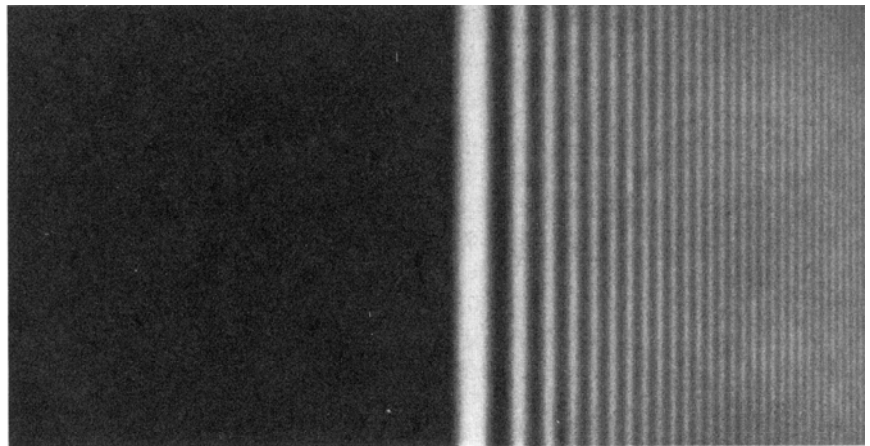


Diffraction

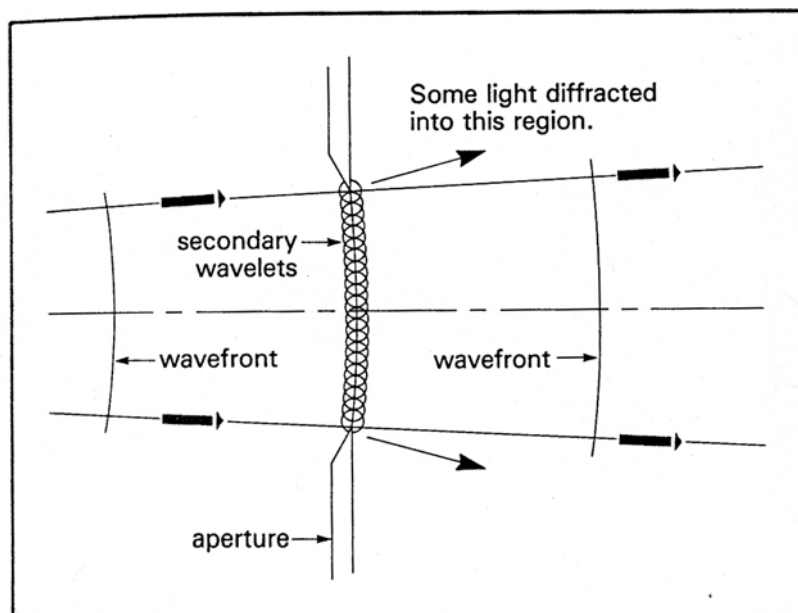
- Diffraction occurs when light waves are passed by an aperture/edge
- Huygen's Principal:
 - each point on wavefront acts as source of another circular wave
- Consider light from point source at infinity or parallel beam (laser)
- Light diffracts around objects
- Consider a slit in a plate: light from edge diffracts
- Edge creates interference effects between the waves at each point
- Thus at edges asymmetric effects results in bending
- Diffractions sets the resolution limits for lens and optics systems

Figure 13-14 Straight-edge diffraction.

(d) Diffraction fringes from a straight line.
(From M. Cagnet, M. Francon, and J. C. Thrierr, *Atlas of Optical Phenomenon*, Plate 32, Berlin: Springer-Verlag, 1962.)



(d)



HUYGEN'S PRINCIPLE states that each point on a propagating wavefront is an emitter of secondary wavelets.

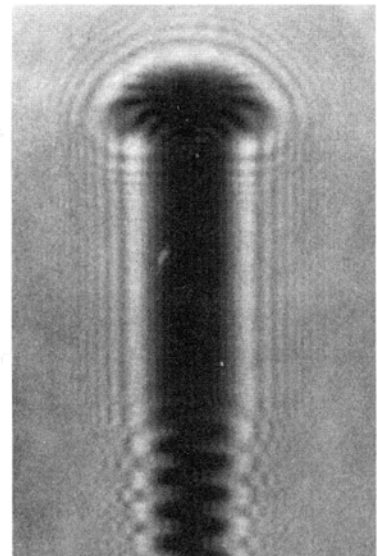


Figure 13-18 Fresnel shadow of a screw.
(From M. Cagnet, M. Francon, and J. C. Thrierr, *Atlas of Optical Phenomenon*, Plate 36, Berlin: Springer-Verlag, 1962.)

Single Slit: Fresnel and Fraunhofer Interference

- Image created depends on distance from slit light is seen
- Strongly seen in interference at the edge or small objects
- Two types: Fresnel and Fraunhofer Interference
- Both assume light at infinity nearly parallel light or a laser beam

Fraunhofer Interference

- Diffracted light sensed at infinity (also a focused image)
- Simpler equations - what is usually used

Fresnel interference

- Pattern created within short distance to the diffraction point
- Much more complex equations: involve near E-mag field
- Pattern very dependent on the distance & slit/object size

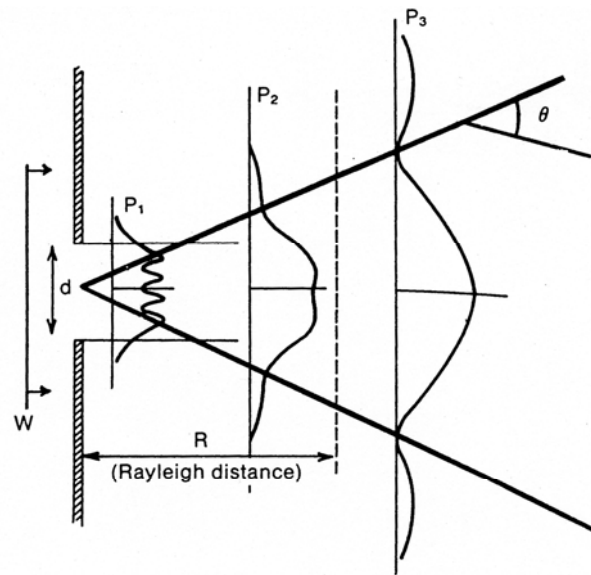
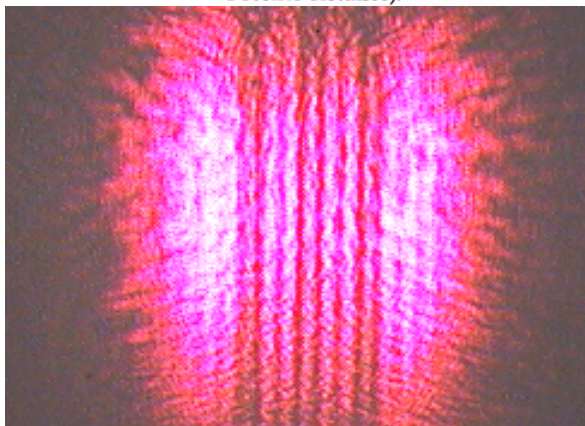
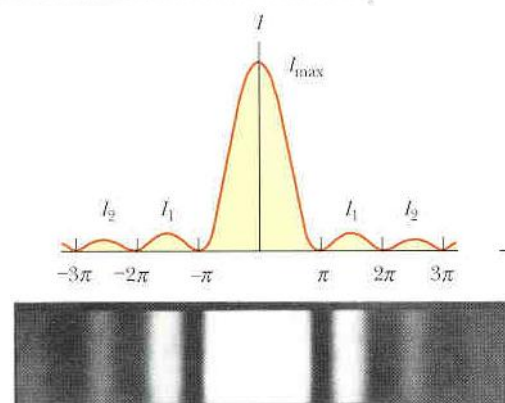


Fig. 2.8.14. Transition from Fresnel to Fraunhofer diffraction. A portion of a wave, W , passes through a slit of width, d . Intensity distributions across the wave are shown for planes P_1 (close to the slit), P_2 (just inside the Fresnel distance), and P_3 (beyond the Fresnel distance).



Fresnel Interference from laser



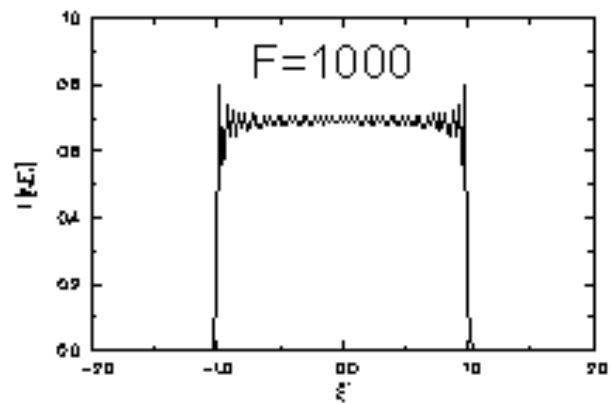
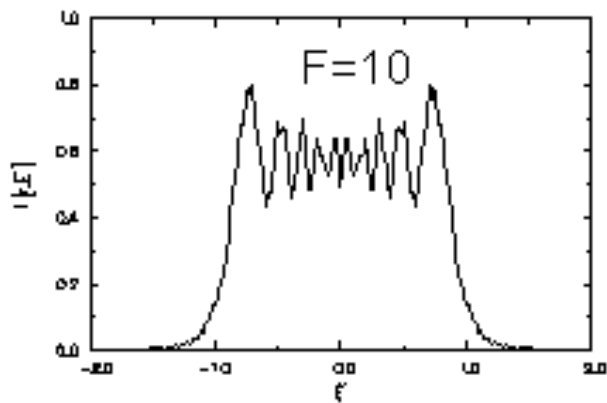
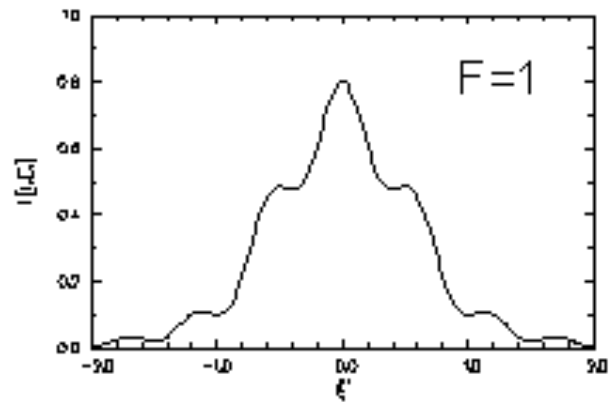
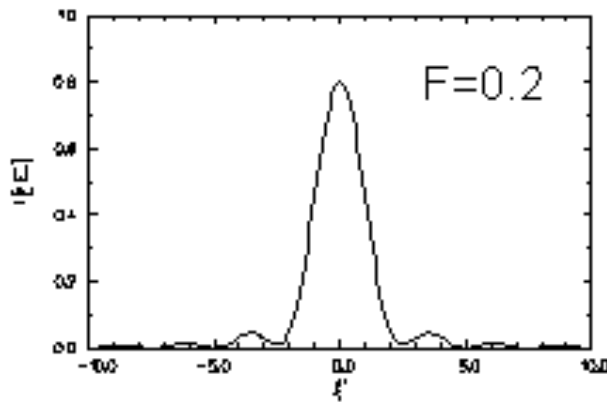
Fraunhofer Interference

Fresnel Number

- Fresnel number measures if the image is close or far field
- Consider a slit width b illuminated by wavelength λ
- Measuring image at a distance R
- Fresnel number is

$$F > \frac{b^2}{R\lambda}$$

- $F \ll 1$ Fraunhofer interference (Far Field)
- $F \gg 1$ Fresnel – complicated image function of distance R



Fraunhofer Interference

- Consider a single slit width b but infinite length in a sheet
- Illuminated by single wavelength parallel source (at infinity)
- Diffraction effects “seen at infinity” which means distances R

$$R > \frac{b^2}{\lambda}$$

- If focus slit with lens get the same as at infinity
- Most common effect
- Intensity follows the pattern of a sinc function

$$I(\beta) = I_0 \left[\frac{\sin(\beta)}{\beta} \right]^2$$

where

$$\beta = \frac{\pi b \sin(\theta)}{\lambda}$$

θ = angular deviation of pattern from minimum

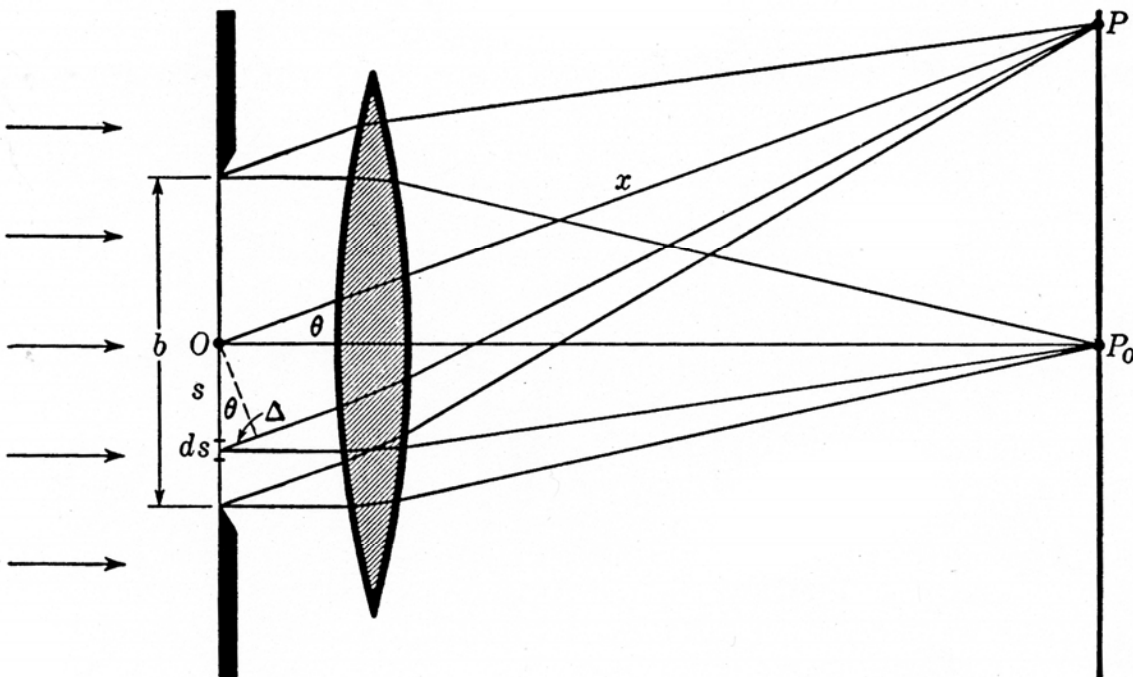


FIGURE 15C

Geometrical construction for investigating the intensity in the single-slit diffraction pattern.

Single Slit Fraunhofer Interference Pattern

- Zeros are at

$$\beta = \pm N\pi$$

where N is any integer

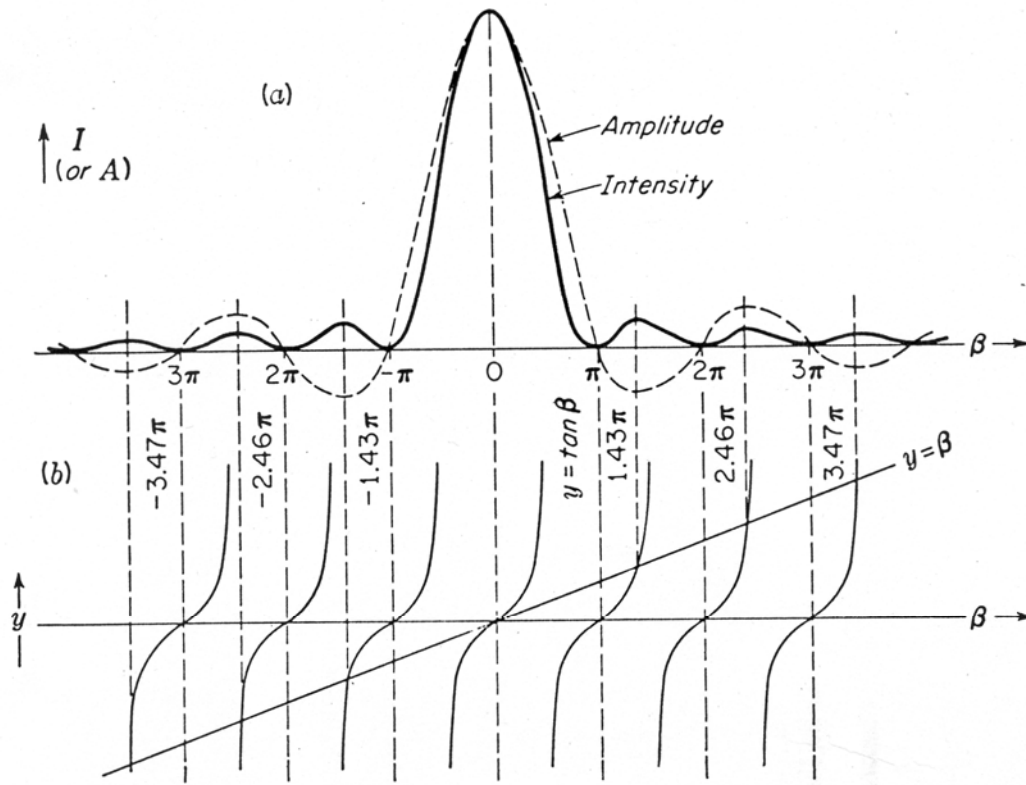


FIGURE 15D

Amplitude and intensity contours for Fraunhofer diffraction of a single slit, showing positions of maxima and minima.

- Large d little pattern (a), small d pattern spreads out (c)

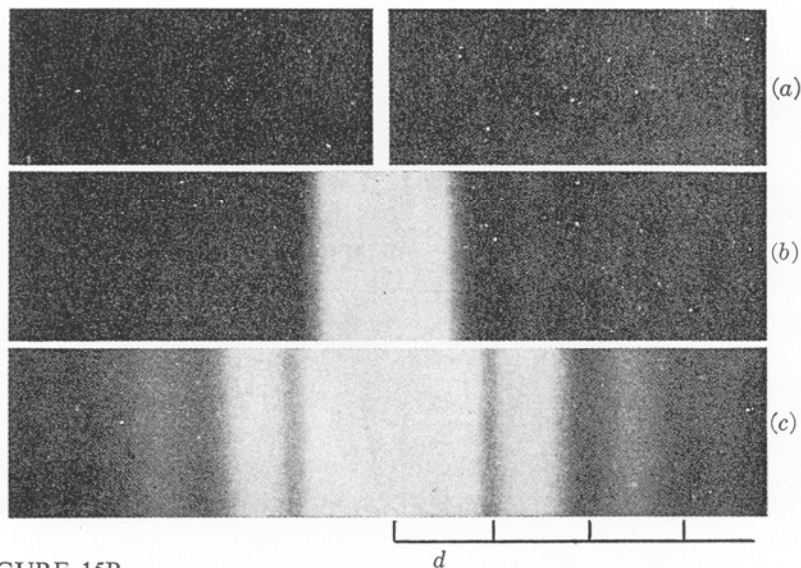


FIGURE 15B

Photographs of the single-slit diffraction pattern.

Circular Fraunhofer Interference

- Interference changes for circular opening – eg circular lenses
- Called an Airy Disk
- Lenses act as circular apertures
- Most important for laser systems
- For single circular aperture diameter D
- Intensity follows the pattern

$$I(\beta) = I_0 \left[\frac{2J_1(\beta)}{\beta} \right]^2$$

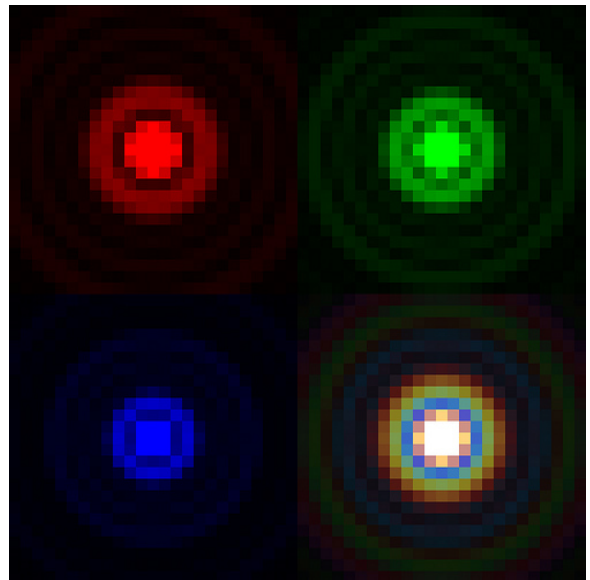
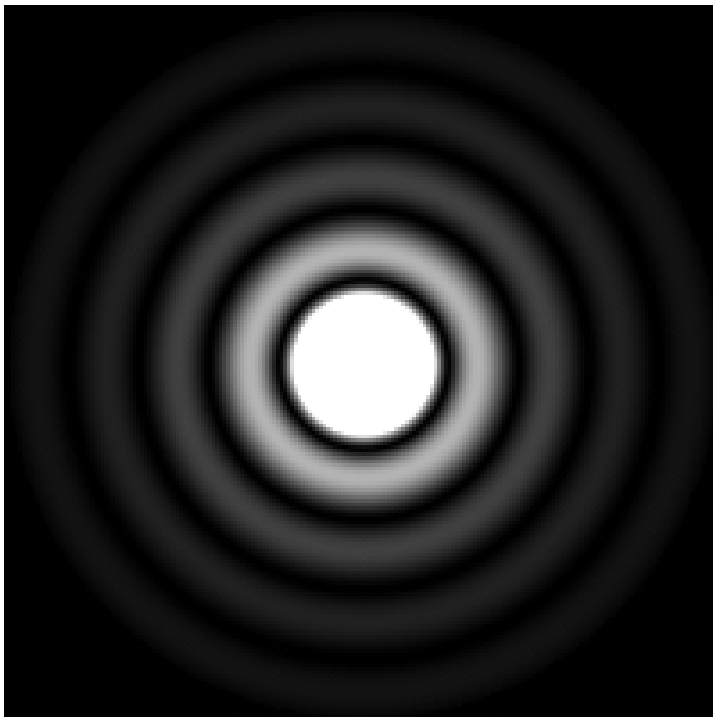
where

$$\beta = \frac{\pi D \sin(\theta)}{\lambda}$$

J_1 = Bessel function of first kind, order 1

θ = angular deviation of pattern from minimum

- Shrinks rings as wavelength decreases
- White light creates multicolour rings



Bessel Functions

- Bessel functions: from solutions to cylindrical coordinate problems
- Comes from solutions to the DE

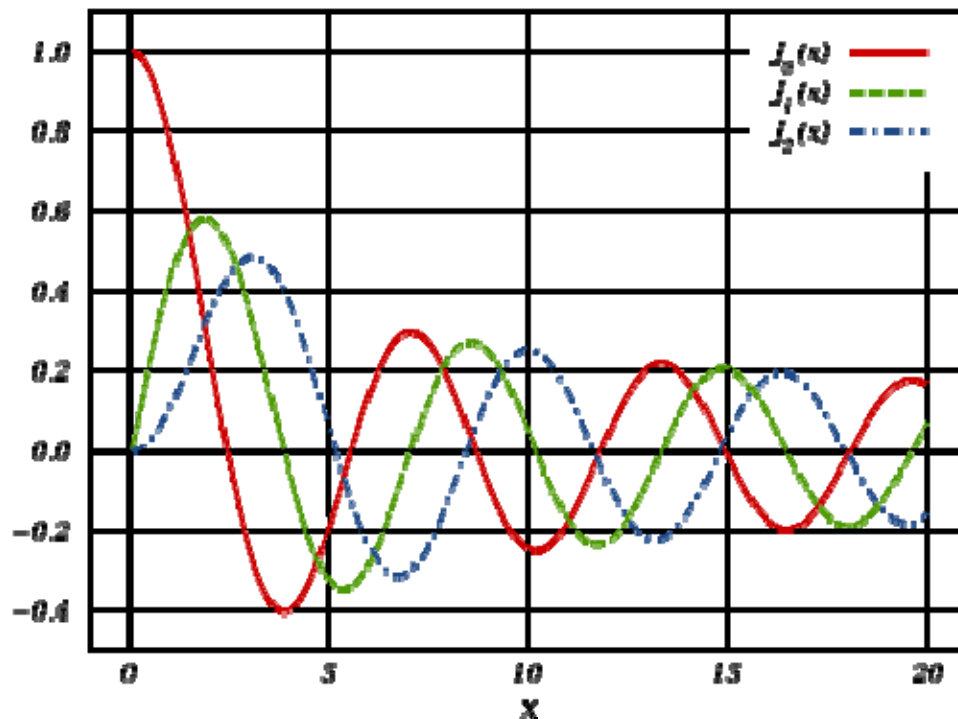
$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + (x^2 - n^2)y = 0$$

- Typical applications Emag waves in cylindrical cavities
- Heat flow in cylinder (eg laser heating of material)
- Bessel functions of first kind order n are $J_n(x)$
- Bessel function $J_1(x)$ are given by the expansion

$$J_1(x) = \frac{x}{2} - \frac{x^3}{2^2 \cdot 4} + \frac{x^5}{2^2 \cdot 4^2 \cdot 6} \dots$$

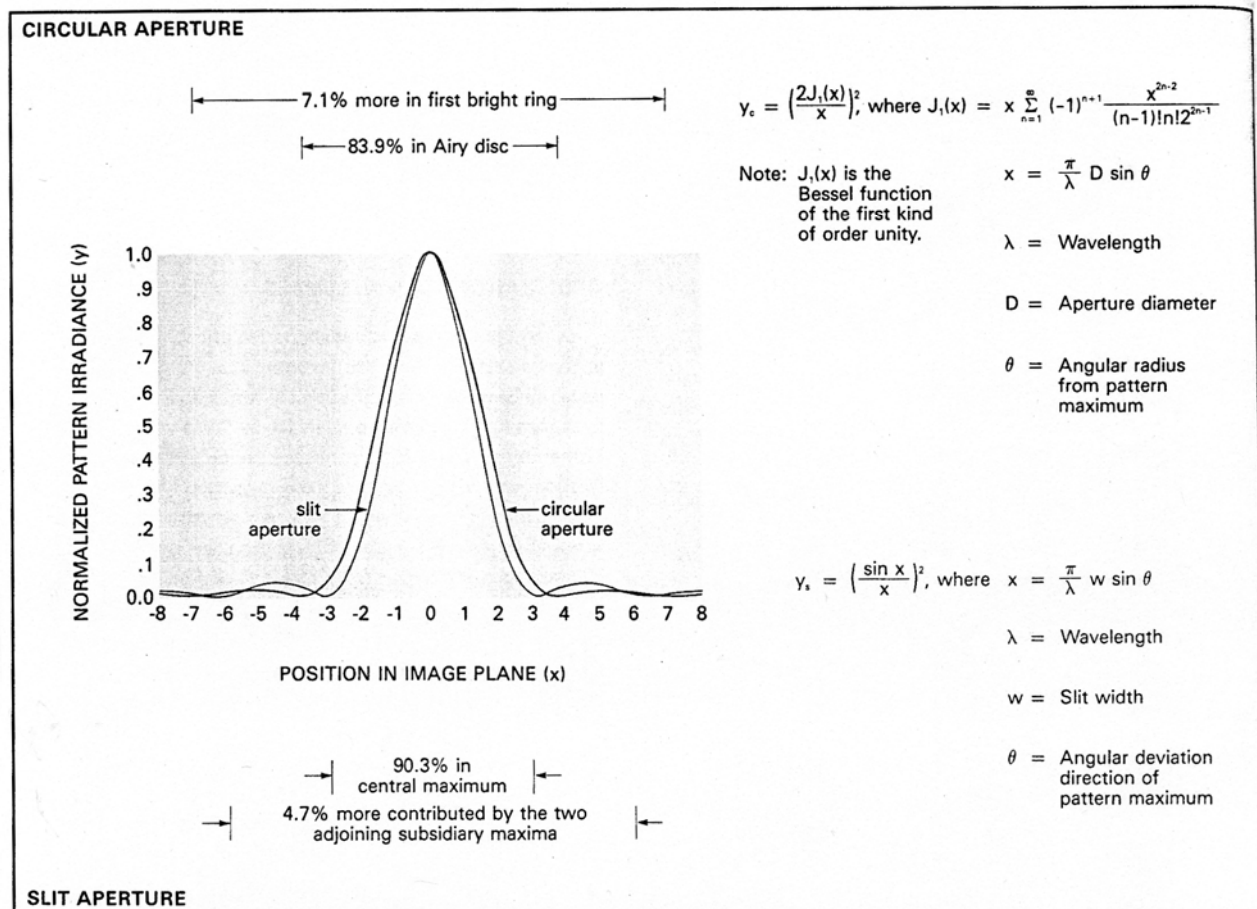
- Look like a decaying sin/cos wave
- Thus for Airy disk at the centre

$$I(\beta \rightarrow 0) \rightarrow I_0 \left[\frac{2J_1(\beta)}{\beta} \right]^2 \approx I_0 \left[\frac{\beta}{2} \frac{2}{\beta} \right]^2 \rightarrow I_0$$



Comparison of Circular and Slit Fraunhofer Interference

- Slit produces smaller width pattern
- Circular Airy disks is wider than slit of same size/wavelength
- Nulls occur further out
- Peaks smaller after nulls than for slit
- Higher % energy in central peaks



FRAUNHOFER DIFFRACTION PATTERN of a singlet slit superimposed on the Fraunhofer Diffraction Pattern of a circular aperture.

Minimum Spotsize of Focused Laser Beam

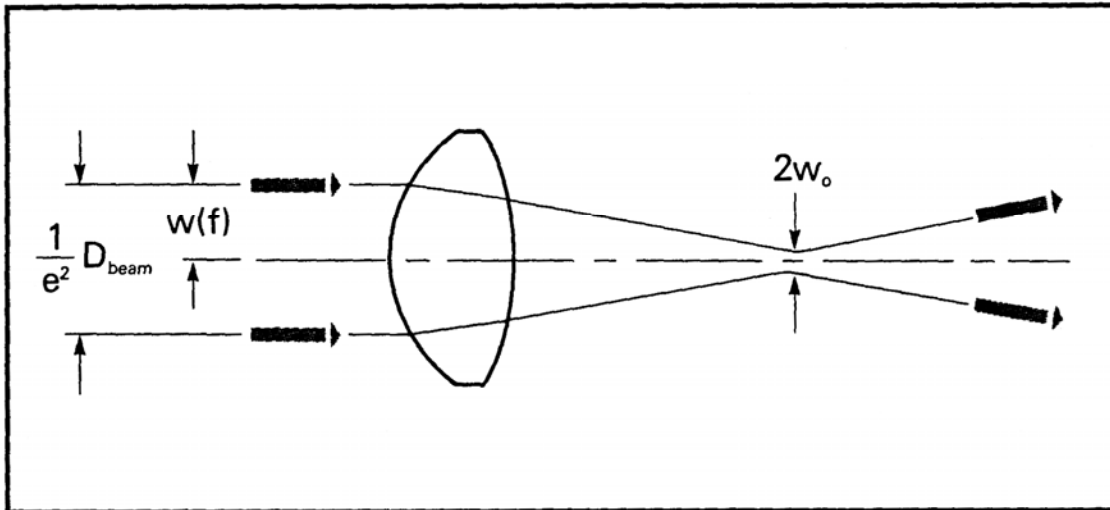
- For beam much smaller than lens
 limited by waist size of input beam
- Hence if waist in at the focus then

$$w_0 = \frac{\lambda f}{\pi w}$$

- NOTE: lens aberration may modify this by a factor
- eg He-Ne laser is focused through a lens with
 $f = 7 \text{ mm}$ $\lambda = 632.8 \text{ nm}$ $w_{\text{out}} = 0.4 \text{ mm}$
- What is the minimum spot produced?
- Assume input waist is at focus

$$w_0 = \frac{\lambda f}{\pi w} = \frac{6.328 \times 10^{-7} (0.007)}{\pi 4 \times 10^{-4}} = 3.5 \times 10^{-6} = 3.5 \mu\text{m}$$

- For singlet lens multiply this by 1.333 for 4.7 micron



CONCENTRATION OF LASER BEAM by a laser line focusing singlet. Size of the focal waist has been greatly exaggerated for illustrative purposes.

Diffraction Limited spot

- If laser beam fills the lens then diffraction limited
- Opening of width D
- Minimum spot is to point of first zero in diffraction pattern

$$I = \frac{b \sin(\theta)}{\lambda} = \frac{b d_{min}}{\lambda 2 f}$$

$$d_{min} = \frac{2 f \lambda}{D}$$

- Since circular effectively Airy diffraction add a factor of 1.22

$$d_{min} = \frac{2.44 f \lambda}{D}$$

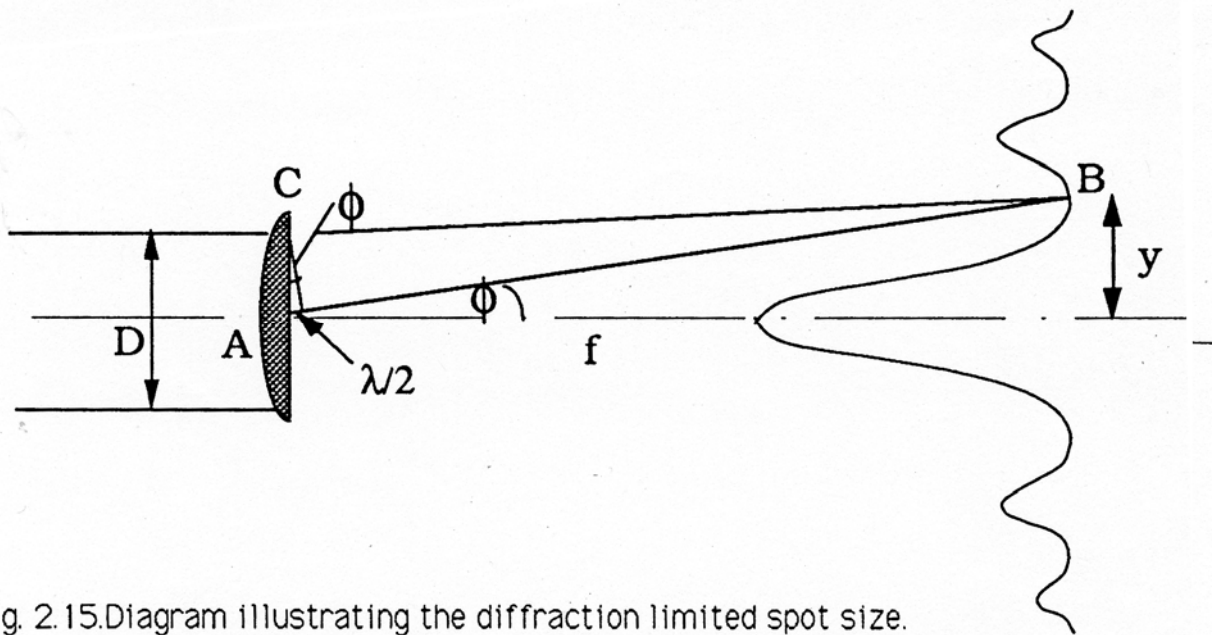


Fig. 2.15. Diagram illustrating the diffraction limited spot size.

Resolution in Optical Systems

- Resolution tells how good is the fine detail on an image
- Simple case: what is the min separation of 2 small, close, spots
- When spots fully separated then can resolve
- In telescopes how close can 2 stars be seen as separate objects
- In astronomy this is called the angular separation
- Bigger telescope mirror – greater angular separation
- However on earth atmosphere density changes limits resolution
- Actually effective atm telescope diameter only 20 cm
- Later can see how to compensate for atmosphere)
- Hence space based telescopes much better (Hubble)

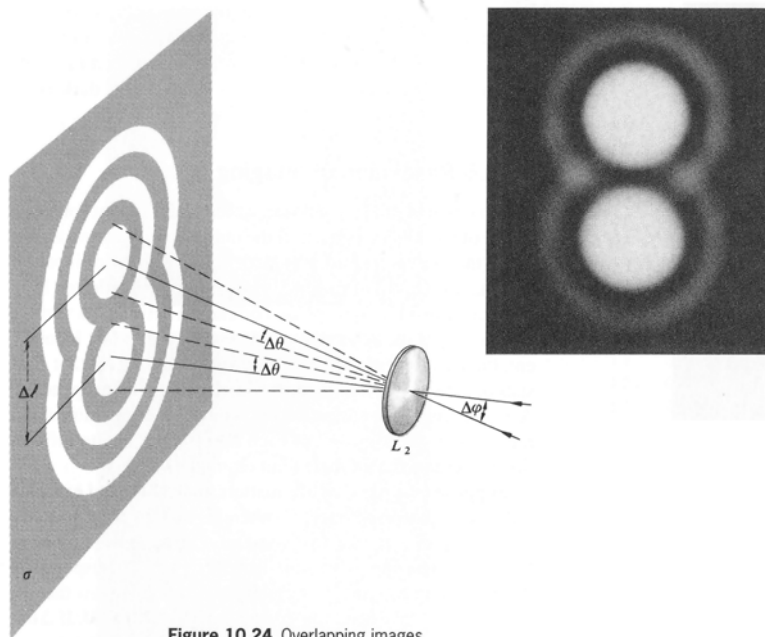
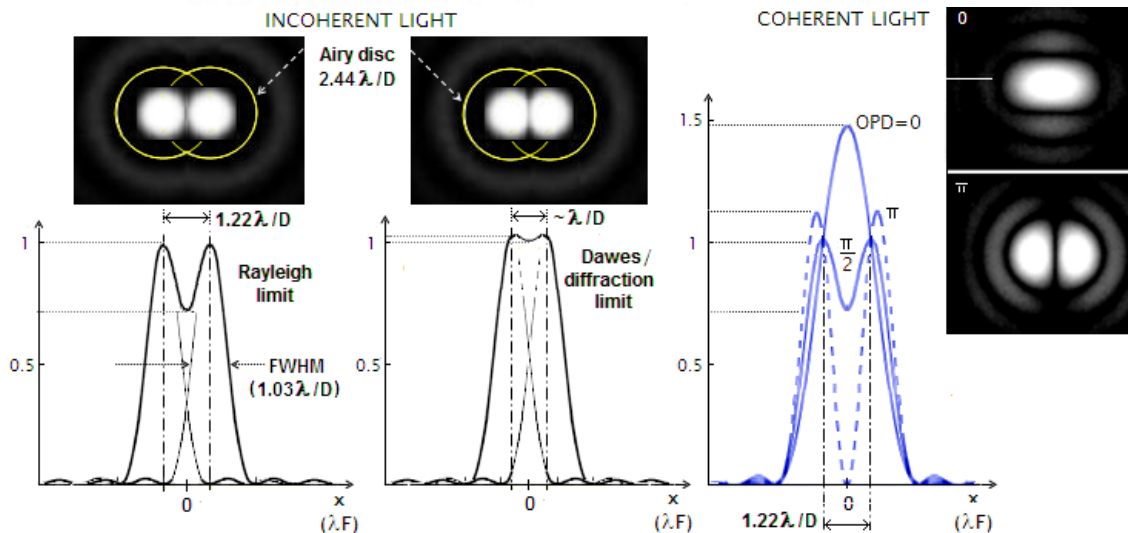


Figure 10.24 Overlapping images.

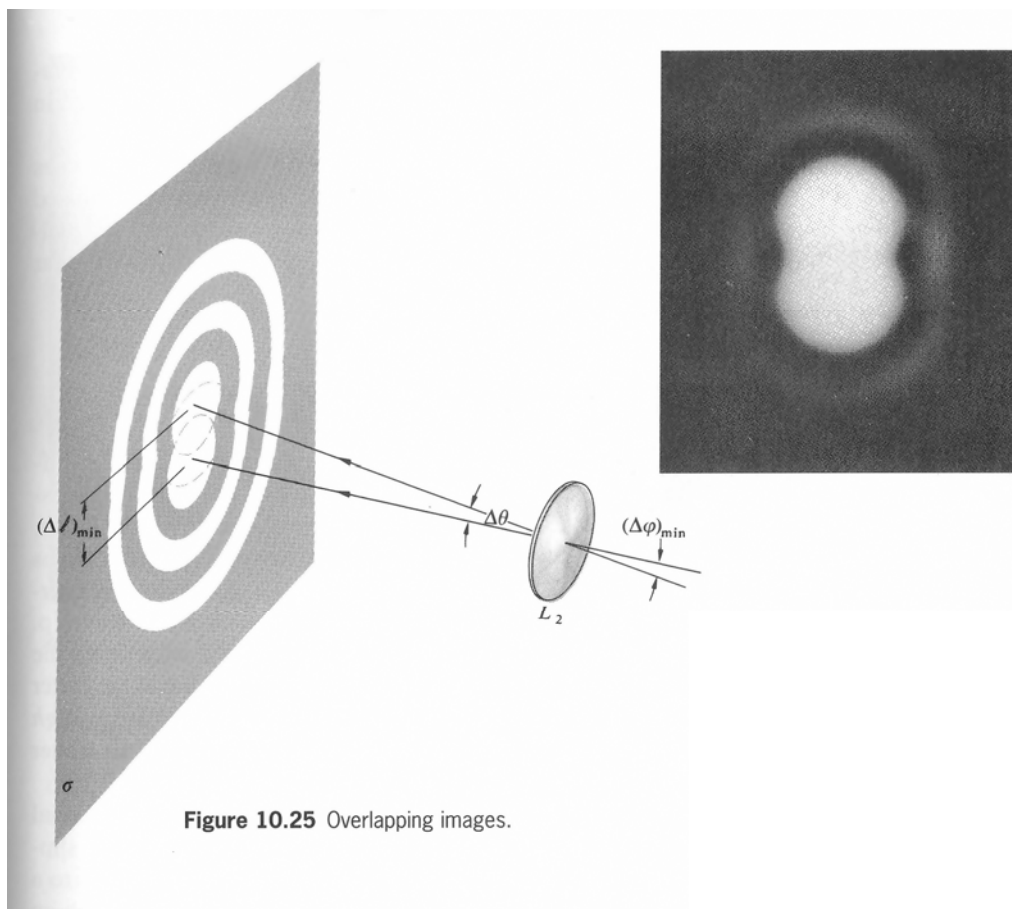


Rayleigh's Criteria Images

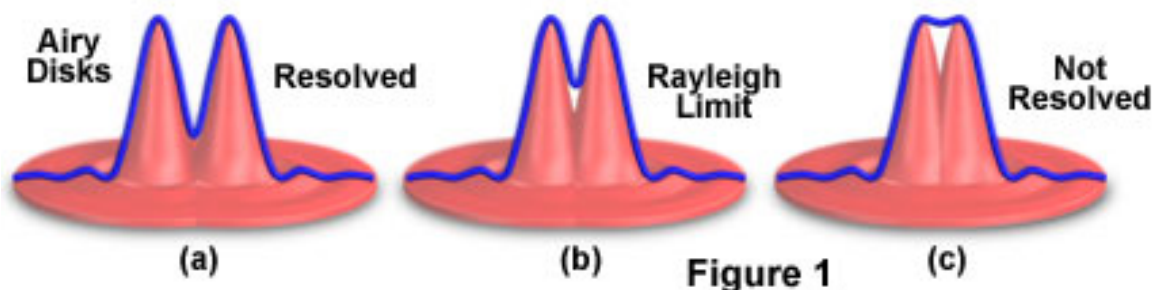
- What determines when we can separate two spots in an image
- When spots overlap enough cannot separate
- Different systems determine how much overlap allowed

$$d_{min} = \frac{1.22 f \lambda}{D}$$

- This is Rayleigh's Criteria most common
- Where two separate peaks can just be separated



Airy Disk Separation and the Rayleigh Criterion



Depth of Focus

- Spot is in focus if waist expands less than 5%
- Using waist formula

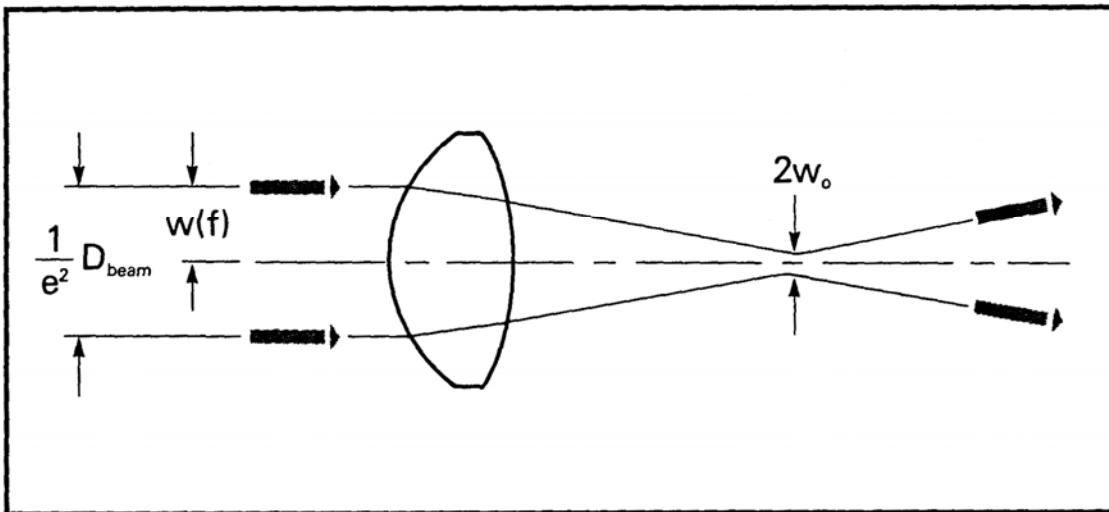
$$w(z) = w_0 \left[1 + \left(\frac{\lambda z}{\pi w_0^2} \right)^2 \right]^{1/2}$$

- Solve this waist formula for 5% change then

$$\Delta z = \pm \frac{0.32 \pi w_0^2}{\lambda}$$

- eg previous He-Ne laser is focused through a lens with
 $f = 7 \text{ mm}$ $\lambda = 632.8 \text{ nm}$ $w_0 = 3.5 \text{ micron}$
 what is the depth of focus of spot produced?

$$\Delta z = \pm \frac{0.32 \pi (3.5 \times 10^{-6})^2}{6.328 \times 10^{-7}} = 1.97 \times 10^{-5} \text{ m} = 19.7 \text{ } \mu\text{m}$$



CONCENTRATION OF LASER BEAM by a laser line focusing singlet. Size of the focal waist has been greatly exaggerated for illustrative purposes.

Diffraction Limit and Gaussian Optics

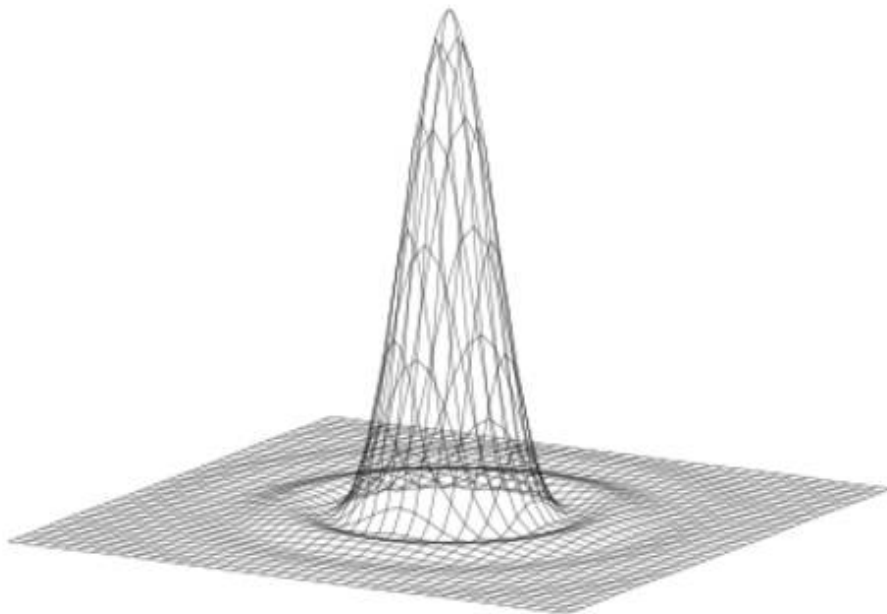
- Both give slightly different answer
because looking at different beam parts
- eg He-Ne laser is focused through a lens with
 $f = 50 \text{ mm}$ $\lambda = 632.8 \text{ nm}$ $D = 10 \text{ mm}$
- What is the minimum spot produced?
- If want first minimum of Airy disk then

$$d_{min} = \frac{2.44 f \lambda}{D} = \frac{2.44 (0.05) (6.328 \times 10^{-7})}{0.01} = 7.7 \times 10^{-6} \text{ m} = 7.7 \mu\text{m}$$

- If assume input waist is at focus using range formulas
- Let spot be typical 1/3 lens so waist $w_{out} = 3.3 \text{ mm}$

$$w'' = \frac{\lambda f}{\pi w_0} = \frac{6.328 \times 10^{-7} (0.05)}{\pi 0.0033} = 3.05 \times 10^{-6} = 3.05 \mu\text{m}$$

- Difference comes from 1st minimum diameter of Airy disk
verses $1/e^2$ radius



Young's Double Slit Experiment

- Thomas Young 1801 experiment to prove light is a wave nature
- Consider 2 slits width b
- Separated by space a (centre to centre of slits)
- Illuminated by monochromatic light at infinity (flat wave front)
- Now the pattern created by one slit creates interference with other
- At time considered the proof that light behaved as a wave

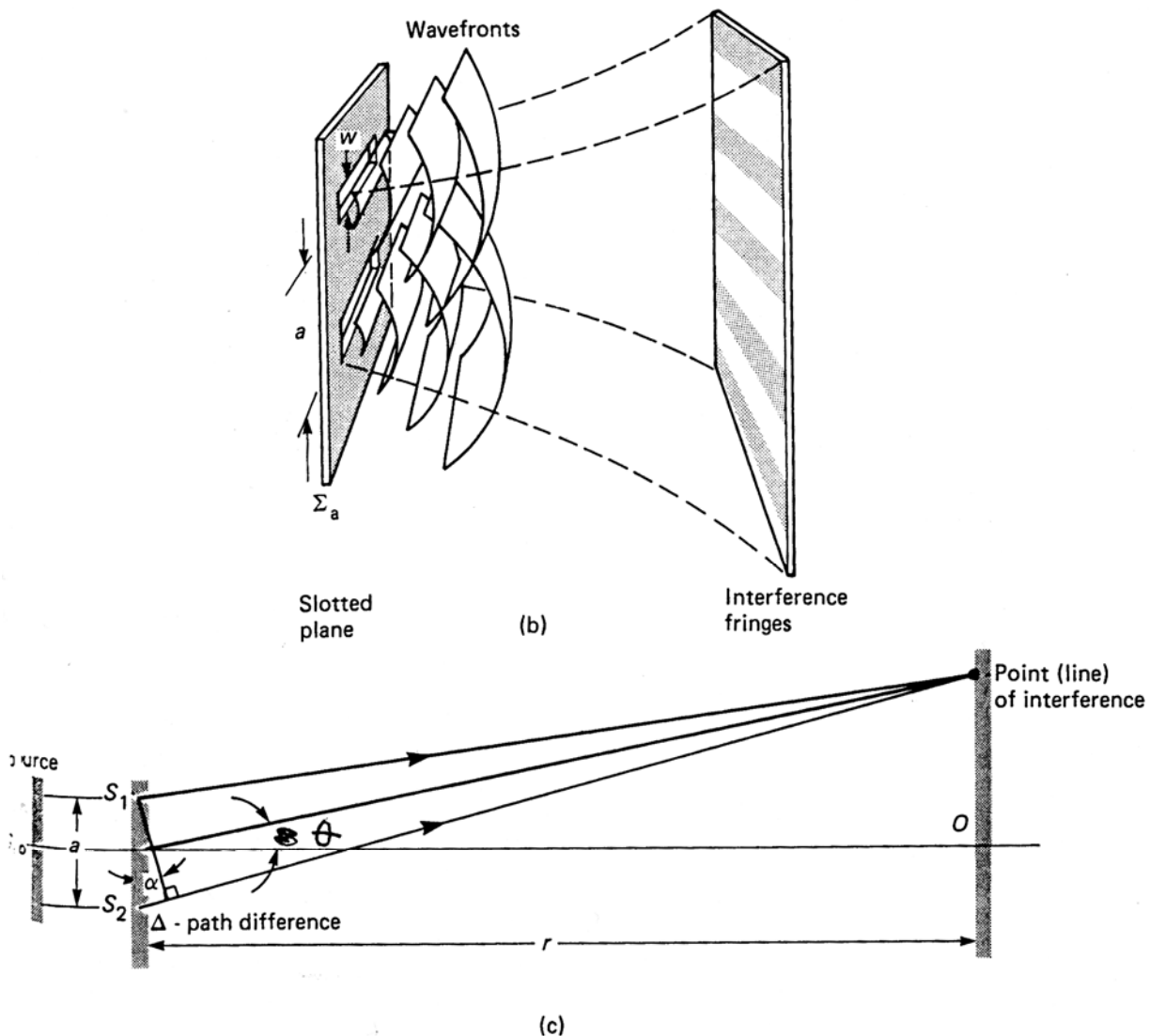


Figure 10.20 (a, b) Interference fringes formed when a monochromatic coherent plane wave strikes a thin opaque plane with two neighbouring narrow slits in it (the Young Experiment). (c) The common source S_0 ensures coherence

Double Slit Interference

- Get the single slit pattern forming envelope
- Interference of two slits modulating that.

$$I(\beta) = 4I_0 \left[\frac{\sin(\beta)}{\beta} \right]^2 \cos(\alpha) \quad \beta = \frac{\pi b \sin(\theta)}{\lambda} \quad \alpha = \frac{\pi a \sin(\theta)}{\lambda}$$

θ = angular deviation of pattern from minimum

- For zeros: $\beta = \pm N\pi$
- Principal Maximums occur at

$$\sin(\theta) = \frac{m\lambda}{a}$$

where m = any integer, order of the diffraction

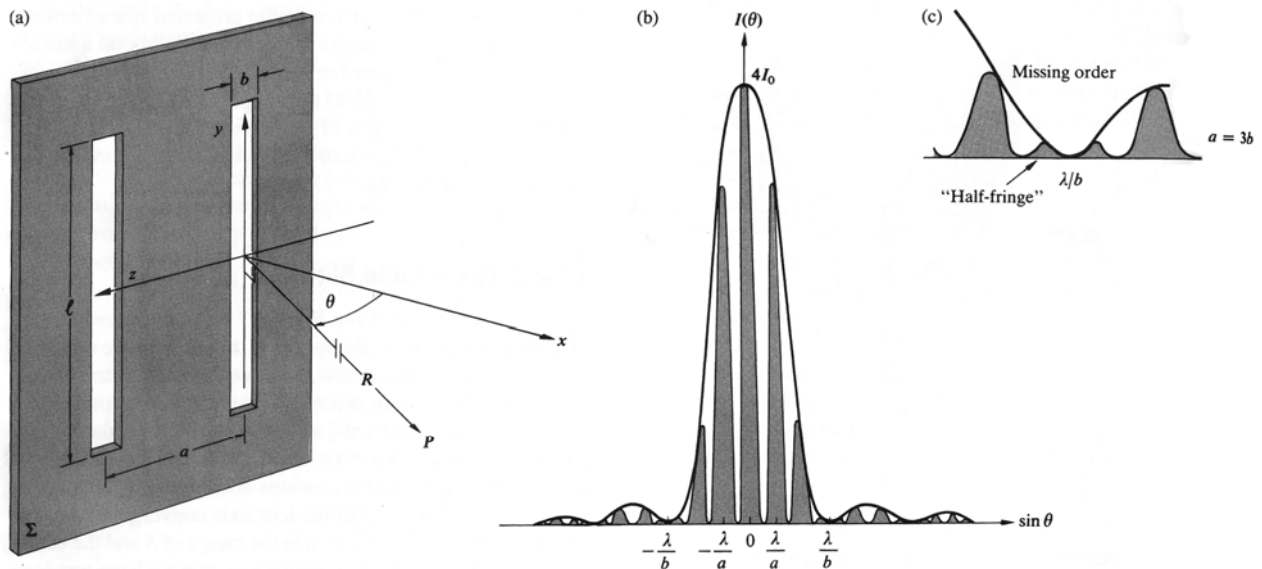


Figure 10.13 (a) Double-slit geometry. Point P on σ is essentially infinitely far away. (b) A double-slit pattern ($a = 3b$).

Young's Double Slit & Single Slit

- If take single slit
- Then add second slit see the one pattern on top of other
- If slits wide see both
- Slits narrow – nearly even patter due to wide null in small slit

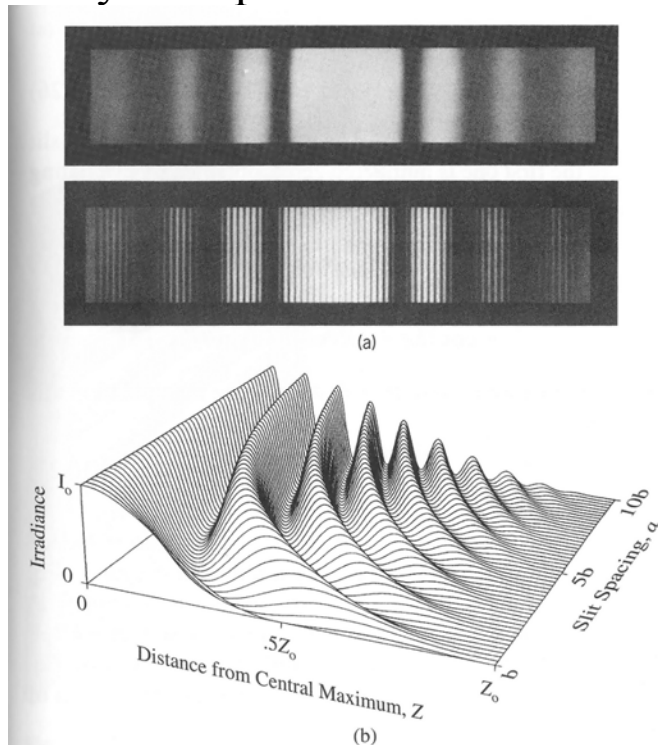
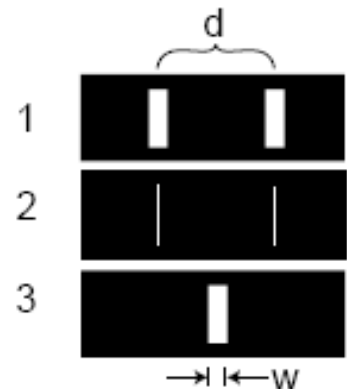
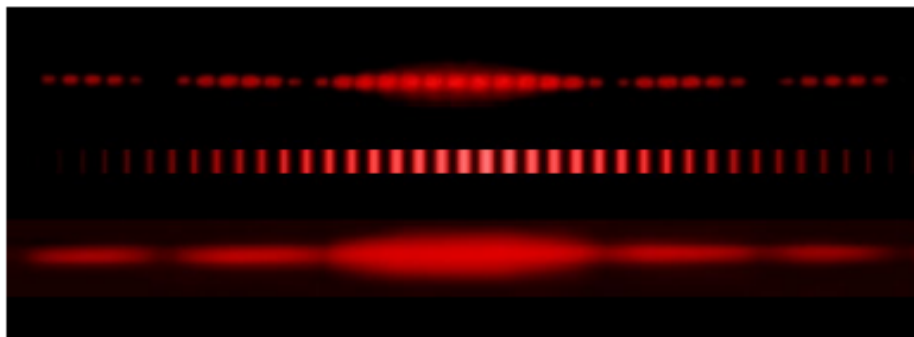
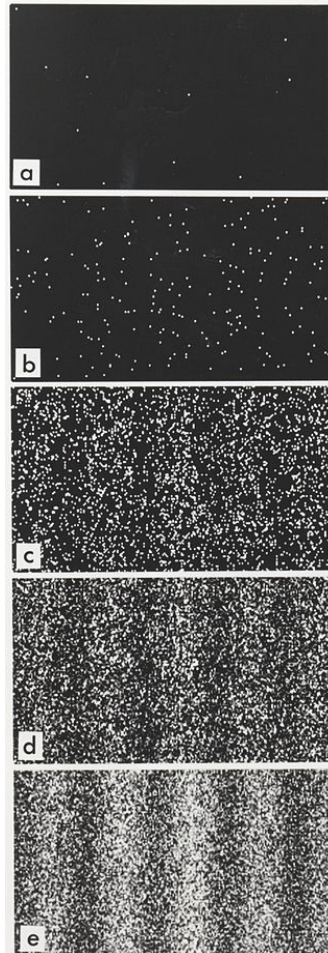


Figure 10.14 Single- and double-slit Fraunhofer patterns. (a) Photographs taken with monochromatic light. The faint cross-hatching arises entirely in the printing process. (Photos courtesy M. Cagnet, M. Francon, and J. C. Thierri: *Atlas optique des phénomènes*, Berlin-Heidelberg-New York: Springer, 1962.) (b) When the slit spacing equals b , the two slits coalesce into one (of width $2b$) and the single-slit pattern appears—that's the first curve closest to you. The farthest curve corresponds to the two slits separated by $a = 10b$. Notice that the two-slit patterns all have their first diffraction minimum at a distance from the central maximum of Z_0 . Note how the curves gradually match Fig. 10.13b as the slit width b gets smaller in comparison to the separation a . (Reprinted from "Graphical Representations of Fraunhofer Interference and Diffraction" *Am. J. Phys.*, 62, 6, (1994), with permission of A. B. Bartlett, University of Colorado and B. Mechtly, Northeast Missouri State University and the American Association of Physics Teachers.)



Young's Double Slit & Quantum Effects

- Classic description of Young's double slit is a wave effect
- But what about light being particles: photons
- Taylor (1909) reduced light so one photon at time at slit
- Still get interference
- Davisson and Germer in 1927 proved particles acted as waves
- Reason is the wave – particle duality first proposed by de Broglie
- The wave of a particle detects both slits
- Claus Jönsson in 1962 showed electrons do same thing
- Because e^- 's have a quantum wave function
- Even if single particle going through one.
- Creates the dual slit pattern due to quantum probability field
- Wave function field of photon extends far enough to see other slit
- Alternative: the many worlds theory of quantum
- See impact of all possible probabilistic events from “all universes



Diffraction Gratings

- Diffraction gratings: extends the 2 slits to a repeated pattern used by many systems eg spectrometers, acousto-optic deflectors
- Recall the Interference from a single width b

$$I(\beta) = I_0 \left[\frac{\sin(\beta)}{\beta} \right]^2 \quad \beta = \frac{\pi b \sin(\theta)}{\lambda}$$

θ = angular deviation of pattern from minimum

- Now consider n slits width b spaced distance d apart
- Extending the 2 slits get the diffraction pattern from each slit
- But the diffraction patterns interfere
- The more slits, the more the interference patterns add up
- If have several slits then waves from each interfere
- More slits narrower beams

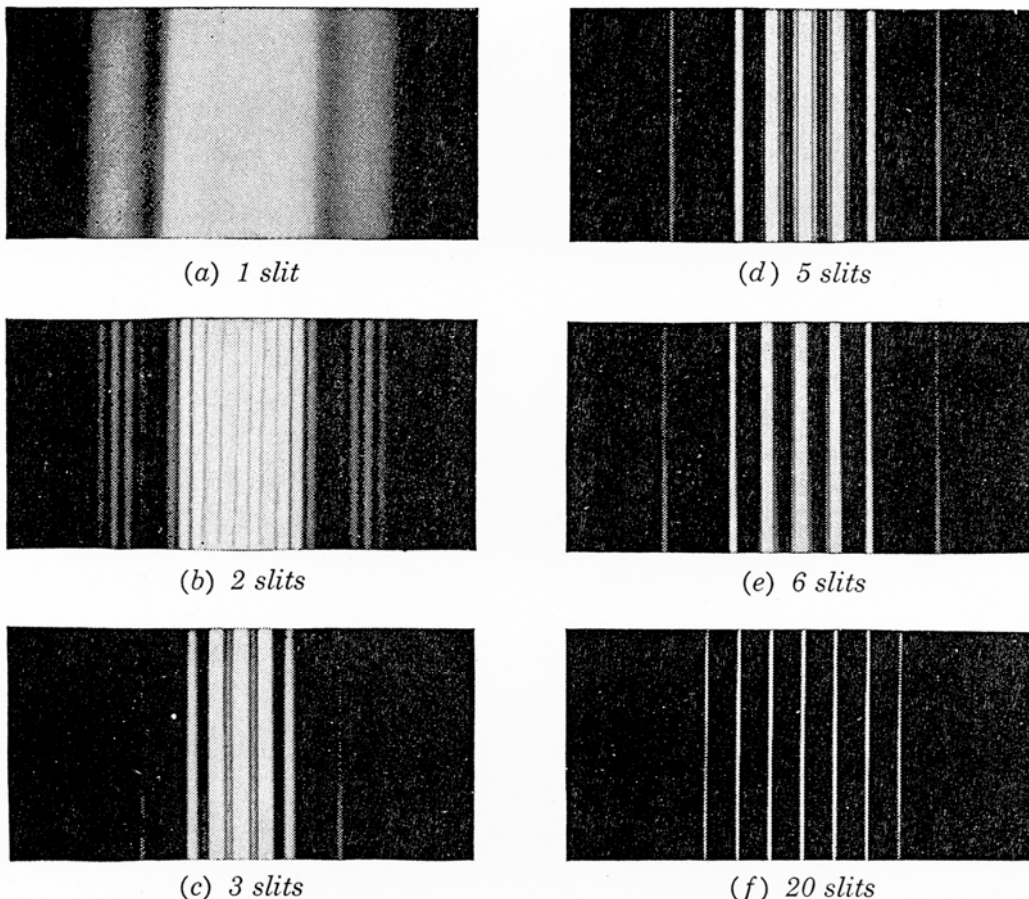


FIGURE 17A
Fraunhofer diffraction patterns for gratings containing different numbers of slits.

Diffraction Gratings Formulas

- Similar to the 2 slit the intensity becomes for n slits

$$I(\beta) = I_0 \left[\frac{\sin^2(\beta)}{\beta^2} \right] \left[\frac{\sin^2(n\gamma)}{\sin^2(\gamma)} \right] \quad \gamma = \frac{\pi d \sin(\theta)}{\lambda}$$

- Principal Maximums occur at

$$\sin(\theta) = \frac{m\lambda}{d}$$

where m = any integer, order of the diffraction

- The maxima vary within the single slit β function
- Result is that large n with small d gives narrow lines
- Peaks of different λ at different angles
- Single λ get peaks: white different peaks for each colour

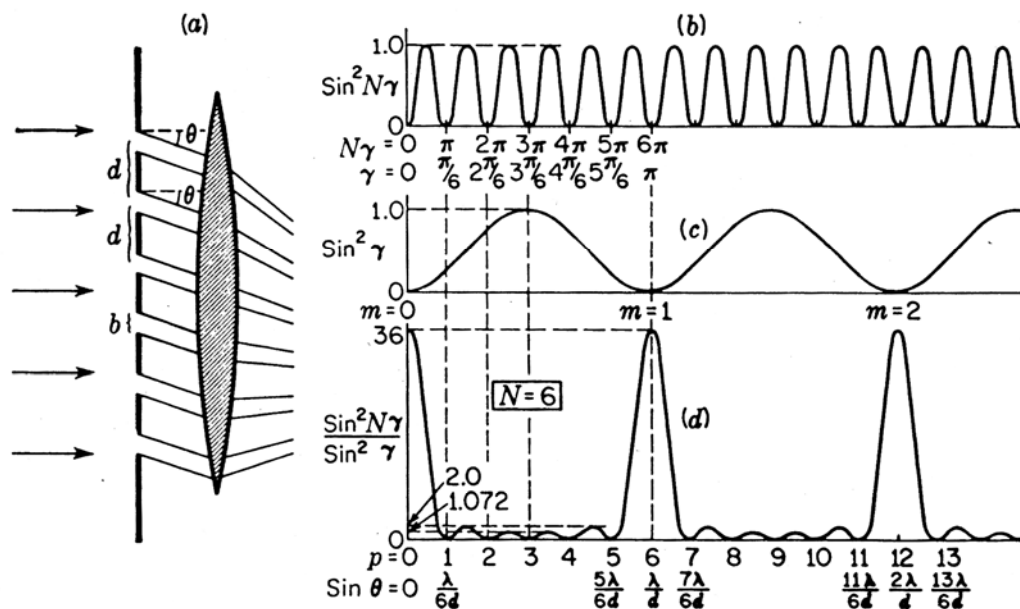
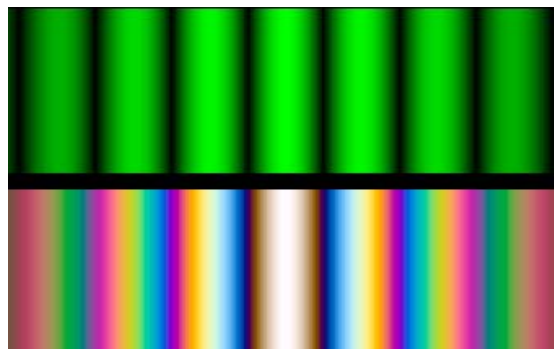


FIGURE 17C

Fraunhofer diffraction by a grating of six very narrow slits and details of the intensity pattern.



Spectrometry

- Want to see the spectral distribution of the light
- Used for detecting presence of elements based on spectrum
- Two ways – prism & diffraction gratings
- Mostly use diffraction gratings
- Generally greater spread than prisms
- Black body continuous spectrum
- Hot glowing gas (plasma) get emission spectrum of material
- Pass black body light through cold gas get absorption spectrum

