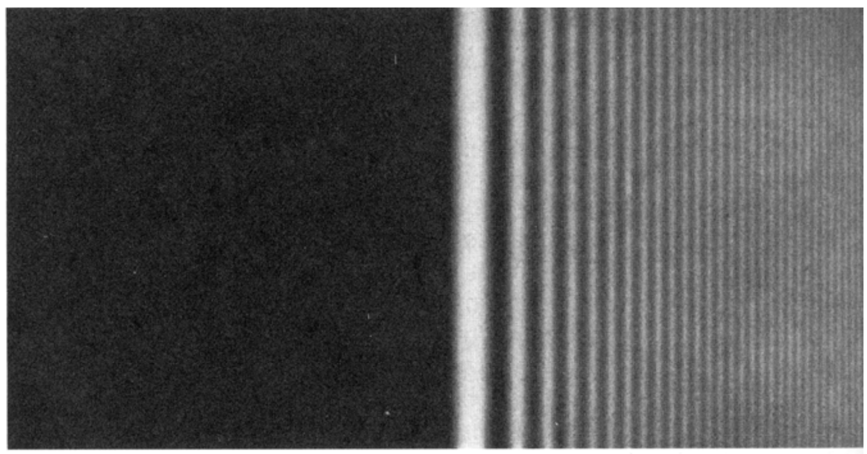
Diffraction

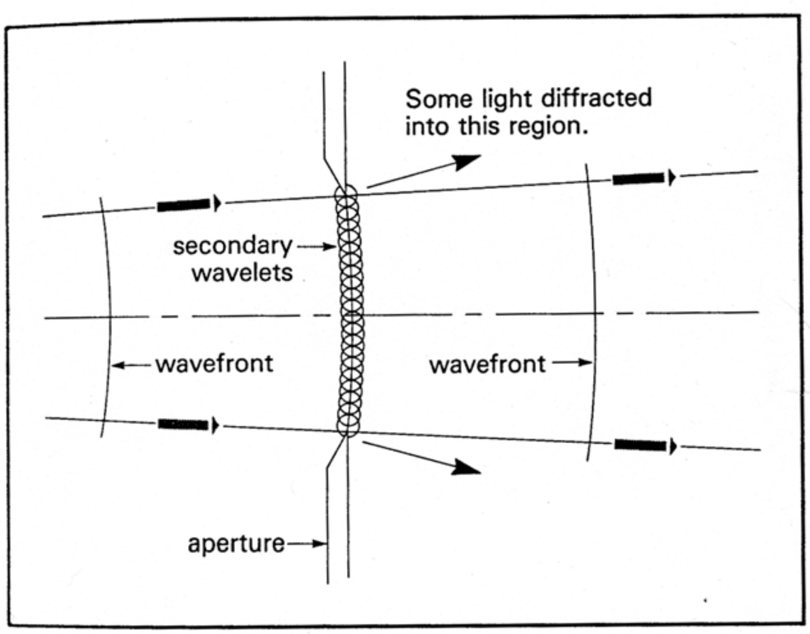
- Diffraction occurs when light waves is passed by an aperture/edge
- Huygen's Principal:
  each point on wavefront acts as source of another circular wave
- Consider light from point source at infinity or parallel beam (laser)
- Light diffracts around objects
- Consider a slit in a plate: light from edge diffracts
- Edge creates interference effects between the waves at each point
- Thus at edges asymmetric effects results in bending
- Diffractions sets the resolution limits for lens and optics systems



(d)

Figure 13-14 Straight-edge diffraction.

(d) Diffraction fringes from a straight line. (From M. Cagnet, M. Francon, and J. C. Thrierr, *Atlas of Optical Phenomenon*, Plate 32, Berlin: Springer-Verlag, 1962.)



HUYGEN'S PRINCIPLE states that each point on a propagating wavefront is an emitter of secondary wavelets.

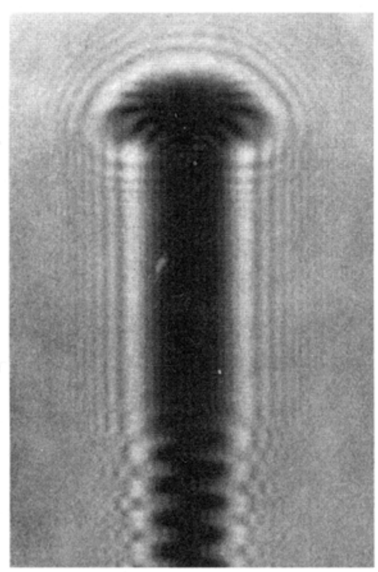


Figure 13-18 Fresnel shadow of a screw. (From M. Cagnet, M. Francon, and J. C. Thrierr, *Atlas of Optical Phenomenon*, Plate 36, Berlin: Springer-Verlag, 1962.)

Fresnel and Fraunhofer Interference

- Image created depends on distance from slit light is seen
- Strongly seen in interference at the edge or small objects
- Two types: Fresnel and Fraunhofer Interference
- Both assume light at infinity nearly parallel light – laser beam

Fraunhofer Interference

- Diffracted light sensed at infinity (ie a focused image)
- Simpler equations

Fresnel interference

- Pattern created near the diffraction point
- Much more complex equations: involve near E-mag field
- Pattern very dependent on the distance & slit/object size

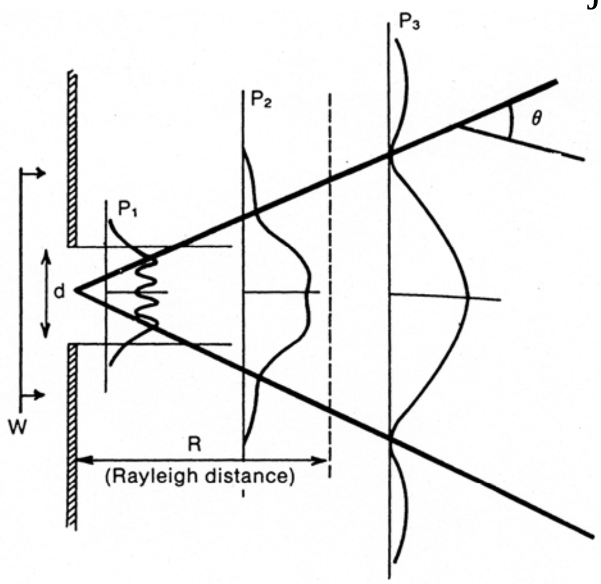
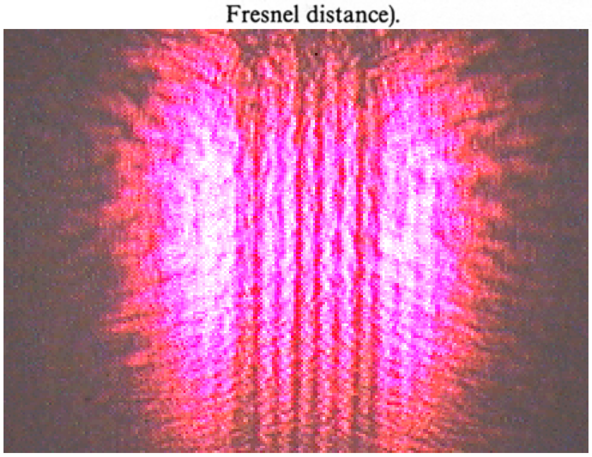
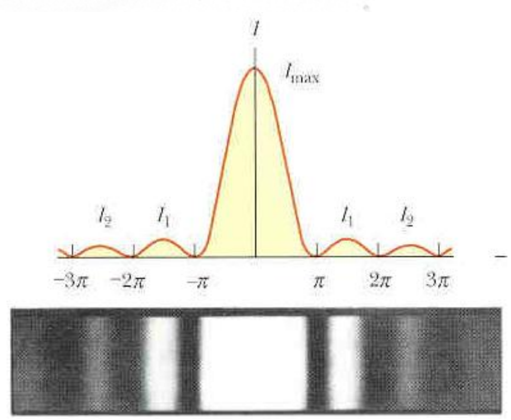


Fig. 2.8.14. Transition from Fresnel to Fraunhofer diffraction. A portion of a wave, *W*, passes through a slit of width, *d*. Intensity distributions across the wave are shown for planes P_1 (close to the slit), P_2 (just inside the Fresnel distance), and P_3 (beyond the Fresnel distance).



Fresnel Interference from laser



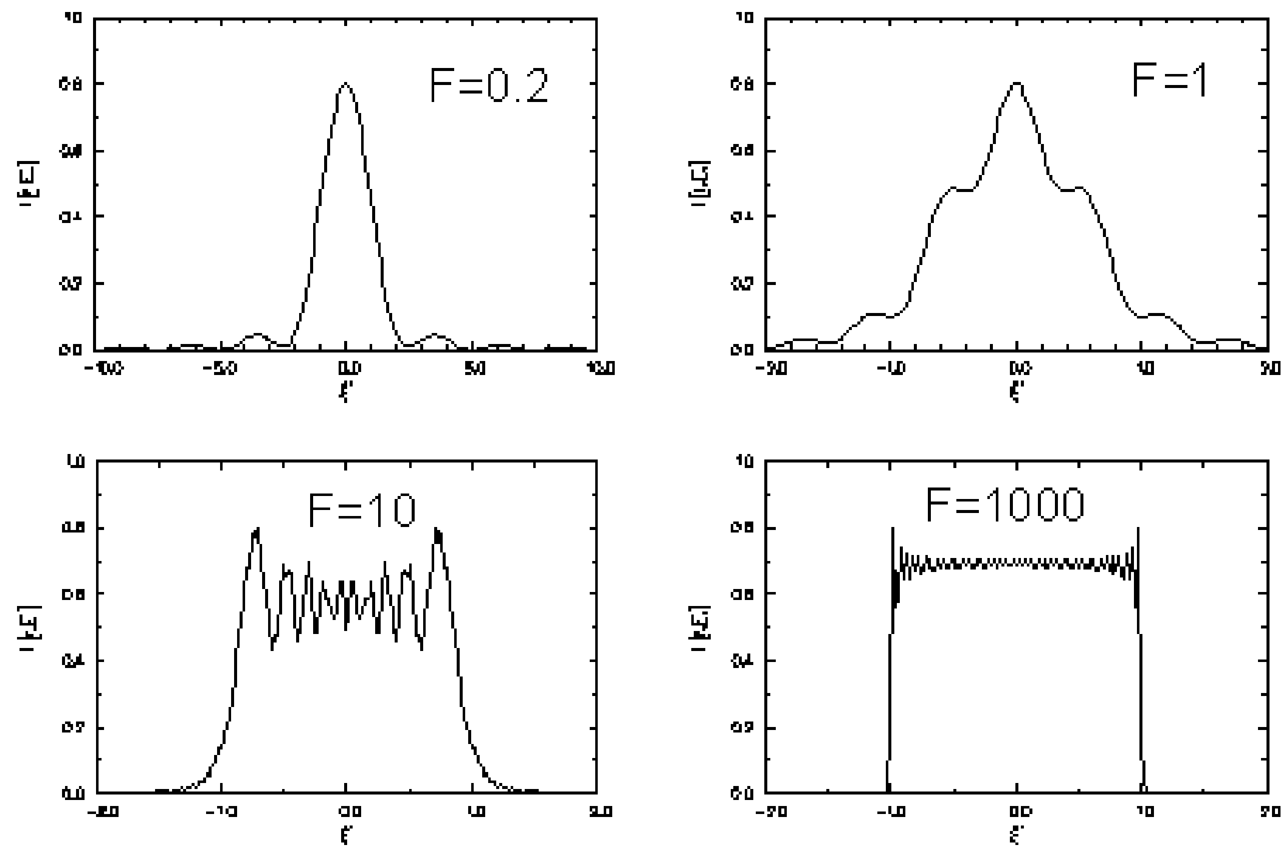
Fraunhofer Interference

Fresnel Number

- Fresnel number measures if the image is close or far field
- Consider a slit width b
- Measuring image at a distance R
- Fresnel number is

$$F > \frac{b^2}{R\lambda}$$

- F << 1 Fraunhofer interference (Far Field)
- F>> 1 Fresnel – complicated image function of distance R



Fraunhofer Interference

- Consider a single slit width b but infinite length
- Diffraction effects "seen at infinity" which means distances R

$$R > \frac{b^2}{\lambda}$$

- If focus slit with lens get the same as at infinity
- Most common effect
- Intensity follows the pattern of a synch function

$$I(\beta) = I_0 \left[\frac{sin(\beta)}{\beta} \right]^2$$

where

$$\beta = \frac{\pi b \sin(\theta)}{\lambda}$$

θ = angular deviation of pattern from minimum

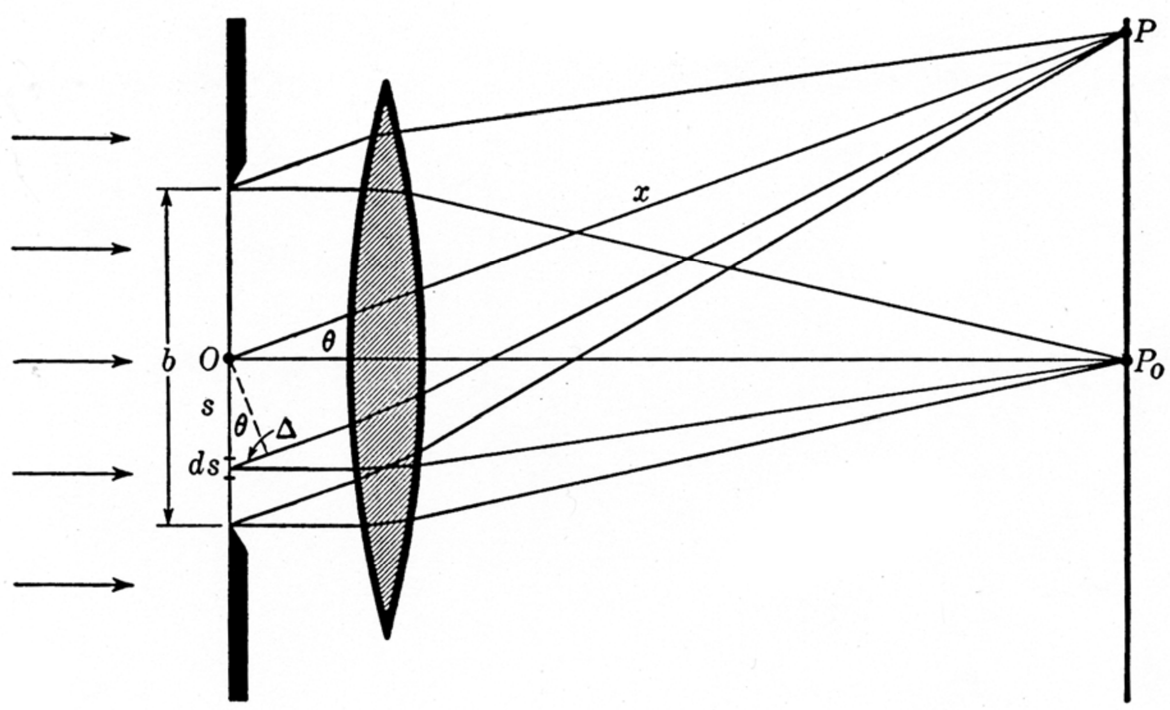


FIGURE 15C
Geometrical construction for investigating the intensity in the single-slit diffraction pattern.

Fraunhofer Interference Pattern

- Zeros are at

$$\beta = \pm N\pi$$

where N is any integer

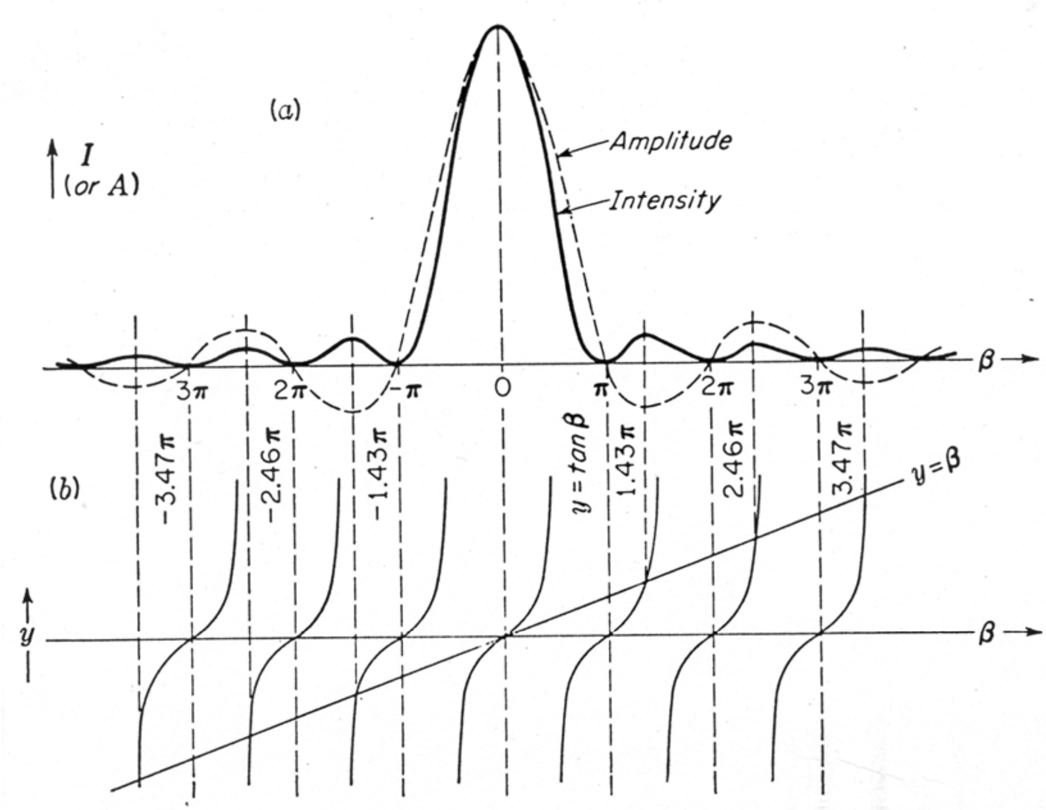


FIGURE 15D
Amplitude and intensity contours for Fraunhofer diffraction of a single slit, showing positions of maxima and minima.

- Large d little pattern (a), small d pattern spreads out (c)

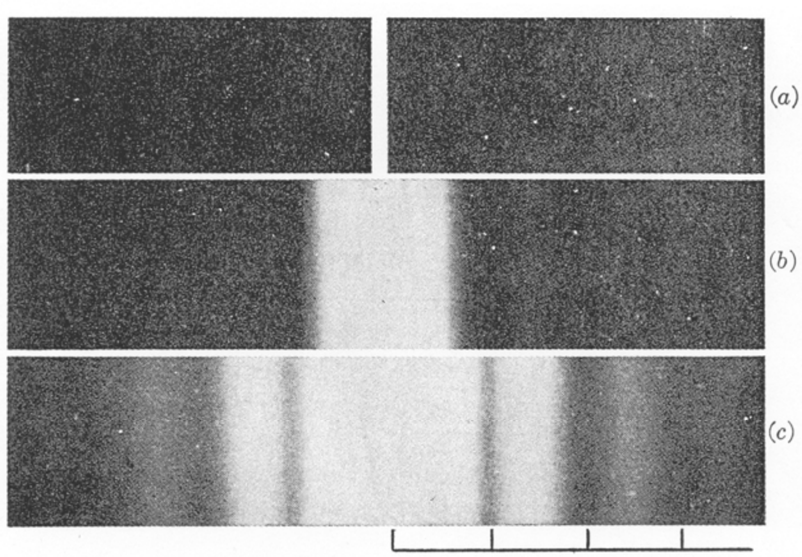


FIGURE 15B
Photographs of the single-slit diffraction pattern.

Circular Fraunhofer Interference

- Interference changes for circular opening
- Called an Airy Disk
- Lenses act as circular apertures
- Most important for laser systems
- For single circular aperture diameter D
- Intensity follows the pattern

$$I(\beta) = I_0 \left[\frac{2J_1(\beta)}{\beta} \right]^2$$

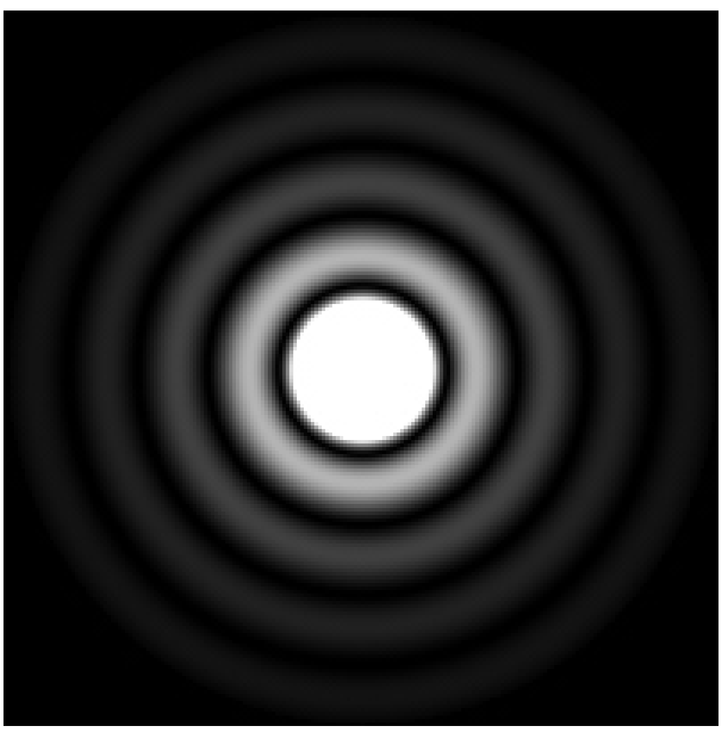
where

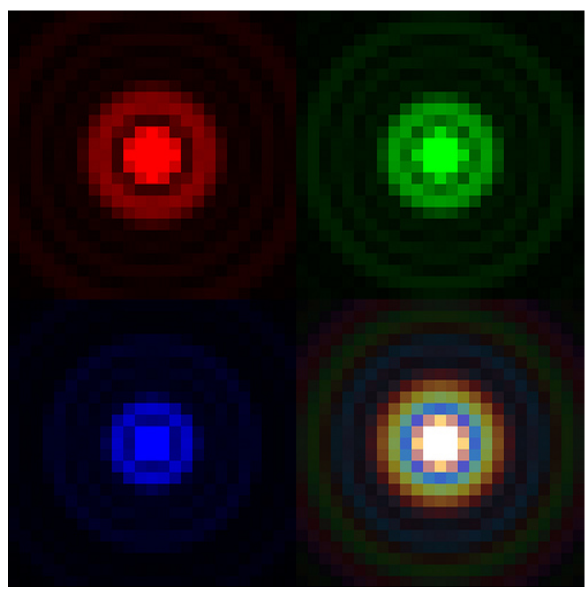
$$\beta = \frac{\pi D \sin(\theta)}{\lambda}$$

J_1 = Bessel function of first kind, order 1

θ = angular deviation of pattern from minimum

- Shrinks rings as wavelength decreases
- White light creates multicolour rings





Bessel Functions

- Bessel functions: from solutions to cylindrical coordinate problems
- Comes from solutions to the DE

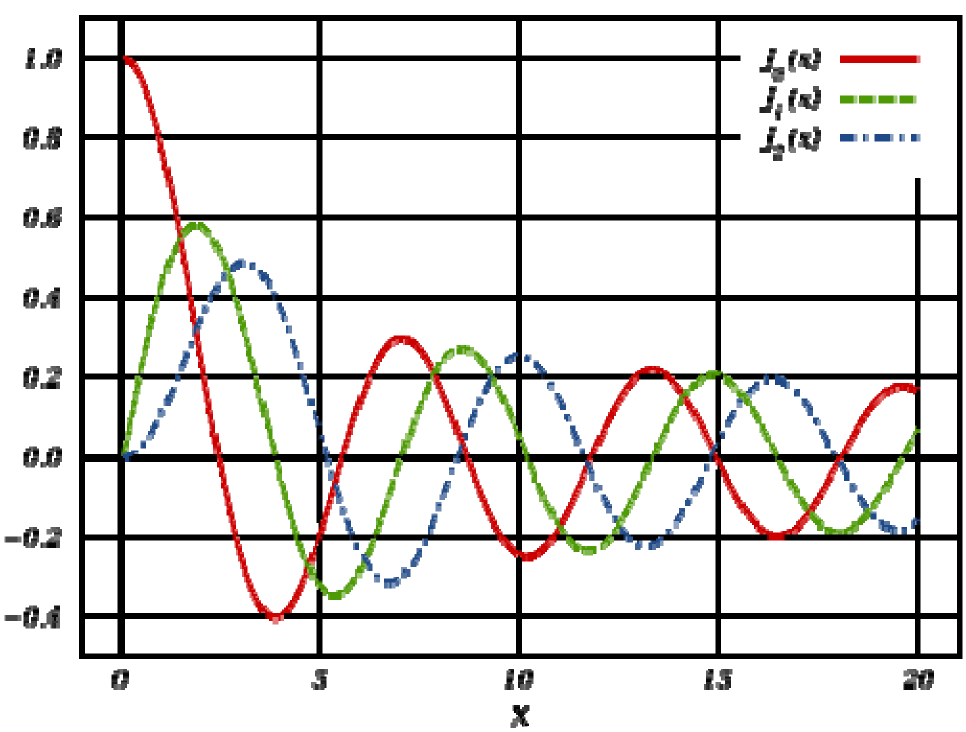
$$x^2 \frac{d^2 y}{dx^2} + x\frac{dy}{dx} + \left(x^2 - n^2\right)y = 0$$

- Typical applications Emag waves in cylindrical cavities
- Heat flow in cylinder (eg laser heating of material)
- Bessel functions of first kind order n are $J_n(x)$
- Bessel function $J_1(x)$ are given by the expansion

$$J_1(x) = \frac{x}{2} - \frac{x^3}{2^2 \bullet 4} + \frac{x^5}{2^2 \bullet 4^2 \bullet 6} \quad \cdots$$

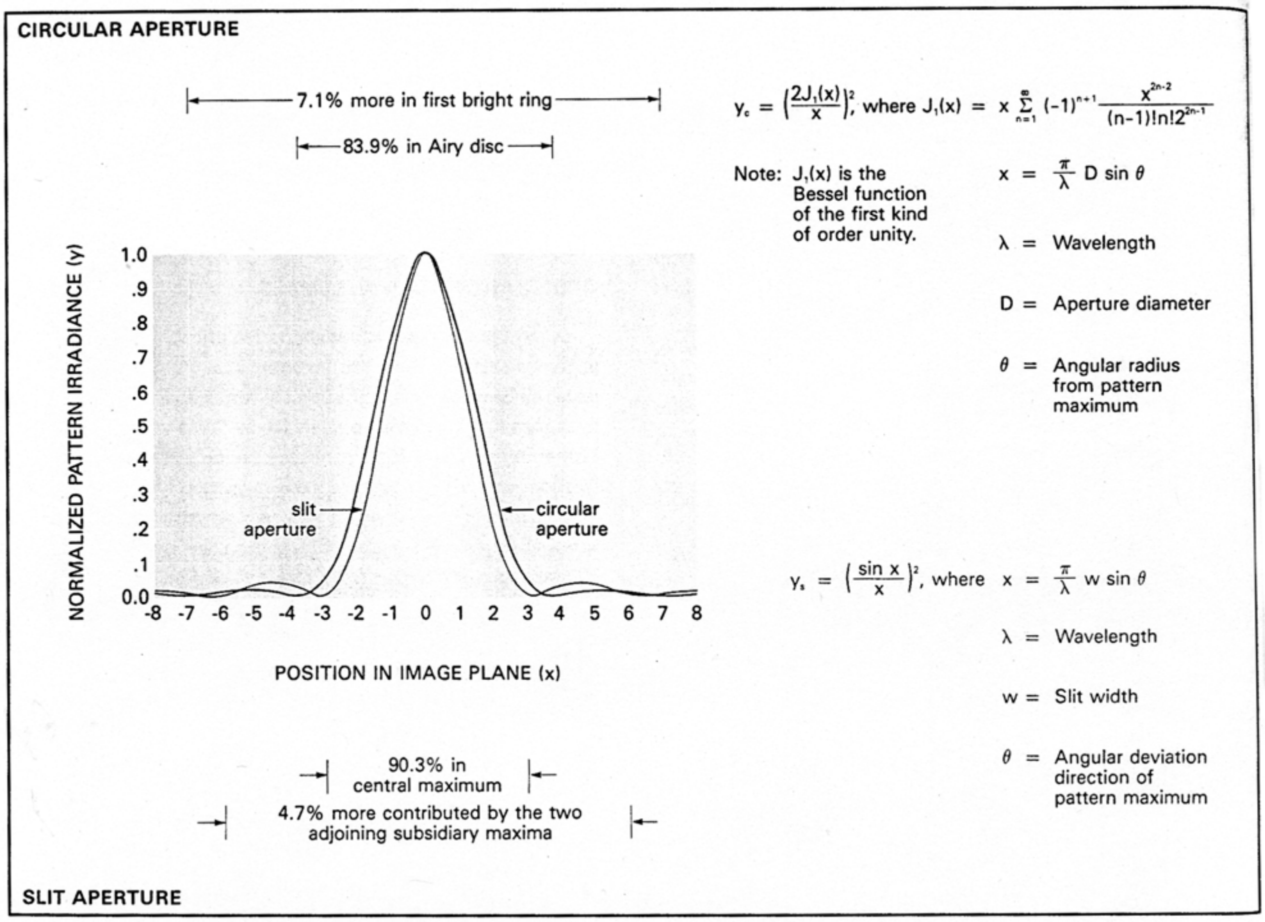
- Look like a decaying sin/cos wave
- Thus for Airy disk at the centre

$$I(\beta \to 0) \to I_0\left[\frac{2J_1(\beta)}{\beta}\right]^2 \approx I_0\left[\frac{\beta}{2}\frac{2}{\beta}\right]^2 \to I_0$$



Comparison of Circular and Slit Fraunhofer Interference

- Slit produces smaller width pattern
- Circular Airy disks is wider than slit of same size/wavelength
- Nulls occur further out
- Peaks smaller after nulls than for slit
- Higher % energy in central peaks



FRAUNHOFER DIFFRACTION PATTERN of a singlet slit superimposed on the Fraunhofer Diffraction Pattern of a circular aperture.

Minimum Spotsize of Focused Laser Beam

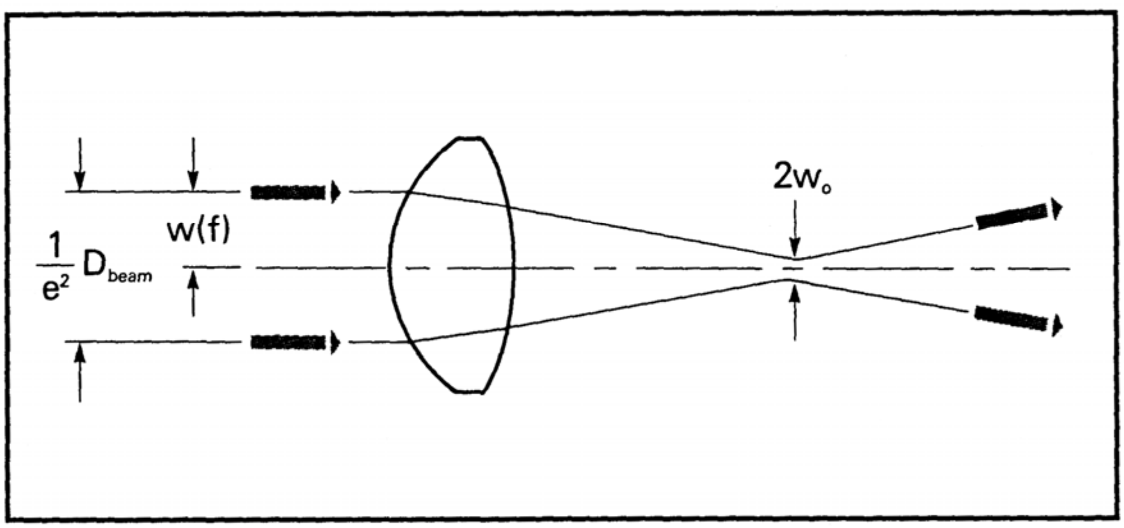
- For beam much smaller than lens
  limited by waist size of input beam
- Hence if waist in at the focus then

$$w_0 = \frac{\lambda f}{\pi w}$$

- NOTE: lens aberration may modify this by a factor
- eg He-Ne laser is focused through a lens with
  f = 7 mm λ = 632.8 nm w_{out} = 0.4 mm
- What is the minimum spot produced?
- Assume input waist is at focus

$$w_0 = \frac{\lambda f}{\pi w} = \frac{6.328x10^{-7}(0.007)}{\pi\ 4x10^{-4}} = 3.5x10^{-6} = 3.5\,\mu m$$

- For singlet lens multiply this by 1.333 for 4.7 micron



CONCENTRATION OF LASER BEAM by a laser line focusing singlet. Size of the focal waist has been greatly exaggerated for illustrative purposes.

Diffraction Limited spot

- If laser beam fills the lens then diffraction limited
- Opening of width D
- Minimum spot is to point of first zero in diffraction pattern

$$I = \frac{b\,sin(\theta)}{\lambda} = \frac{bd_{min}}{\lambda 2f}$$

$$d_{min} = \frac{2f\lambda}{D}$$

- Since circular effectively Airy diffraction
  add a factor of 1.22

$$d_{min} = \frac{2.44\,f\lambda}{D}$$

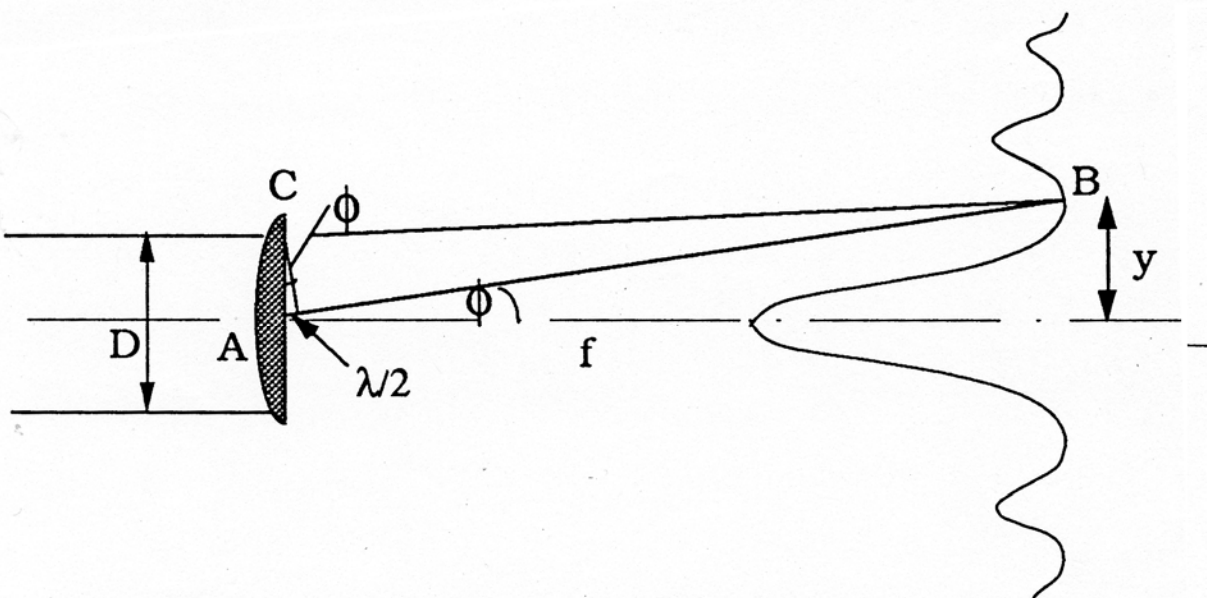


Fig. 2.15.Diagram illustrating the diffraction limited spot size.

Resolution of Spots

- Resolution tells how good is the fine detail on an image
- Simple case: what is the min separation of 2 small, close, spots
- When spots fully separated then can resolve
- In telescopes how close can 2 stars be seen as separate objects
- In astronomy this is called the angular separation
- Bigger telescope mirror – greater angular separation
- However on earth atmosphere density changes limits resolution
- Actually effective atm telescope diameter only 20 cm
- Later can see how to compensate for atmosphere)
- Hence space based telescopes much better (Hubble)

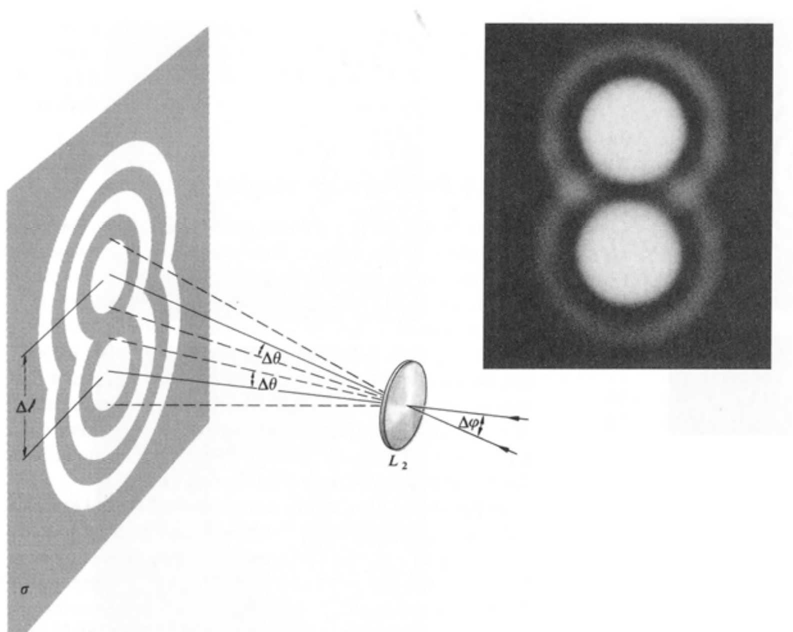
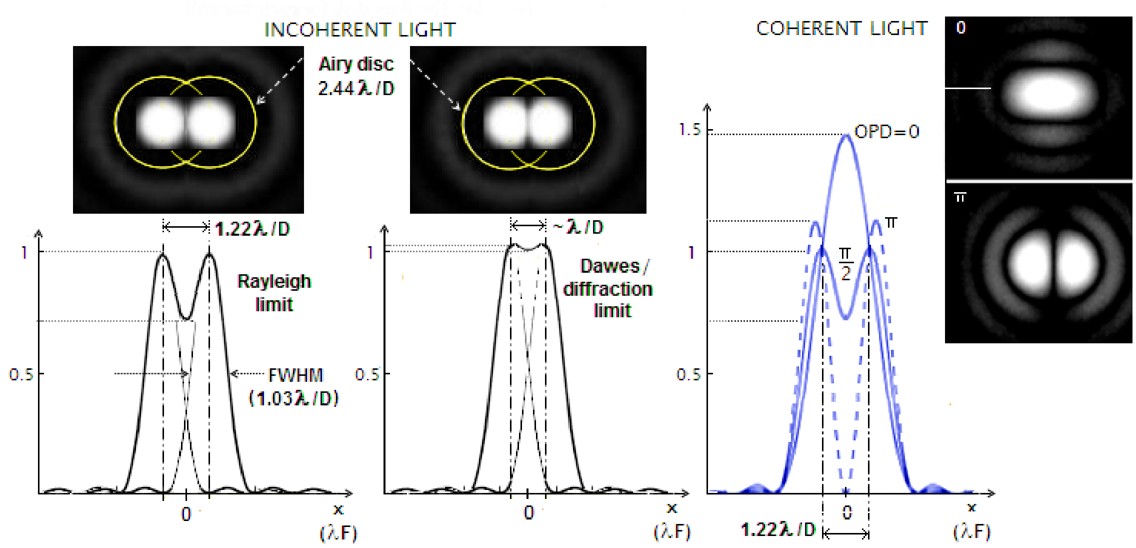


Figure 10.24 Overlapping images.



Rayleigh's Criteria Images

- What determines when we can separate two spots in an image
- When spots overlap enough cannot separate
- Different systems determine how much overlap allowed

$$d_{min} = \frac{1.22\, f\lambda}{D}$$

- This is Rayleigh's Criteria most common
- Where two separate peaks can just be separated

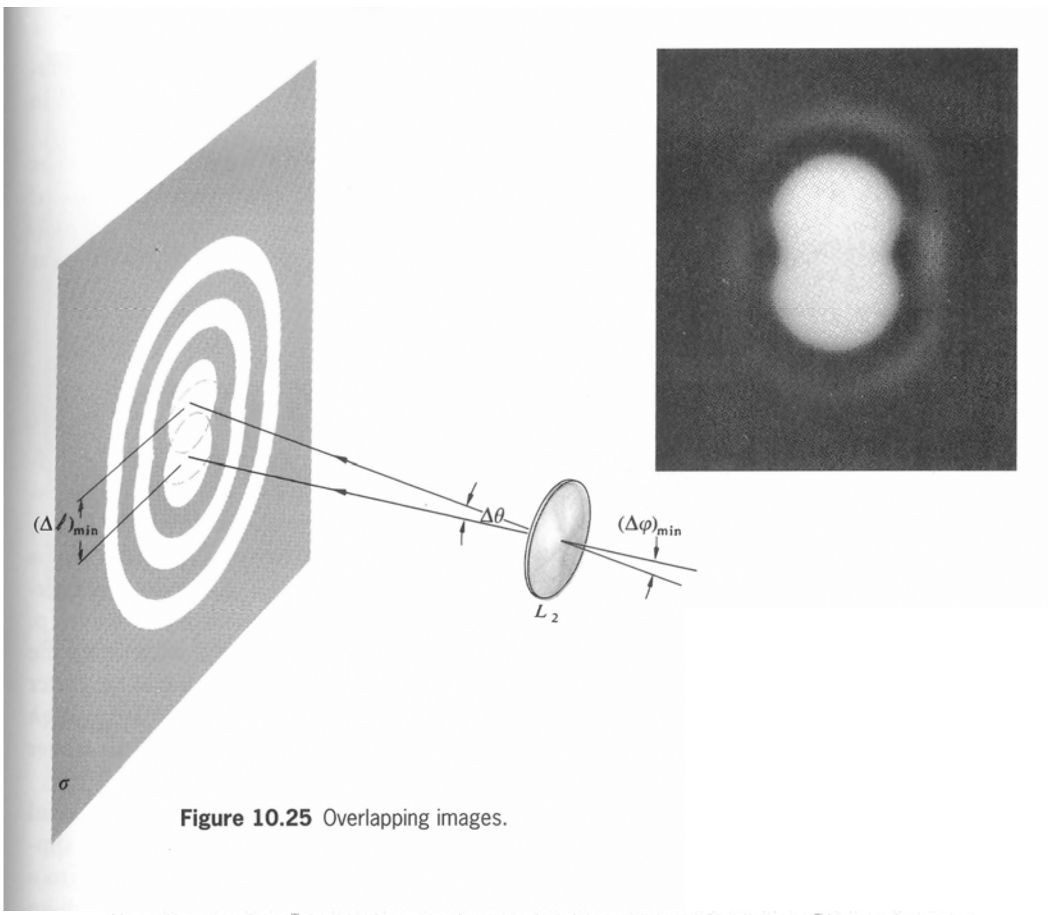


Figure 10.25 Overlapping images.

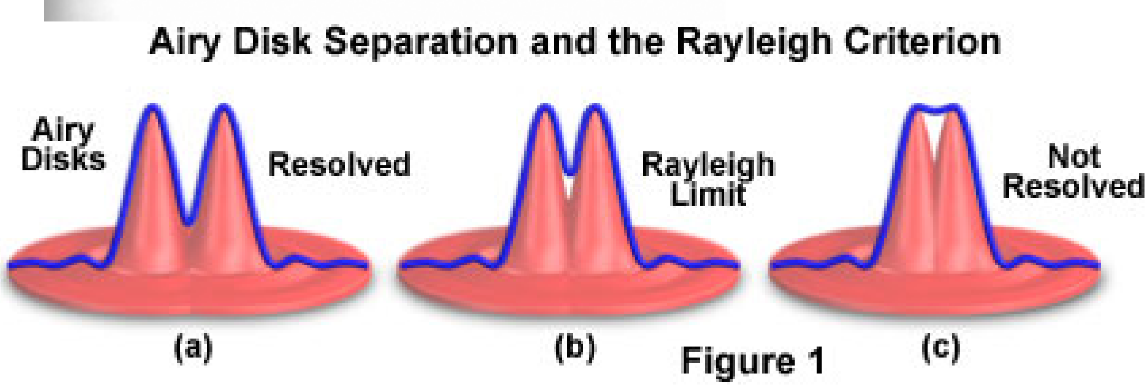


Figure 1

Depth of Focus

- Spot is in focus if waist expands less than 5%
- Using waist formula

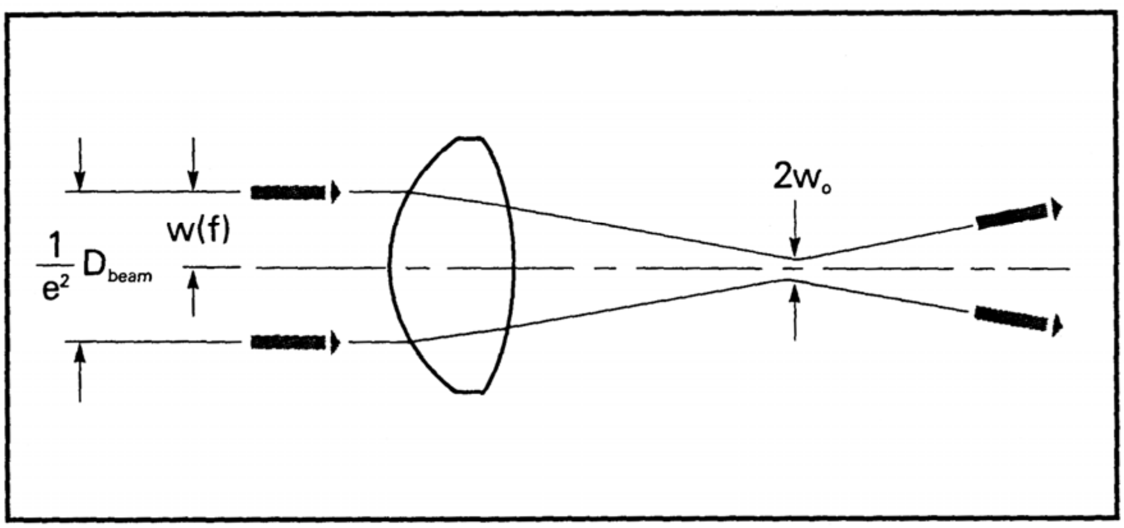
$$w(z) = w_0\left[1+\left(\frac{\lambda z}{\pi w_0^2}\right)^2\right]^{1/2}$$

- Solve this waist formula for 5% change then

$$\Delta z = \pm\frac{0.32\pi w_0^2}{\lambda}$$

- eg previous He-Ne laser is focused through a lens with
  f = 7 mm λ = 632.8 nm w_0 = 3.5 micron
  what is the depth of focus of spot produced?

$$\Delta z = \pm\frac{0.32\pi\left(3.5x10^{-6}\right)^2}{6.328x10^{-7}} = 1.97x10^{-5}\ m = 19.7\ \mu m$$



CONCENTRATION OF LASER BEAM by a laser line focusing singlet. Size of the focal waist has been greatly exaggerated for illustrative purposes.

Diffraction Limit and Gaussian Optics

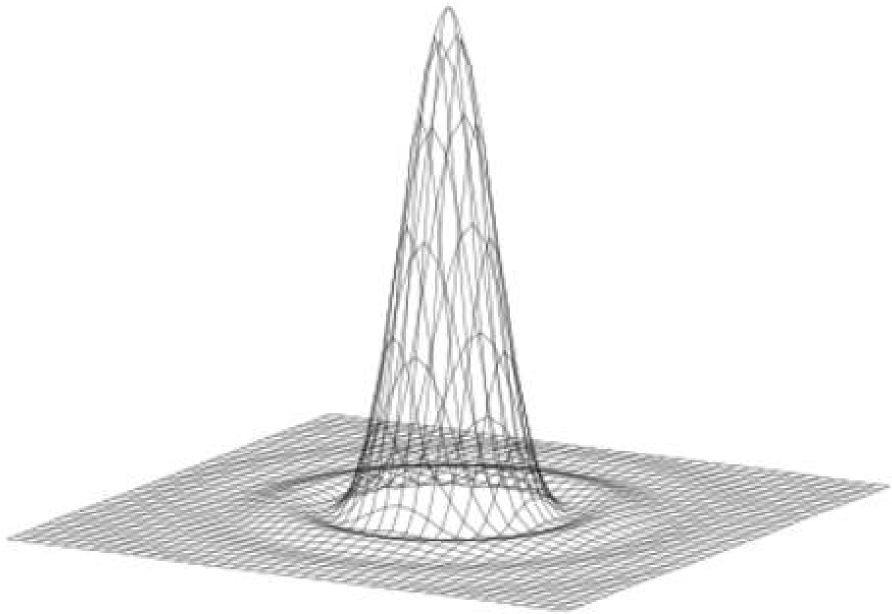
- Both give slightly different answer
  because looking at different beam parts
- eg He-Ne laser is focused through a lens with
  f = 50 mm   λ = 632.8 nm   D = 10 mm
- What is the minimum spot produced?
- If want first minimum of Airy disk then

$$d_{min} = \frac{2.44\, f\lambda}{D} = \frac{2.44(0.05)(6.328x10^{-7})}{0.01} = 7.7\,X\,10^{-6}\,m = 7.7\,\mu m$$

- If assume input waist is at focus using range formulas
- Let spot be typical 1/3 lens so waist w_{out} = 3.3 mm

$$w'' = \frac{\lambda\, f}{\pi\, w_0} = \frac{6.328x10^{-7}(0.05)}{\pi\, 0.0033} = 3.05x10^{-6} = 3.05\,\mu m$$

- Difference comes from 1st minimum diameter of Airy disk
  verses $1/e^2$ radius



Young's Double Slit Experiment

- Thomas Young 1801 experiment to prove light is a wave nature
- Consider 2 slits width b
- Separated by space a (centre to centre of slits)
- Now the pattern created by one slit creates interference with other
- At time considered the proof that light behaved as a wave

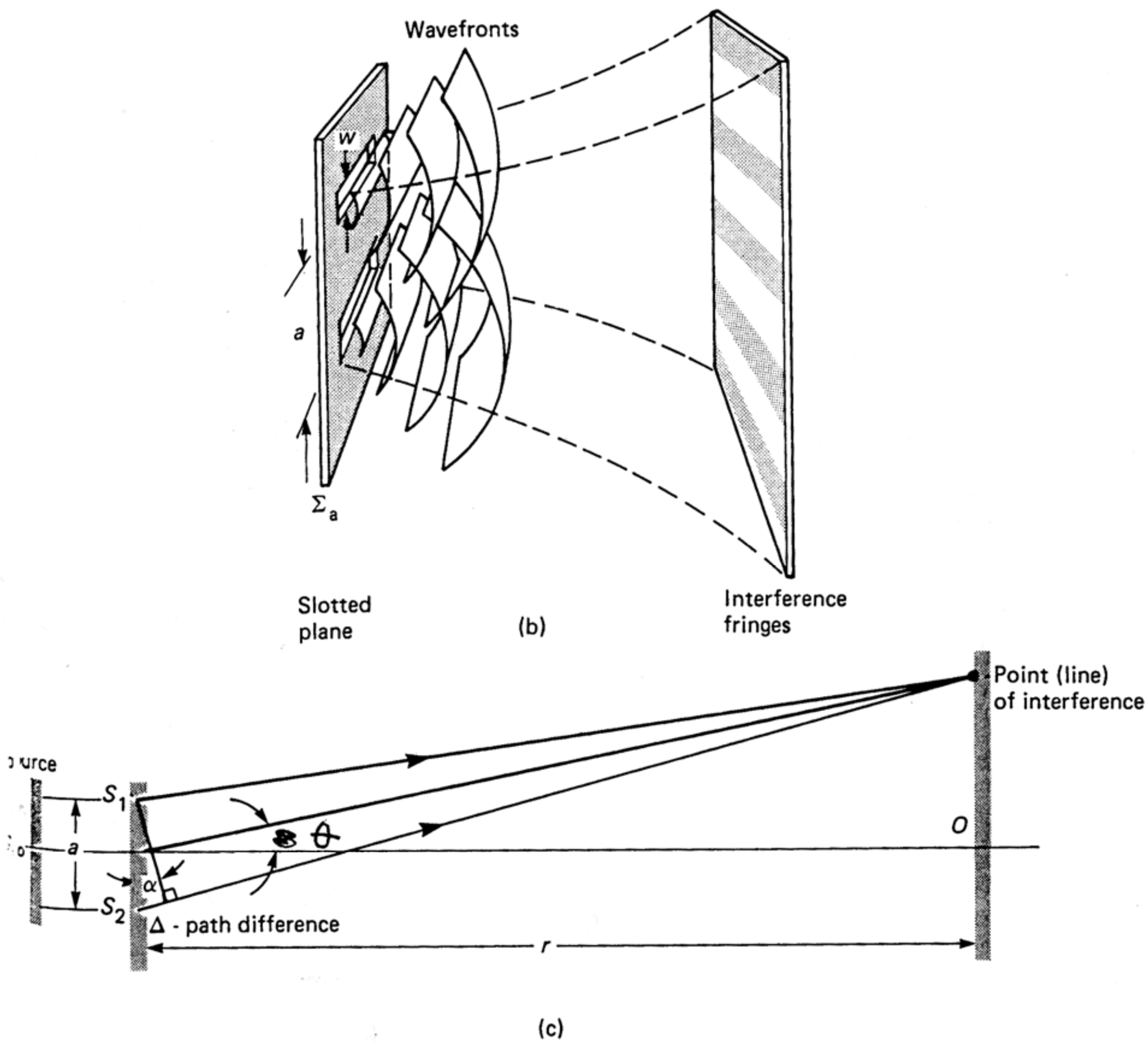


Figure 10.20 (a, b) Interference fringes formed when a monochromatic coherent plane wave strikes a thin opaque plane with two neighbouring narrow slits in it (the Young Experiment). (c) The common source S_o ensures coherence

Double Slit Interference

- Get the single slit pattern forming envelope
- Interference of two slits modulating that.

$$I(\beta) = 4I_0 \left[\frac{sin(\beta)}{\beta} \right]^2 cos(\alpha) \qquad \beta = \frac{\pi\, b\, sin(\theta)}{\lambda} \qquad \alpha = \frac{\pi\, a\, sin(\theta)}{\lambda}$$

θ = angular deviation of pattern from minimum

- For zeros: $\beta = \pm N\pi$
- Principal Maximums occur at

$$sin(\theta) = \frac{m\lambda}{a}$$

where m = any integer, order of the diffraction

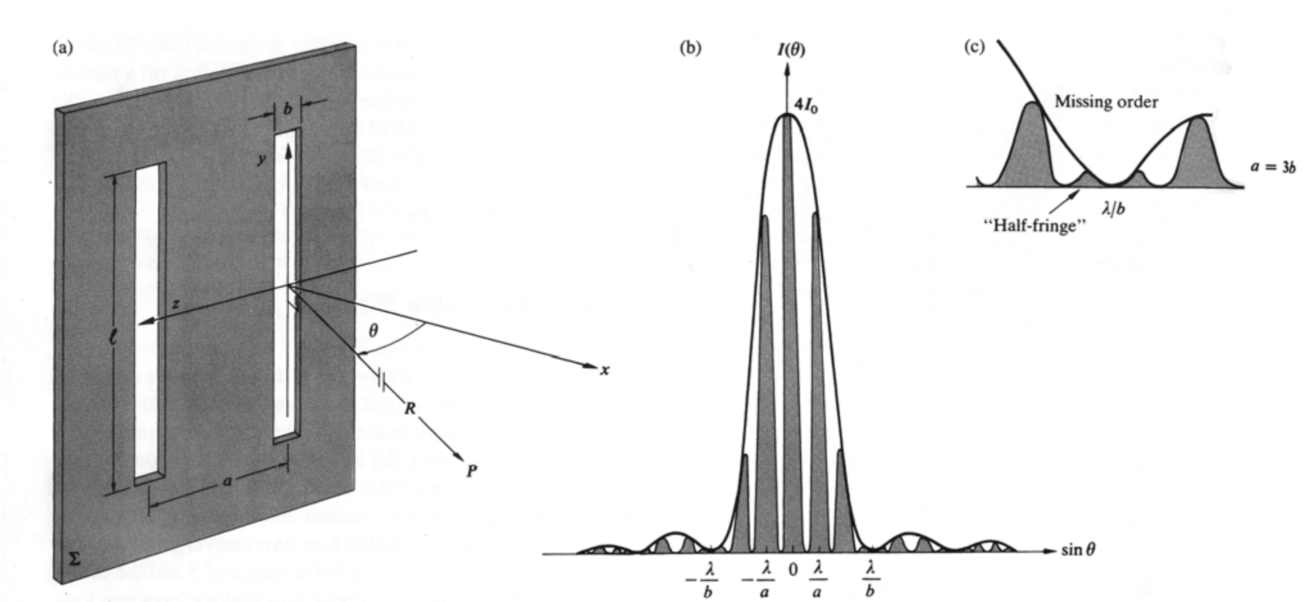


Figure 10.13 (*a*) Double-slit geometry. Point *P* on σ is essentially infinitely far away. (*b*) A double-slit pattern ($a = 3b$).

Young's Double Slit & Single Slit

- If take single slit
- Then add second slit see the one pattern on top of other
- If slits wide see both
- Slits narrow – nearly even patter due to wide null in small slit

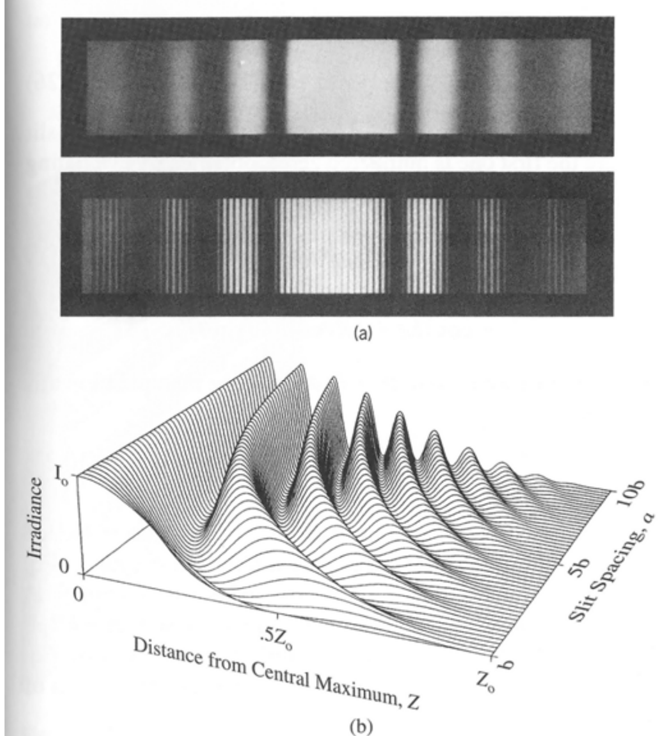
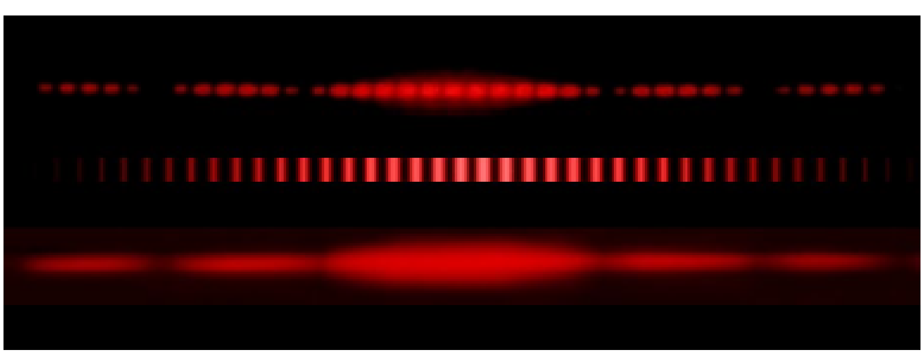
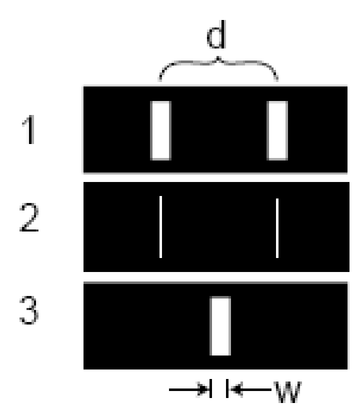


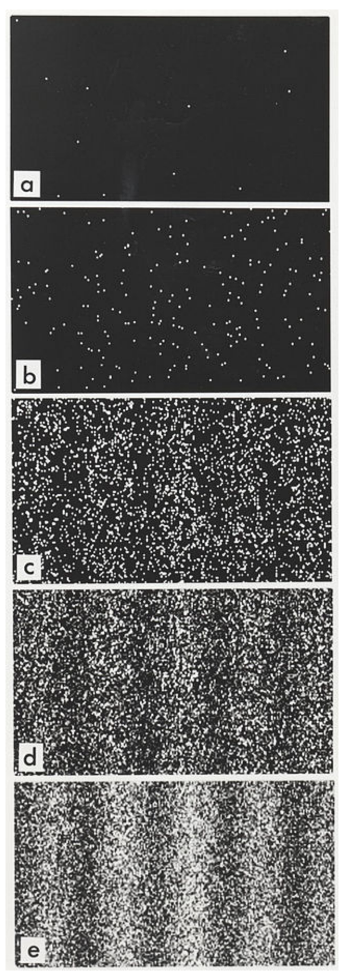
Figure 10.14 Single- and double-slit Fraunhofer patterns. (*a*) Photographs taken with monochromatic light. The faint cross-hatching arises entirely in the printing process. (Photos courtesy M. Cagnet, M. Francon, and J. C. Thrierr: *Atlas optischer Erscheinungen*, Berlin–Heidelberg–New York: Springer, 1962.) (*b*) When the slit spacing equals *b*, the two slits coalesce into one (of width 2*b*) and the single-slit pattern appears—that's the first curve closest to you. The farthest curve corresponds to the two slits separated by $a = 10b$. Notice that the two-slit patterns all have their first diffraction minimum at a distance from the central maximum of Z_0. Note how the curves gradually match Fig. 10.13*b* as the slit width *b* gets smaller in comparison to the separation *a*. (Reprinted from "Graphical Representations of Fraunhofer Interference and Diffraction" *Am. J. Phys.*, 62, 6, (1994), with permission of A. B. Bartlett, University of Colorado and B. Mechtly, Northeast Missouri State University and the American Association of Physics Teachers.)





Young's Double Slit & Quantum Effects

- Classic description of Young's double slit is a wave effect
- But what about light being particles: photons
- Taylor (1909) reduced light so one photon at time at slit
- Still get interference
- Davisson and Germer in 1927 proved particles acted as waves
- Reason is the wave – particle duality first proposed by de Brogile
- The wave of a particle detects both slits
- Claus Jönsson in 1962 showed electrons do same thing
- Because e's have a quantum wave function
- Even if single particle going through one.
- Creates the dual slit pattern due to quantum probability field
- Wave function field of photon extends far enough to see other slit
- Alternative: the many worlds theory of quantum
- See impact of all possible probabilistic events from "all universes



Diffraction Gratings

- Diffraction gratings: extends the 2 slits to a repeated pattern used by many systems eg spectrometers, acousto-optic deflectors
- Recall the Interference from a single width b

$$I(\beta) = I_0 \left[\frac{sin(\beta)}{\beta} \right]^2 \qquad \beta = \frac{\pi b \, sin(\theta)}{\lambda}$$

θ = angular deviation of pattern from minimum

- Now consider n slits width b spaced distance d apart
- Extending the 2 slits get the diffraction pattern from each slit
- But the diffraction patterns interfere
- The more slits, the more the interference patterns add up
- If have several slits then waves from each interfere
- More slits narrower beams

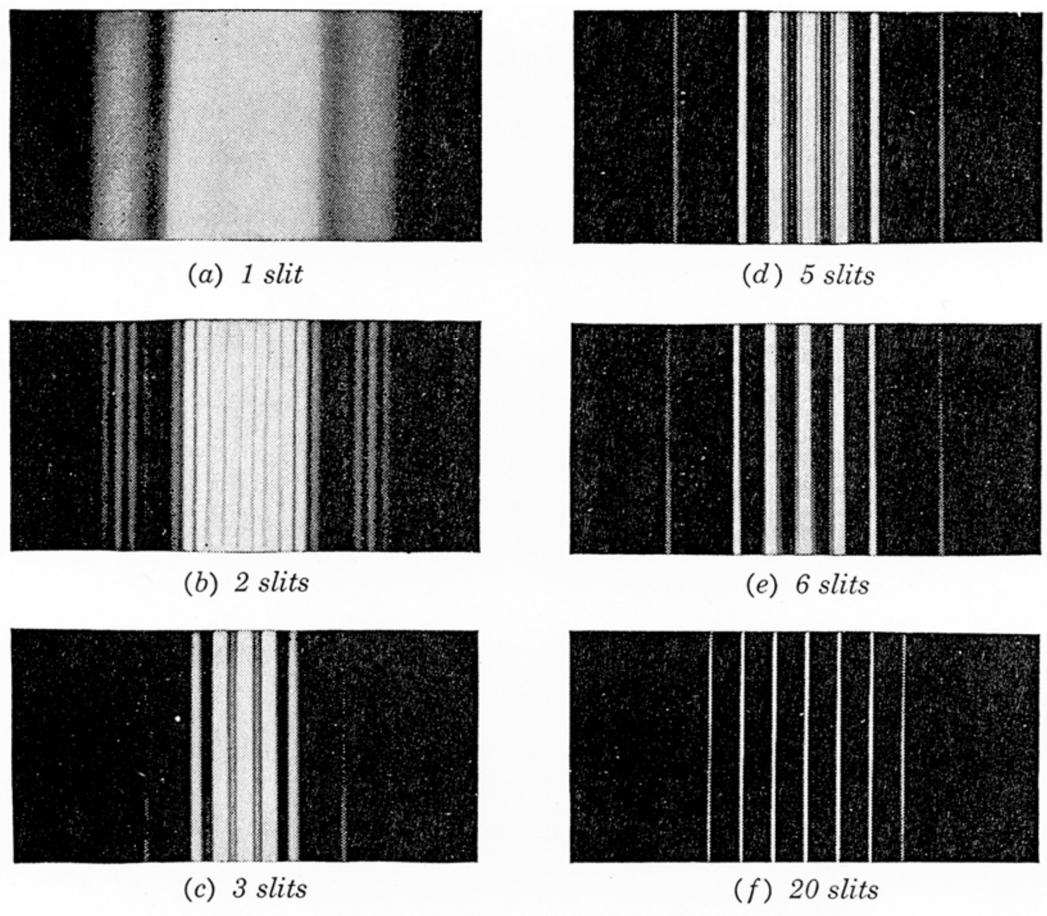


FIGURE 17A
Fraunhofer diffraction patterns for gratings containing different numbers of slits.

Diffraction Gratings Formulas

- Similar to the 2 slit the intensity becomes for n slits

$$I(\beta) = I_0 \left[\frac{\sin^2(\beta)}{\beta^2} \right] \left[\frac{\sin^2(n\gamma)}{\sin^2(\gamma)} \right] \qquad \gamma = \frac{\pi d \sin(\theta)}{\lambda}$$

- Principal Maximums occur at

$$\sin(\theta) = \frac{m\lambda}{d}$$

where m = any integer, order of the diffraction

- The maxima vary within the single slit β function
- Result is that large n with small d gives narrow lines
- Peaks of different λ at different angles
- Single λ get peaks: white different peaks for each colour

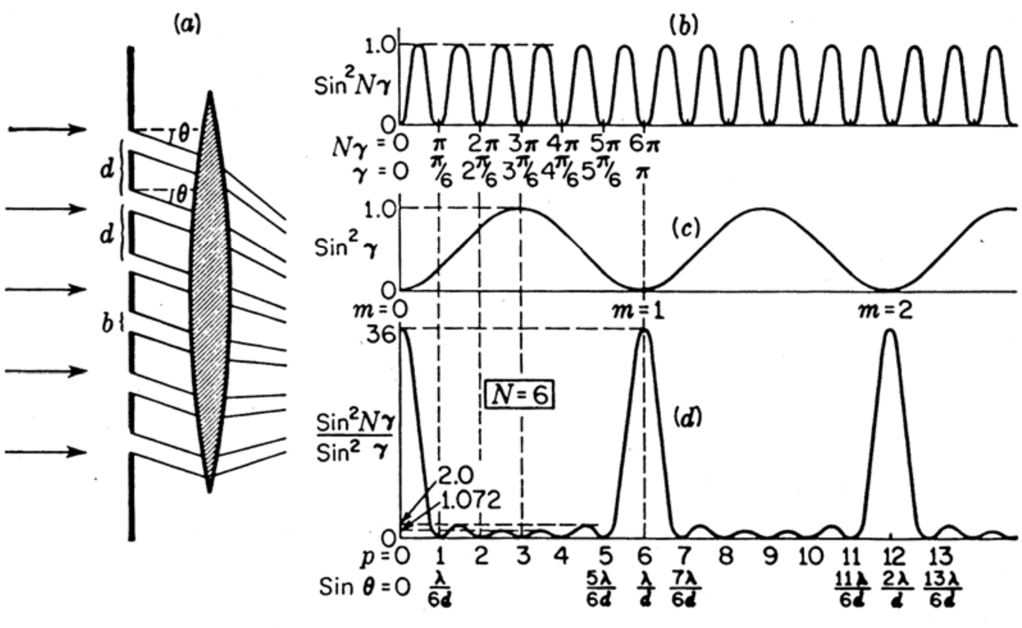
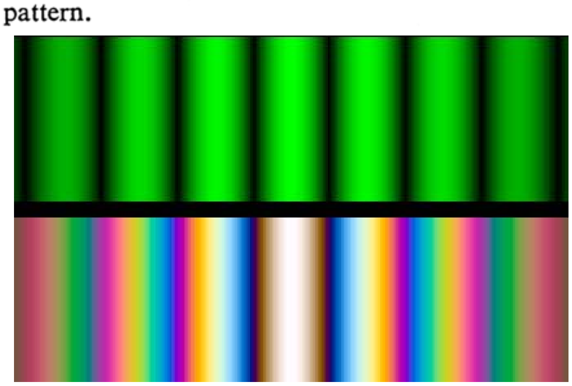


FIGURE 17C
Fraunhofer diffraction by a grating of six very narrow slits and details of the intensity pattern.



Spectrometry

- Want to see the spectral distribution of the light
- Used for detecting presence of elements based on spectrum
- Two ways – prism & diffraction gratings
- Mostly use diffraction gratings
- Generally greater spread than prisms
- Black body continuous spectrum
- Hot glowing gas (plasma) get emission spectrum of material
- Pass black body light through cold gas get absorption spectrum