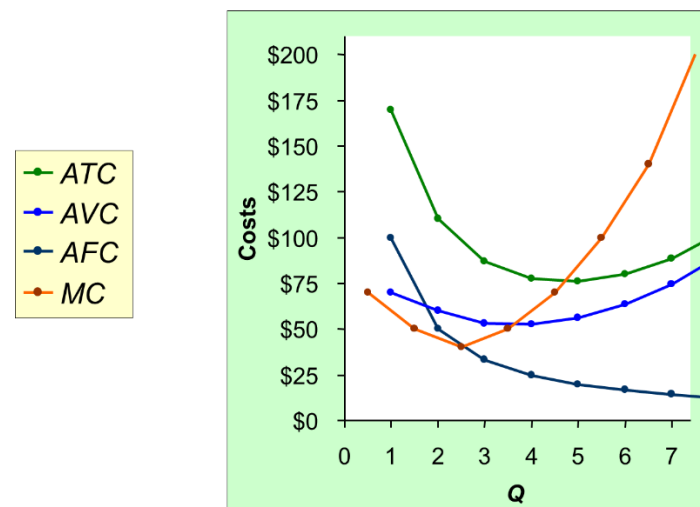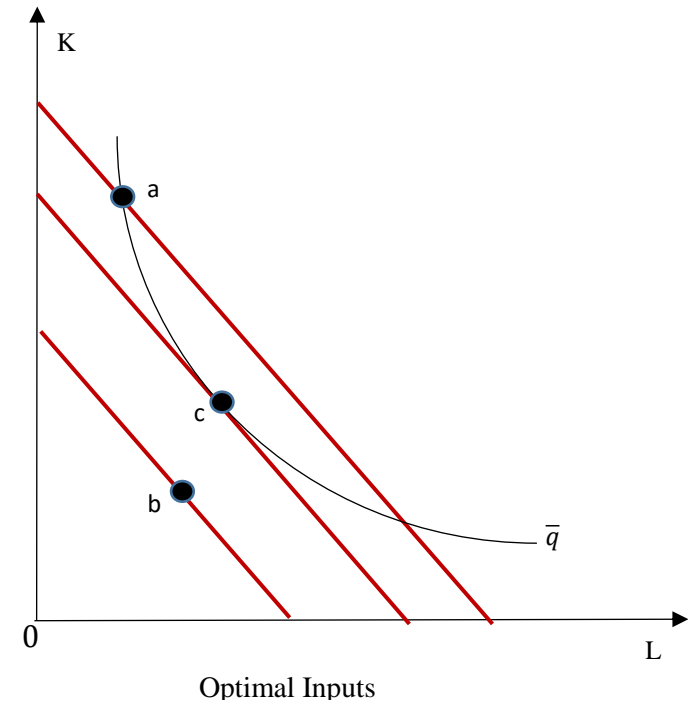
II. Long-Run Costs

TC = rK(q) + wL(q)

1. Isocost Line: all the points on the line lead to the same cost.

$$\overline{TC} = rK + wL$$

$$K = \frac{\overline{TC}}{r} - \frac{w}{r}L$$

K

15

w=5, r=10

10

5

-5/10

C=150

C=100

C=50

0   10   20   30   L

Isocost lines

2. Optimal Inputs (given an output q): At the optimal bundle of K and L, the total cost $TC = rK + wL$ must be minimized.

K

a

c

b

$\overline{q}$

0   L

Optimal Inputs

- At the optimal bundle, the Isocost line is tangent to the Isoquant curve of the given $\overline{q}$.
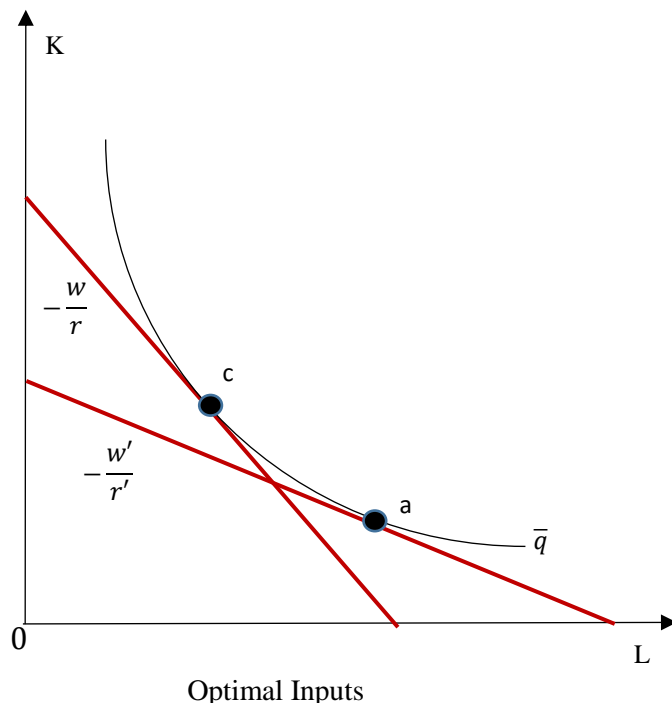
Therefore the optimal point has two properties:

1) slope of the isocost line = slope of the isoquant at the point:

$$\frac{w}{r} = MRTS = \frac{MP_L}{MP_K} \quad (1)$$

2) The optimal point is on the isoquant curve:
$$f(K,L) = \bar{q} \quad (2)$$

Both the equation (1) and (2) have the variables K and L. So we can solve these two equations for the optimal K and L.

When the price ratio changes:

Optimal Inputs

Example:
$$w = 10, r = 5, q = \sqrt{KL} = 100$$
$$MP_L = \frac{\sqrt{K}}{2\sqrt{L}}, \quad MP_K = \frac{\sqrt{L}}{2\sqrt{K}}$$

What are the optimal K and L that produces an output of 100?

3. **Long-run Cost Function: $TC = C(q)$**

$\bar{C} = 4000$

$\bar{C} = 3000$

Expansion path

$\bar{C} = 2000$

a

200

c

150

b

100

$\bar{q} = 200$

$\bar{q} = 150$

$\bar{q} = 100$

0   50   75   100   L

LRTC   Long-run cost curve

4000   a

3000   c

2000   b

0   100   150   200
                              q

- The long-run cost function is determined by the production function and the prices of the two inputs.

Example questions:
1. It takes 1 unit of labor and 1 unit of capital to produce 1 unit of output. What is the long-run cost function?

2. What the long-run cost function if K and L are perfect substitutes?