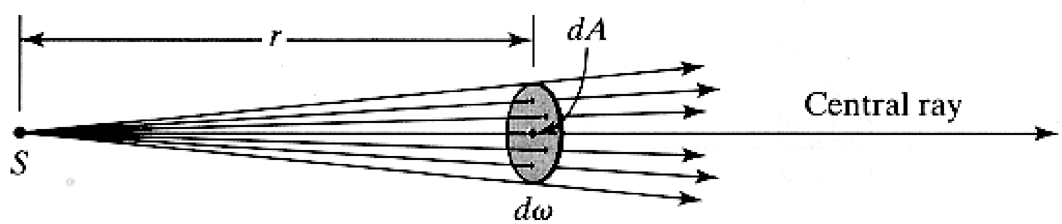
Radiometry
(From Intro to Optics, Pedrotti 1-4)

- Radiometry is measurement of Emag radiation (light)
- Consider a small spherical source
- Assume a black body type emitter: uniform emission
- Total energy radiating from the body over some time is
  Q = total **Radiant Energy** (Joules or Watt sec)
- Rate of energy radiated is
- **Radiant Flux** or **Radiant Power** = Watts

$$\Phi = \frac{dQ}{dt}$$

- I **= Irradiance** or **Light Intensity** is Flux Φ per area A (W/m^2)

$$I = \frac{d\Phi}{dA}$$



Irradiance and Distance

- A sphere radiates energy symmetrically in all directions
- At distance r light can be thought of hitting spherical surface
- Surface of sphere is $4\pi r^2$ thus Irradiance is

$$I = \frac{d\Phi}{dA} = \frac{\Phi}{4\pi r^2}$$

- Thus light falls with the square of the distance
- Thus a typical lightbulb (15 Watt compact fluorescent)
- Only <30% efficient so produces ~5 W of light
- At 1 meter light on surface is 400 mW/m^2 (milliWatts/sq m) or 40 $\mu W/cm^2$ (microwatts/sq cm)
- At 10 meter drops by 100x to 4 mW/m^2 or 0.4 $\mu W/cm^2$
- Lasers are directed beams – lose much less with distance

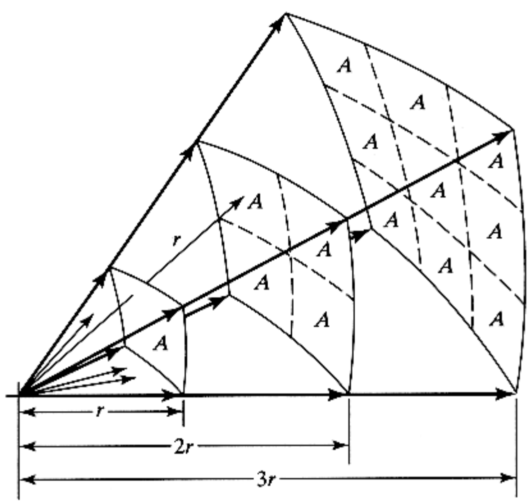


Figure 1-3 Illustration of the inverse-square law. The flux leaving a point source within any solid angle is distributed over increasingly larger areas, producing an irradiance that decreases inversely with the square of the distance.

R   2R   3R

Raiant Intensity

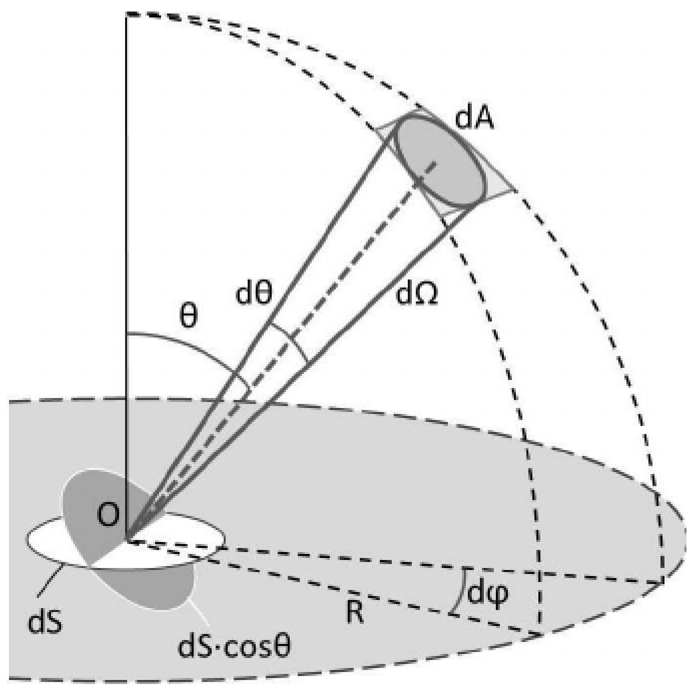
- Define a unit called the **Solid Angle** dω or dΩ in Steradians (sr)

$$d\omega = \frac{dA}{r^2}$$

- 4π Steradians in a sphere around any body
- **Radiant Intensity** I_e is solid angle equivalent of Irradiance

$$I = \frac{d\Phi}{d\omega}$$

- Units Watts/steradian = W/sr
- Often use Radiant Intensity about body
- Most useful because things are not spherically symmetric



Radiance

- What happens when surface is not a sphere
- Consider light at distance r from and angle θ from a flat surface
- Now use **Radiance** L_e or Radiant intensity per unit projected area

$$L_e = \frac{dI_e}{dA\cos(\theta)} = \frac{d^2\Phi}{d\omega(dA\cos(\theta))}$$

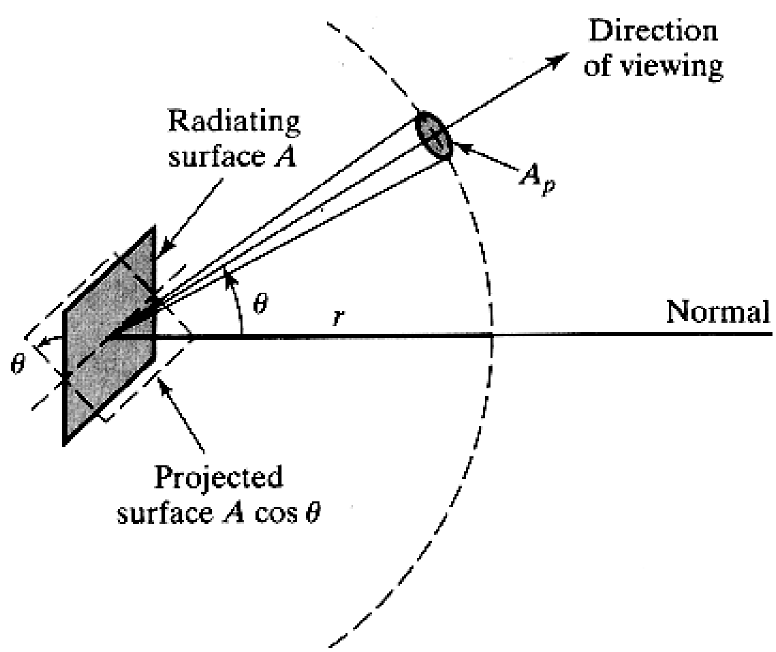
- Units Watts/sr/m^2
- In practice most light emits as a Lambert cosine Law

$$I(\theta)=I(0)\cos(\theta)$$

- Called a Lambertian source
- Consider looking at a flat plate
- Because the project emission area changes with angle then

$$L_e = \frac{I(\theta)}{dA\cos(\theta)} = \frac{I(0)\cos(\theta)}{dA\cos(\theta)} = \frac{I(0)}{A} = C$$

- Thus L_e is a constant
- Objects look equally bright when view in all directions



Emitter and Detector at Different Angles

- Consider an emitting surface 1, and detection surface 2
- Emitting surface is at angle θ_1 to radius connecting
- Receiving surface is at angle θ_2 to radius
- Thus the solid angle created by surface A_2 is

$$d\omega_1 = \frac{dA_2 \cos(\theta_2)}{r^2}$$

- Then Radiance received at surface 2 is

$$L_e = \frac{d^2\Phi_1}{d\omega_1[dA_2 \cos(\theta_2)]} = \frac{d^2\Phi_1}{\left(\frac{dA_1 \cos(\theta_1)}{r^2}\right)[dA_2 \cos(\theta_2)]}$$

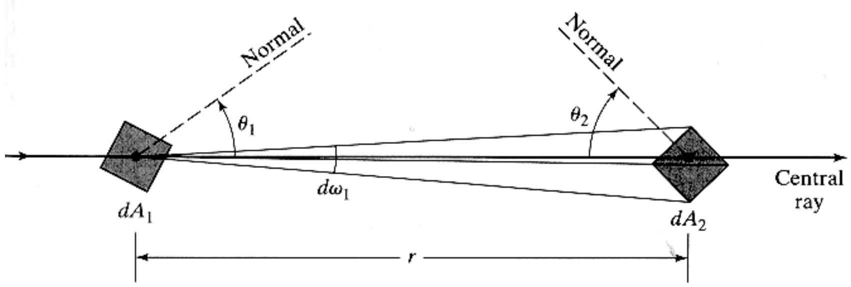


Figure 1-5 Geometry used to show the invariance of the radiance in a uniform, lossless medium.

Two General Radiating Objects

- Consider an emitting object 1, and detection object 2
- Both objects are of arbitrary shape
- Then the radiant power Φ_{12} emitted from object 1 surface area A_1
- Received by object 2 surface area A_2 is

$$d^2\Phi_{12} = \frac{L_1 dA_1 dA_2 \cos(\theta_1)\cos(\theta_{21})}{r_{12}^2}$$

- The total flux received by object 2 from object 1 is

$$\Phi_{12} = \int_{A_1}\int_{A_2} \frac{L_1 dA_1 dA_2 \cos(\theta_1)\cos(\theta_{21})}{r_{12}^2}$$

- Note this is for classically emitting objects
- Adding optical elements (lens or mirror) changes this
- Also lasers do not follow this classic emission
- Because they are Guassian emitters with optical elements

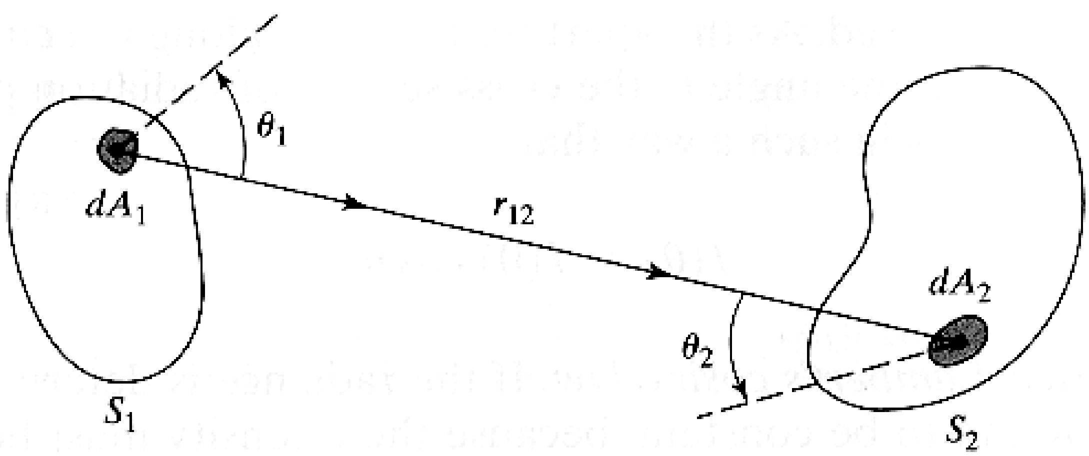


Figure 1-6 General case of the illumination of one surface by another radiating surface. Each elemental *radiating area* dA_1 contributes to each elemental *irradiated area* dA_2.

Basic Optics: Index of Refraction

- Denser materials have lower speeds of light
- Index of Refraction n

$$n = \frac{c}{v}$$

where c = speed of light in vacuum
v = velocity in medium

- Even small changes can create difference in n
- Higher index shortens the wavelength

$$\lambda' = \frac{\lambda}{n'}$$

- Use this reduction if to get higher resolution in microfabrication
- Immersion lithography: put lens in water so it has smaller λ'
- 195 nm light acts as 147 nm light

Substance	Index of refraction	Substance	Index of refraction
Solids:		Liquids at 20°C:	
Ice (H_2O)	1.309	Methyl alcohol (CH_3OH)	1.3290
Fluorite (CaF_2)	1.434	Water (H_2O)	1.3330
Rock salt (NaCl)	1.544	Ethyl alcohol (C_2H_5OH)	1.3618
Quartz (SiO_2)	1.544	Carbon tetrachloride (CCl_4)	1.4607
Zircon ($ZrO_2 \cdot SiO_2$)	1.923	Turpentine	1.4721
Diamond (C)	2.417	Glycerine	1.4730
Fabulite ($SrTiO_3$)	2.409	Benzene	1.5012
Rutile (TiO_2)	2.616 2.903	Carbon disulfide (CS_2)	1.6276

Light Refraction Through Glass and Water

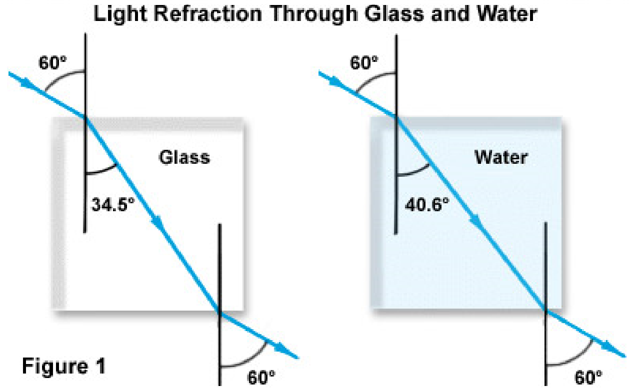


Figure 1

Index of Refraction and Emag waves

- The velocity in a material is set by the emag parameters
- Velocity of light is given by

$$v = \frac{1}{\sqrt{\varepsilon\mu}}$$

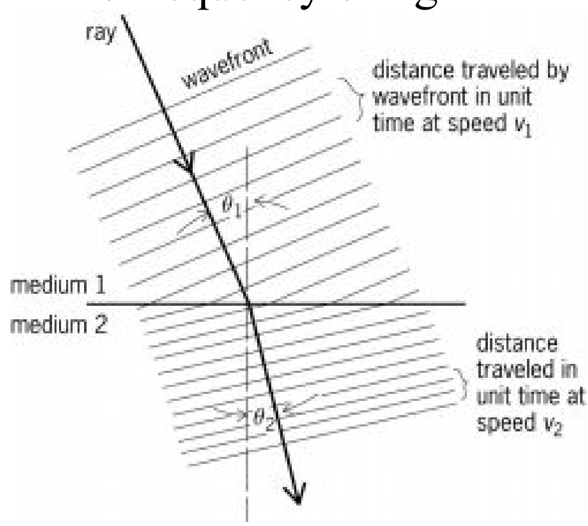
- ε = dielectric constant (permittivity) of material
- μ = magnetic permeability of material
- Thus the index of refraction is

$$n = \frac{c}{v} = \sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}} = \sqrt{\varepsilon_r\mu_r}$$

- Where ε_r and μ_r are relative to vacuum values
- Most optical materials are non magnetic so μ = free space value
- ie $\mu_r = 1$
- For non magnetic materials this means

$$n \propto \sqrt{\varepsilon_r}$$

- Note it must be ε_r at the frequency of light



What Happens to Slow Light Down

- In material the E wave of affects the electrons of material
- Energy is transferred to the e's and back to light
- Results in a drag on the photons moving through material
- In an insulator (eg. glass) electrons almost do not flow
- Energy is returned but with a time delay
- Hence little absorption (ie transparent) but slows down

- Metals have many free electrons (electron sea)
- In metals energy to the free electrons causes them to move
- This results in a loss of energy to the materials
- Light dies out and gets slower

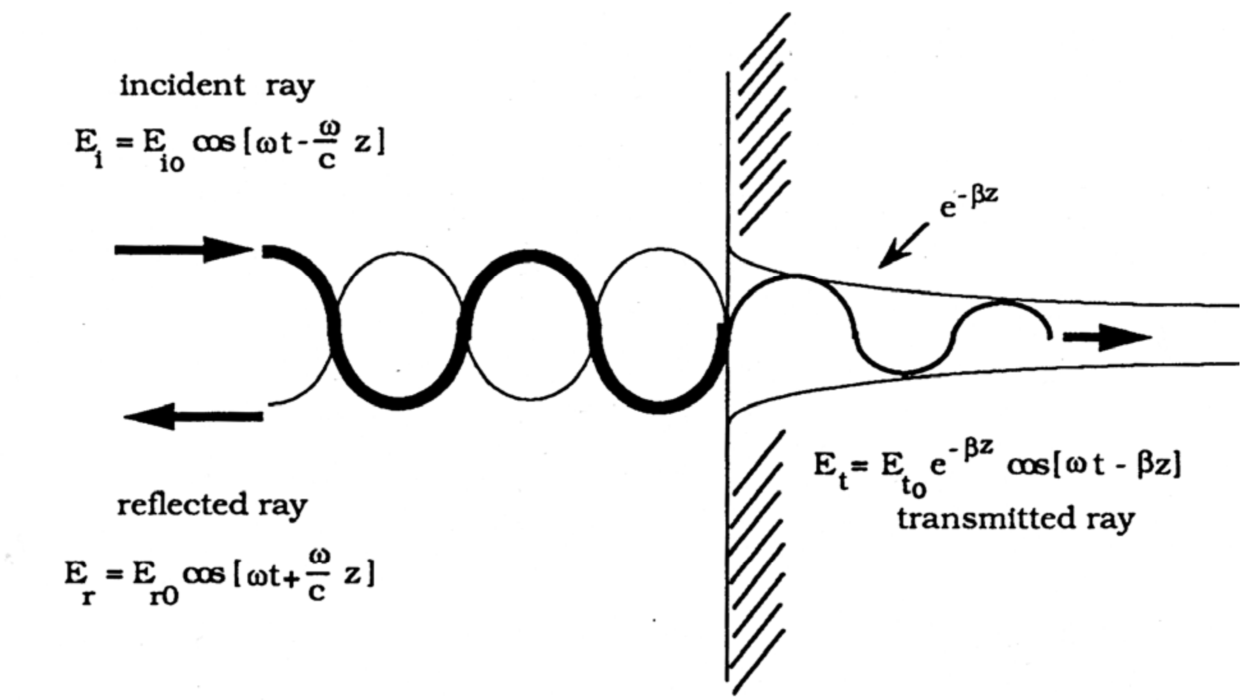


Fig. 2.1. The phase and amplitude of an electromagnetic ray striking an air/solid interface and undergoing reflection and transmission.

Polarization of Light

- Polarization is where the E and B fields aligned in one direction
- All the light has E field in same direction
- Black Body light is not polarized
- But if pass though a material with parallel structure may polarize
- Causes mostly light with same alignment to pass through
- Caller polarizing filters
- Changes unpolarized light into polarized light
- Polarization also often occurs on reflection or on scattering
- Polarizing filters have direction
- If have two filters at 90^o then almost no light passes through

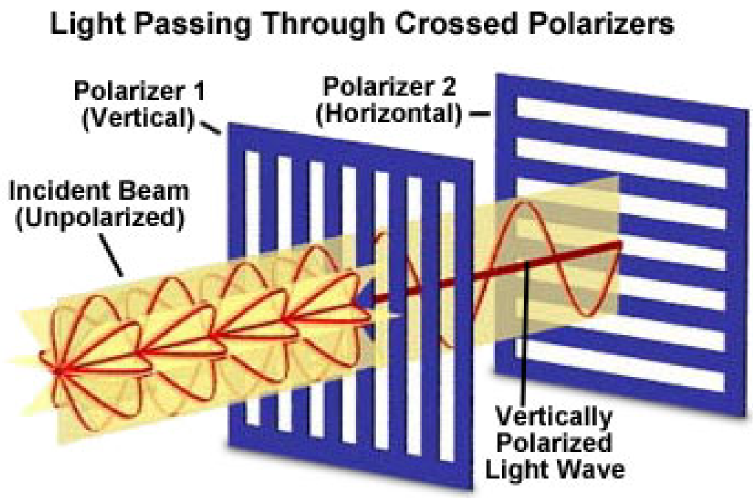
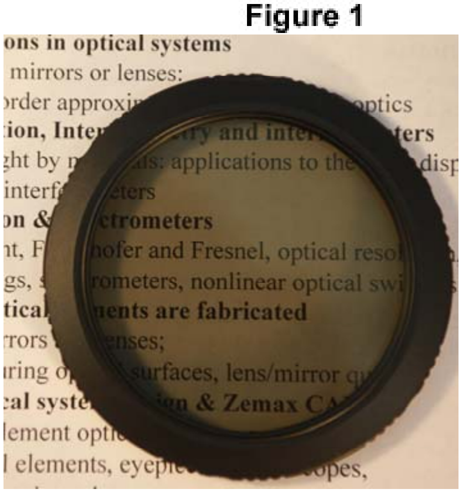


Figure 1



Aligned polarizer: 2x reduction   Crossed polarizer: 256x reduction

Basic Optics: Reflection

- Consider a light beam incident on a surface
- If surface reflects the ray then
  incident angle ϕ_i = reflected angle ϕ_r

$$\varphi_i = \varphi_r$$

- Angles given relative to surface normal
- Surface reflective with index of refraction change
- Metals have very high n, thus very reflective

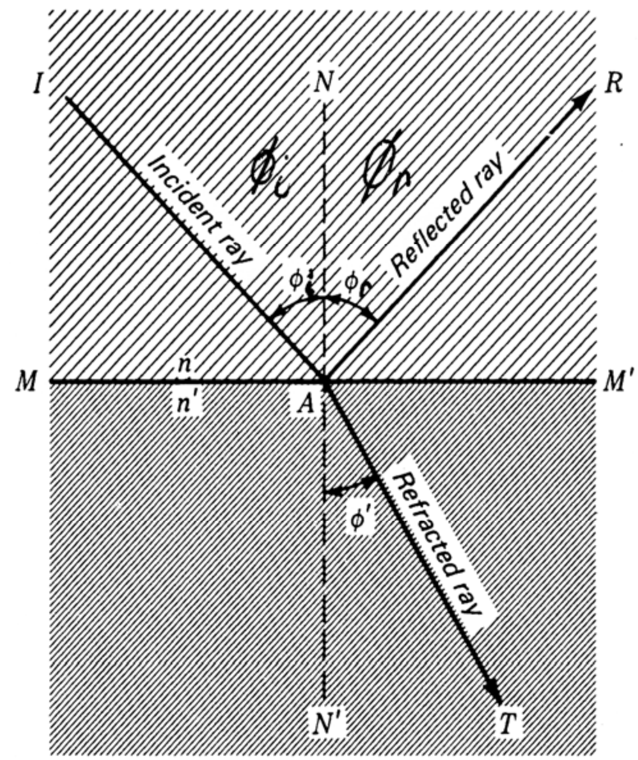


FIGURE 1F
Reflection and refraction at the boundary separating two media with refractive indices *n* and *n'*, respectively.

Basic Optics: Refraction

- Light incident on a surface with change in index n
- Again measure angle with respect to normal
- Then transmitted light will be refracted to new angle φ’
- Refraction angle is given by Snell's law

$$\frac{\sin(\varphi)}{\sin(\varphi')} = \frac{n'}{n}$$

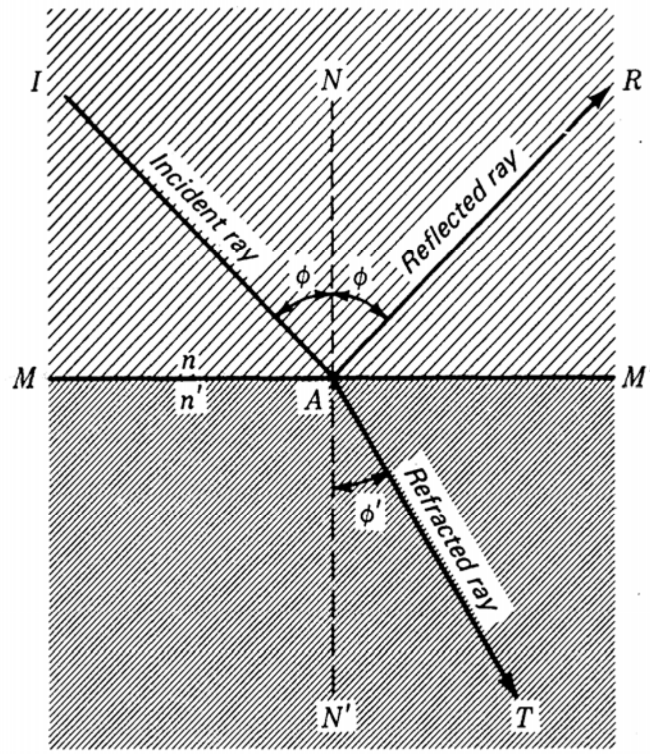


FIGURE 1F
Reflection and refraction at the boundary separating two media with refractive indices *n* and *n′*, respectively.

Measuring Reflection and Refraction

If make half disk of transparent material see both
Place disk on gynaometer – allows you to set angle φ_i
Then send in line of light so hits center of disk
See reflected light at same angle φ_r
Refracted light angle φ' seen
Can calculate index of refraction of materials with this

$$\frac{sin(\varphi)}{sin(\varphi')} = \frac{n'}{n}$$

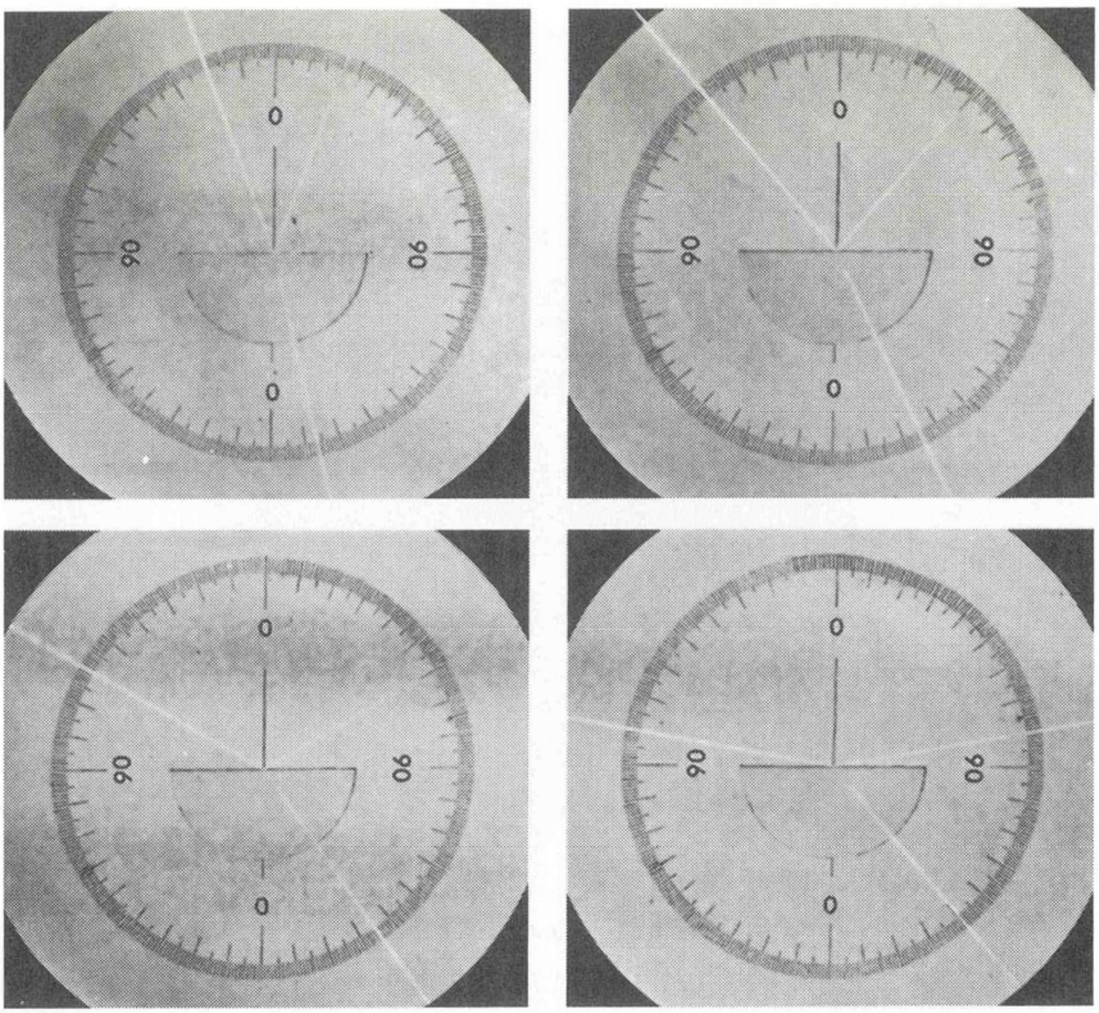
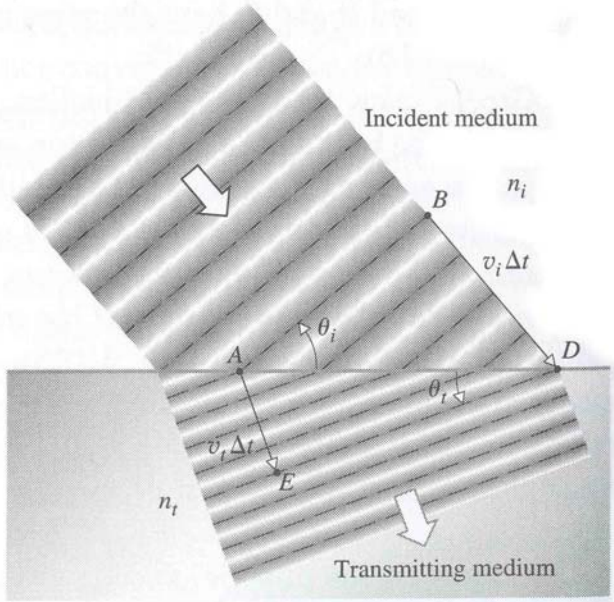
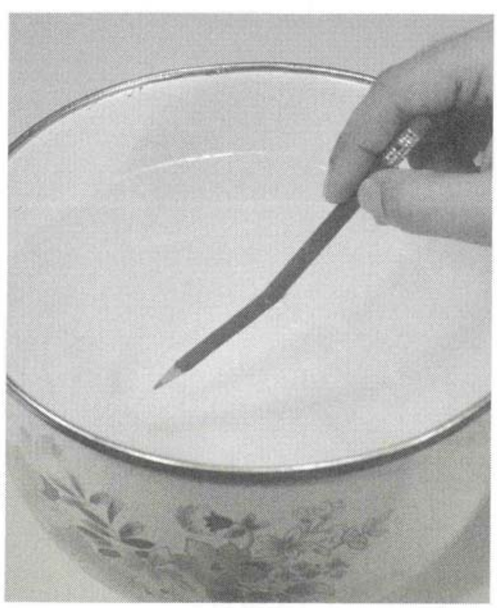


Figure 4.22 Refraction at various angles of incidence. Notice that the bottom surface is cut circular so that the transmitted beam within the glass always lies along a radius and is normal to the lower surface in every case. (Photos courtesy *PSSC College Physics*, D. C. Heath & Co., 1968.)

Waves and Refraction

- Waves entering index n material get refracted
- Waves compress, and are bent
- Atoms at surface absorb light waves and radiate them
- Create the different angle of the light when waves combined
- See same thing with object entering water
- Appears to bend
- If want to hit object in water aim at actual position not apparent





Rays from the submerged portion of the pencil bend on leaving the water as they rise toward the viewer. (Photo by E.H.)

Total Internal Reflection

- When going from a high index n to a low index n'
- At a critical angle ϕ_c the beam is refracted 90^o

$$\frac{sin(\varphi_c)}{sin(90^o)} = \frac{n'}{n}$$

$$sin(\varphi_c) = \frac{n'}{n}$$

- All larger angles (shallower to surface) reflected
- Called "Total Internal Reflection"

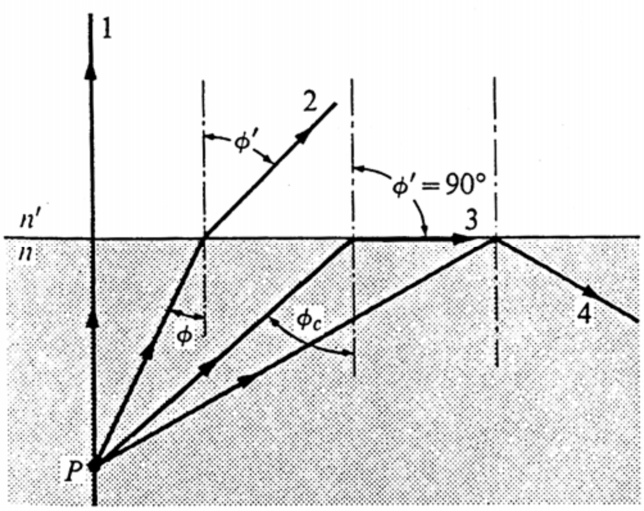
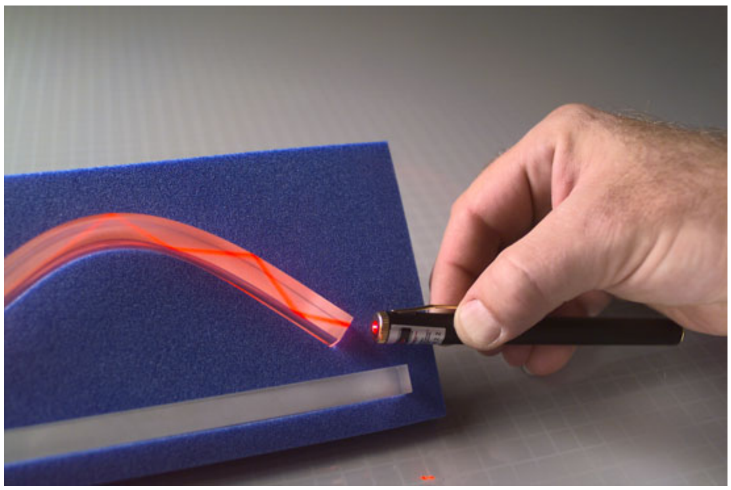
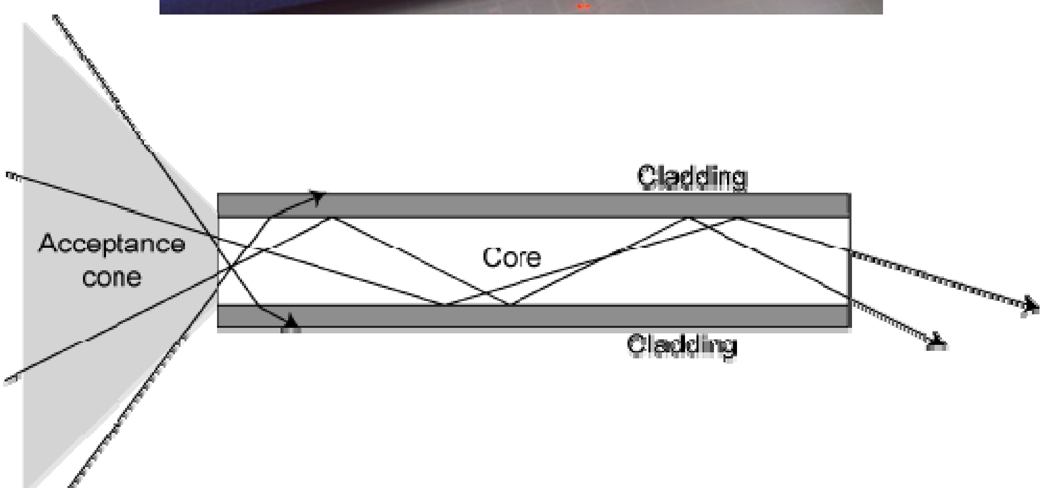


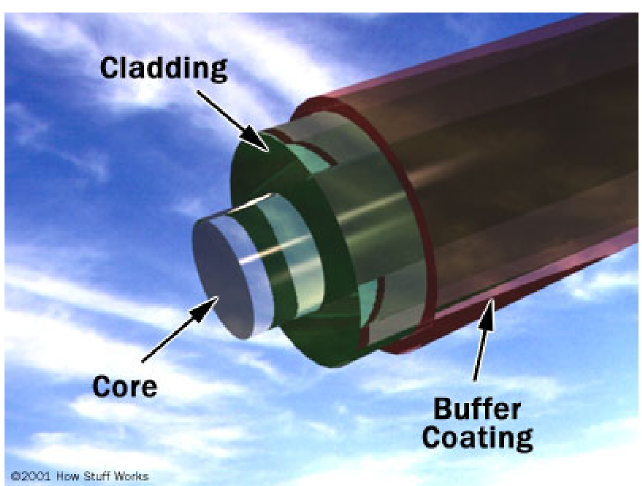
Fig. 38–7. Total internal reflection. The angle of incidence ϕ_c, for which the angle of refraction is 90°, is called the critical angle.

Optical Waveguide

- Fiber optic: confining light within an Optical Waveguide
- High index material surrounded by low index material
- Then get beam confined by Total Internal Refection
- Optical Confinement or Waveguide
- These are integral part of Semiconductor lasers & Fiber optic communications.





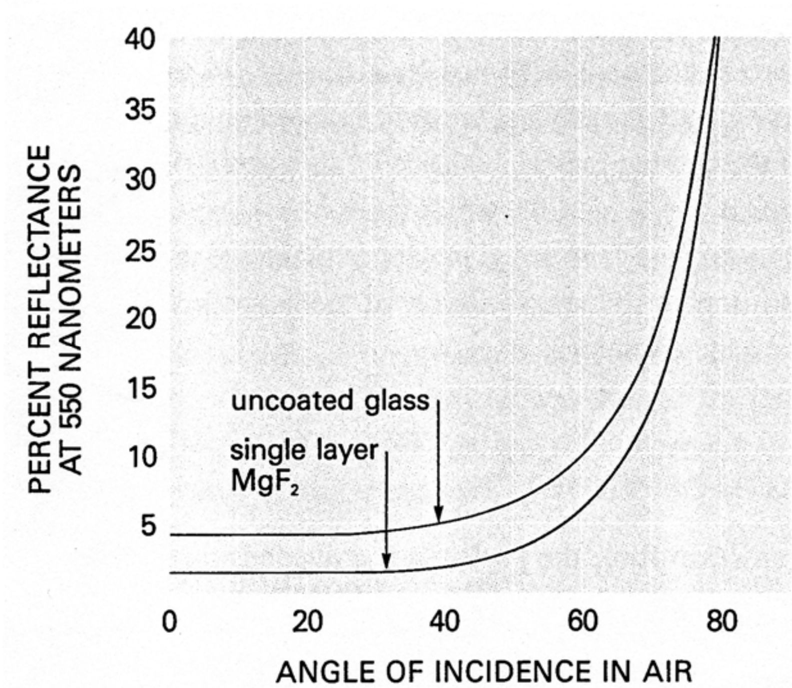


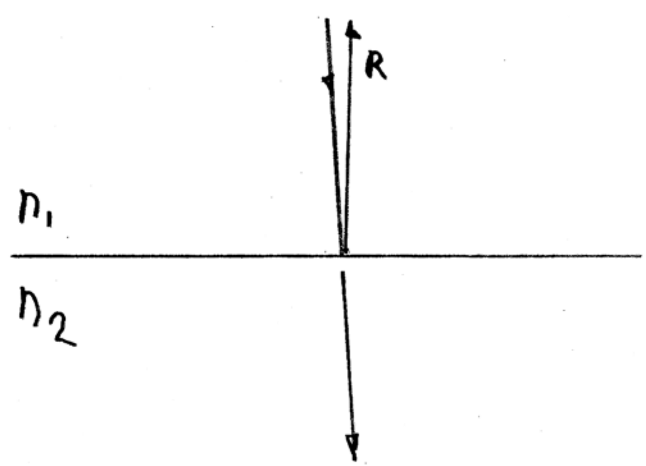
Basic Laser Optics: Reflection Normal to a Surface

- Light normal incident on optical surface
- Reflectance R fraction reflected from surface

$$R = \left(\frac{n_1 - n_2}{n_1 + n_2} \right)^2$$

where surface 1 is the incident side



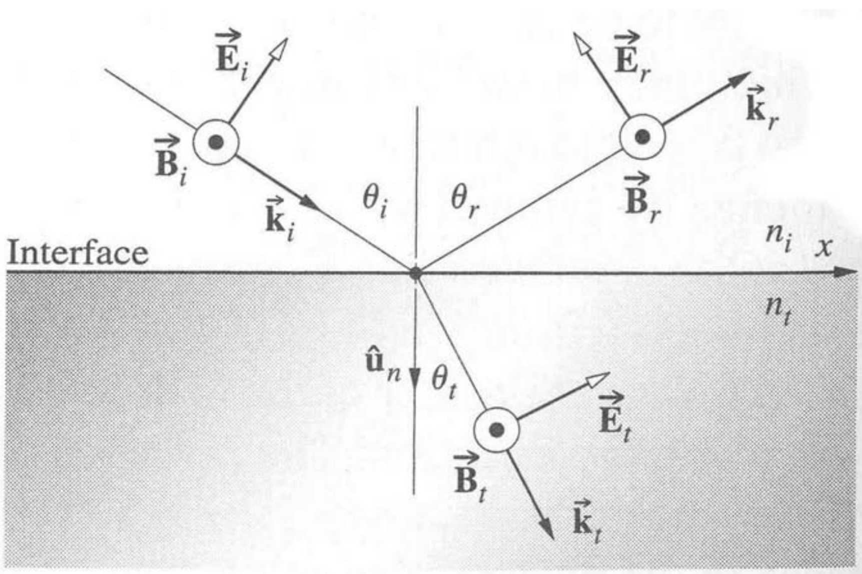


Reflection at Angle to a Surface

- If light comes in at an angle then
  different equations for parallel and perpendicular polarizations
- Formulas from E-Mag theory
- Called the Fresnel Formulas
- Must look at Electric field vectors
- Define r as the values for reflection of E fields
- Let θ_1 be angle light from outside surface (n_1)
- Let θ_2 be angle light is refracted to inside surface (n_2)
- Then the reflection of the E fields gives

$$r_{parallel} = r_p = \left[\frac{n_2 \cos(\theta_1) - n_1 \cos(\theta_2)}{n_2 \cos(\theta_1) + n_1 \cos(\theta_2)} \right]$$

$$r_{perpendicular} = r_s = \left[\frac{n_1 \cos(\theta_1) - n_2 \cos(\theta_2)}{n_1 \cos(\theta_1) + n_2 \cos(\theta_2)} \right]$$



mirage – shallow reflection

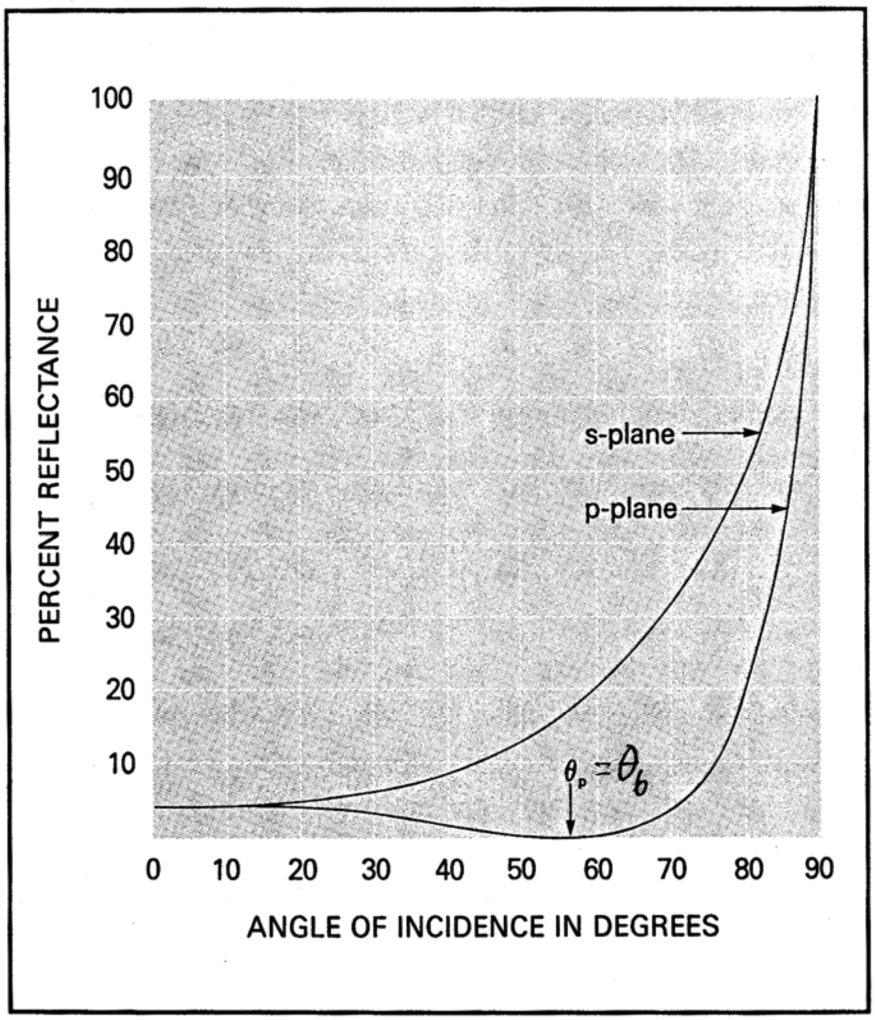
Intensity of Reflection at Angle to a Surface

- Uses Fresnel Formulas for :

$$R_{parallel} = R_p = \left[\frac{tan(\theta_1 - \theta_2)}{tan(\theta_1 + \theta_2)}\right]^2$$

$$R_{perpendicular} = R_s = \left[\frac{sin(\theta_1 - \theta_2)}{sin(\theta_1 + \theta_2)}\right]^2$$

- Important point – as θ_1 angle approaches 90^o for smooth surface
- Reflectivity near 1 ie perfect reflection no matter what material is
- Only fails when surface rough



EXTERNAL REFLECTION at a glass surface (n = 1.52)

Brewster's Law

- When reflected and refracted rays are 90° apart
  reflected light: polarized perpendicular to surface
  transmitted light: polarized parallel to surface
- From Fresnel Formulas
  Reflected parallel polarization goes to zero when

$$\theta_1 + \theta_2 = 90^o = \frac{\pi}{2}$$

- Using Snell's law then at Brewster angle

$$n_1 \sin(\theta_b) = n_2 \sin(\theta_b - 90^o) = n_2 \cos(\theta_b)$$

$$\theta_b = \tan^{-1}\left(\frac{n_2}{n_1}\right)$$

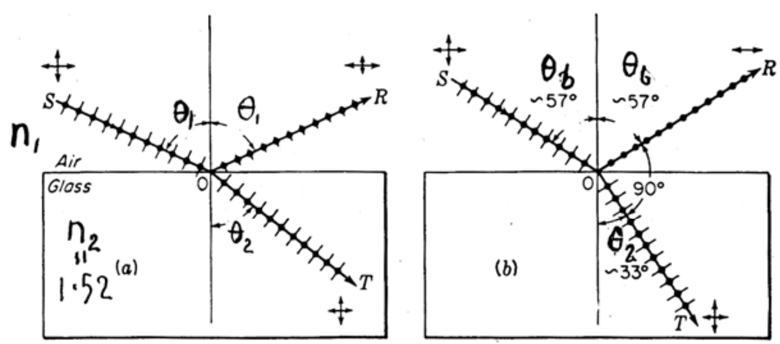
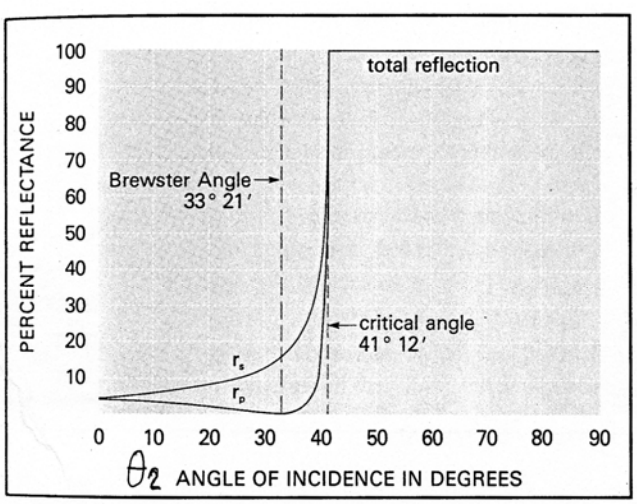


FIGURE 24D
(*a*) Polarization by reflection and refraction. (*b*) Brewster's law for the polarizing angle.



INTERNAL REFLECTION at a glass surface (n = 1.52) showing s- and p-polarized components.

Scattering from Surface

- If surface is smooth we get "specular" reflection
- If surface is not perfectly smooth get scattering
- Called Diffuse reflection
- In practice to reduce scattering surface roughness must be < $\lambda/4$
- Thus for optics needs typically < 120 nm

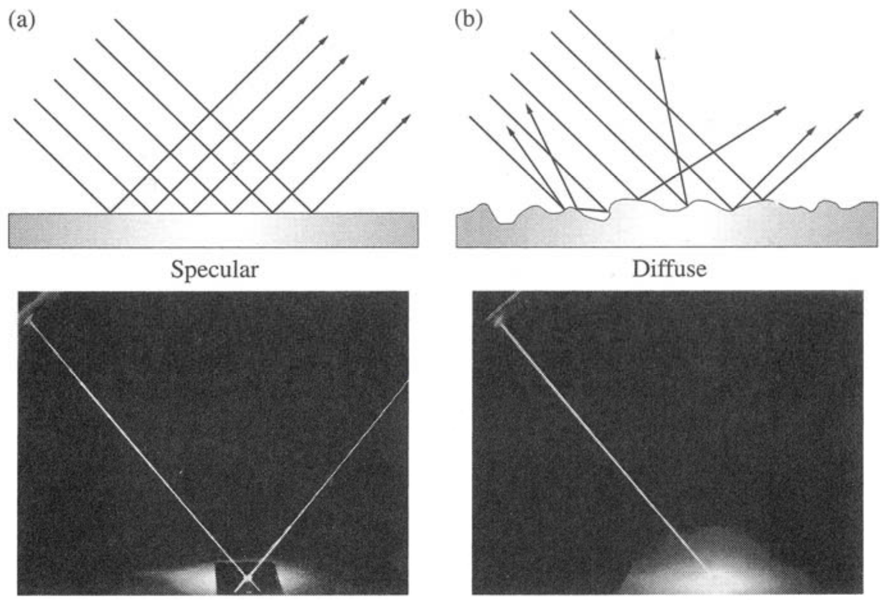
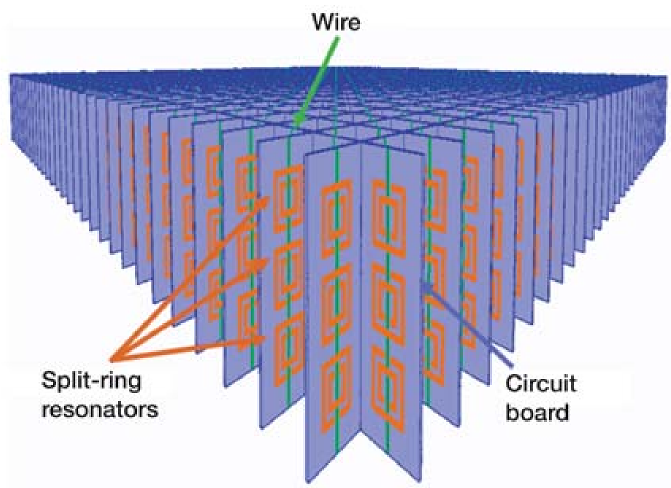
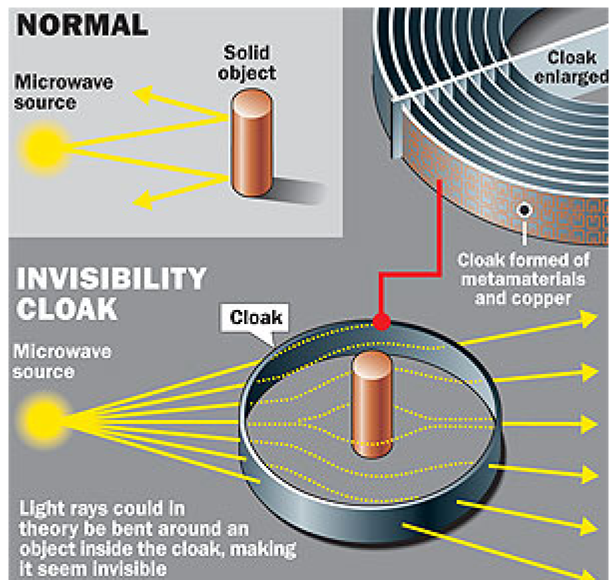


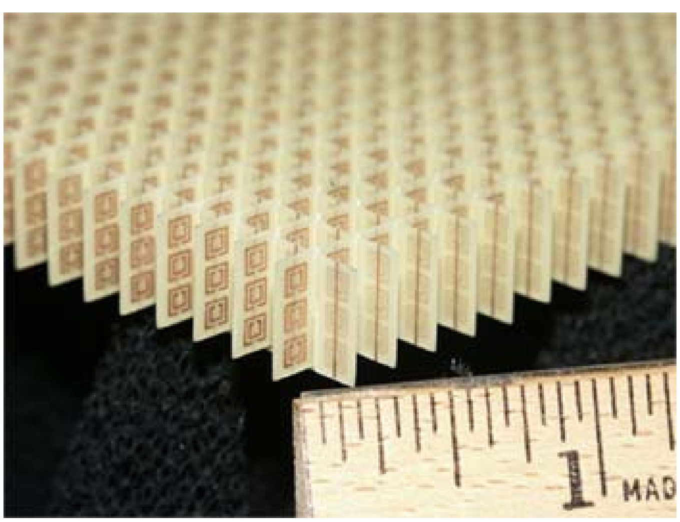
Figure 4.18 (*a*) Specular reflection. (*b*) Diffuse reflection.
(Photos courtesy Donald Dunitz.)

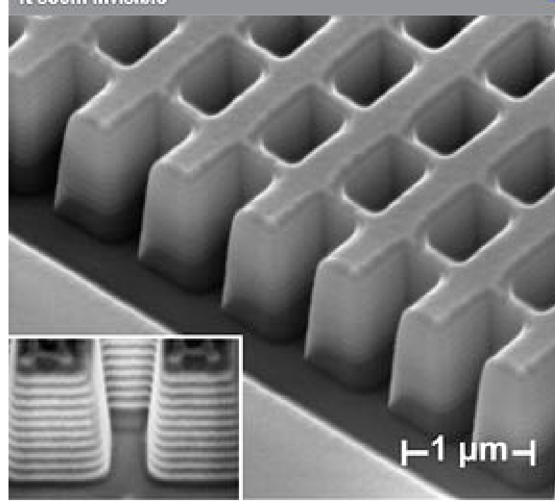
Negative Index of Refraction: Meta Materials

- Normal materials cannot have n<1
- However can create a metamaterial that does
- Proposed by Victor Veselago in 1997
- Actually create material that interacts with Emag wave
- Causes phase delay in emission so index is negative
- First done in microwaves – 13Ghz
- Now achieved optical with nanolayer materials
- Creates “Invisibility cloak” by bending light around object
- Bends light so passes around object
- Works in only specific wavelength &directions









Optics and Parallax Assumption

- Often assume dealing with small angles in optics as 1st approx
- Called the Parallax assumption
- For angles less than 5 degrees then can assume
- Comes from truncating Taylor's series

$$sin(\theta) = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} \cdots \approx \theta \qquad when\ \theta < 0.09$$

(angles in radians)

- Cause of many corrections to lenses/mirrors

Angle	Sin(x)	Approx	%error
5	0.08716	0.08727	0.127
10	0.17365	0.17453	0.510
15	0.25882	0.26180	1.152
20	0.34202	0.34907	2.060
25	0.42262	0.43633	3.245
30	0.50000	0.52360	4.720
35	0.57358	0.61087	6.501
40	0.64279	0.69813	8.610
45	0.70711	0.78540	11.072
50	0.76604	0.87266	13.918
55	0.81915	0.95993	17.186
60	0.86603	1.04720	20.920
65	0.90631	1.13446	25.174
70	0.93969	1.22173	30.014
75	0.96593	1.30900	35.517
80	0.98481	1.39626	41.780
85	0.99619	1.48353	48.920

Basic Optics: Mirrors (Hecht 5.4)

- Mirrors basic optical device: simpler reflectors of light
- A smooth surface: quality of reflection = polished surface
- Either a polished reflecting material (metals: copper, silver)
- More often glass surface coated with metal (silver, aluminum)
- Oldest optics: polished copper mirrors 4000BC in Mesopotamia
- As reflectors can be nearly wavelength independent
- Flat surface mirrors widely used
- But high quality optically flat ones hard to make

Flat Mirror Concave Mirror

Basic Laser Optics: Mirrors (Hecht 5.4)

- Most optical devices are circularly symmetric
- Note some are not eg cylindrically symmetric (discuss later)
- Define distances relative to the axis of the optical device
- Vertex is the point A where axis intersects the mirror
- Radius of curvature of the mirror is r located at point C
- Assume parallel light (a plane wave) aligned with the axis
- Then light ray hitting mirror at T is reflected
- Normal at T is radius to C so light makes angle ϕ to normal
- It is reflected at angle $\phi' = \phi$ to point F
- As all points on mirror have normal to radius C
- Then all focused through F
- Concave mirror focuses light at Focal Point F
- Convex mirror light radiates as if from Focal Point
- Focal length f is

$$f = -\frac{r}{2}$$

where r = radius of curvature of mirror

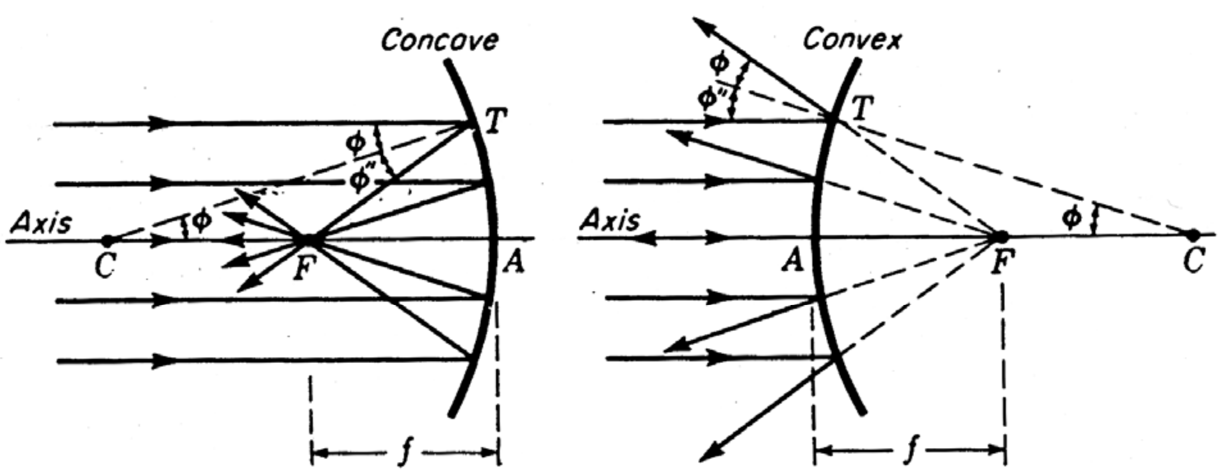


FIGURE 6A
The primary and secondary focal points of spherical mirrors coincide.

Distances in Mirrors & Lenses

- Define distances relative to the axis of the optical device
- Vertex is the point A where axis intersects the mirror
- Measure all distances in cm or m
- Radius r or R of curvature of the mirror is r located at point C
- Then parallel light ray from object
- Assume object is place in front of the mirror (point 3)
- Height of object is M

s or s_o is the object distance from object 3 to vertex A

- Observe an image at point 9

s' or s_i is the image distance from vertex A to image 9

- Height of image is M'

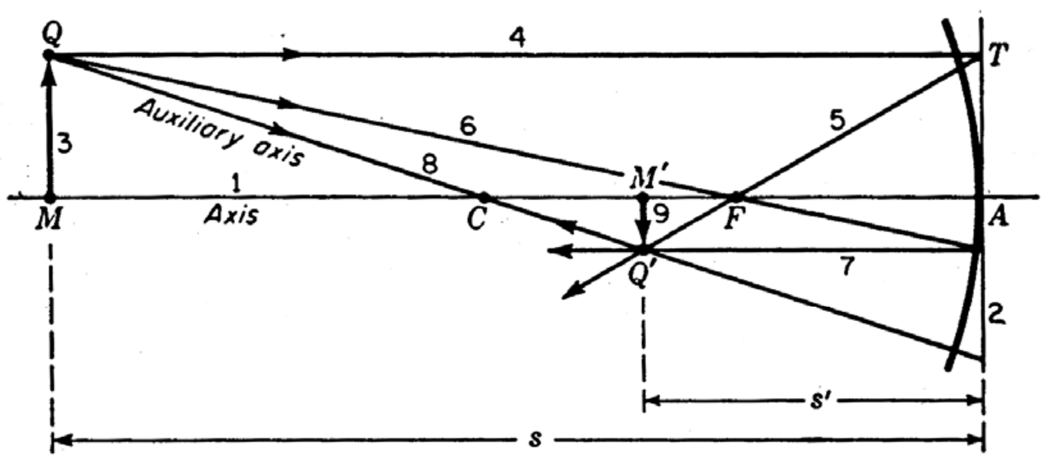
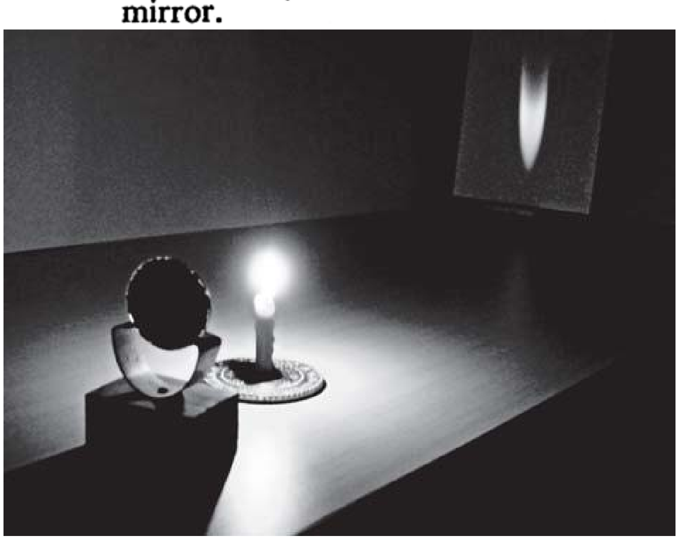
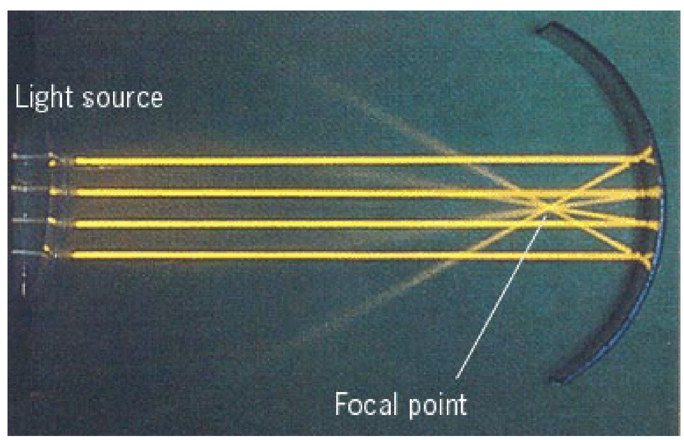


FIGURE 6E
Parallel-ray method for graphically locating the image formed by a concave mirror.





Mirror Conventions

- From Jenkins & White: Fundamentals of Optics
- Distance + if left to right, - if right to left
- Incident rays travel left to right
- Reflected rays travel right to left
- Focal length measured from focal pt. to vertex
  f positive for concave, negative for convex
- Radius from vertex to centre of Curvature
  r negative for concave, positive for convex
- Object distance s and image s' measured relative to vertex
  s & s' positive & real if to left of mirror (concave)
  s & s' negative & virtual if to right of mirror (concave)

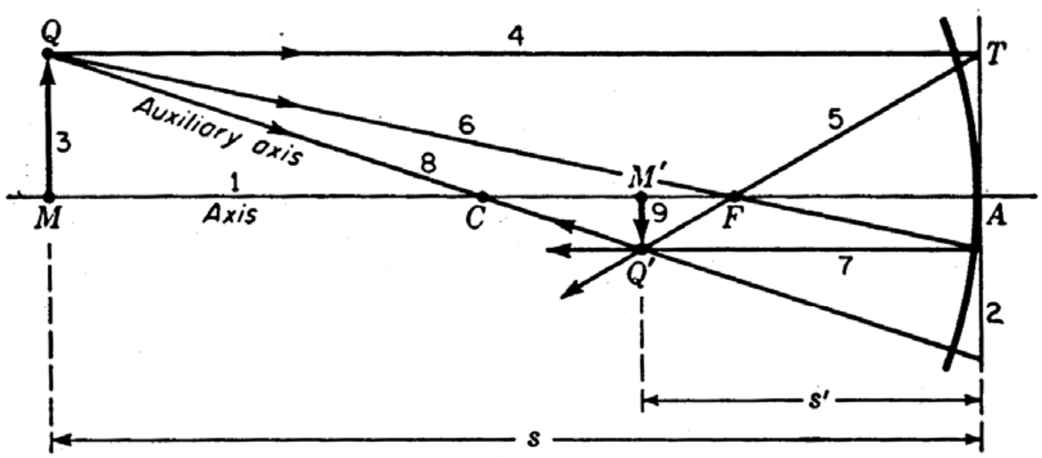


FIGURE 6E
Parallel-ray method for graphically locating the image formed by a concave mirror.

Reflection from Convex and Concave Surfaces

Figure 3

Mirror Formulas

- Basic Mirror Formula

$$\frac{1}{s}+\frac{1}{s'}=-\frac{2}{r}$$

- Primary focal point: image at infinity and s given by

$$\frac{1}{s}+\frac{1}{\infty}=-\frac{2}{r} \quad or \quad s=-\frac{r}{2}$$

- Secondary focal point: object at infinity

$$\frac{1}{\infty}+\frac{1}{s'}=-\frac{2}{r} \quad or \quad s'=-\frac{r}{2}$$

- Primary and secondary the same: Thus mirror formula

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \qquad f=-\frac{r}{2}$$

where f = the focal length

- Mirror Magnification m

$$m=\frac{M'}{M}=-\frac{s'}{s}$$

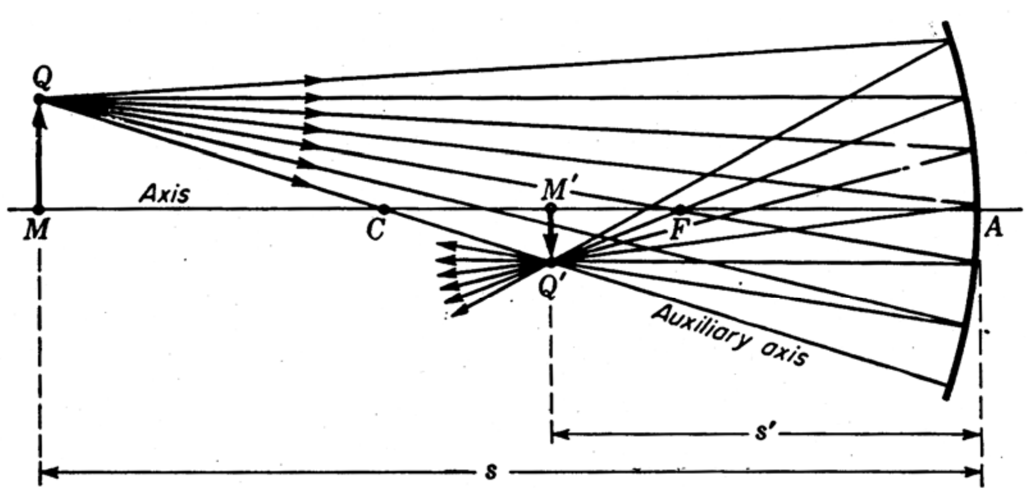


FIGURE 6C
Real image due to a concave mirror.

Real Mirrors: Spherical Aberration

- For large mirrors parallax formula fails
- Get not all light focused at same point
- Called circle of confusion & creates Spherical Aberration
- Creates fuzzy images

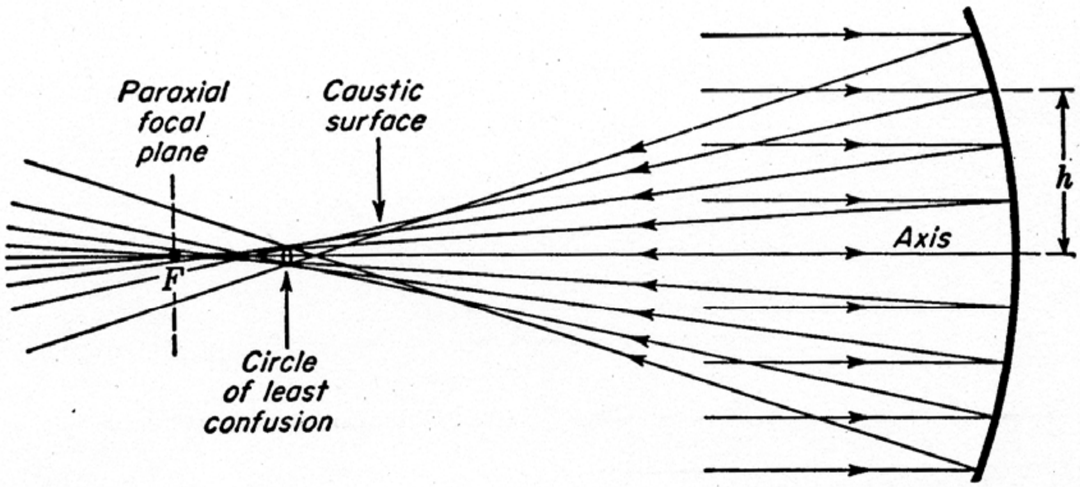


FIGURE 6K
Spherical aberration of a concave spherical mirror.

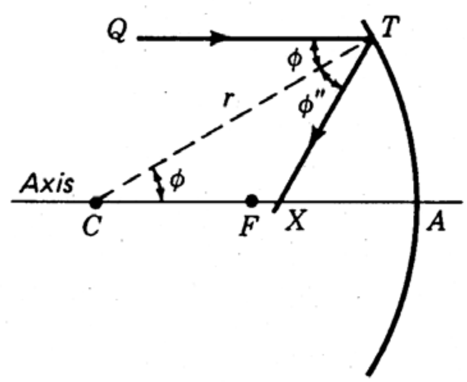


FIGURE 6L
Geometry showing how marginal rays parallel to the axis of a spherical mirror cross the axis inside the focal point.

Spherical Aberration & Parabolic Mirrors

- Corrected using parabolic mirror shape
- Focuses all light to focal point
- Parabola is curve with points equidistant from focus and straight line, the directex

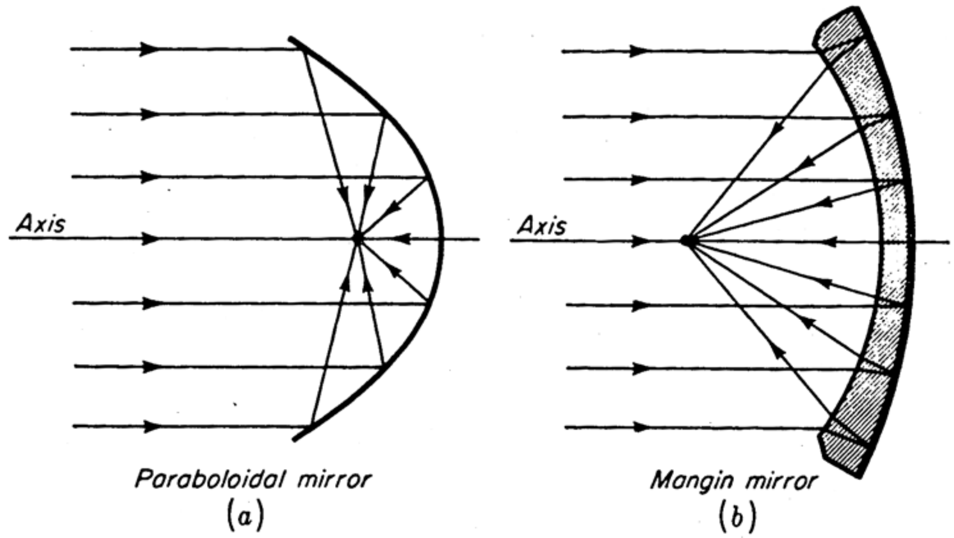


FIGURE 6M
(*a*) Concave parabolic mirror and (*b*) concave spherical mirror, corrected for spherical aberration.

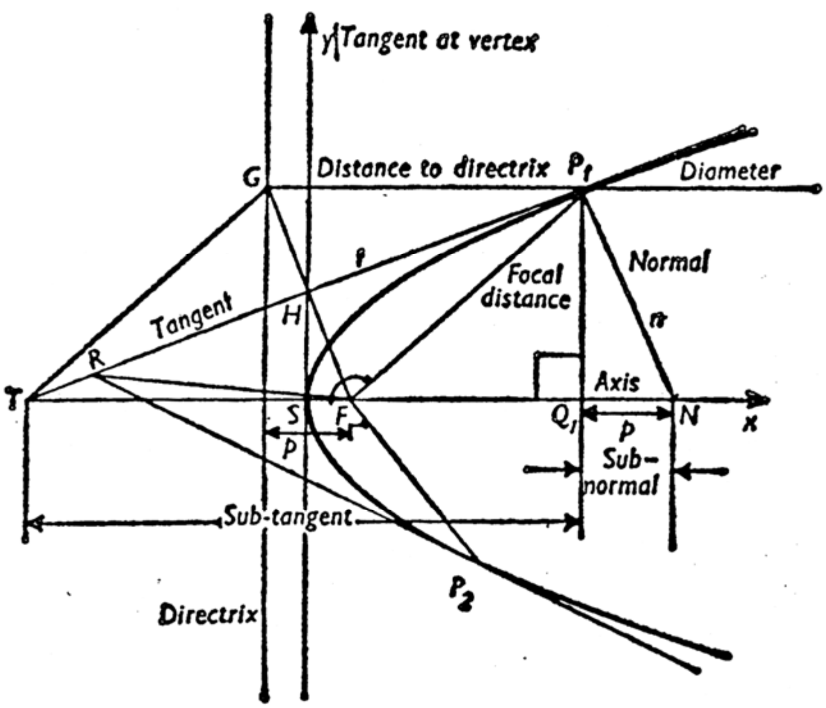


FIG. 210

Focal Ratio or F number

- F number (F#) is important measure of optical system speed
- Focal Ratio is related to the mirror diameter ϕ

$$F\# = \frac{f}{\varphi}$$

- Smaller F#, more light gather power
- When F# changes by $\sqrt{2}$ then amount of light focused doubles
- Hence camera F# change as 16, 11, 8, 5.6, 4, 2.8, 2, 1.4
- These are called F stops
- For cameras: light gathered for proper exposure is constant
- Thus when $F\#_2 = \sqrt{2}\, F\#_1$ then for exposure time t_2 is

$$t_2 = \left[\frac{F\#_2}{F\#_1}\right]^2 t_1 = 2t_1$$

- Rough rule F# >>12 then parabola & sphere nearly same
- Important for lens and mirror fabrication
- Easier to make spherical optics than parabolic
- Do not need correction in that case

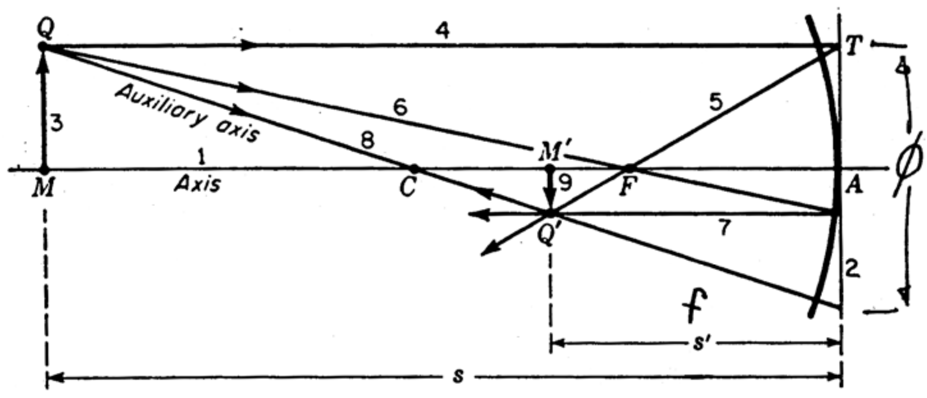


FIGURE 6E
Parallel-ray method for graphically locating the image formed by a concave mirror.

Real Mirrors: Surface quality

- Real optical mirrors must be very smooth
- Measure defects relative to wavelength of light
- Defects create scattered light
  Hence defects must be smaller than wavelength
- Minimum needed λ/4 - creates only small deviations
- λ/10 or λ/20 often specified for better quality
- This is true for either flat or curved mirrors
- Testing mirrors eg telescope making, shows these defects