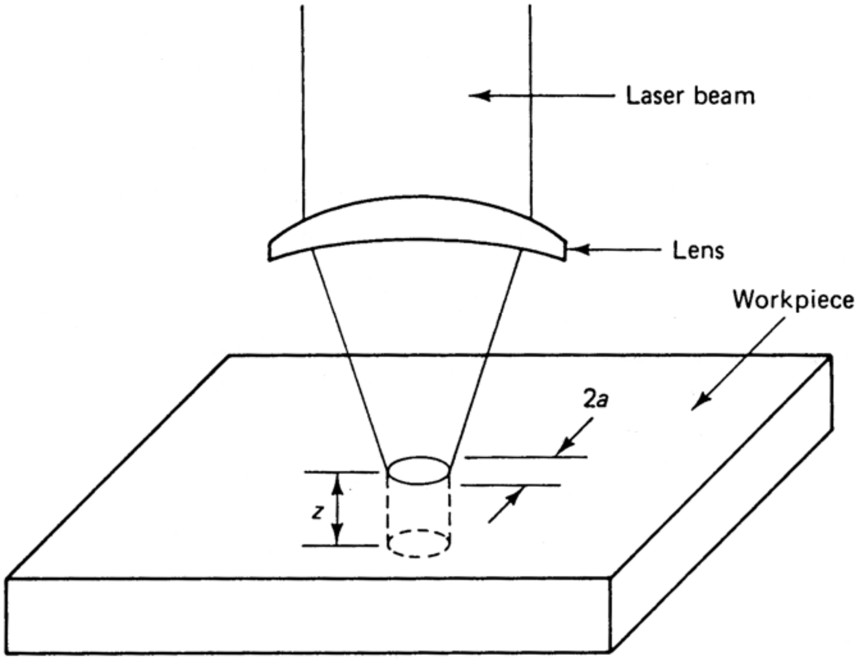
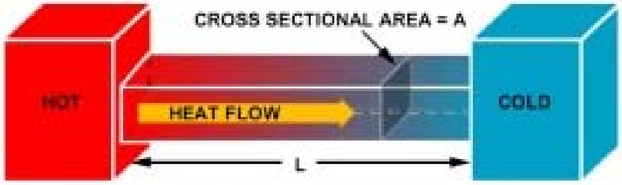
Laser Heat Processing: Advantages

- Laser radiation very "clean"
  no containments with other materials
- Working atmosphere can be controlled as needed
- Lasers can be focused to small spots
  very localized heating at very high power
- Careful control of heating power
- Beam easy to direct into hard to access points
  Can pass beam through glass to isolated areas
- Energy generally deposited near the surface





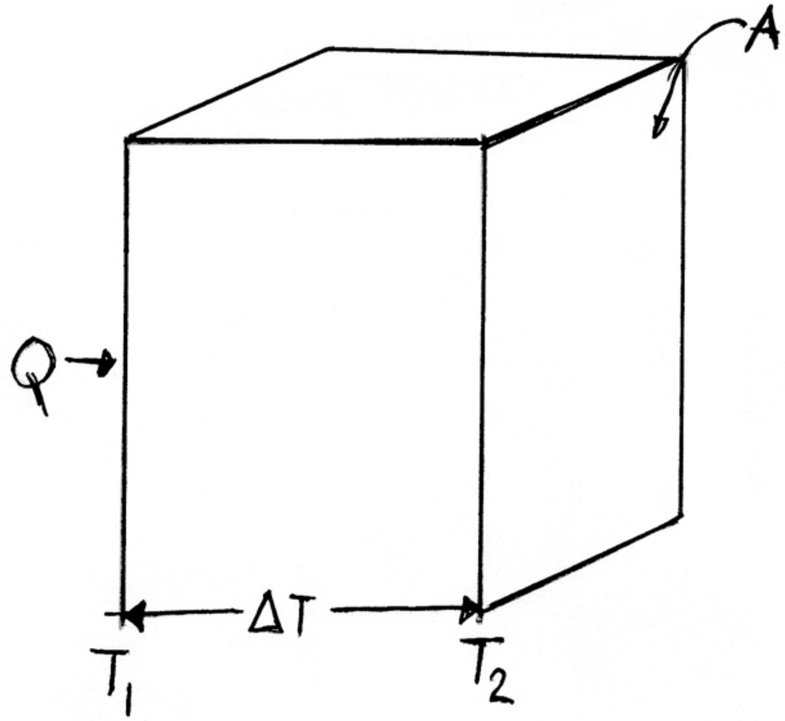
General Heat Flow

- Laser heating is just like other heat flow problems
- For the time being assume heat source
  note that this is not always possible for laser
- Heat flows by Fourier Law of Heat Conduction

$$Q = -kA\frac{\partial T}{\partial x} = -kA\nabla T$$

where Q = heat generated: Watts (some books use H)
k = Thermal conductivity W/m°C or W/cm°C or W/m°K
heat flow per unit area per unite distance
A = cross sectional area
T = temperature in °C
z = depth into the material

- This is actually the definition of thermal conductivity
- Note this assumes steady state conditions
- laser heating stabilizes in 10^{-12} to 10^{-13} sec
  Thus assume steady state conditions



Thermal Values of Materials

- Thermal Conductivity changes with temperature
  Hence k may change with position

Specific Heat

- When heat enters a volume temperature change depends
  Specific Heat C of material, J/kg/^{o}K
- Specific Heat determine how much heat needed to raise
  temperature of unit mass one degree C.

Density

- Density of material ρ Kg/m^3

Latent Heats

- Latent heats give the energy required for a phase change
  units of J/kg
- L_f Latent Heat of Fusion: energy for melting
- L_v Latent Heat of Vaporization: energy to vaporize

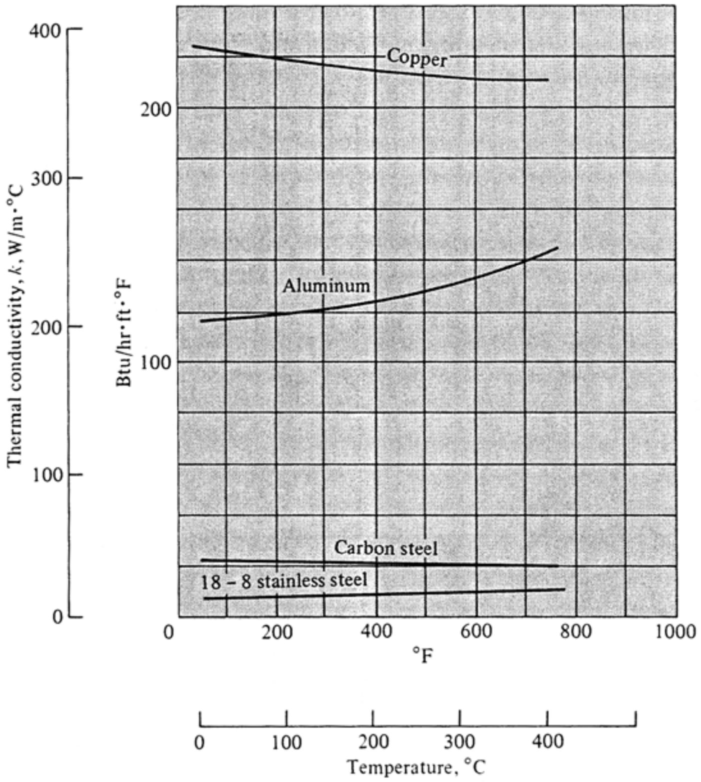


Fig. 1-6 Thermal conductivities of some typical solids.

Basic Heat Flow Differential Equations

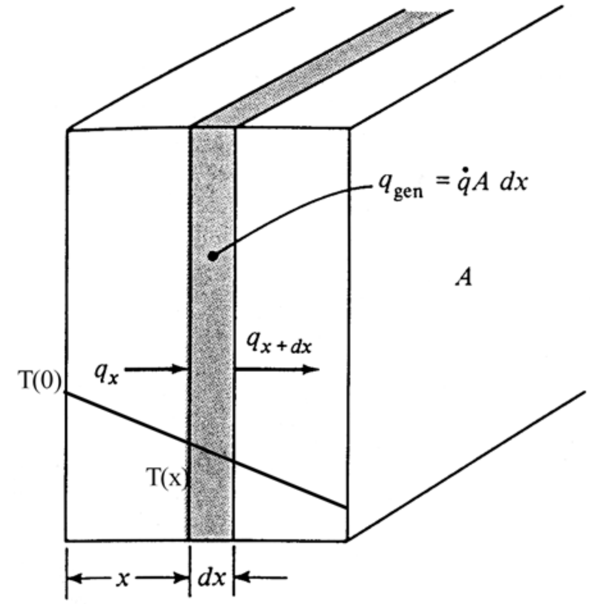
- let $q^{\cdot}$ be the energy deposited per unit volume
  (use H in some books) W/m^3
- Consider heat flowing through a volume
- Then: energy at left face
  = change in internal energy + energy out right face
- Then energy per unit time in left face (Fourier's law)

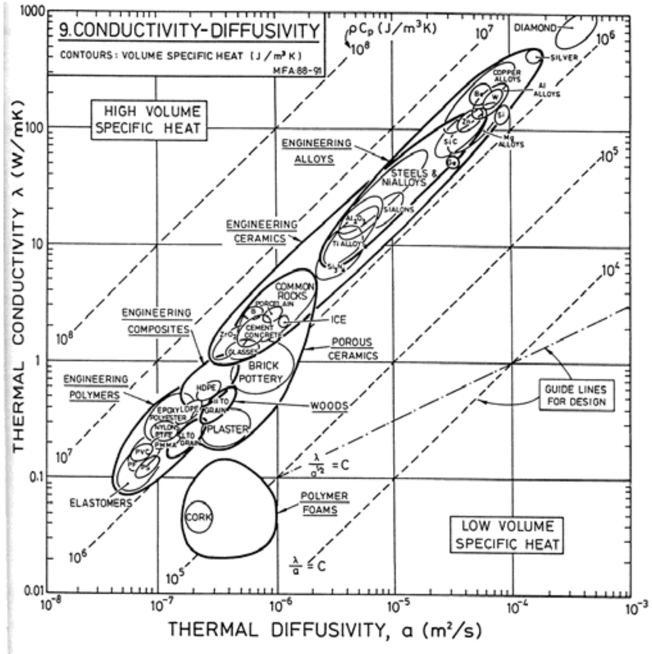
$$q_x = -kA\frac{\partial T}{\partial x}$$

- **Thermal Conductivity** k in W/m/^{o}K
- If uniform k, steady state get Newton's Law of Cooling

$$q_x = -kA\frac{\Delta T}{\Delta x}$$

- Hence can calculate heat loss if know thermal conductivity





Thermal Values of Materials

- Thermal Conductivity changes significantly with materials
- Inverse of thermal conductivity like resistance in electric circuit
- Newton heat flow means slope of conductive varies as 1/k

$$\frac{\Delta T}{\Delta x} = -\frac{q_x}{kA}$$

- In construction use R values, which are 1/k but in British units hr-ft^{o}F/BTU
- Higher the R the less heat flow

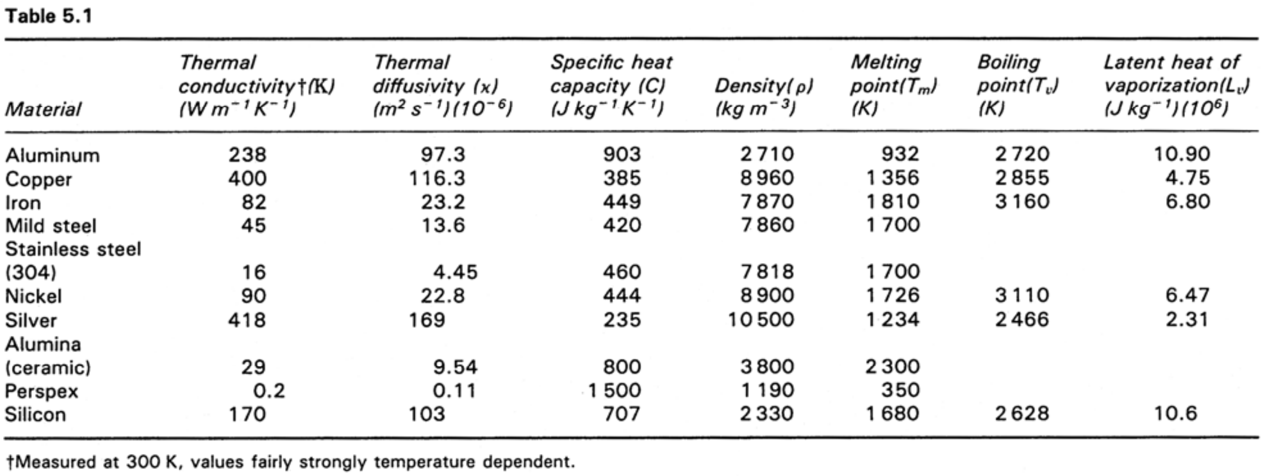


Table 5.1

Material	Thermal conductivity†(K) ($W m^{-1} K^{-1}$)	Thermal diffusivity (x) ($m^2 s^{-1}$)(10^{-6})	Specific heat capacity (C) ($J kg^{-1} K^{-1}$)	Density(ρ) ($kg m^{-3}$)	Melting point(T_m) (K)	Boiling point(T_v) (K)	Latent heat of vaporization(L_v) ($J kg^{-1}$)(10^6)
Aluminum	238	97.3	903	2 710	932	2 720	10.90
Copper	400	116.3	385	8 960	1 356	2 855	4.75
Iron	82	23.2	449	7 870	1 810	3 160	6.80
Mild steel	45	13.6	420	7 860	1 700		
Stainless steel (304)	16	4.45	460	7 818	1 700		
Nickel	90	22.8	444	8 900	1 726	3 110	6.47
Silver	418	169	235	10 500	1 234	2 466	2.31
Alumina (ceramic)	29	9.54	800	3 800	2 300		
Perspex	0.2	0.11	1 500	1 190	350		
Silicon	170	103	707	2 330	1 680	2628	10.6

†Measured at 300 K, values fairly strongly temperature dependent.

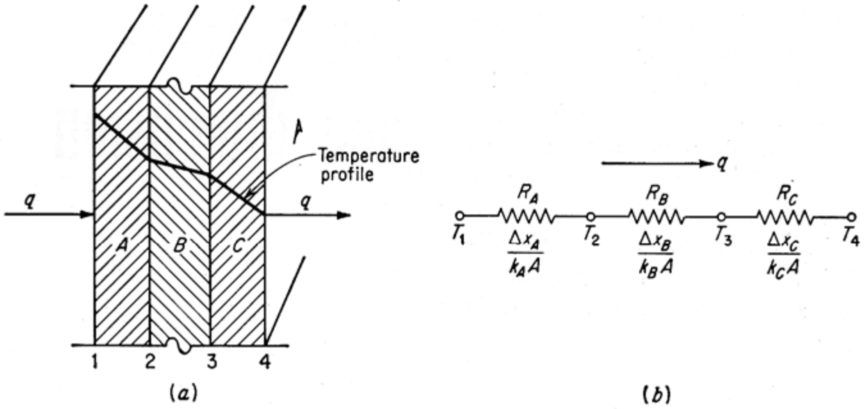


Fig. 2-1 *One-dimensional heat transfer through a composite wall and electrical analogue.*

Basic Heat Flow Differential Equations

- Now consider 1 dimensional heat flowing through a volume
- In the general case there q deposited and q generated within
- Then: energy at left face + heat generated
  = change in internal energy + energy out right face
- Energy per unit time in left face

$$q_x = -kA\frac{\partial T}{\partial x}$$

- Energy out of right face of x+dx is

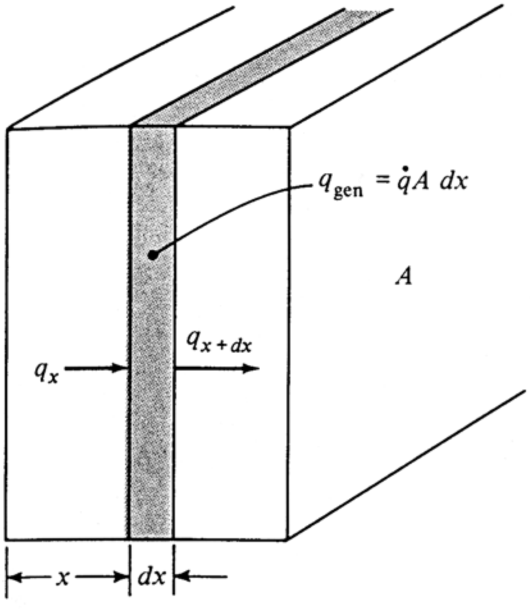
$$q_{x+dx} = q_x + \frac{\partial q_x}{\partial x}dx = -A\left[k\frac{\partial T}{\partial x} + \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)dx\right]$$

- Energy generated within the element per unite volume & time

$$q_{gen} = \dot{q}Adx$$

- Change (loss) in internal energy due to heating of material

$$q_{internal} = \rho CA\frac{\partial T}{\partial t}dx$$



Basic Heat Flow Differential Equations

- Thus writing the energy balance

$$q_x + q_{gen} = q_{int\,ernal} + q_{x+dx}$$

$$-kA\frac{\partial T}{\partial x} + \dot{q}Adx = \rho CA\frac{\partial T}{\partial t}dx - A\left[k\frac{\partial T}{\partial x} + \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)dx\right]$$

- Combining gives heat flow DE

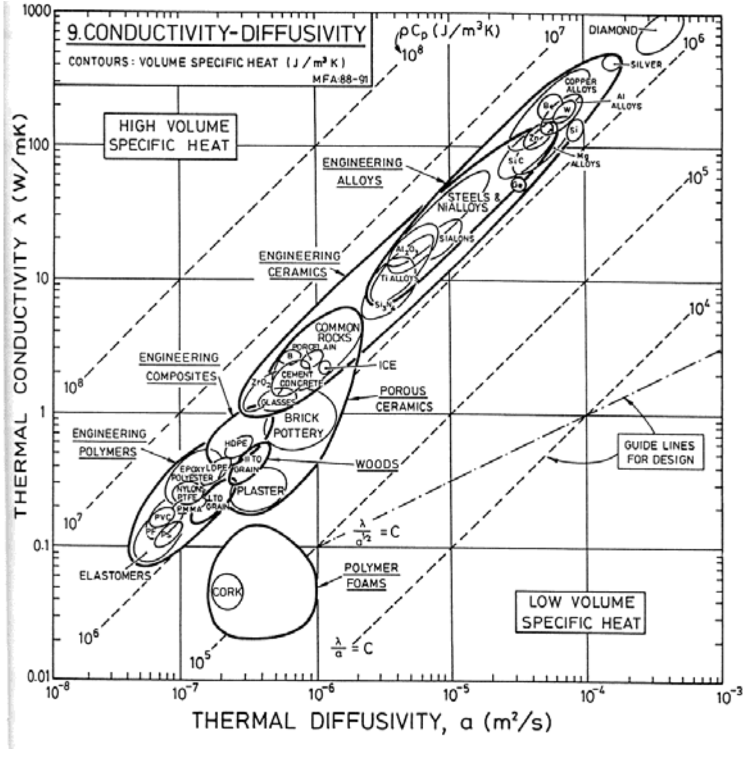
$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \dot{q} = \rho C\frac{\partial T}{\partial t}$$

- If assume thermal conductivity constant with position then

$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

- where α **Thermal Diffusivity** in m^2/s

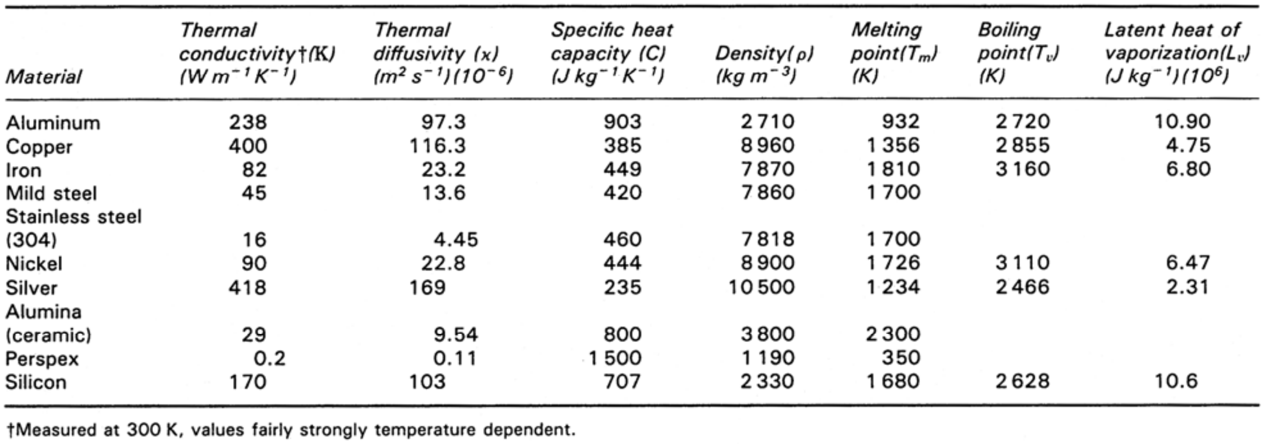
$$\alpha = \frac{k}{\rho C}$$



α Thermal Diffusivity

- Thermal Diffusivity gives T rise caused by an applied heat pulse or how rapidly heat diffuses through the material
- High thermal diffusivity: low surface temperature rise
  deep penetration of heat pulse
- Low thermal diffusivity: high surface temperature rise

Table 5.1



Material	Thermal conductivity† (K) (W m^{-1} K^{-1})	Thermal diffusivity (κ) (m^2 s^{-1})(10^{-6})	Specific heat capacity (C) (J kg^{-1} K^{-1})	Density(ρ) (kg m^{-3})	Melting point(T_m) (K)	Boiling point(T_v) (K)	Latent heat of vaporization(L_v) (J kg^{-1})(10^6)
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†Measured at 300 K, values fairly strongly temperature dependent.

- In three dimensional the heat flow DE becomes

$$\nabla^2 T + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

- Use ∇^2 for coordinate system that notes symmetry of problem
- For Cartesian

$$\nabla^2 T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}$$

- Often useful to use cylindrical coordinates for laser
- Example laser spots are circularly symmetric
- As angle θ is often uniform reduces from 3D to 2D problem values
- Reduces to a problem in radius r and depth z
- But must use proper cylindrical polar equations to get right

$$\nabla^2 T = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}$$

- Generally use angular symmetric conditions so θ term eliminated

Temperature Change for Uniform Illumination

- Assume that the surface is uniformly illuminated by the laser
- Energy absorbed at the surface in a very small depth

$$H=I(1-R)$$

where R = reflectivity
I = light intensity

- The heat 1 Dim DE has been solved for depth z and time t (by Carslaw & Jaeger, 1959)

$$\Delta T(z,t)=\frac{2H}{k}\sqrt{\alpha t}\ ierfc\left[\frac{z}{2\sqrt{\alpha t}}\right]$$

- The ierfc is the integral of the complementary error function

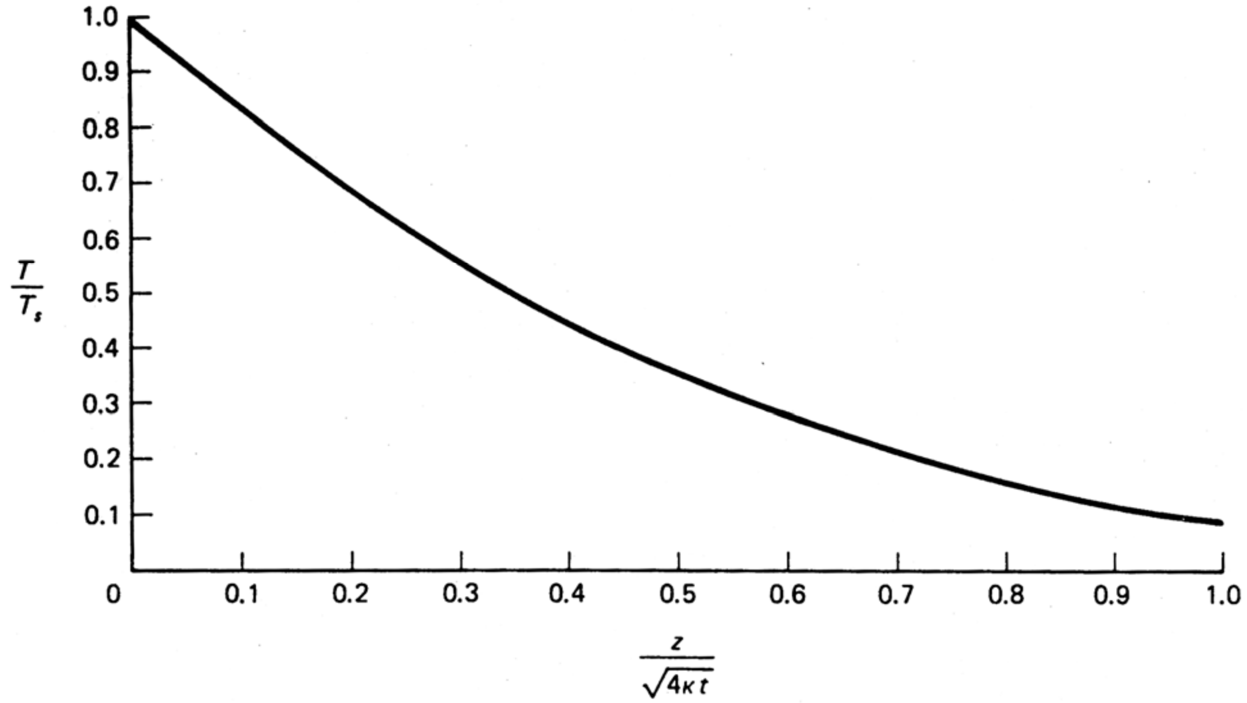


Figure 12-1 Ratio of temperature T at depth z to the surface temperature T_s versus $z/\sqrt{4\kappa\tau}$ for uniform, constant irradiance.

Error Function Related Equations

- Heat flow equations are related to the Error Function erf

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-s^2} ds$$

- This is the integral of a Gaussian between 0 and x
- The Complementary Error Function erfc

$$erfc(x) = 1 - erf(x)$$

- erfc is the error function but integred from x to infinity
- The ierfc is related to the error function by

$$ierfc(x) = \int_{\infty}^{x} erfc(s) ds = \frac{1}{\sqrt{\pi}} exp\left(-x^2\right) - x\left[1 - erf(x)\right]$$

- ierfc(1) = 0.05 and is falling rapidly

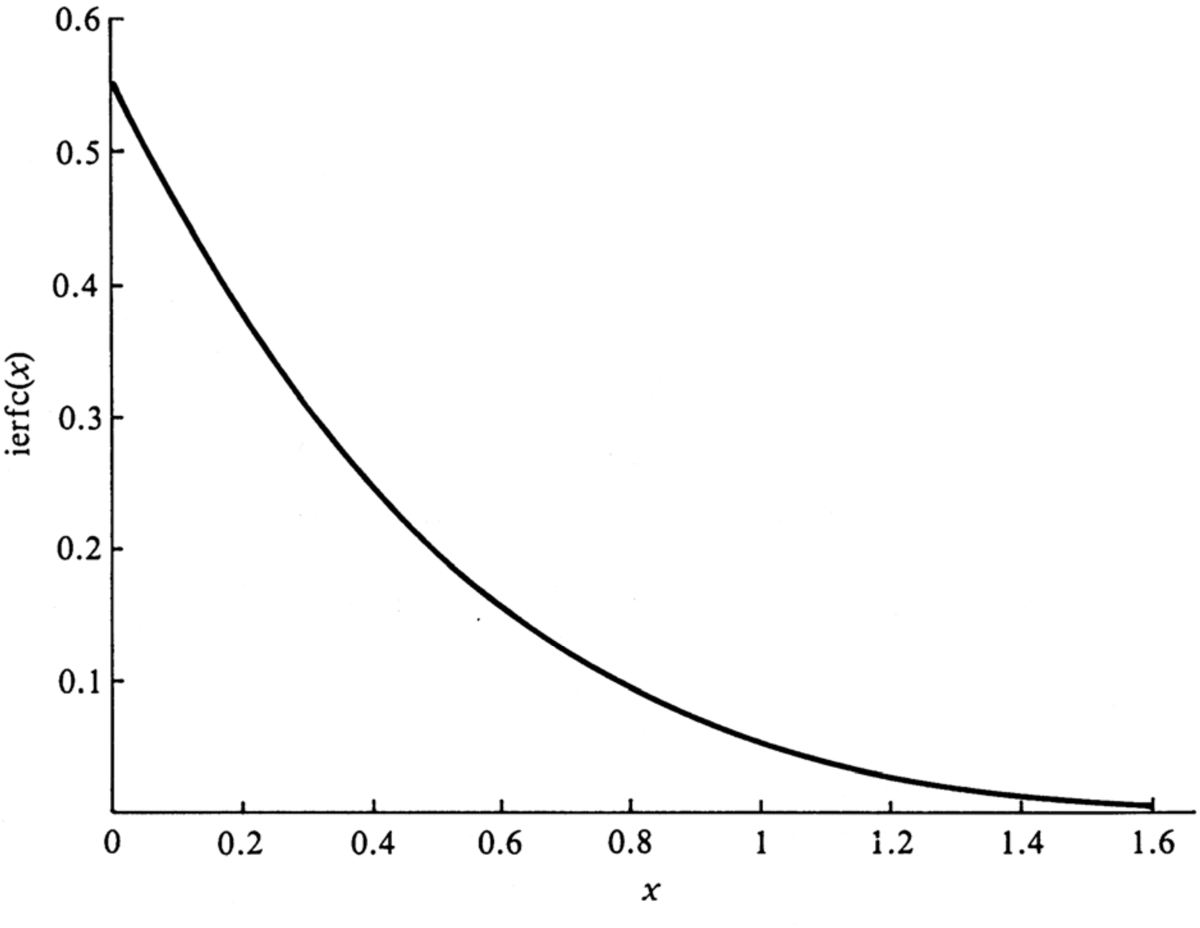


Fig. 5.3 Graph of the function ierfc(x)

Useful Error Function erfc(x) Approximations

- Error function erf(x), Complementary Error Function erfc(x) are

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-s^2} ds$$

$$erfc(x) = 1 - erf(x) = \frac{2}{\sqrt{\pi}} \int_\infty^x e^{-s^2} ds$$

- erf(x) hard to find but easy to approximate with

$$erf(x) = 1 - \left(a_1 t + a_2 t^2 + a_3 t^3\right) e^{-x^2}$$

$$t = \frac{1}{1 + px} \quad where \quad p = 0.47047$$

$a_1 = 0.3480242$, $a_2 = -0.0958798$, $a_3 = 0.7478556$

- See Abramowitz & Segun (Handbook of Mathematical Functions)
- Error on this is $< 2.5 \times 10^{-5}$ for all x (<2% error for x << 5.5)
- We are using complementary error function

$$erfc(x) = 1 - erf(x) \qquad erfc(0) = 1 \qquad erfc(\infty) = 0$$

- Asymptotic approximation

$$erfc(x) \rightarrow \frac{e^{-x^2}}{x\sqrt{\pi}} \left[1 - \frac{1}{2x^2}\right] \quad as \quad x \rightarrow \infty$$

- For x>3.5 this has <1% error (use plot page following for x<3.5)
- Excel & Quatropro spreadsheet have erf and erfc built in.
  Must activate analysis toolpack & solver first
  but become inaccurate for x>5.4 – then use asymptotic
- Matlab & maple OK for higher x's
- For x > 5.4 then ierfc(x) becomes

$$ierfc(x) \rightarrow \frac{e^{-x^2}}{2x^2\sqrt{\pi}} \quad as \quad x \rightarrow \infty$$

- Especially useful for approx. solving for x when have ierfc value

Temperature Rise for Uniform Illumination

- From DE solution since ierfc is small for x>1
- Thus find that T rise is small when

$$\frac{z}{2\sqrt{\alpha t}} > 1$$

- Hence small rise when

$$z^2 > 4\alpha t$$

- Heat will diffuse a depth L in time of order

$$t = \frac{L^2}{4\alpha}$$

- Change in surface temperature with time substitute z = 0 and note

$$ierfc(0) = \frac{1}{\sqrt{\pi}}$$

- Thus surface temperature change is:

$$\Delta T(0,t) = \frac{2H}{k}\sqrt{\alpha t}\ ierfc[0] = \frac{2H}{k}\sqrt{\frac{\alpha t}{\pi}}$$

- Thus temp increases with $\sqrt{\alpha t}$

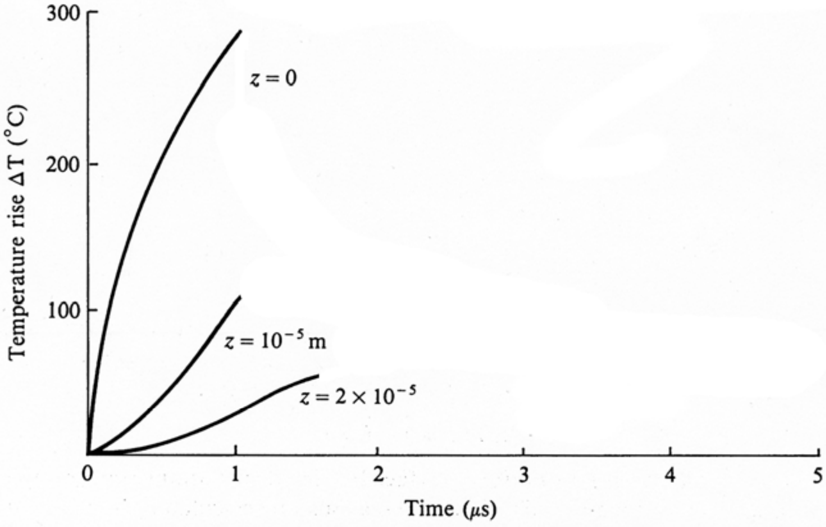


Fig. 5.4 Calculated temperature increases at various depths (z) below the surface of a semi-infinite copper block when irradiated with a constant heat pulse of 10^{10} Wm^{-2}

Comparison of Normalized Gaussian & ERFC

- erfc(x) much steeper than Gaussian
- For erfc use plot for x<3.5, asymptotic formula for > 3.5
- Use for getting inverse of erfc
- Use asymptotic eqn for inverse values of erfc(x)<10^{-6}

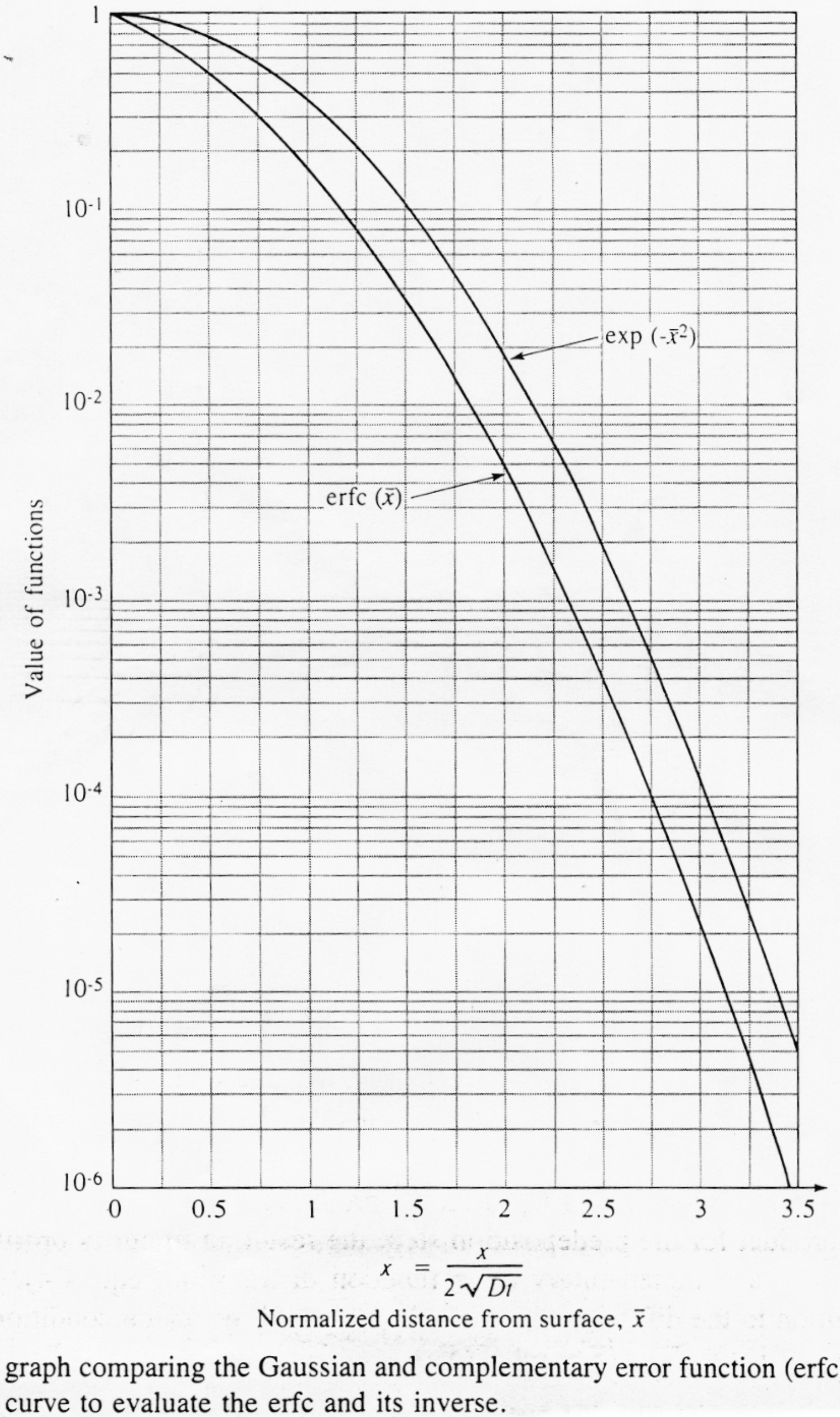


Fig. 4.4 A graph comparing the Gaussian and complementary error function (erfc) profiles. We will use this curve to evaluate the erfc and its inverse.

Temperature Change with Finite Time Laser Pulse

- If have a square pulse of duration t_p
- The for $t < t_p$ follow the previous formula
- For Time greater than the pulse

$$\Delta T(z,t)_{t>t_p} = \delta T(z,t) - \delta T(z,t-t_p)$$

- eg. Consider Cooper with $H = 10^{10}$ W/m^2 for $t_p = 10^{-6}$ sec
- From table $\alpha = 1.16\text{x}10^{-4}$ m^2/s
- T rises highest at surface (z=0) and changes fastest
- At pulse end heat has diffused about L

$$L \approx \sqrt{4\alpha t} \approx \sqrt{4(1.16x10^{-4})(10^{-6})} = 2.15x10^{-5}\ m$$

- At depth peak T occurs much later, and lower values

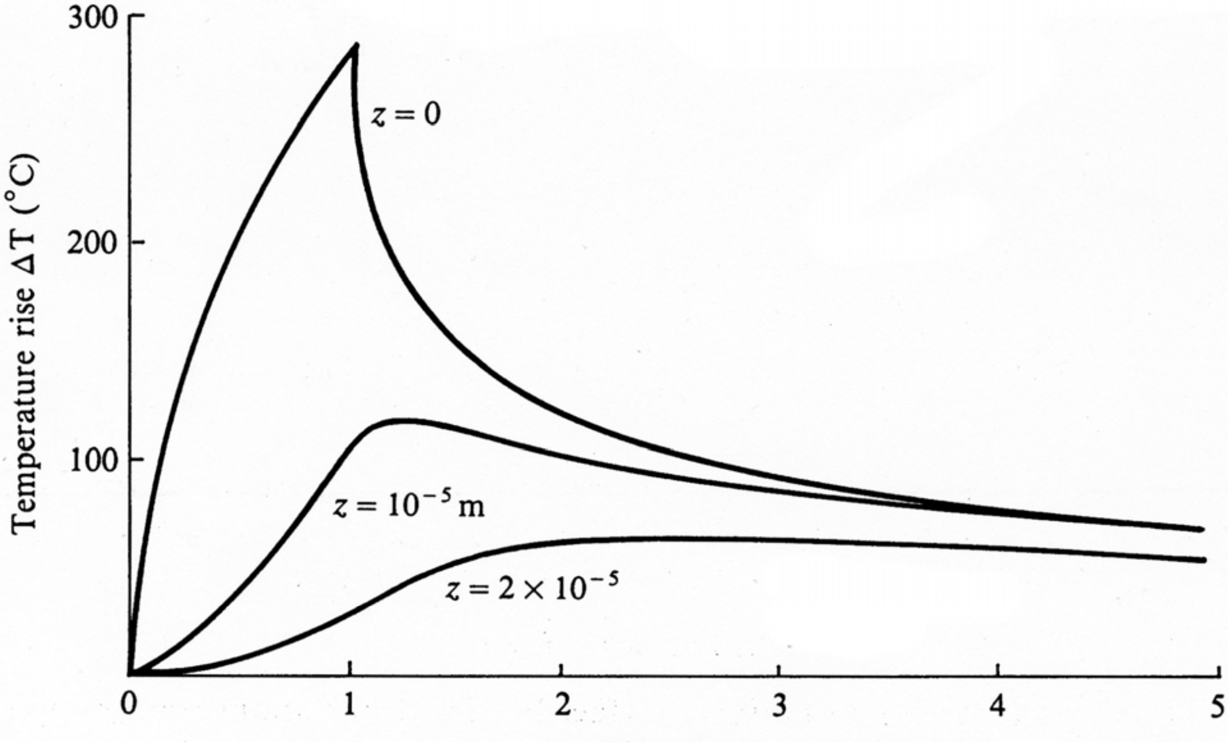


Fig. 5.4 Calculated temperature increases at various depths (z) below the surface of a semi-infinite copper block when irradiated with a constant heat pulse of 10^{10} Wm^{-2} for a time of 1 μs.

Laser Focused into a Spot

- If laser focused into uniform spot radius a
  then formula changes to (by Carslaw & Jaeger, 1959)

$$\Delta T(z,t) = \frac{2H}{k}\sqrt{\alpha t}\left\{ ierfc\left[\frac{z}{2\sqrt{\alpha t}}\right] - ierfc\left[\frac{\sqrt{z^2 + a^2}}{2\sqrt{\alpha t}}\right]\right\}$$

- Term on right caused by sideways diffusion
- At the centre of the spot (z = 0)

$$\Delta T(z,t) = \frac{2H}{k}\sqrt{\alpha t}\left\{ \frac{1}{\sqrt{\pi}} - ierfc\left[\frac{a}{2\sqrt{\alpha t}}\right]\right\}$$

- This gives same as uniform heating if

$$ierfc\left[\frac{a}{2\sqrt{\alpha t}}\right] << 1$$

- **This is true for ierfc(>1) thus**

$$t < \frac{a^2}{4\alpha}$$

- eg for Copper with a = 1 mm and $\alpha = 1.16 \times 10^{-4}$ m^2/s,

$$t < \frac{\left(10^{-3}\right)^2}{4\left(1.16 x 10^{-4}\right)} \quad or \quad t < 2.16 x 10^{-3} \ s$$

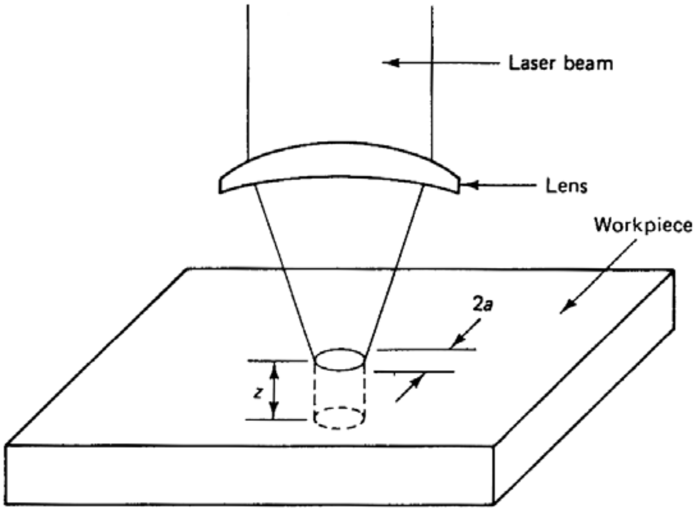


Figure 12-2 Laser beam focused on a workpiece.

Laser Focused into a Spot

- As t goes to infinity (very long times) it can be shown

$$\Delta T(z,t) = \frac{H}{k}\left[\sqrt{z^2 + a^2} - z\right]$$

- Thus for finite spot temperature reaches a limit
- Highest surface temp

$$\Delta T(0,\infty) = \frac{Ha}{k}$$

- Effects of beam Gaussian distribution is not that different
- Choose set diameter to FWHM with same total power
- In practice as thermal conductivity k, reflectance R, thermal diffusivity α all vary with temperature
- Thus tend to use numerical simulations for real details

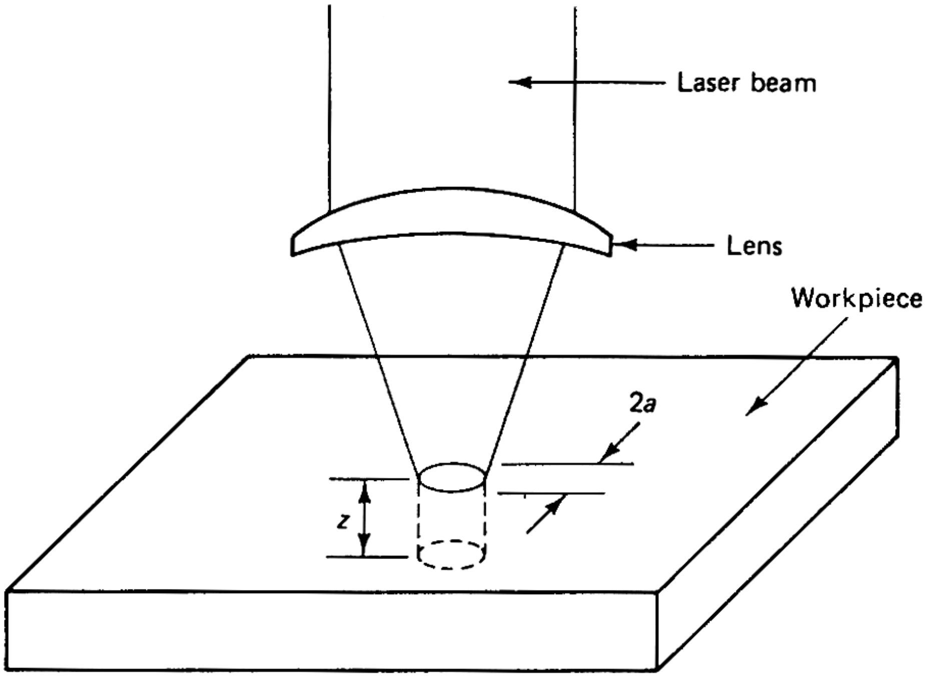
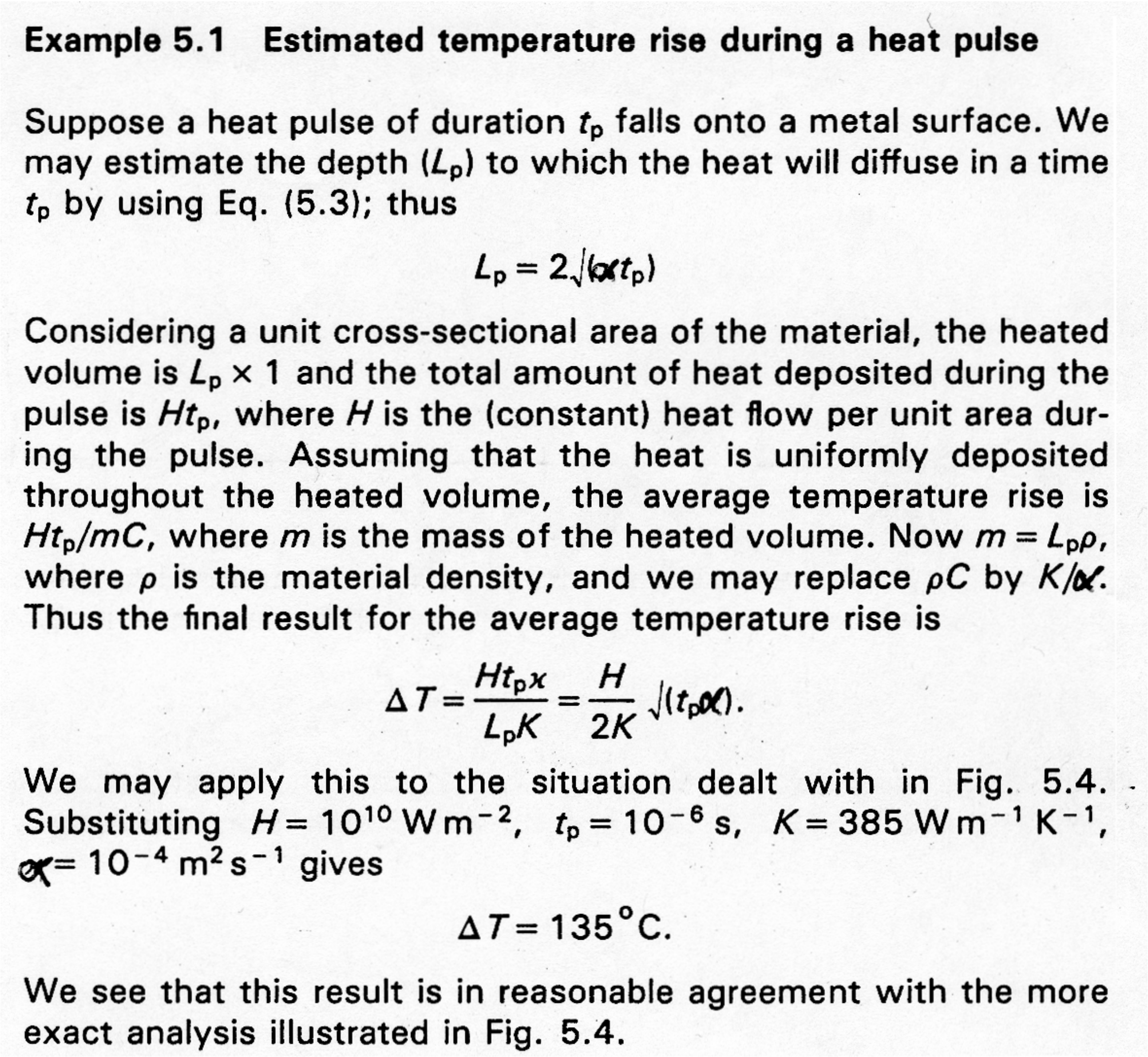


Figure 12-2 Laser beam focused on a workpiece.

Example Focused Laser Spot Calculation



Example 5.1 Estimated temperature rise during a heat pulse

Suppose a heat pulse of duration t_p falls onto a metal surface. We may estimate the depth (L_p) to which the heat will diffuse in a time t_p by using Eq. (5.3); thus

$$L_p = 2\sqrt{(\alpha t_p)}$$

Considering a unit cross-sectional area of the material, the heated volume is $L_p \times 1$ and the total amount of heat deposited during the pulse is Ht_p, where H is the (constant) heat flow per unit area during the pulse. Assuming that the heat is uniformly deposited throughout the heated volume, the average temperature rise is Ht_p/mC, where m is the mass of the heated volume. Now $m = L_p\rho$, where ρ is the material density, and we may replace ρC by K/α. Thus the final result for the average temperature rise is

$$\Delta T = \frac{Ht_p x}{L_p K} = \frac{H}{2K}\sqrt{(t_p \alpha)}.$$

We may apply this to the situation dealt with in Fig. 5.4. Substituting $H = 10^{10}\ \mathrm{W\,m^{-2}}$, $t_p = 10^{-6}\ \mathrm{s}$, $K = 385\ \mathrm{W\,m^{-1}\,K^{-1}}$, $\alpha = 10^{-4}\ \mathrm{m^2\,s^{-1}}$ gives

$$\Delta T = 135\,^\circ\mathrm{C}.$$

We see that this result is in reasonable agreement with the more exact analysis illustrated in Fig. 5.4.

Phase Changes and Energy Balance

- Energy Balance:

  Energy in = Energy to raise temp + heat flow

- Note: a rough rule of thumb if near steady state

  half the energy goes into heat flow

  so energy required is twice that to raise temperature
- As heating increase will get melting of the surface
- Eventually also get vaporization point
- All requires energy to heat
- In general the specific heat of the material changes

  C_s = specific heat of solid

  C_l = specific heat of liquid phase

  L_f = Latent Heat of Fusion: energy for melting

  L_v = Latent Heat of Vaporization: energy to vaporize
- Energy required to melt a unite volume of material

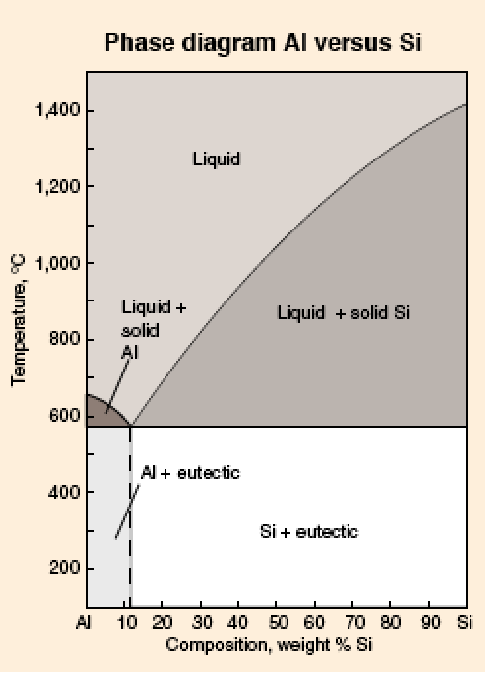
$$E_m = \rho \left\lfloor C_s \left(T_m - T \right) + L_f \right\rfloor$$

where T_m is the melting point,

T the starting temp.

ρ = density of material

- Note: this does not include energy lost to heat flow



Phase Changes and Energy Balance

- When vaporization occurs

$$E_v = \rho \left\lfloor C_s(T_m - T) + C_v(T_v - T_m) + L_f + L_v \right\rfloor$$

- Generally true that heat capacity does not change much with T

$$C_m \approx C_v \approx C$$

- Generally Latent heat of vapourization >heat of fusion

$$L_f < L_v$$

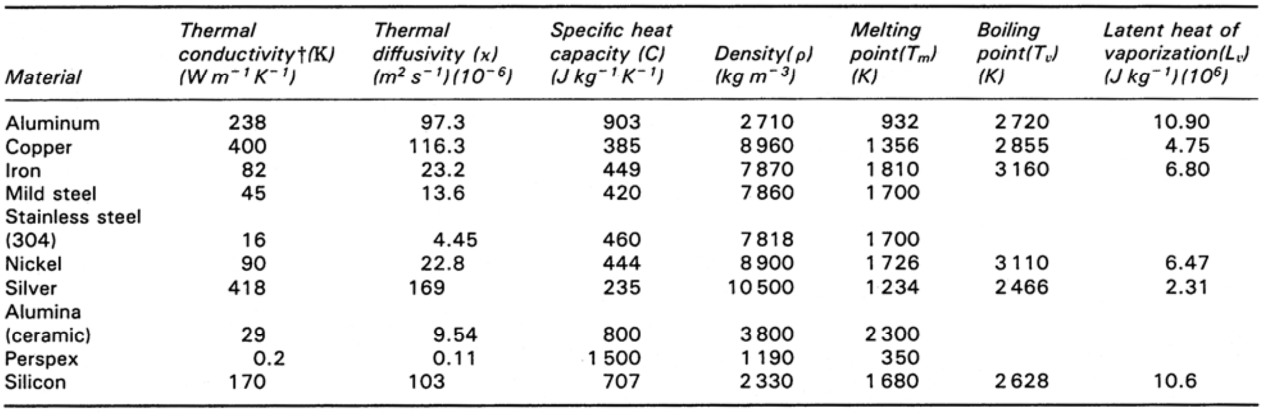
- Vapourization temperature is much > base or melting

$$T<T_m<T_v$$

- Energy input required is approximately

$$E_v \approx \rho \left(CT_v + L_v\right)$$

Table 5.1



Material	Thermal conductivity† (K) ($W\,m^{-1}\,K^{-1}$)	Thermal diffusivity ($\varkappa$) ($m^2\,s^{-1}$)(10^{-6})	Specific heat capacity (C) ($J\,kg^{-1}\,K^{-1}$)	Density(ρ) ($kg\,m^{-3}$)	Melting point(T_m) (K)	Boiling point(T_v) (K)	Latent heat of vaporization(L_v) ($J\,kg^{-1}$)(10^6)
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Silicon	170	103	707	2 330	1 680	2 628	10.6

†Measured at 300 K, values fairly strongly temperature dependent.

Melting Depths

- Consider light pulse on surface
- Will get melting to some depth
- Eventually also raise surface to vaporization point
- This is the laser welding situation
- Can estimate the depth of melt front after some time t
- Recall temperature distribution

$$\Delta T(z,t) = \frac{2H}{k}\sqrt{\alpha t}\ \mathrm{ierfc}\left[\frac{z}{2\sqrt{\alpha t}}\right]$$

- Ratio of the Temperatures changes with depth are

t

$$\frac{\Delta T(z,t)}{\Delta T(0,t)} = \sqrt{\pi}\ \mathrm{ierfc}\left[\frac{z}{2\sqrt{\alpha t}}\right]$$

- Eventually surface raises to vaporization point
- It cannot rise higher without vapourization thus stays at T_v
- Hence can calculate the melt pool depth with this.

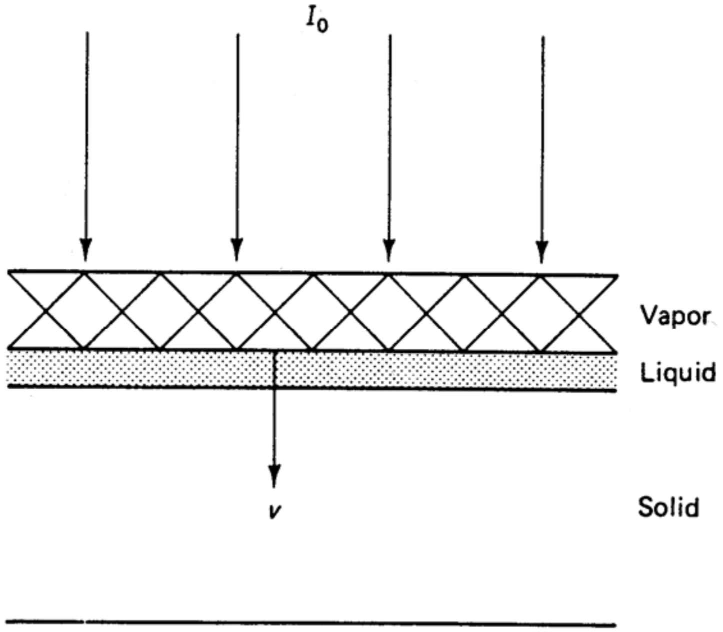


Figure 12-3 Vapor front propagation in a solid.

Melt Depth Estimate

- Estimate the depth of melt front for that after some time t
- Bottom of melt is at melting point,
- Top at vapourization point
- Assume base temperature is near 0°C (ie ~room temp)

$$\Delta T(z,t) = T_m \quad \Delta T(0,t) = T_v$$

$$\frac{T_m}{T_v} = \sqrt{\pi}\ ierfc\left[\frac{z}{2\sqrt{\alpha t}}\right]$$

- Recall that at the surface

$$\Delta T(0,t) = \frac{2H}{k}\sqrt{\frac{\alpha t}{\pi}}$$

- Thus time can be eliminate by solving for

$$T_v \frac{k\sqrt{\pi}}{2H} = \sqrt{\alpha t}$$

- Depth of melt is given by

$$ierfc\left[\frac{z_m H}{T_v k\sqrt{\pi}}\right] = \frac{T_m}{T_v\sqrt{\pi}}$$

- Note: for a given material Hz_m is fixed
- Thus large welding depths given by low heat intensities
  applied for long time
  provided that there is sufficient energy in the beam

Example of Melting Calculations

- What is the heat flow required for
  weld depth of 0.1 mm in copper
- From the table for copper
  T_m = 1060 °C
  T_v = 2570 °C
  K = 400 W/m°C
- Thus

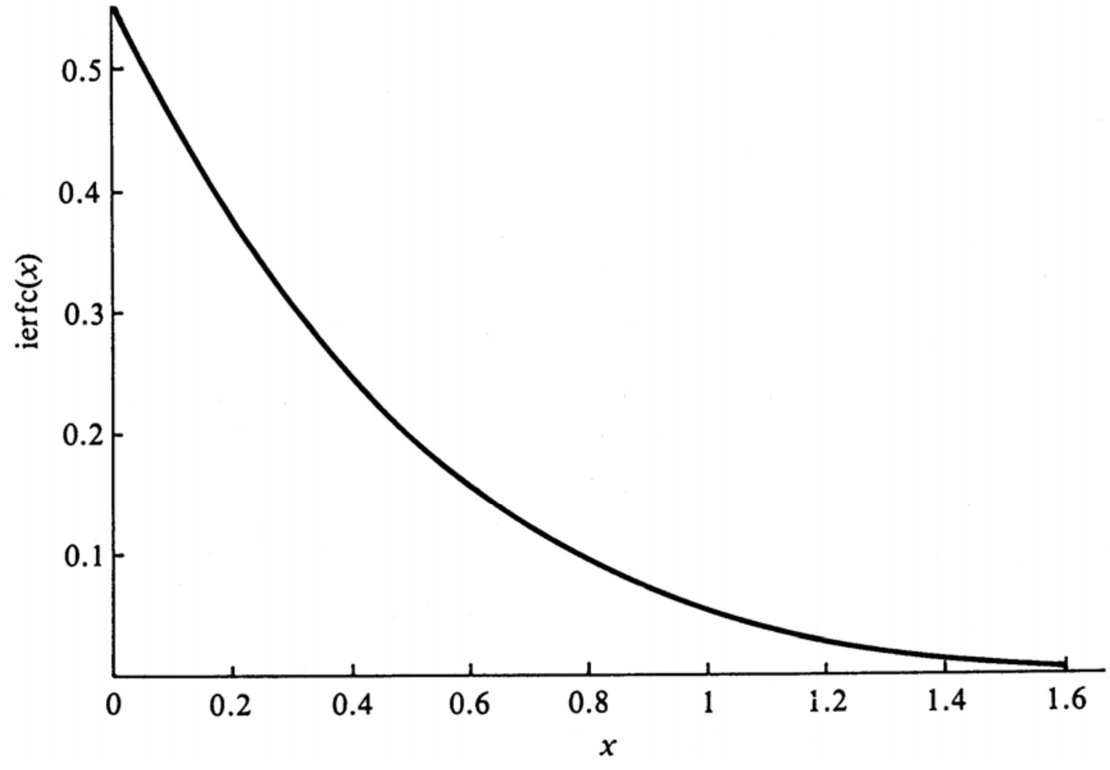
$$ierfc\left[\frac{z_m H}{T_v k\sqrt{\pi}}\right] = \frac{T_m}{T_v\sqrt{\pi}} = \frac{1060}{2570\sqrt{\pi}} = 0.232$$

- From the graph or calculation

$$ierfc(x{=}0.44) = 0.232$$

- Thus

$$H = \frac{T_v k\sqrt{\pi}(x)}{z_m} = \frac{400(2570)\sqrt{\pi}(0.44)}{0.0001} = 8x10^9\, W/m^2$$



Vaporization of Material

- When material removed by vaporization
- Get a melt front and a heated front
- liquid front moves with velocity v_s
- From the heat balance,
  assume that all power goes into heating
- Then the melt front should be

$$H \approx v_s \, \rho \left(C T_v + L_v \right)$$

where H is power density per square area

- Note this is the minimum power value
- Good rule of thumb is actual power required twice this
  loss about the same by heat flow to substrate
- Can calculate depth d_v of holes by knowing
  laser pulse duration t_p and front velocity

$$d_v = v_s \, t_p$$

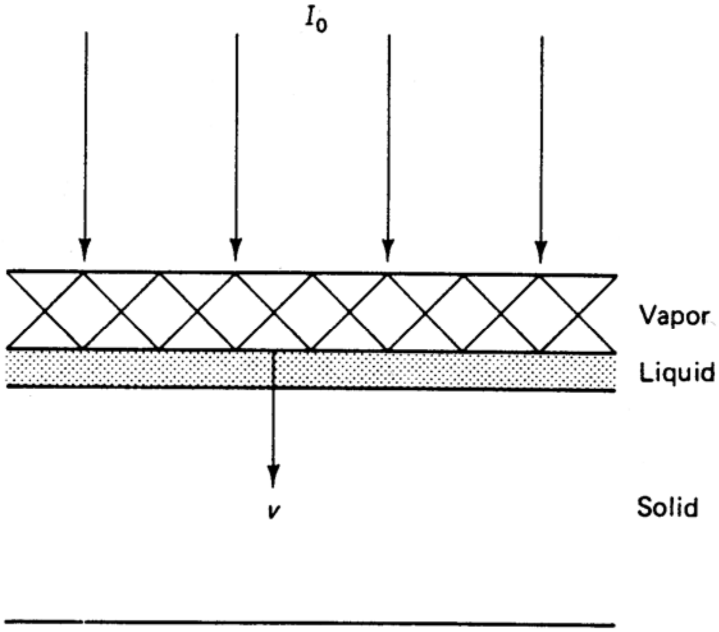


Figure 12-3 Vapor front propagation in a solid.

Example Depth of Hole with Vaporization

- Heat pulse of H = 10^{11} W/m^2 and t = 500 microsec hits copper. What will be the resulting max hole depth
- From the tables

  $T_v = 2570\ ^{o}C$

  $\rho = 8960$ kg/m^3

  $C = 385$ J/kg^{o}C

  $L_v = 4.75x10^6$ J/kg

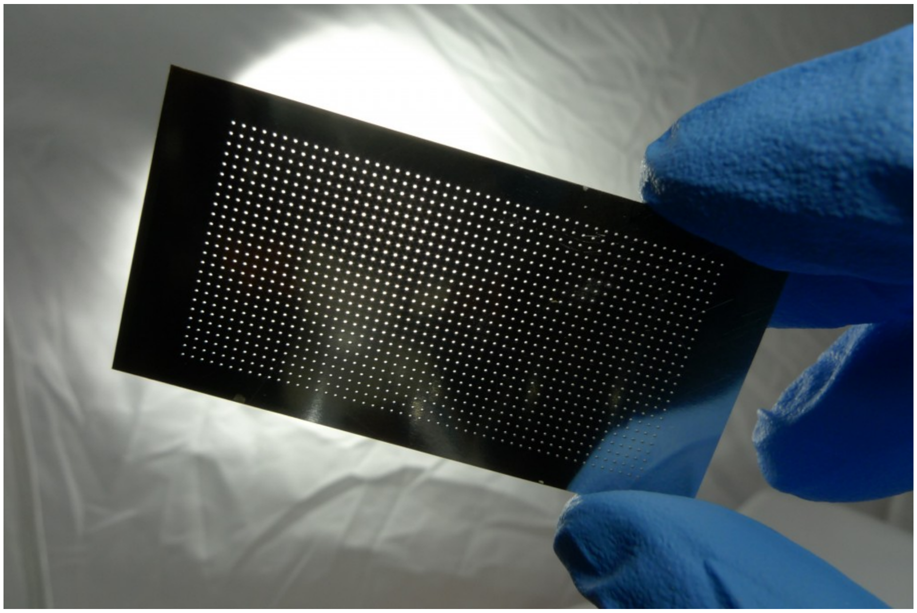
$$d_v = v_s t_p = \frac{Ht_p}{\rho\,(CT_v + L_v)} = \frac{10^{11}\left(5x10^{-4}\right)}{8960\left(385[2570] + 4.75x10^{6}\right)}$$

$$= 0.95x10^{-3}m = 0.95mm$$

Table 5.1

Material	Thermal conductivity†(K) (W m^{-1} K^{-1})	Thermal diffusivity (x) (m^2 s^{-1})(10^{-6})	Specific heat capacity (C) (J kg^{-1} K^{-1})	Density(ρ) (kg m^{-3})	Melting point(T_m) (K)	Boiling point(T_v) (K)	Latent heat of vaporization(L_v) (J kg^{-1})(10^6)
Aluminum	238	97.3	903	2 710	932	2 720	10.90
Copper	400	116.3	385	8 960	1 356	2 855	4.75
Iron	82	23.2	449	7 870	1 810	3 160	6.80
Mild steel	45	13.6	420	7 860	1 700		
Stainless steel (304)	16	4.45	460	7 818	1 700		
Nickel	90	22.8	444	8 900	1 726	3 110	6.47
Silver	418	169	235	10 500	1 234	2 466	2.31
Alumina (ceramic)	29	9.54	800	3 800	2 300		
Perspex	0.2	0.11	1 500	1 190	350		
Silicon	170	103	707	2 330	1 680	2 628	10.6

†Measured at 300 K, values fairly strongly temperature dependent.



Keyholes and Increased Welding/Cutting Depths

- When the laser forms hole in material
- Beam penetrates to much greater depth
- Creates a large deep melt pool behind moving beam
- Melt fills in hole behind moving beam
- If not true welding limited 1 mm in steel

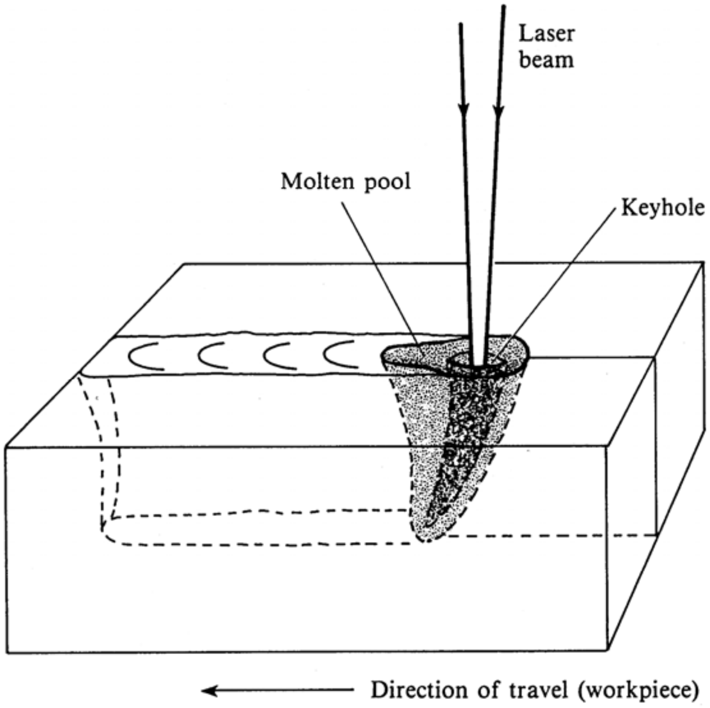


Fig. 5.15 Formation of a 'keyhole' during high-power laser welding.

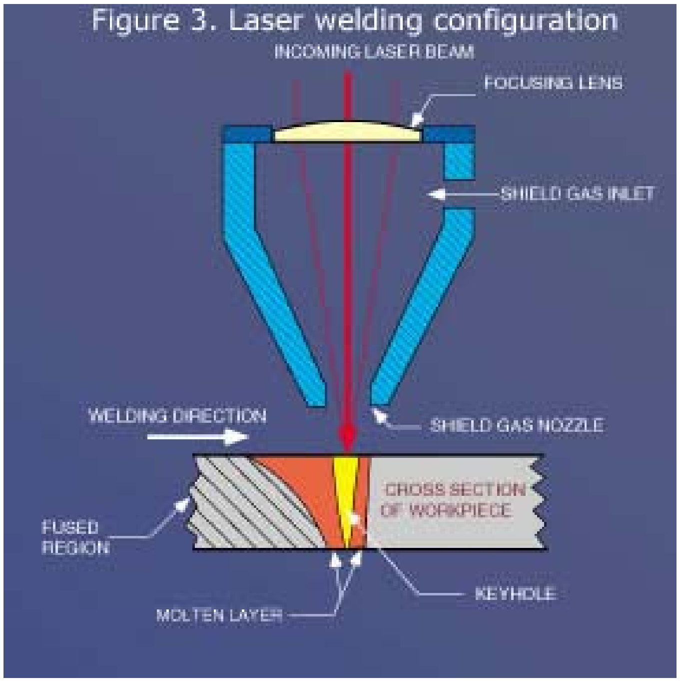
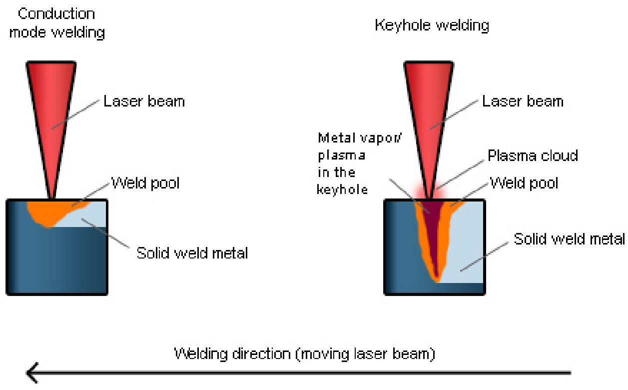


Figure 3. Laser welding configuration



Keyholes Formulas

- Modeled by Swift, Hook and Gick, 1973
- Assume linear heat source power P (W)
- Note P is total power while H is unreflected power because keyhole absorbs all the power (reflections do no escape)
- Extends into metal depth a
- Moving forward with velocity *v* (weld speed) in direction y direction across weld is x (centred on the heating point)
- Temperature distribution becomes

$$T = \frac{P}{2\pi ak} exp\left(\frac{vx}{2\alpha}\right) K_0\left(\frac{v\alpha\sqrt{x^2+y^2}}{2}\right)$$

- where K_0 is the Bessel function 2nd kind order 0
- Width w of the weld is given by the point where T=melting

$$w \approx 0.484 \frac{\alpha}{v} \frac{P}{akT_m}$$

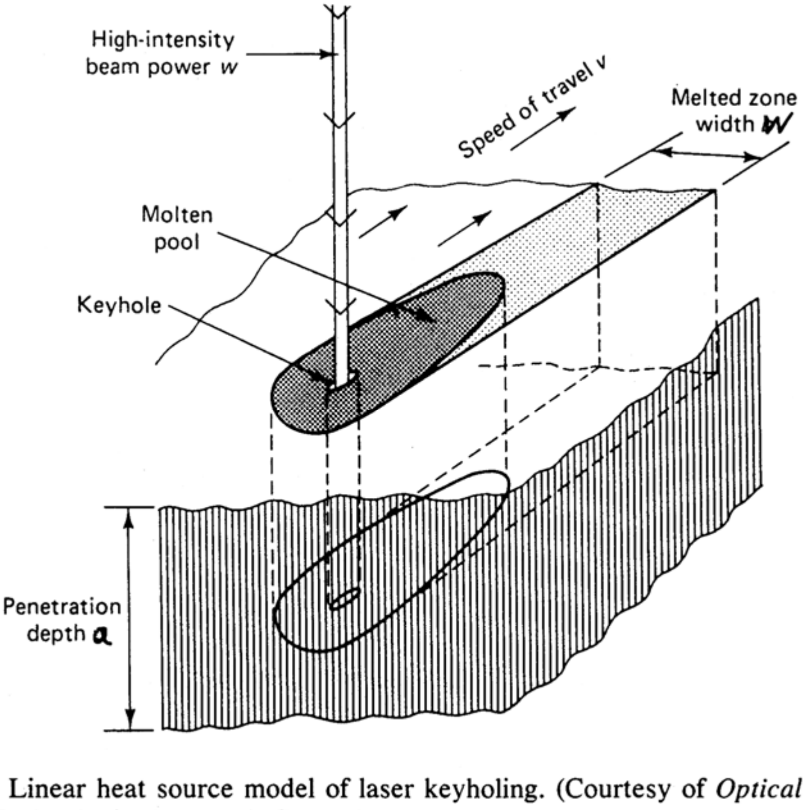


Figure 12-5 Linear heat source model of laser keyholing. (Courtesy of *Optical Engineering* and United Technologies Research Center.)