Holography & Coherence

- For Holography need coherent beams
- Two waves coherent if fixed phase relationship between them for some period of time

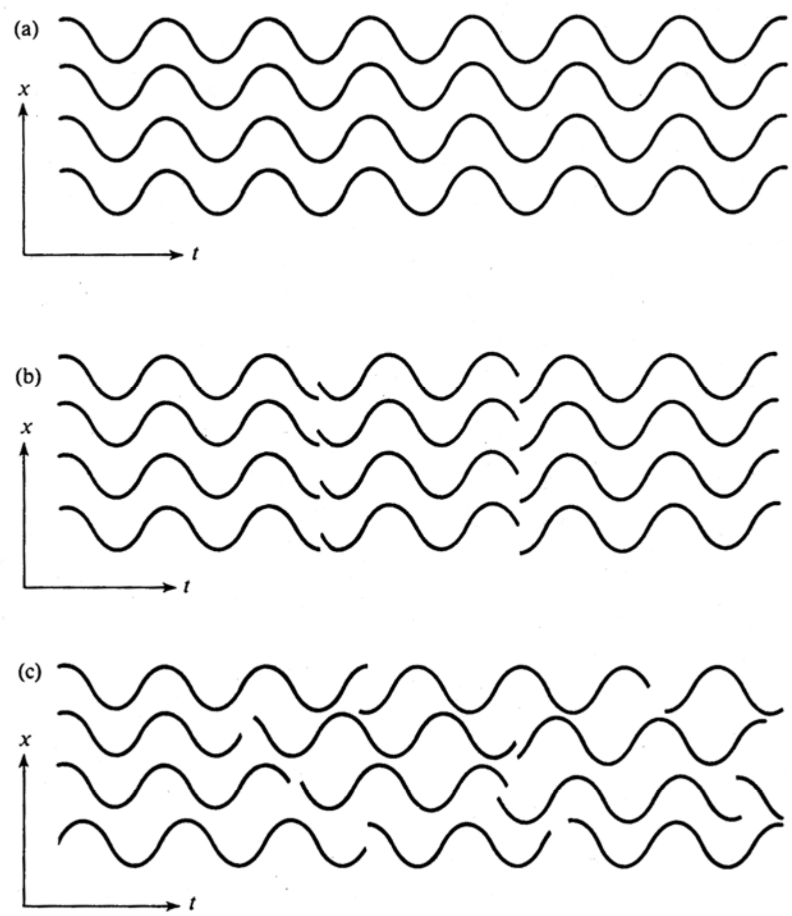
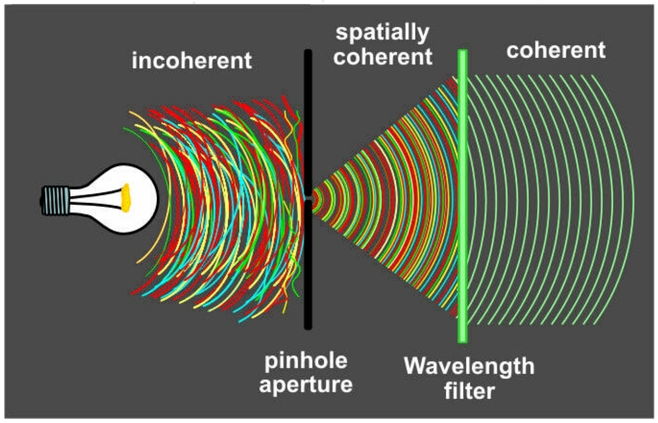


Fig. 3.8 An illustration of coherence. In (a) we show a perfectly coherent beam. All the constituent waves are in phase at all times. In (b) we have a beam which is spatially coherent, but which exhibits only partial temporal coherence. This is because the waves simultaneously change their phases by an identical amount every few oscillations. In (c) we show an almost completely incoherent beam where the phases of each wave change randomly at random times. Note however that even in this case some small degree of temporal coherence remains, since over very short time intervals the phases are to some extent predictable.



Coherence

• Coherence appear in two ways

Spatial Coherence

• Waves in phase in time,
  but at different points in space
• Required for interference and diffraction
• Before lasers need to place slits far from source
  or pass light through slit so only part of source seen

Temporal Coherence

• Correlation of phase at the same point
  but at different times
• Regular sources rapidly change phase relationships
• Single atom on 10^{-8} sec coherent
  lifetime of atom in an excited state
• Much shorter for groups of atoms
• For lasers in single mode much longer time

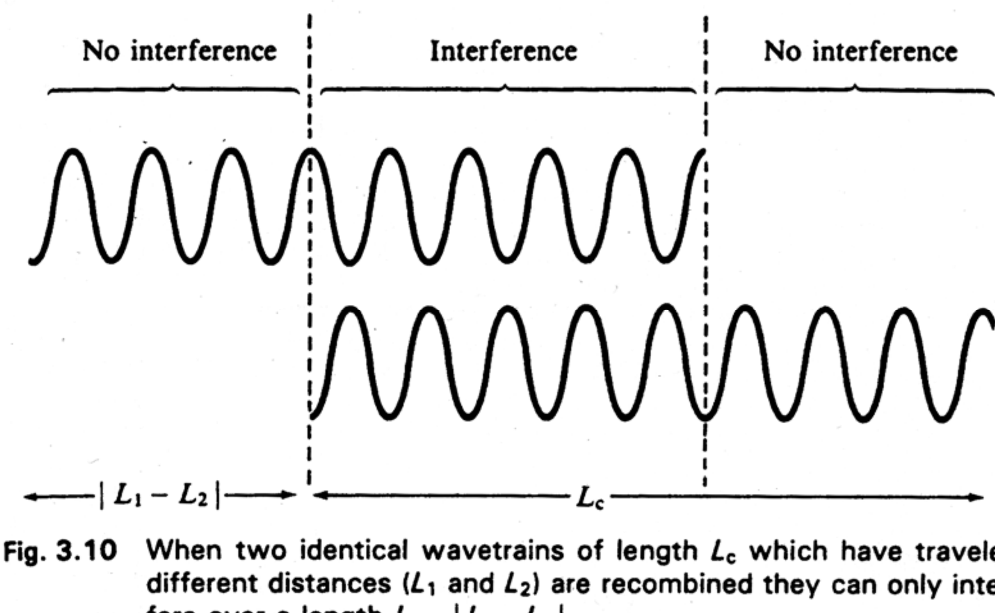


Fig. 3.10 When two identical wavetrains of length L_c which have traveled different distances (L_1 and L_2) are recombined they can only interfere over a length $L_c - |L_1 - L_2|$.

Coherence Length and Time

- Time of coherence given by τ_{coh}
- Coherence time about time taken for photon to pass a given distance (Coherence length) in space
- Coherence length is

$$L_{coh} = c\tau_{coh}$$

- Best seen in Michelson-Morley experiment
- Beam is split into two beam paths, reflected and combine
- If get interference pattern then within Coherence lengths
- Before lasers paths needed to be nearly equal
- With lasers only require

$$2(L_1 - L_2) < L_{coh}$$

- Coherence last 50 - 100 m with lasers

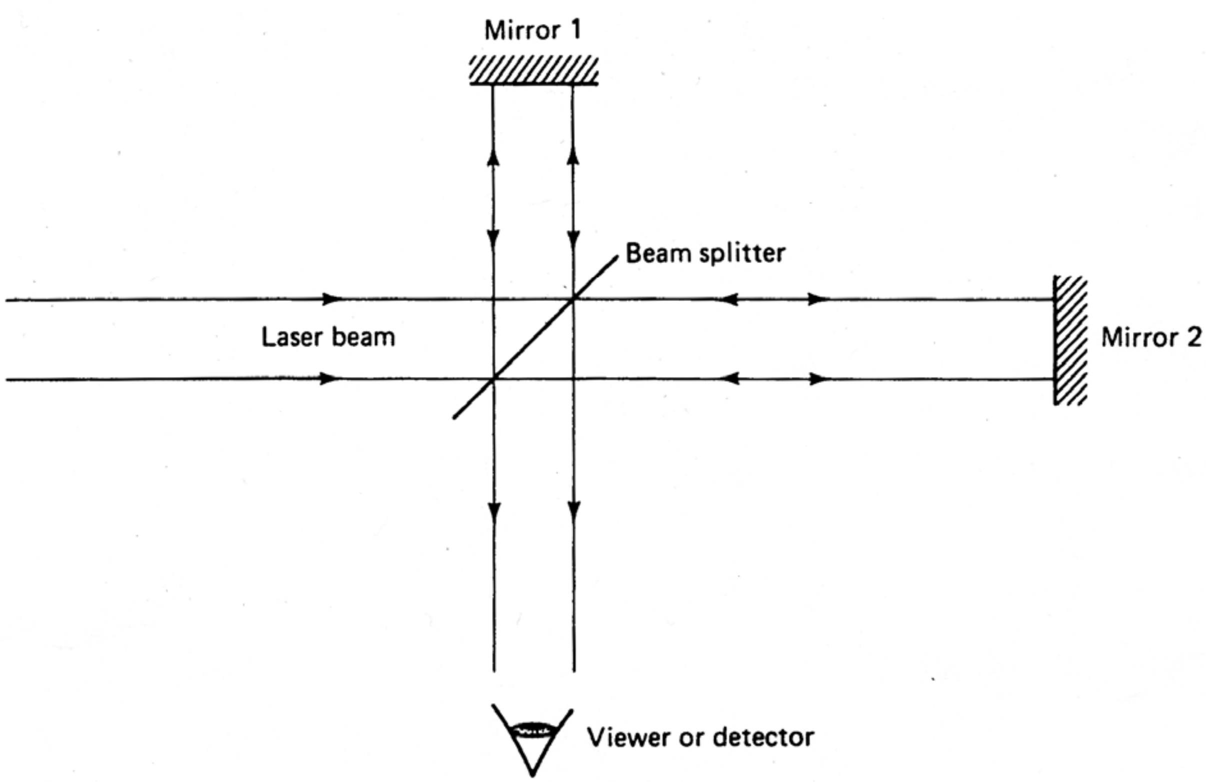


Figure 1-24 Michelson interferometer.

Coherence Length and Lasers

- It can be shown Coherence time related to laser frequency width $\Delta\nu$ (linewidth)

$$\tau_{coh} = \frac{1}{\Delta\nu}$$

- As the coherence length is

$$L_{coh} = c\tau_{coh}$$

- For holography setup distances must be < coherence length
- Long coherence lasers have small linewidth, thus high stability
- Ideal systems operate in TEM_{00} mode
- If have M modes then $\Delta\nu_M = M\Delta\nu$
- Thus coherence time and length is reduced by M

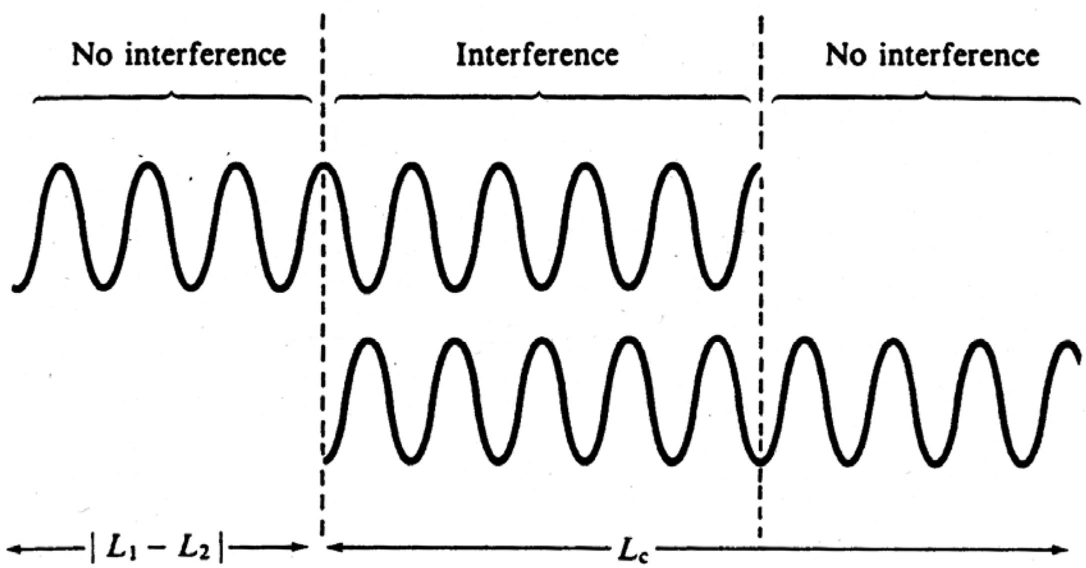


Fig. 3.10 When two identical wavetrains of length L_c which have traveled different distances (L_1 and L_2) are recombined they can only interfere over a length $L_c - |L_1 - L_2|$.

Example of Coherence Length

Sodium vapour lamp yellow "D" line

- λ = 589 nm and linewidth $5.1x10^{11}$ Hz
- Thus coherence time and length is

$$\tau_{coh} = \frac{1}{\Delta\nu} = \frac{1}{5.1x10^{11}} = 1.96x10^{-12}\ sec$$

$$L_{coh} = c\tau_{coh} = 2.98x10^{8}\left(1.96x10^{-12}\right) = 5.88x10^{-4}\ m = 0.59mm$$

- Coherence small hence hard to create holograms

- HeNe laser in multimode operation
- λ = 632.8 nm and linewidth 1500 MHz (1500 modes)
- Thus coherence time and length is

$$\tau_{coh} = \frac{1}{\Delta\nu} = \frac{1}{1.5x10^{9}} = 6.67x10^{-10}\ sec$$

$$L_{coh} = c\tau_{coh} = 2.98x10^{8}\left(6.67x10^{-10}\right) = 0.2\ m$$

- If single mode HeNe operation linewidth goes to 1 Mz
  and cohrence time is 1 microsec, cohrence length 300 m

Interferometers

- Can use interference effects to precisely measure distance
- First example Michelson Interferometer
- Have 2 mirrors (M_1 & M_2) placed on arms at 90 degrees
- Splitting mirror O (half silvered mirror) at intersection
- Splitter mirror – reflects part (~50%) of light 90°
  Lets part pass directly through
- Of a thin film of Aluminium ~100 nm, not full absorbing
- Monochormatic & coherent light source along path of one arm
- Detector at other arm
- Light to M_1 is reduced, reflected by M_1 then by splitter to detector
- Light at splitter reduced but reflected to M_2
- The passed through splitter O to detector

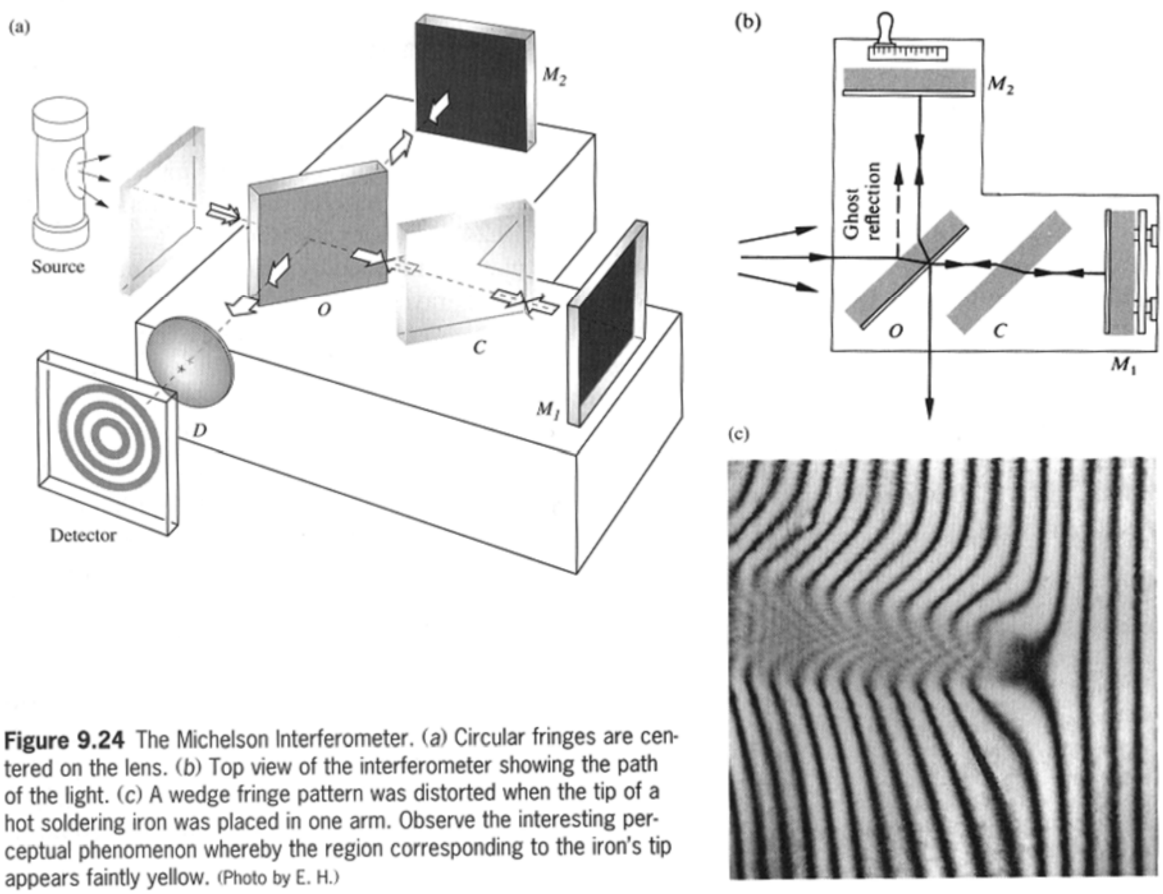


Figure 9.24 The Michelson Interferometer. (*a*) Circular fringes are centered on the lens. (*b*) Top view of the interferometer showing the path of the light. (*c*) A wedge fringe pattern was distorted when the tip of a hot soldering iron was placed in one arm. Observe the interesting perceptual phenomenon whereby the region corresponding to the iron's tip appears faintly yellow. (Photo by E. H.)

Michelson Interferometer

- At detector two beams combine to create interference
- Let path length difference be Δl
- Then if Δl = Nλ/2 get constructive interference – bright
- Dark if

$$\Delta l = \frac{2N+1}{4}\lambda$$

- Now can measure very small distance changes
- Eg if put glass plate C in can see small defects in glass
- Interferometers used in measuring distance
- Digitize light level and measure changes – can get λ/64 or 256
- Measure 2 nm distance
- Need extremely stable laser

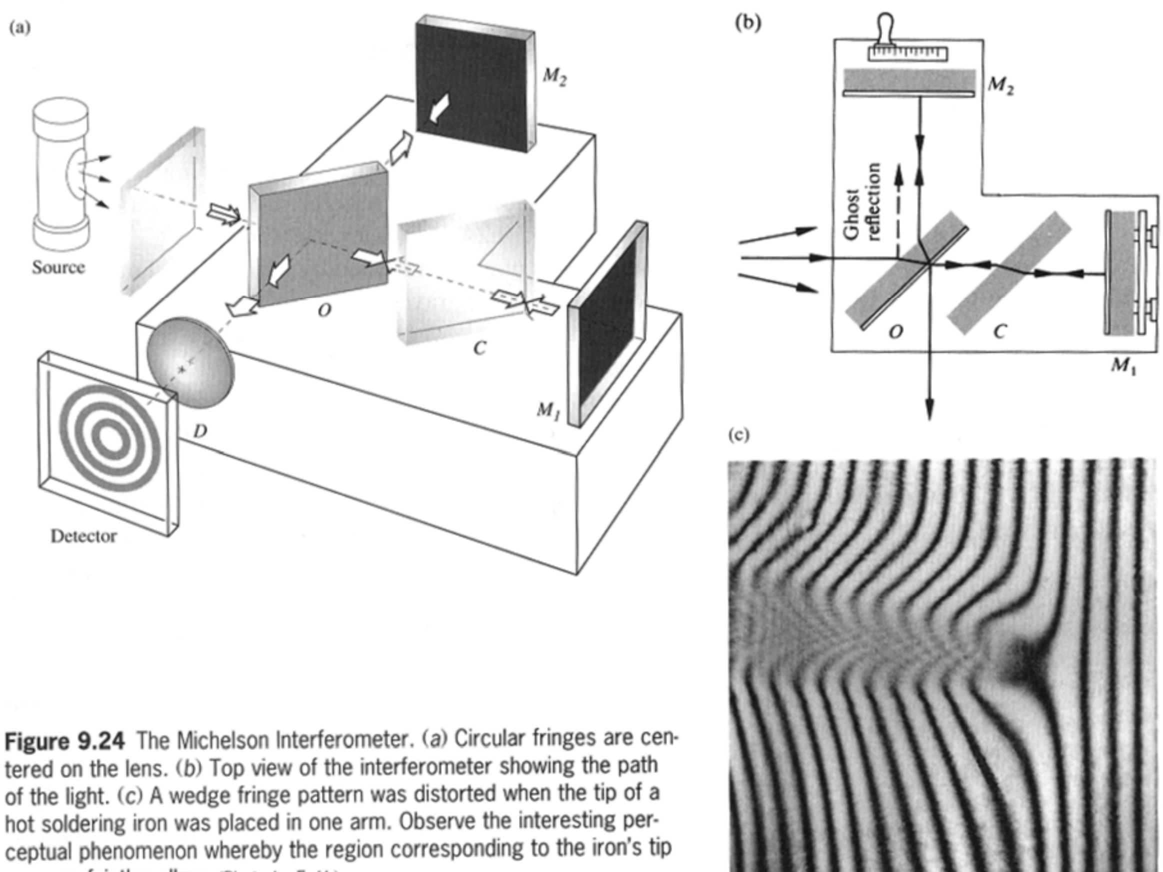


Figure 9.24 The Michelson Interferometer. (*a*) Circular fringes are centered on the lens. (*b*) Top view of the interferometer showing the path of the light. (*c*) A wedge fringe pattern was distorted when the tip of a hot soldering iron was placed in one arm. Observe the interesting perceptual phenomenon whereby the region corresponding to the iron's tip appears faintly yellow. (Photo by E. H.)

Michelson Interferometer

- Actually see circular interference at the detector
- Reason distance from detector to splitter is d
- The angle θ at the detector when destructive interference is

$$2d\cos(\theta_m) = m\lambda$$

- Result is rings of interference

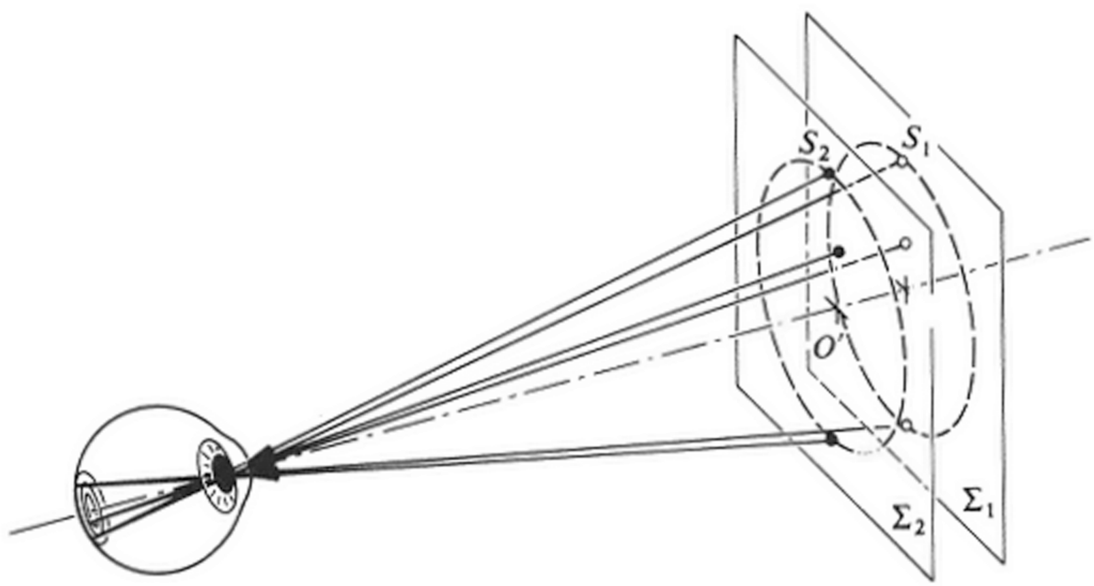
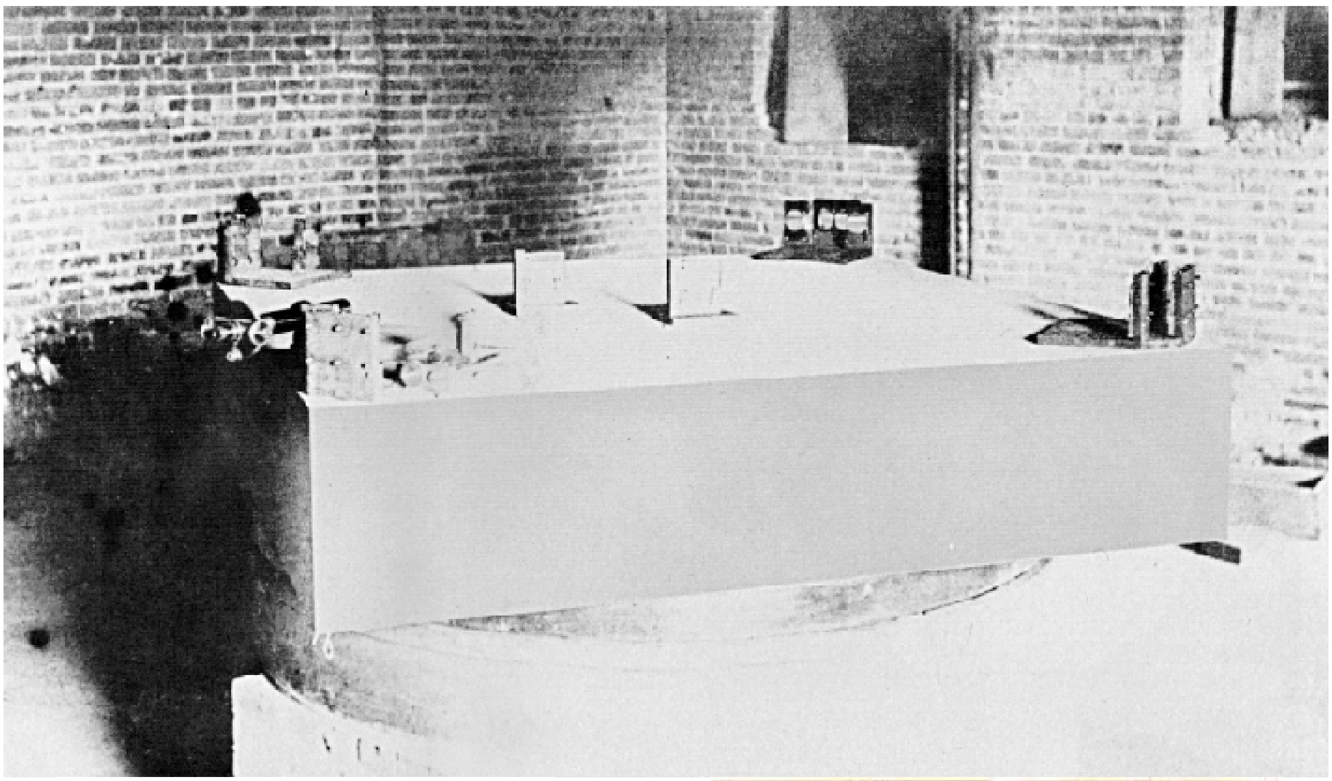
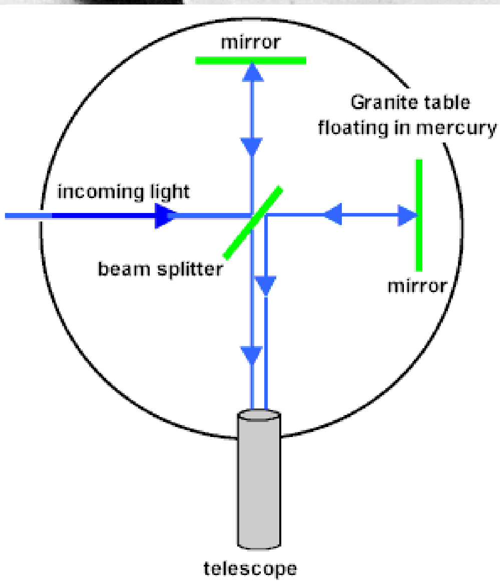


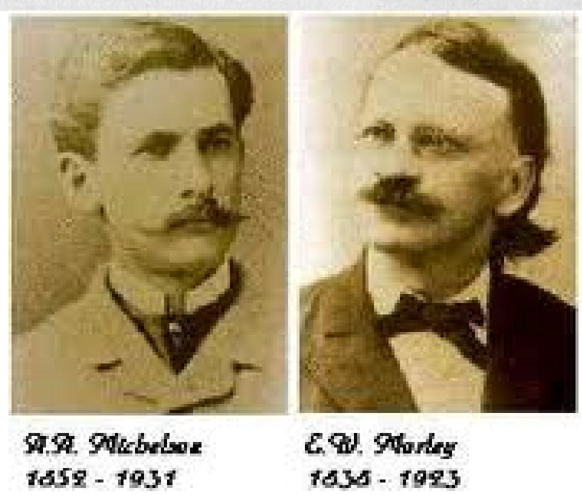
Figure 9.26 Formation of circular fringes.

Michelson Morley Experiment

- Use Michelson interferometer floating on mercury
- Align so path along direction of earth around sun
- Other path at along radius to sun
- Then rotate by 90 degrees
- Classic physics: Along the path in direction of motion
- light should arrive sooner to addition of velocities
- But no difference found – first indication of relativity







Mach- Zehnder & Sagnac Interferometer

- Mach- Zehnder Interferometer
- 2 splitters and 2 mirrors
- Detects small changes in one path relative to other
- Used to detect small events in one path
- Sagnac is a ring interferometer
- Used to small deviations in both directions

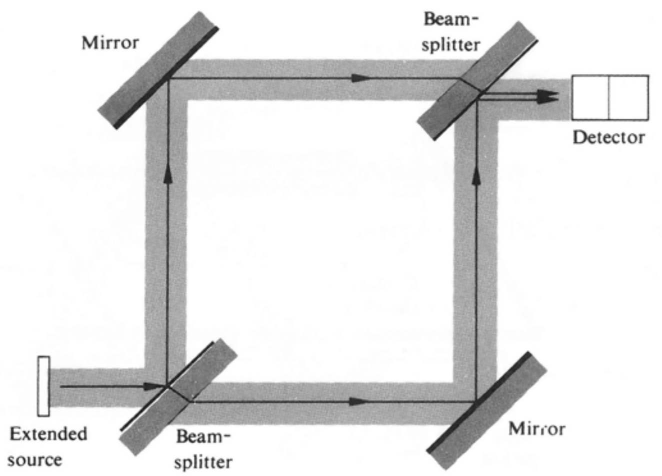


Figure 9.27 The Mach–Zehnder Interferometer.

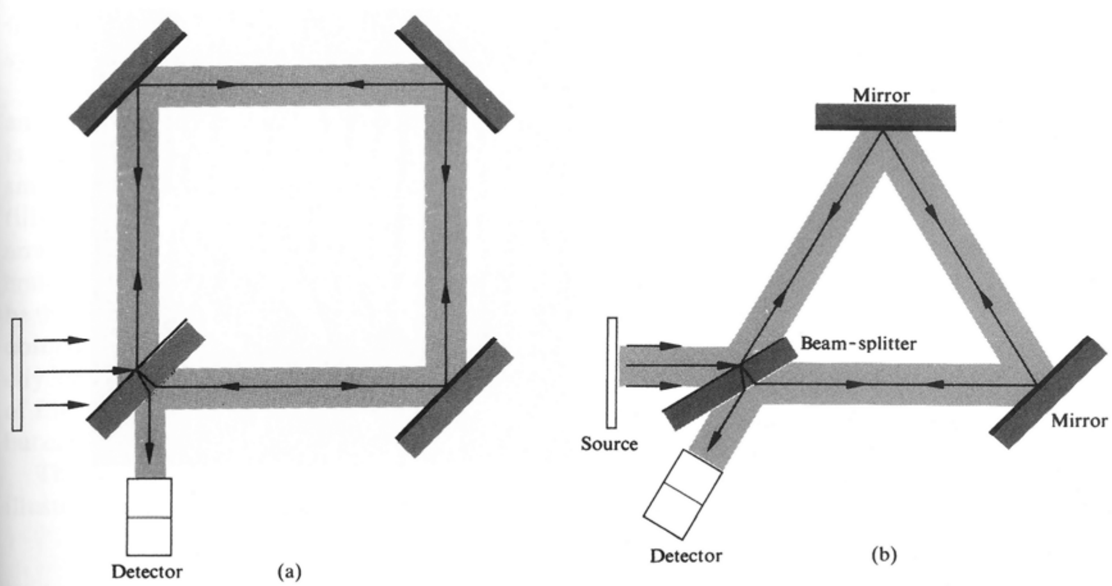
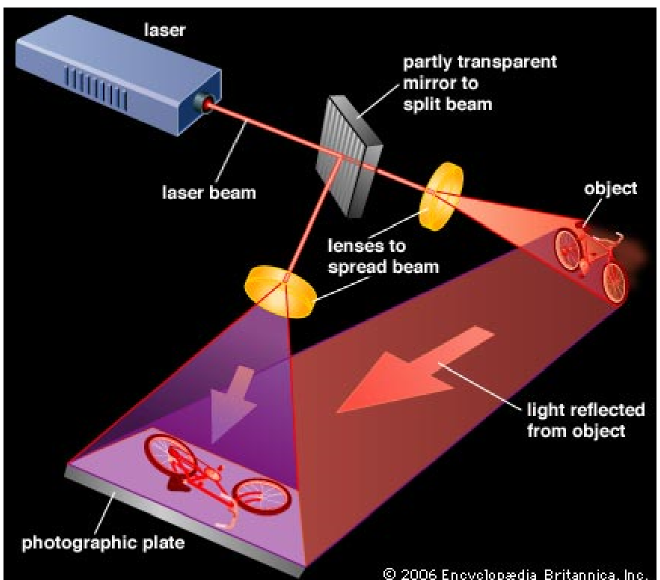
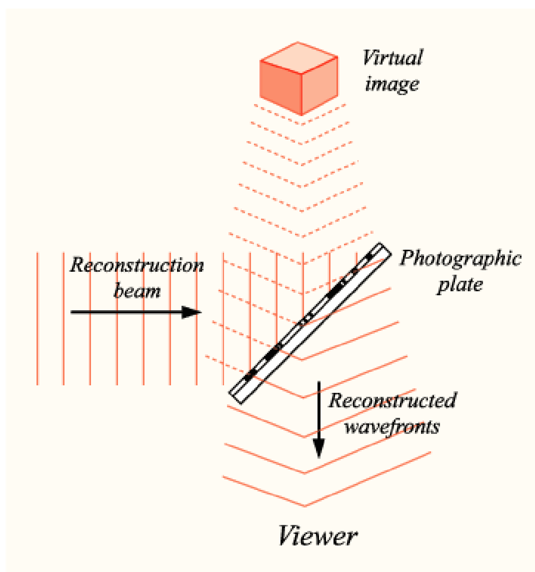


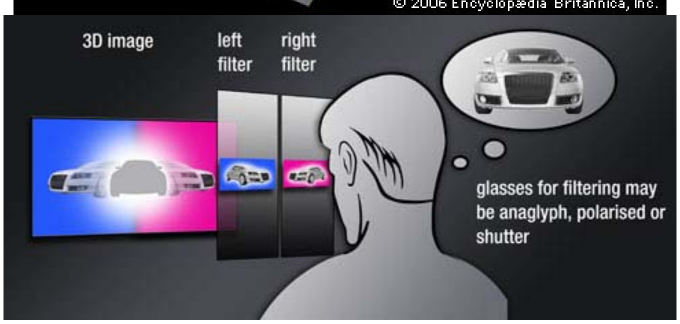
Figure 9.29 (***a***) A Sagnac Interferometer. (***b***) Another variation of the Sagnac Interferometer.

Holography

- Holography mathematics proposed by Dennis Gabor 1948
  received Nobel prize in 1972
- Not very practical before laser
  needs long coherence length to expose any sizable object
- Emmett Leith & Juris Upatnieks produced first laser hologram
- Hologram: an optical device using photographic techniques
  and laser light to create a 3 dimensional image
- Holos Greek for complete, and Gram for message
- Makes a complete Intensity and phase copy
  of the light from a scene
- Regular photograph makes only intensity copy
- Regular 3D uses only 2 images: right & left eye from one point
- With phase added has the same depth information as original
- Depth info changes as the view moves around
- Setup must have very stable table and lasers







Basic Holographic Setup

- Must have extremely stable, air bearing table: changes < λ
- Laser must operate in TEM_{00} mode
- He-Ne, Argon and Krypton most common
- Beam split into two by splitter
- Reference beam reflected by mirror through
  converging lens to illuminate Photographic Plate
- photographic film, photoresist, thermoplastic or dicromated gelatin
- Object beam also spread by converging lens
  to uniformly illuminate object so light reflects on plate
- Pinholes spatial filters to remove optical noise dust & lens defect
- object & reference path length must be with coherence length

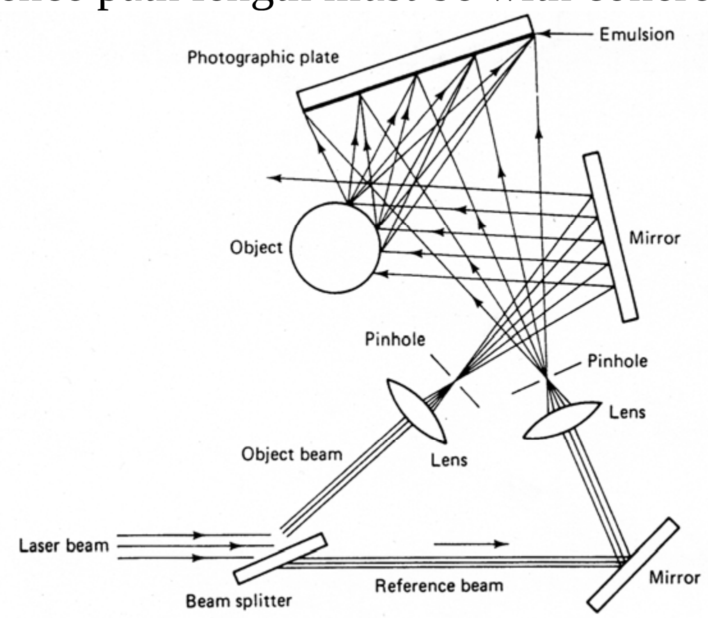
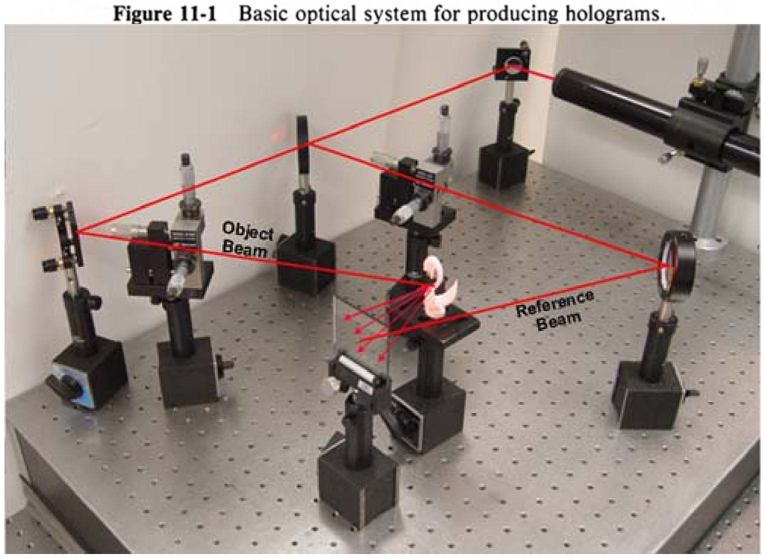


Figure 11-1 Basic optical system for producing holograms.



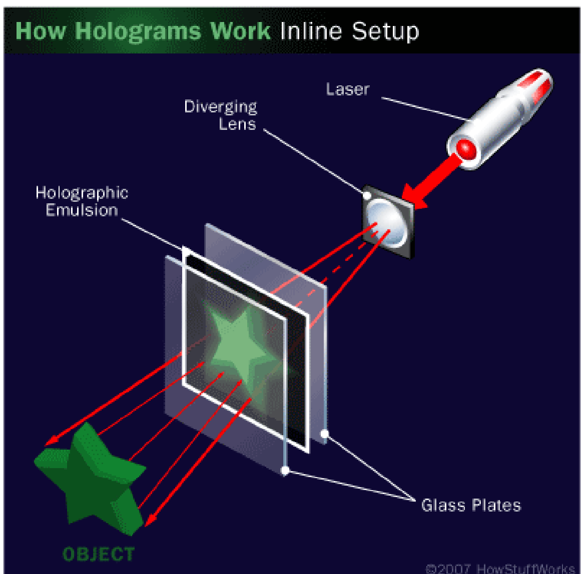
Holograms Very Different from Photographs

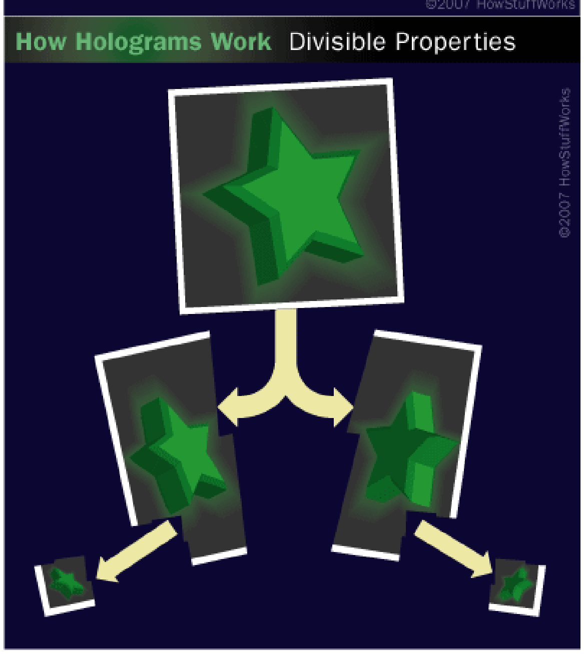
Light coming from Hologram

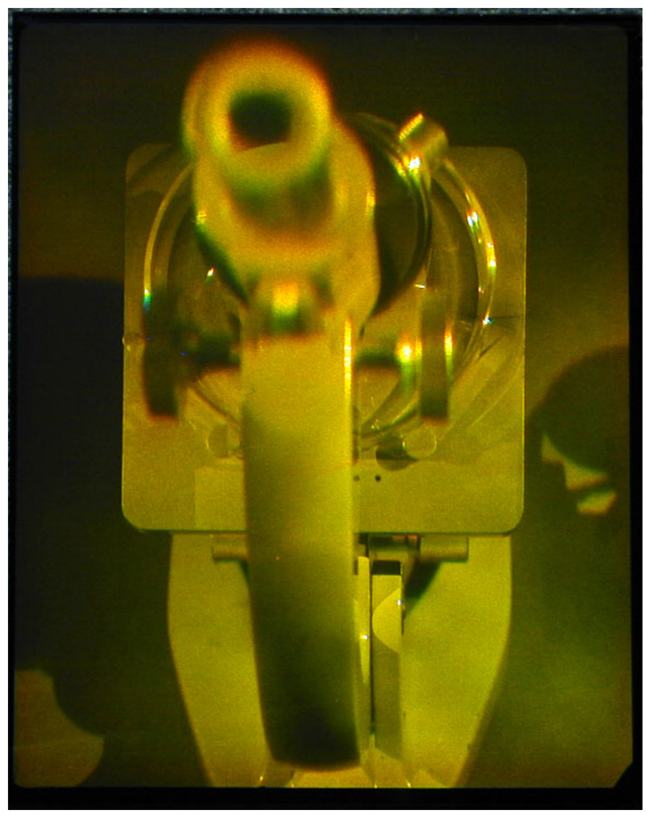
- Light from hologram reproduces light from original scene
- Get 3D image (different image at each eye)
- Thus if focus with lens get same depth of focus behaviour

Information Storage

- Information is stored everywhere on the hologram
- Hence if cut hologram in half do not get only half the picture
- Get two holograms which can see object but from different angles
- Also reduced resolution of object
- Note does not work on simple metal holograms







Rotation of Holograms

- If take a hologram and then rotate plate can take another
- Playback image depends on the angle of reference beam then
- Playback of image same as with original reference beam
- Hence rotating hologram can store a movie

Commercial Holograms

Silver Halid Films

- Silver Halid photographic emulsion first holgrams
- Glass plate holograms expensive but best quality
- Hologram there adds a filter that lets regular bright light be used
- Holograms on film plates next best

Photoresist Holograms

- Best now photoresist holograms
- Create patters in photoresist
  (organic material whose thickness depends on exposures level)
- Can create colour holograms that way

Metal Film Holograms

- Metal film holograms: make master plate with 3D structure
- Press metal foil (or metal film on plastic) onto master
- Creates hologram with the reflected light (from height of metal)
- Also can do that with press printing

Computer Generated Holograms

- Use computer calculations to get phase/intensity at all points
- Then write with laser in photoresist to create master
- Often used for logo type holograms in metal films

Mathematical Description of Light for Holograms

- Recall that light's electric field vector is written as

$$\vec{E}(z,t) = \hat{i}E_0 exp\left[j\left(-\omega t + kz + \varphi\right)\right]$$

where

$$k = \frac{2\pi}{\lambda} \qquad j = \sqrt{-1}$$

- The time independent amplitude of the wave is

$$U = \hat{i}E_0 exp\left[j\left(kz + \varphi\right)\right]$$

- The time averaged light intensity is given by

$$I = \langle E \bullet E \rangle = \frac{U \bullet U^*}{2Z}$$

- Note: complex conjugate U^*
  is beam traveling in opposite direction

where Z is the intrinsic impedance of the medium

$$Z = \sqrt{\frac{\mu}{\varepsilon}}$$

Z = intrinsic impedance
μ = permittivity of the medium
ε = dielectric constant

- For dielectrics it can be shown that

$$Z = \frac{377}{\sqrt{\varepsilon_r}} = \frac{377}{n}\Omega$$

Holography Equations

- In the Holographic set up the complex amplitude at the photographic plate

$$U = U_r + U_o$$

- where U_r = the reference beam and U_o = the object beam
- Intensity at the plate is

$$I = \frac{1}{2Z}\left(U_r U_r^* + U_o U_o^* + U_o U_r^* + U_r U_o^*\right)$$

- Which becomes

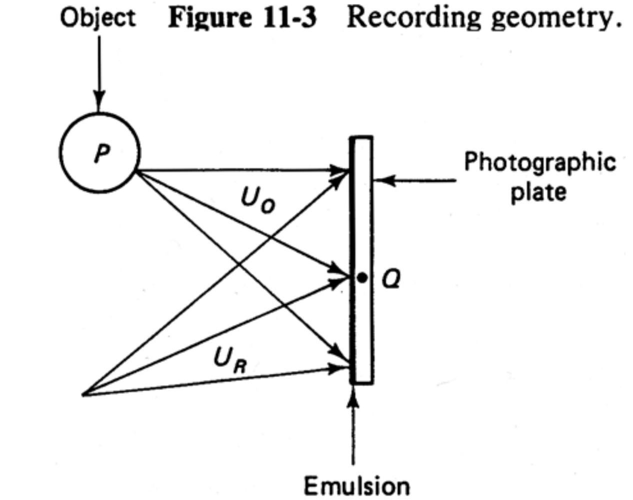
$$I = I_r + I_o + \frac{1}{2Z}\left(U_o U_r^* + U_r U_o^*\right)$$

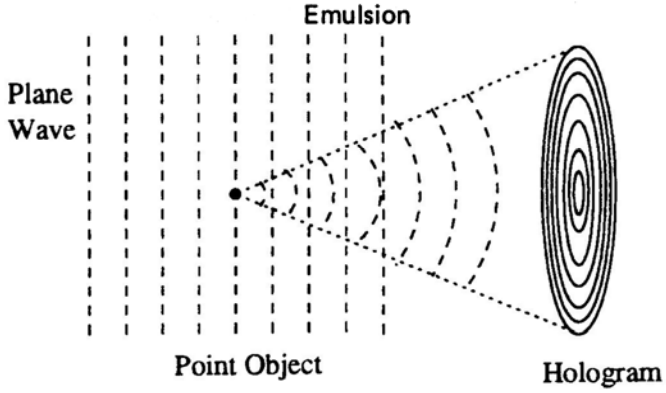
where I_r = intensity (irradiance) from reference

I_o = intensity (irradiance) from object

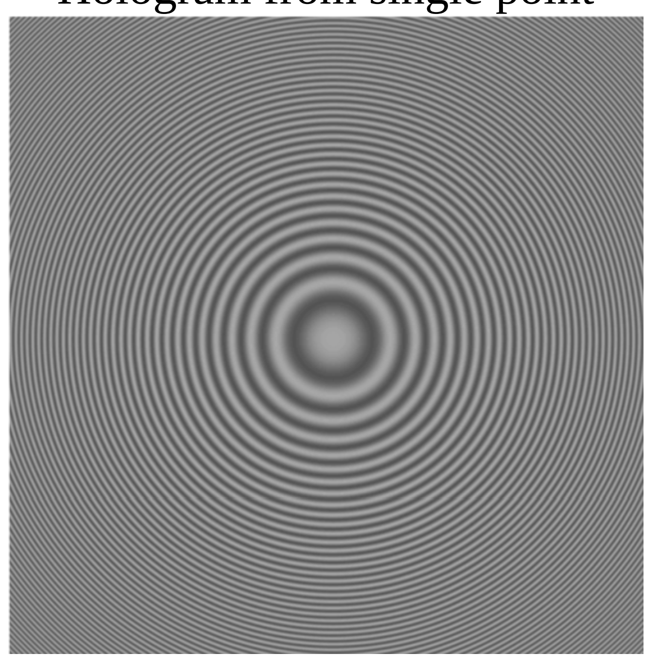
- As this is a photograph must have the Exposure of the plate = IT where T = time

Figure 11-3 Recording geometry.





Hologram from single point



Absorption Hologram

- Two basic types of Holograms
- Absorption Hologram: darkness in photographic plate
- Phase Hologram: thickness variation in plate

- In Photographic material response curve has linear region
  bias reference intensity to linear region
- After development transmittance t_p from plate is

$$t_p = t_0 + \beta \left[I_0 + \frac{1}{2Z}\left(U_o U_r^* + U_r U_o^*\right) \right]$$

where t_o = bias point of plate transmittance curve
β = slope of emulsion transmittance curve

- β is negative for negative emulsions

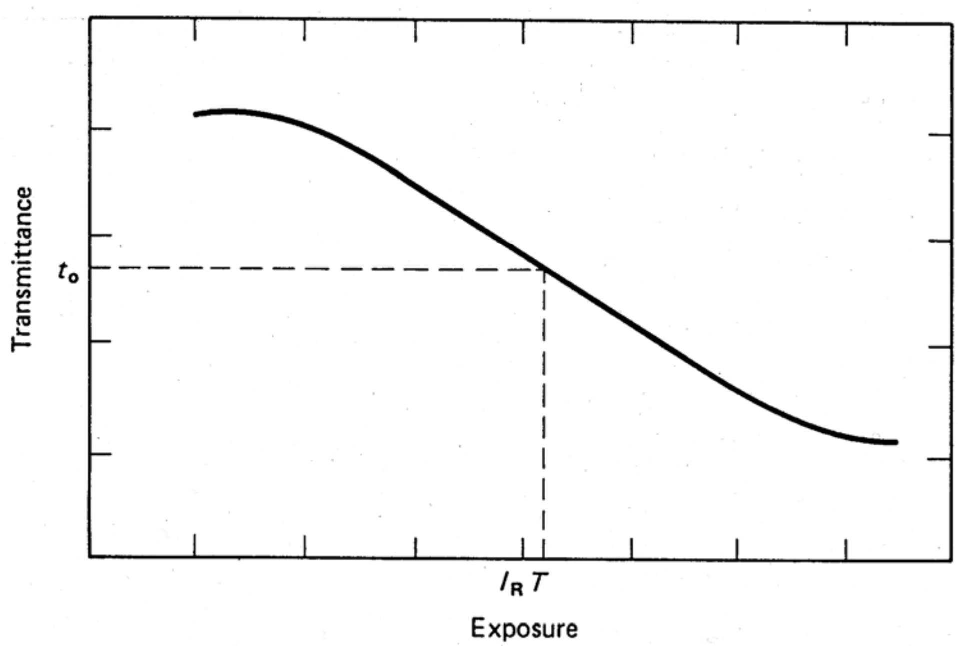


Figure 11-4 Light-amplitude transmittance versus exposure for typical photographic plate used for holography.

Image Creation in Absorption Holograms

- If absorption plate illuminated by a playback laser U_p
  light transmitted through beam is written as

$$U_p t_p = U_1 + U_2 + U_3 + U_4$$

$$U_1 = U_p t_o$$

$$U_2 = U_p \beta \, I_o$$

$$U_3 = \frac{U_p U_r^* \bullet U_o}{2Z}$$

$$U_4 = \frac{U_p U_r \bullet U_o^*}{2Z}$$

- Note: each of these is in different direction
- U_1 is light transmitted straight through hologram
- U_2 is light diffracted to small angle
  from low frequency interference of light at plate from object
- U_3 & U_4 light forming holographic images

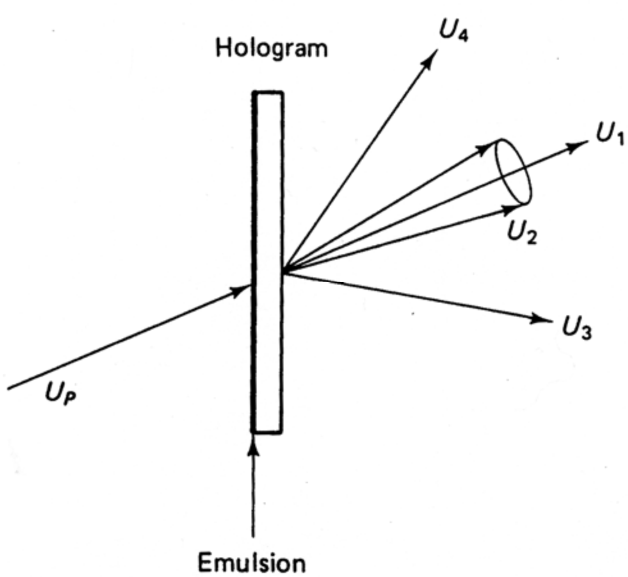


Figure 11-5 Light transmitted and diffracted by a hologram when illuminated with a playback beam.

Hologram Virtual Image Reconstruction

- Virtual image (appears behind hologram) if
  playback beam differs from reference beam only by intensity

$$U_p = bU_r$$

where b = some constant

- Diffracted light is then

$$U_3 = b\beta\, I_r U_o$$

where I_r = reference beam intensity
U_o = object beam intensity

- Thus output is exact duplicate of original
  see light as though it comes from the object
- Result is true 3D image
- If change the playback or record reference then get different image
- Can record many images on the same file with different angles
- Several hundred images

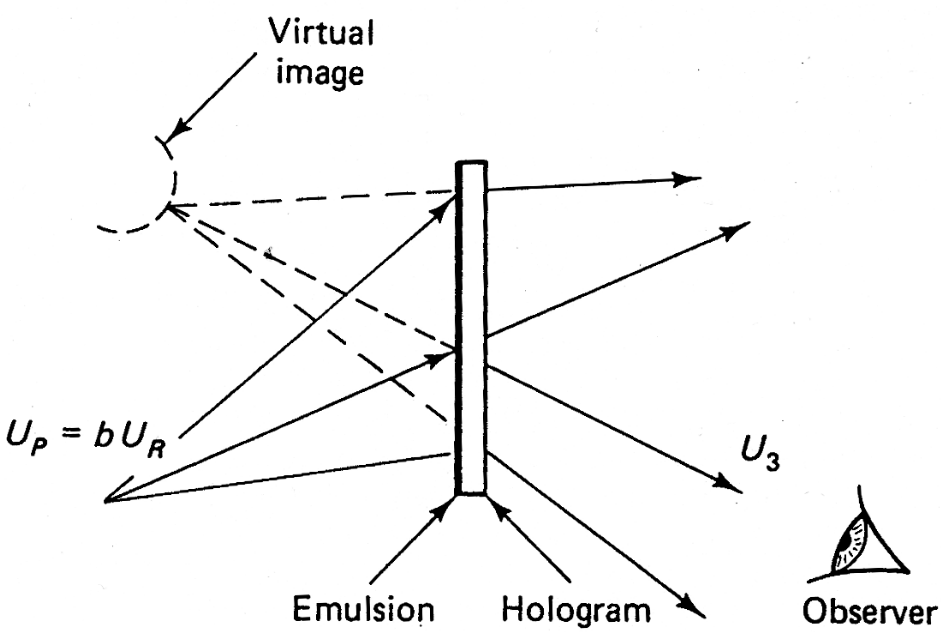


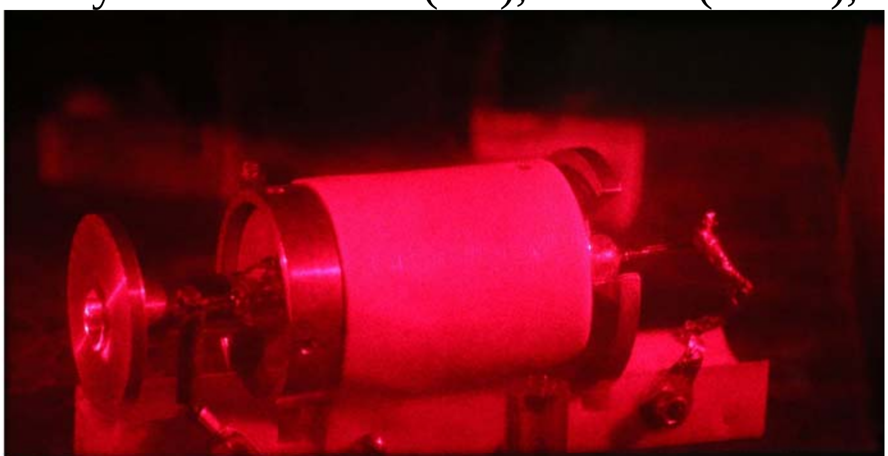
Figure 11-6 Virtual image reconstruction.

Wavelength and Hologram Playback

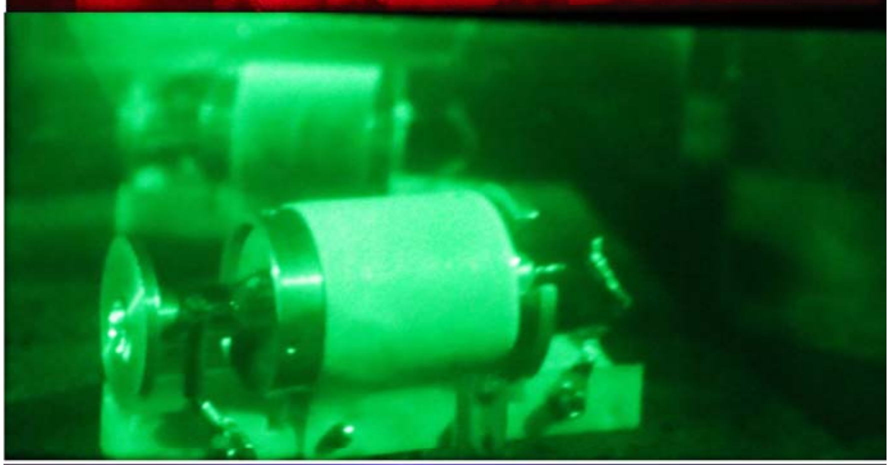
- Hologram playback unlike regular images
- Image seen depends on wavelength of illumination
- If playback wavelength λ_p different than original hologram λ_h
- Image magnified by

$$M = \frac{\lambda_p}{\lambda_h}$$

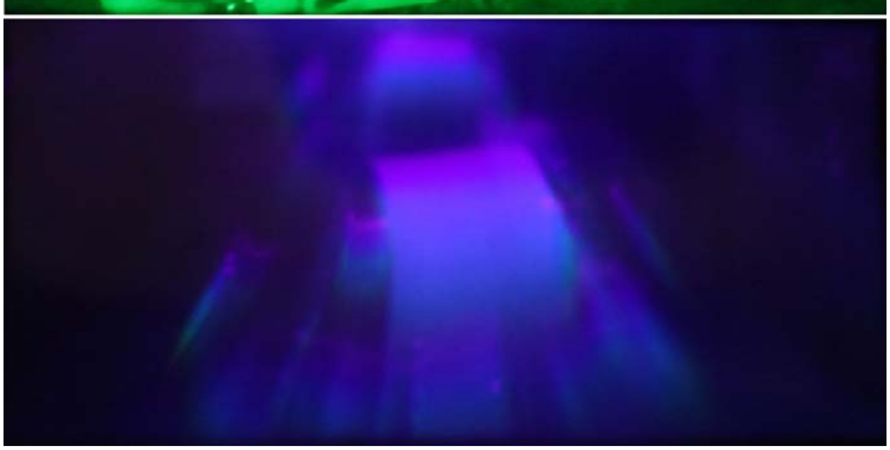
- Eg Worlds first laser hologram taken at Nd:Yag 1060nm IR
- Playback at 670 nm (red), 533 nm (Green), 406 nm (Violet)



670 nm (red)
(diode)



533 nm (Green)
Nd:Yag 2nd

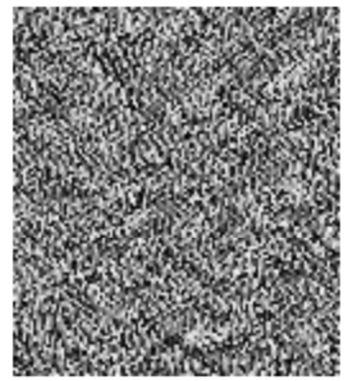


406 nm (Violet)
diode

What Does a Hologram Look Like

- Consider a hologram of a simple image (letters)
- Hologram as viewed in regular light has noisy looking image
- In playback with laser illumination get original image

Amplitude



Phase Hologram

Playback image from hologram

Hologram Real Image Reconstruction

- If run reference in opposite direction to original then

$$U_p = bU_r^*$$

where b = some constant

- Then reference is complex conjugate to original
- Diffracted light is now

$$U_4 = b\beta\, I_r U_o^*$$

where I_r = reference beam intensity

U_{o*} = object beam travelling backward

- Thus output is original object beam travelling backward becomes a real image (can be projected on surface)
- Again result is true 3D image

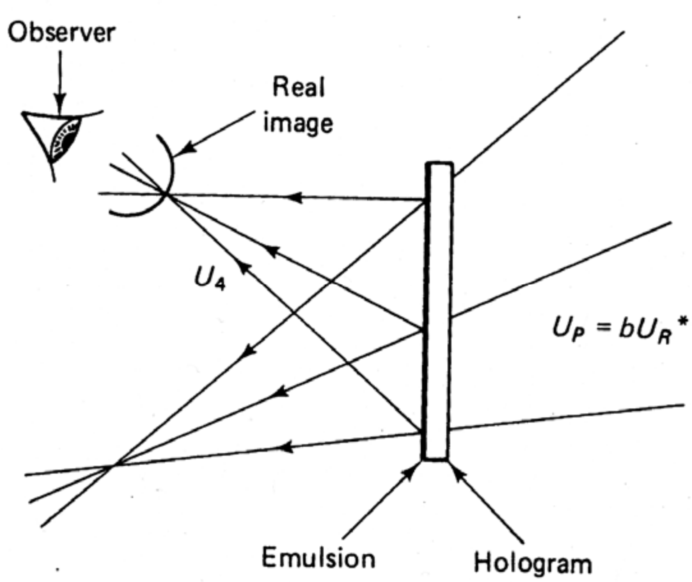


Figure 11-7 Real image reconstruction.

Phase Holograms

- Consist of clear plate where thick varies
- Created from thermoplastic (affected by heat)
  dichromatic gelatin - thickness depends on intensity
- Consider the reference and object beams again:
  They differ only in intensity and phase
- Let the phase difference from a give point be ϕ

$$U_r = A \quad U_o = a\ exp(j\varphi)$$

- Then the intensity becomes

$$I = I_r + I_o + \frac{1}{2Z}\left[Aa\ exp(j\varphi) + Aa\ exp(-j\varphi)\right] = I_r + I_o + \frac{Aa}{2Z}cos(\varphi)$$

- Assume intensity becomes a variation in thickness

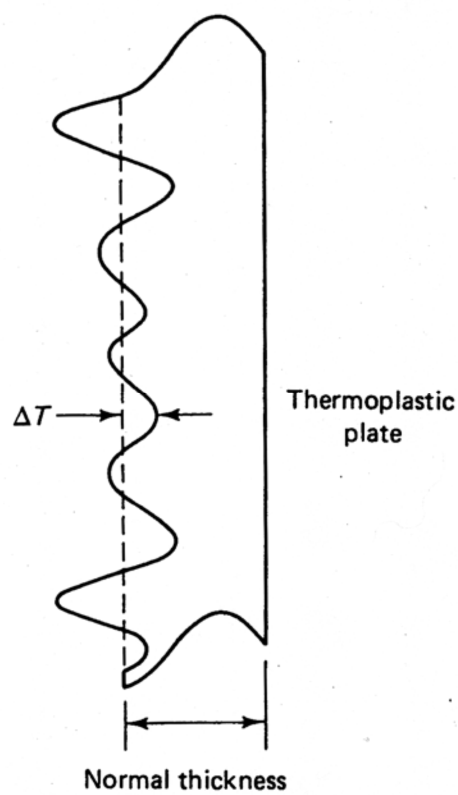


Figure 11-8 Variation of thermoplastic plate thickness due to holographic recording.

Phase Holograms Playback

- Effective thickness of the plate depends only on the changing term

$$\Delta T = -\gamma\,\Delta I = -\gamma\frac{Aa}{2Z}cos(\varphi)$$

where γ = photosensitivity constant of the plastic

- The phase change caused for light passing through the plate

$$\Delta\psi = k(n-1)\Delta T = -k(n-1)\gamma\frac{Aa}{2Z}cos(\varphi)$$

- Applying the playback beam it can be shown that

$$U_3 = -jk(n-1)\gamma\,AaI_r\,exp(j\varphi) = jCU_o$$

where C = a constant

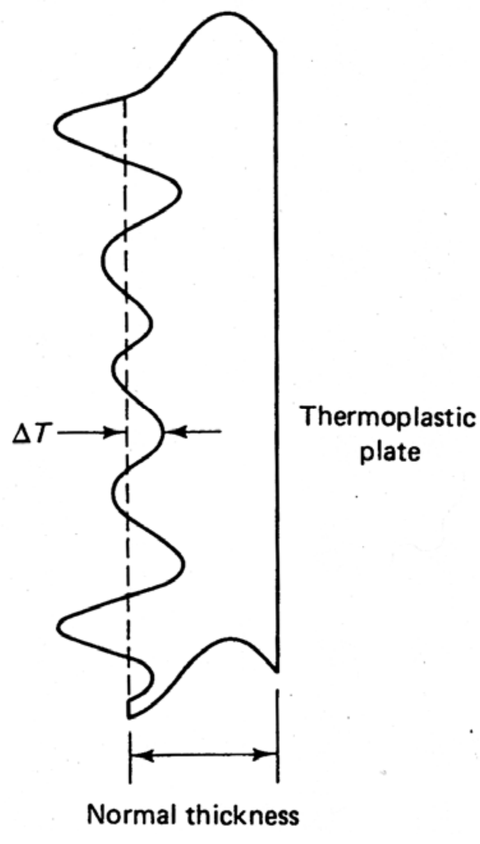


Figure 11-8 Variation of thermoplastic plate thickness due to holographic recording.

Holographic Interferometry

- If take a hologram of an object
  then move, distort, displace or change index of refraction
- Then viewing object with hologram creates interference image
- Called a double exposure hologram
- Creates an interference pattern of moved object
- Eg consider a diaphragm displaced from its original position

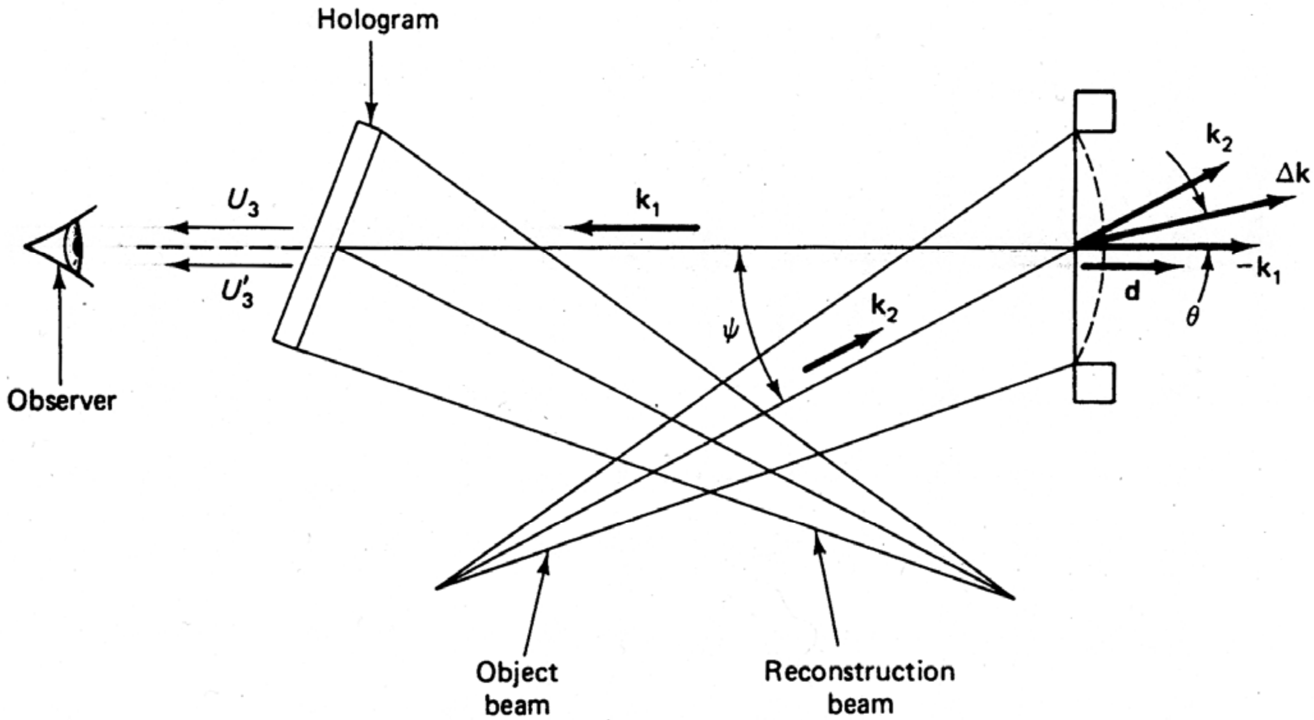


Figure 11-13 System geometry for holographic interferometry.

Holographic Interferometry

- Light from the original hologram playback is

$$U_3 = -U_o$$

- While light from the displaced object

$$U_3' = -U_o \exp(j\varphi)$$

- the phase change is related to the displacement d_p by

$$\Delta\varphi = 2kd_p \cos\left(\frac{\psi}{2}\right)\cos(\theta)$$

where ψ = object beam to reflected beam angle

θ = vector difference between object and reflected beam

- Usually ψ/2 = θ

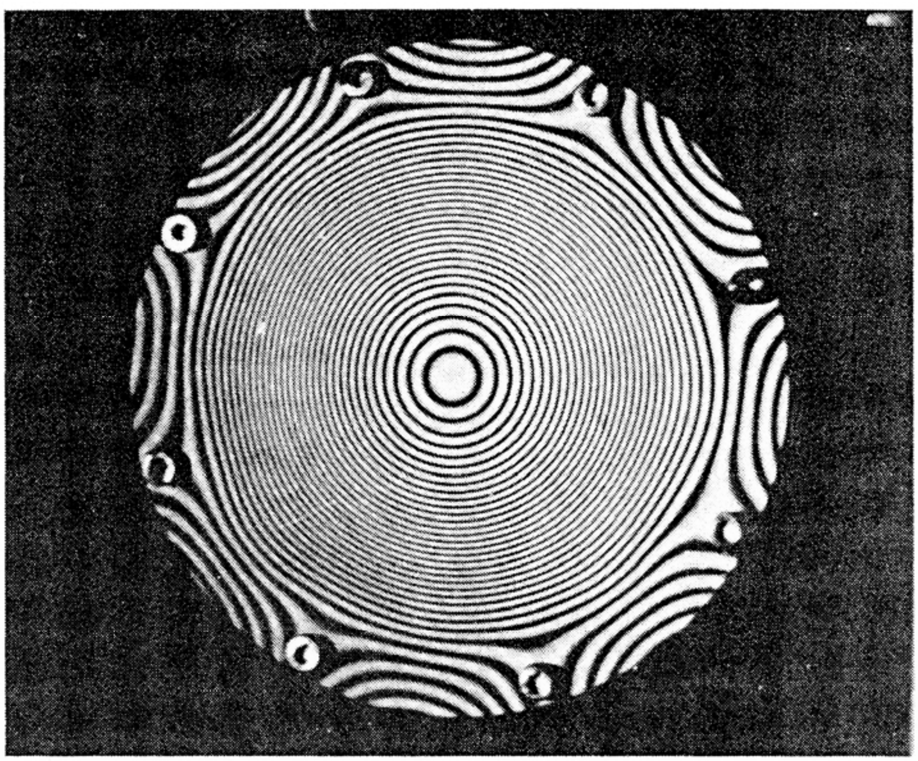


Fig. 6.8 A double exposure holographic interferogram showing the deformation of a circular membrane which has been deformed by uniform pressure. (Photograph courtesy of W. Braga and C. M. Vest, The University of Michigan)

Holographic Interferometry

- The combination of playback and displaced beam is

$$I = 4I_o \cos^2\left(\frac{\Delta\varphi}{2}\right)$$

- Like any interference these have peaks at

$$\Delta\varphi = 2\pi m$$

where m = integers

- A displaced diaphragm will produce concentric rings with displacements

$$d_p = \frac{m\lambda}{2\cos\left(\frac{\psi}{2}\right)\cos(\theta)}$$

- Note again $\psi/2 = \theta$
- Can use this to trace the change in a structures shape

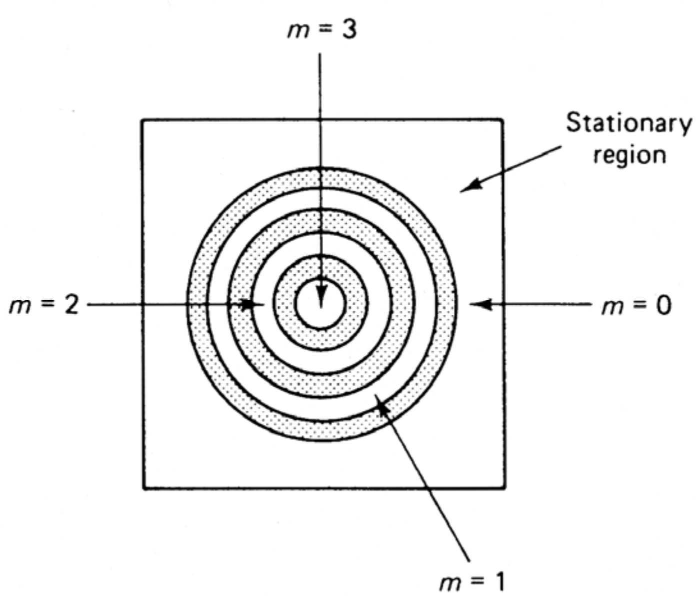


Figure 11-14 Double-exposure holographic image with interference fringes due to normal displacement of the center of a circular diaphragm that is clamped around its perimeter.

Plate Holographic Interferometry

- Bent beam: both vertical an sideways bending seen
- Widely used to determine effects of force on objects
- Compare results to simulations for verifications

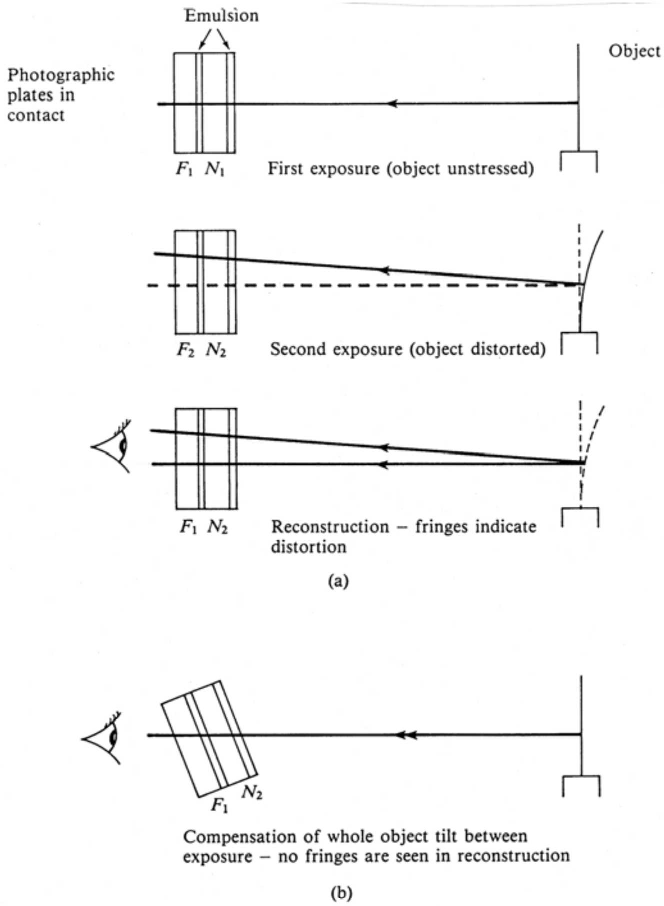


Fig. 6.9 (a) Diagram showing the principles of sandwich holograph: (a) illustrates how the deformation of an object may be determined from the fringe patterns produced by a simultaneous reconstruction of holograms produced at different stages in the deformation of the object; (b) illustrates how a movement of the whole of the object can be compensated for by manipulation of the holograms relative to the reconstructing beam so that no fringes are produced. The identical

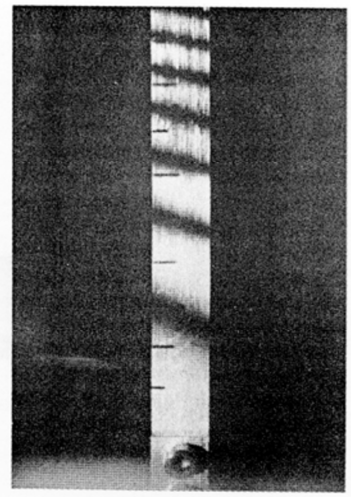
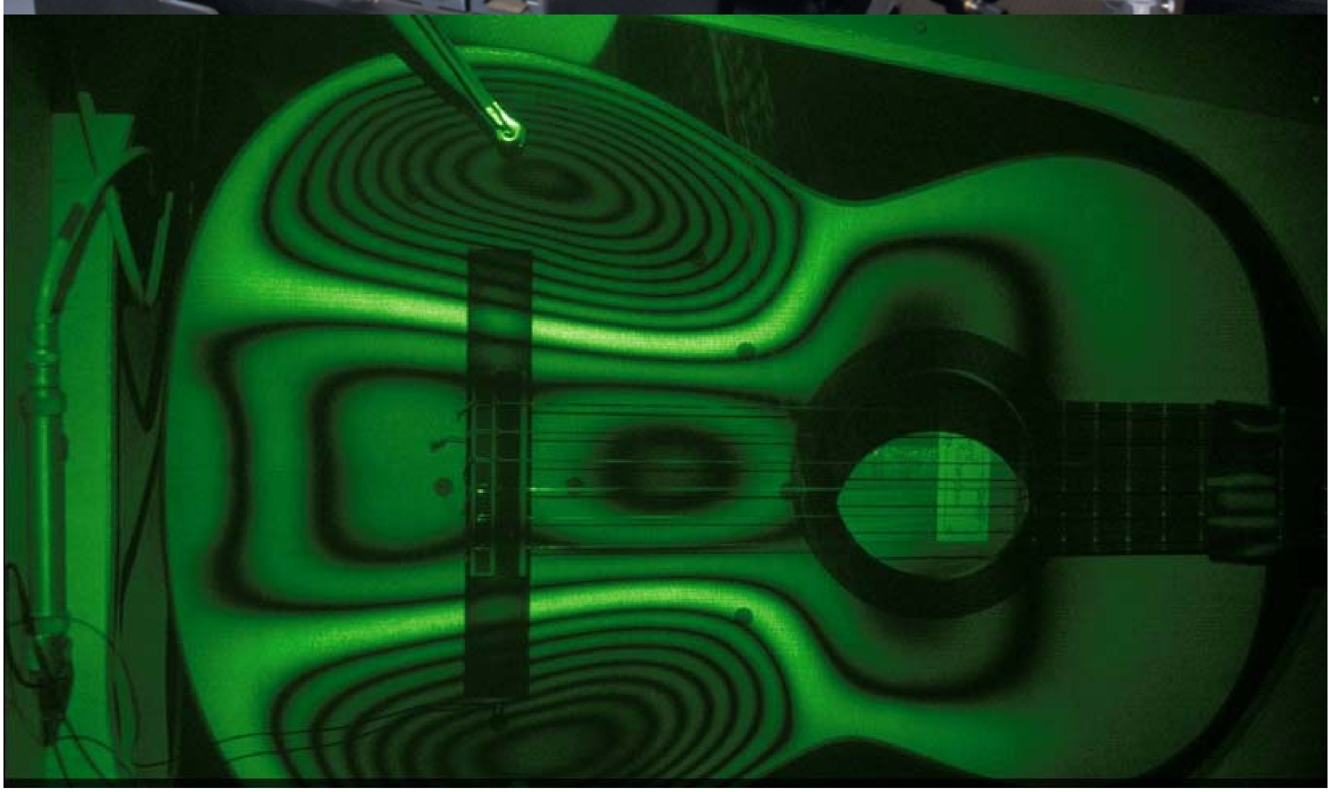


Fig. 6.10 An illustration of real time or single exposure holographic interferometry. Interference of the actual object wave with the reconstructed object wave shows the deformation of the bar. The fact that the fringes are not horizontal indicates that the bar suffers a twist in addition to bending. (From O'Shea/Callen/Rhodes *Introduction to Lasers and their Applications* © 1977 Addison-Wesley, Reading, MA. Fig. 7.14. Reprinted with permission.)

Holographic Interferometry

- For complex systems very difficult to simulate
- eg Vibration movement on musical instruments
- Now compare results for verifications of the models used



Holographic Optical Elements

- Create hologram of optical element, eg lens, diffraction grating
- Laser light changed by the holographic element in same way as real element
- Because hologram changes direction and phase as in real one
- Only problem is loss of some light
- Eg Holographic diffraction gratings
- Holographic scanners - focus light to a point then rotate scanner to scan point

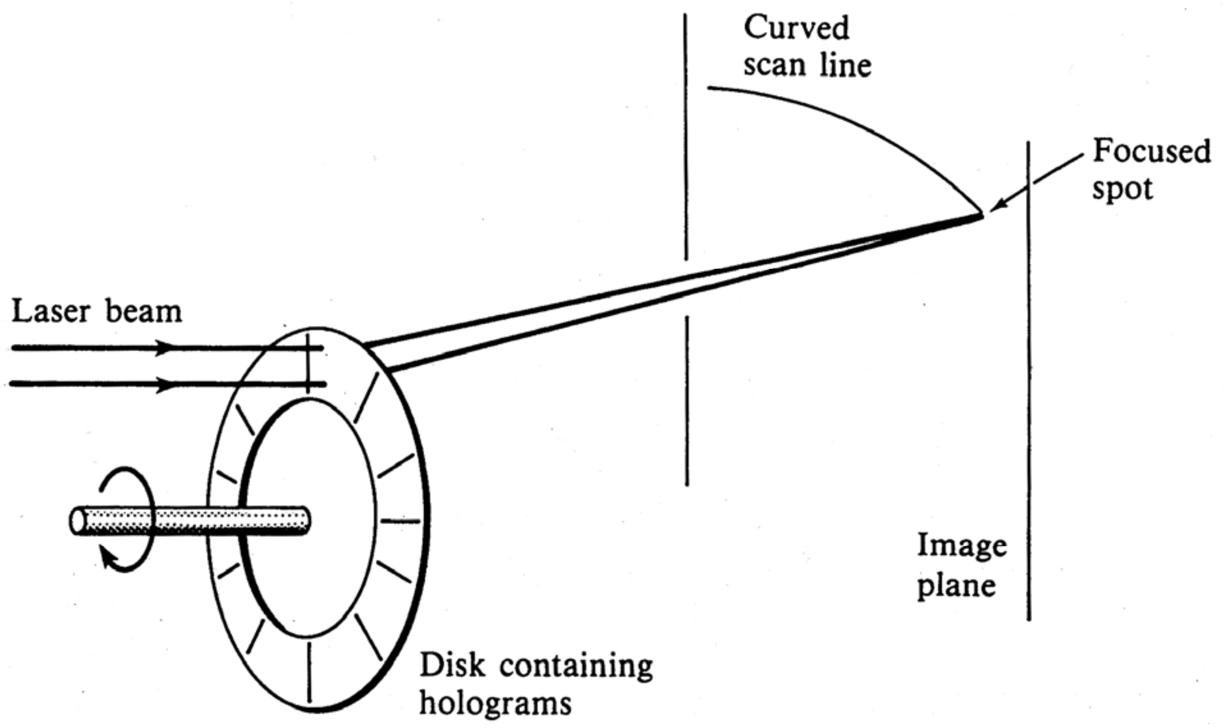


Fig. 6.14 Schematic diagrams of holographic beam scanner, which produces a curved scan line.