Lens Conventions

- From Jenkins & White: Fundamentals of Optics, pg 50
- Incident rays travel left to right
- Object distance s + if left to vertex, - if right to vertex
- Image distance s' + if right to vertex, - if left to vertex
  Opposite to mirrors
- Focal length measured from focal point to vertex
  f positive for converging, negative for diverging
- r positive for convex surfaces
  r negative for concave
- Object and Image dimension
  + if up, - if down from axis

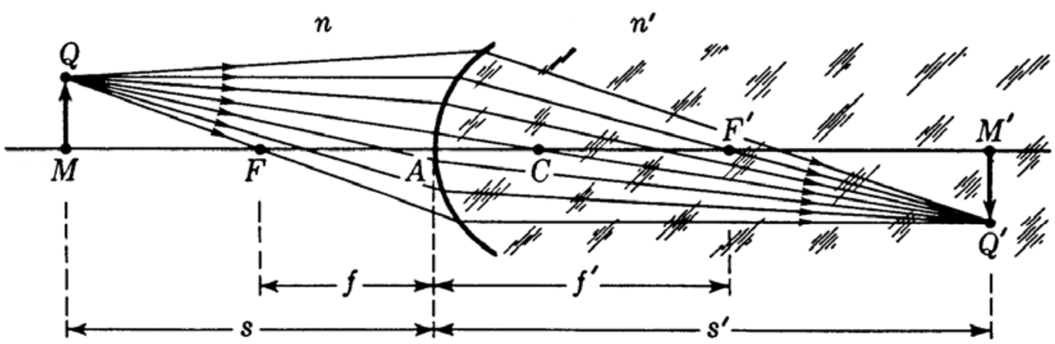


FIGURE 3D
All rays leaving the object point Q and passing through the refracting surface are brought to a focus at the image point Q'.

Gaussian Formula for a Spherical Surface

- The radius of curvature r controls the focus
- Gaussian Lens formula

$$\frac{n}{s} + \frac{n'}{s'} = \frac{n' - n}{r}$$

where *n* index on medium of light origin
n' index on medium entered
r = radius of curvature of surface

- Clearly for s' infinite (parallel light output) then s = f (primary focal length)

$$\frac{n}{s} + \frac{n'}{\infty} = \frac{n}{f} = \frac{n' - n}{r}$$

$$f = \frac{nr}{n' - n}$$

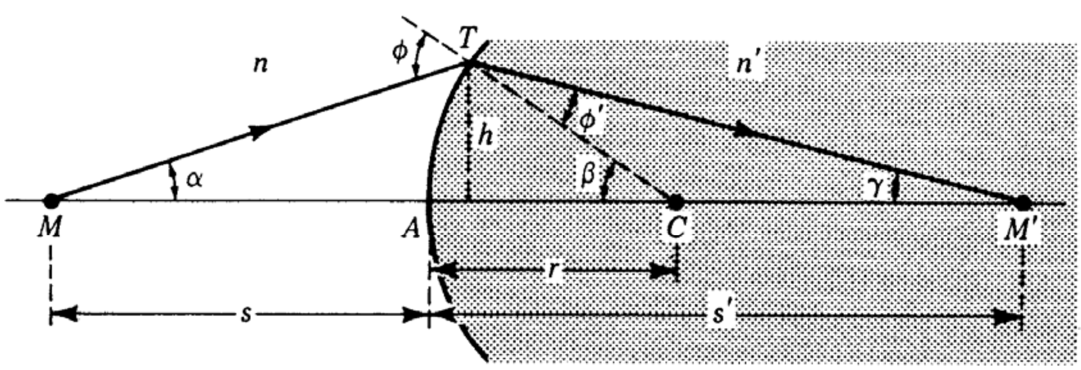


FIGURE 3K
Geometry for the derivation of the paraxial formula used in locating images.

Thin Lens

- Assume that thickness is very small compared to s, s' distances
- This is often true for large focal length lenses
- Primary focus f on left convex lens, right concave
- Secondary focus f' on right convex, left concave
- If same medium on both sides then thin lens approximation is

$$f = f'$$

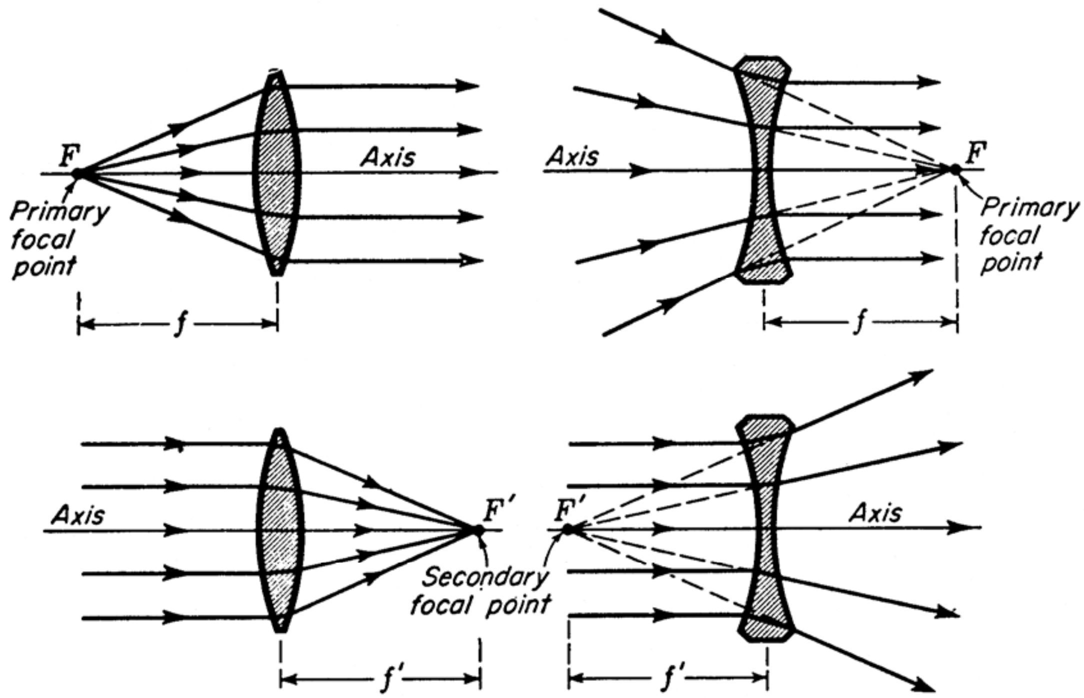


FIGURE 4A
Ray diagrams illustrating the primary and secondary focal points F and F' and the corresponding focal lengths f and f' of thin lenses.

Basic Thin Lens formula

- Basic Thin Lens formula

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

- Lens Maker's formula for calculating f based on lens r_1, r_2 & n

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

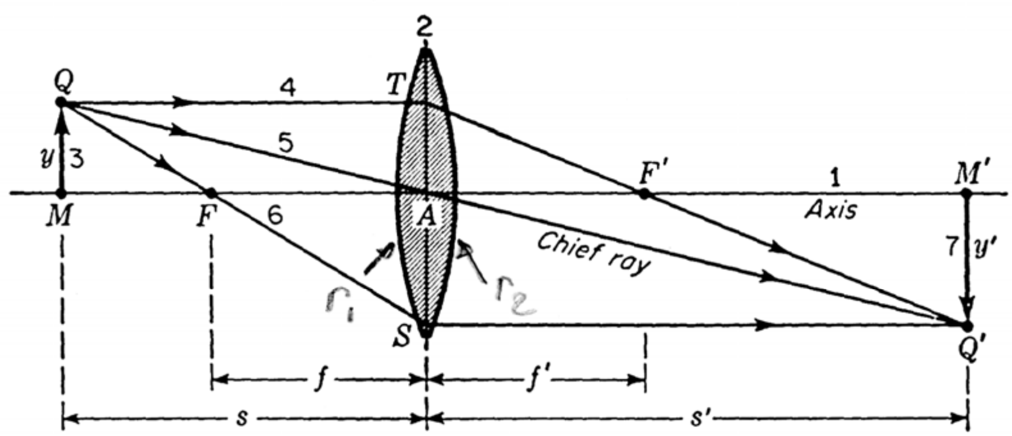


FIGURE 4D
The parallel-ray method for graphically locating the image formed by a thin lens.

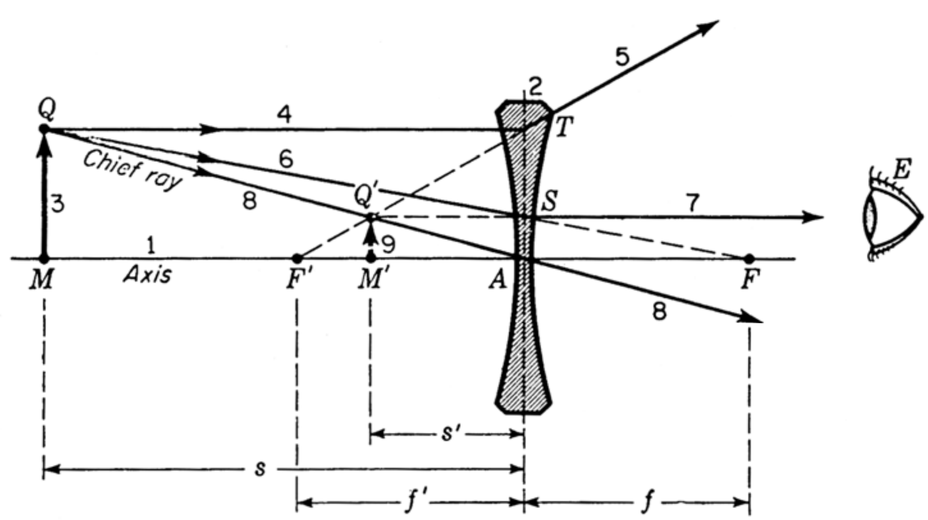


FIGURE 4G
The parallel-ray method for graphically locating the virtual image formed by a negative lens.

Magnification and Thin Lenses

- f positive for convex, negative for concave
- Magnification of a lens is given by

$$m = \frac{M'}{M} = -\frac{s'}{s} = \frac{f}{f-s} = \frac{f-s'}{f}$$

- Magnification is negative for convex, positive for concave

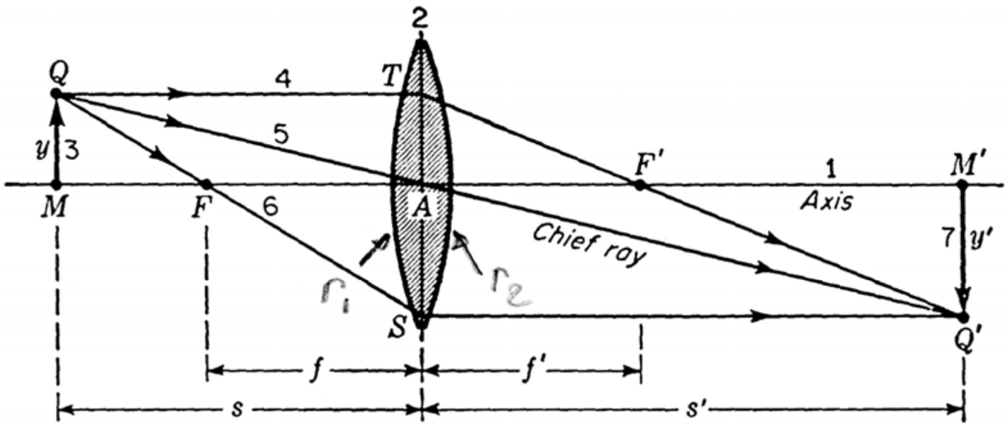


FIGURE 4D
The parallel-ray method for graphically locating the image formed by a thin lens.

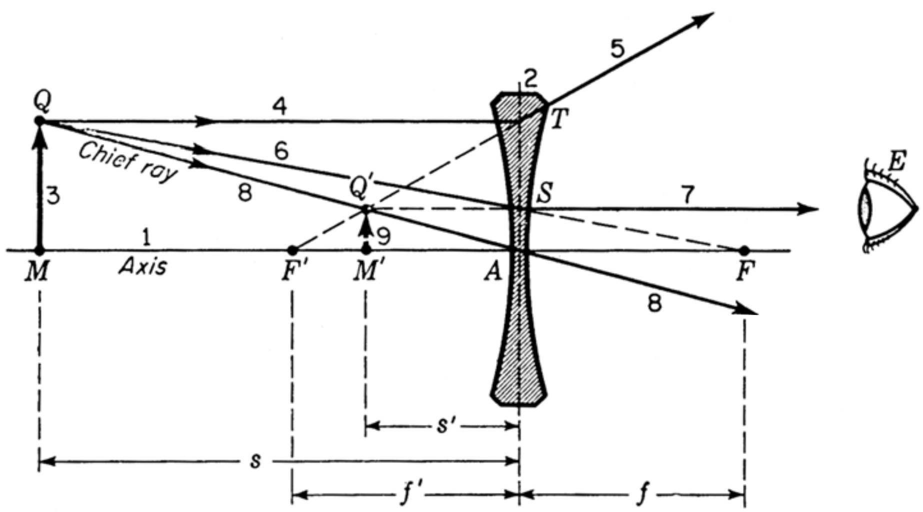


FIGURE 4G
The parallel-ray method for graphically locating the virtual image formed by a negative lens.

Why is Light Focus by a Spherical Lens

- Why does the light get focused by a lens
- Consider a curved glass surface with index n' on right side
- Radius of curvature r is centered at C
- Let parallel light ray P at height h from axis hit the curvature at T
- Normal at T is through C forming angle ϕ to parallel beam
- Beam is refracted by Snell's law to angle ϕ' to the normal

$$n \sin(\varphi) = n' \sin(\varphi')$$

- Assuming small angles then sin(ϕ)~ϕ and

$$\sin(\varphi) = \frac{n'}{n}\sin(\varphi') \quad or \quad \varphi \cong \frac{n'}{n}\varphi'$$

- From geometry for small angles

$$\sin(\phi) = \frac{h}{r} \quad or \quad \phi \cong \frac{h}{r}$$

- Angle θ' the beam makes to the axis is by geometry

$$\theta' = \phi' - \phi = \frac{n'}{n}\phi - \phi = \frac{n'-n}{n}\phi \cong \frac{h}{r}\left[\frac{n'-n}{n}\right]$$

- Thus the focus point is located at

$$f = \frac{h}{\sin(\theta')} \cong \frac{h}{\theta'} \cong h\frac{r}{h}\left[\frac{n}{n'-n}\right] \cong \frac{nr}{n'-n}$$

- Thus all light is focused at same point independent of h position
- This fails when small angle approx exceeded thus lens aberrations

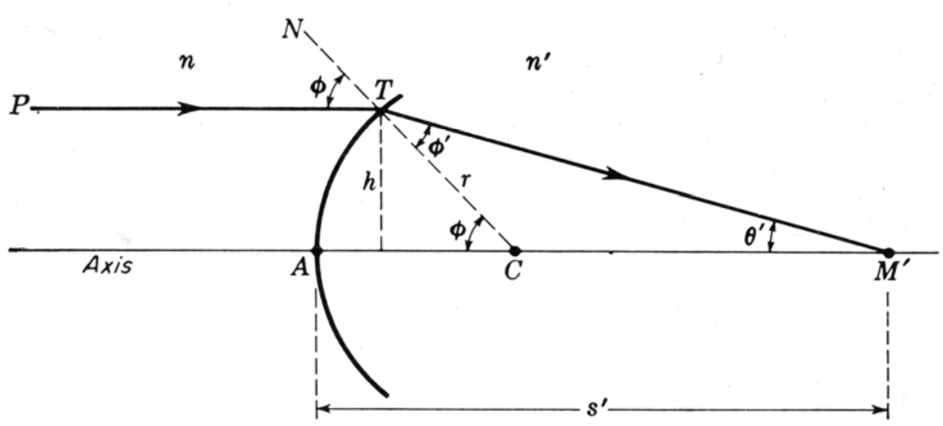


FIGURE 8E
Geometry for ray tracing with parallel incident light.

Simple Lens Example

- Consider a glass (n=1.5) plano-convex lens radius r_1 = 10 cm
- By the Lens Maker's formula

$$\frac{1}{f}=(n-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)=(1.5-1)\left(\frac{1}{10}-\frac{1}{\infty}\right)=\frac{0.5}{10}=0.05$$

$$f=\frac{1}{0.05}=20\ cm$$

- Now consider a 1 cm candle at s = 60 cm from the vertex
- Where is the image

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$$

$$\frac{1}{s'}=\frac{1}{f}-\frac{1}{s}=\frac{1}{20}-\frac{1}{60}=0.03333 \qquad s'=\frac{1}{0.0333}=30\ cm$$

- Magnification $m=\frac{M'}{M}=-\frac{s'}{s}=-\frac{30}{60}=-0.5$
- Image at 30 cm other side of lens inverted and half object size
- What if candle is at 40 cm (twice f)

$$\frac{1}{s'}=\frac{1}{f}-\frac{1}{s}=\frac{1}{20}-\frac{1}{40}=0.05 \quad s'=\frac{1}{0.05}=40\ cm \qquad m=-\frac{s'}{s}=-\frac{40}{40}=-1$$

- Image is at 40 cm other side of lens inverted and same size (1 cm)

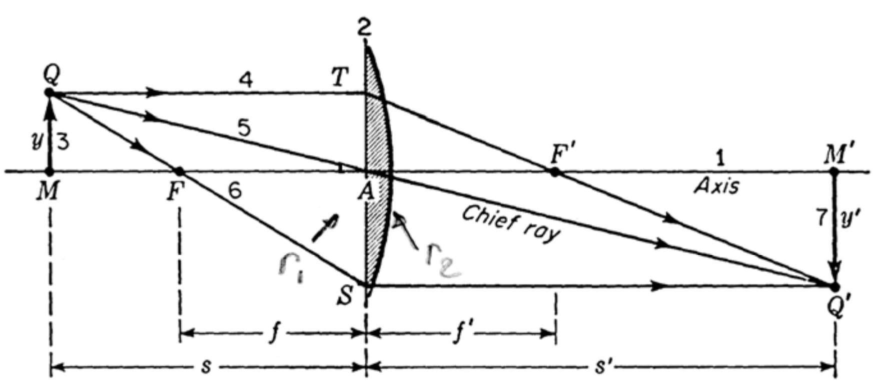


FIGURE 4D
The parallel-ray method for graphically locating the image formed by a thin lens.

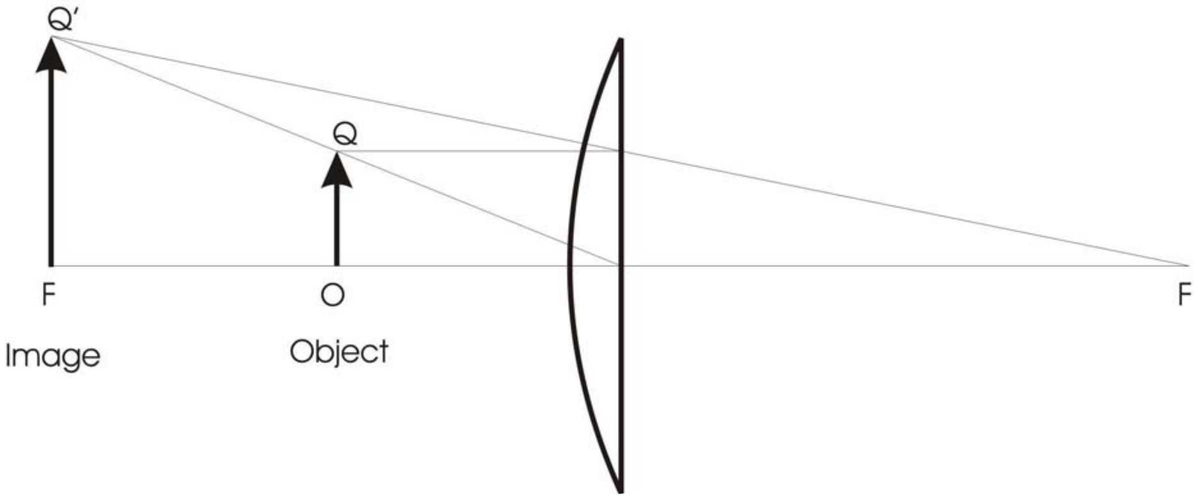
Lens with Object Closer than Focus f

- Now place candle at 10 cm (s <f condition)

$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{1}{20} - \frac{1}{10} = -0.05 \quad s' = \frac{1}{0.05} = -20\ cm$$

$$m = -\frac{s'}{s} = -\frac{-20}{10} = 2$$

- Now image is on same side of lens at 20 cm (focal point)
- Image is virtual, erect and 2x object size
- Virtual image means light appears to come from it



Graphic Method of Solving Lens Optics

- Graphic method is why this is called Geometric Optics
- Use some scale (graph paper good)
- Place lens on axis line and mark radius C & focal F points
- Draw line from object top Q to lens parallel to axis (ray 4)
- Hits vertex line at T
- Then direct ray from T through focus point F and beyond
- Because parallel light from object is focused at f
- Now direct ray from object top Q through lens center (ray 5)
- This intersects ray 4 at image Q' (point 7)
- This correctly shows both position and magnification of object
- This really shows how the light rays are travelling
- Eg Ray through the focal point F (ray 6) becomes parallel
- Intersects ray 5 again at image Q'

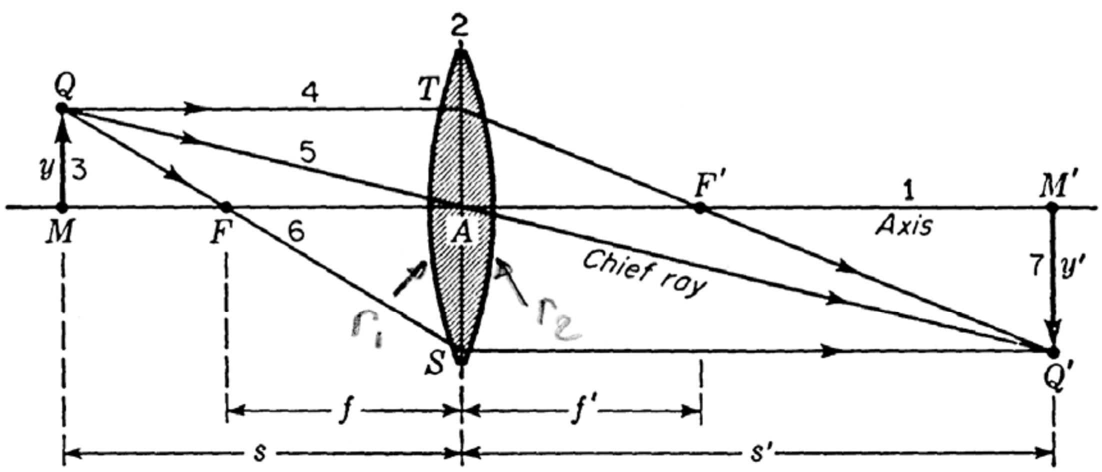


FIGURE 4D
The parallel-ray method for graphically locating the image formed by a thin lens.

Graphical Method with Negative Lenses

- Same general idea as with positive lens
- Draw line (ray 4) from object top Q to lens parallel to axis
- Hits vertex line at T
- Now direct ray from T through focus point F on left side of lens
- Extend this ray beyond lens on right (ray 5)
- This is the diverging ray a parallel beam would make from neg lens
- Now direct ray from object top Q through lens center (ray 8)
- This intersects ray 5 at image Q' (point 9)
- Virtual Image is now to right and is erect & smaller
- This is where and eye on the right would see the object
- Graphics methods great for quick understanding of optics systems
- Really good for understanding what happens in multi-lens optics

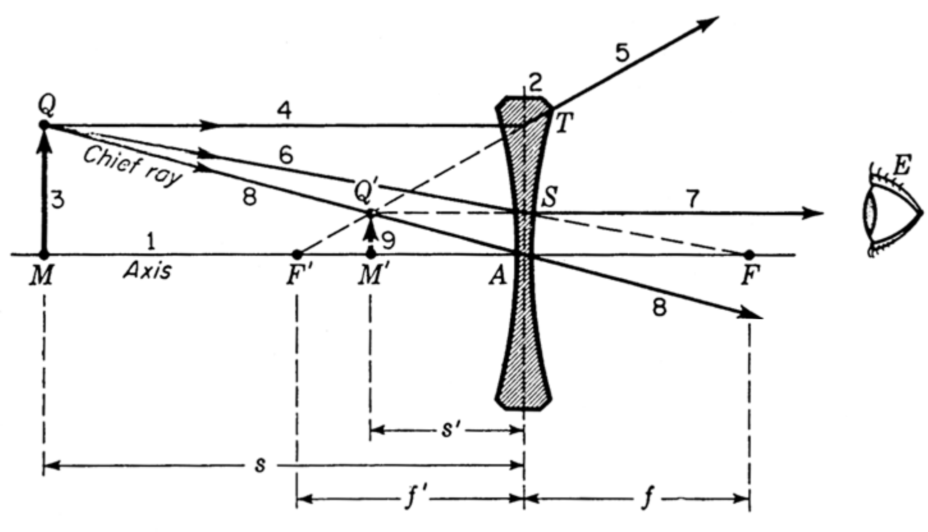
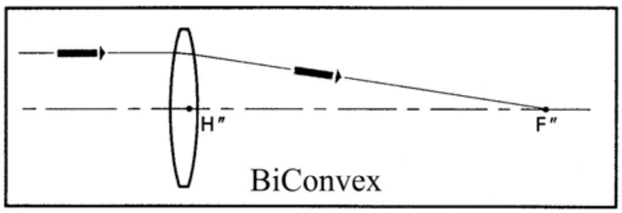
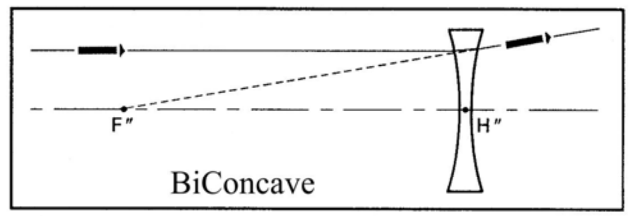


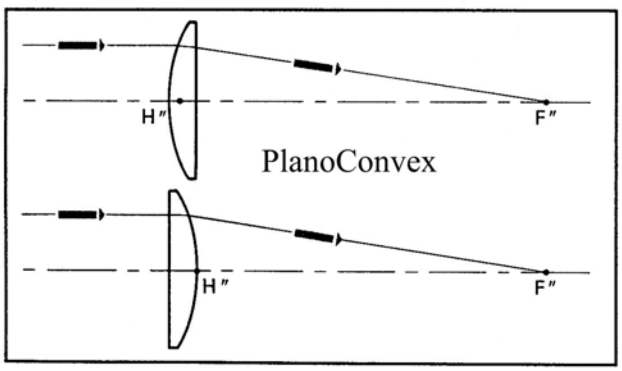
FIGURE 4G
The parallel-ray method for graphically locating the virtual image formed by a negative lens.

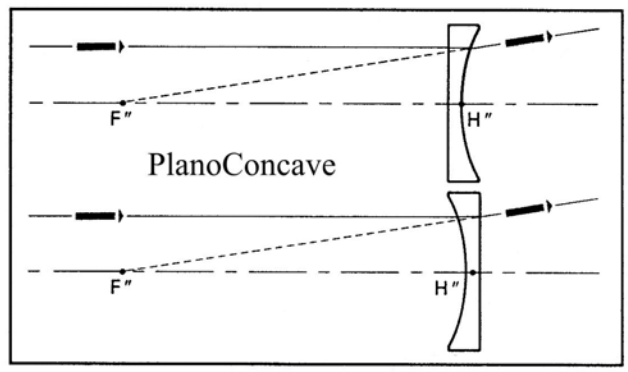
Thin Lens Principal Points

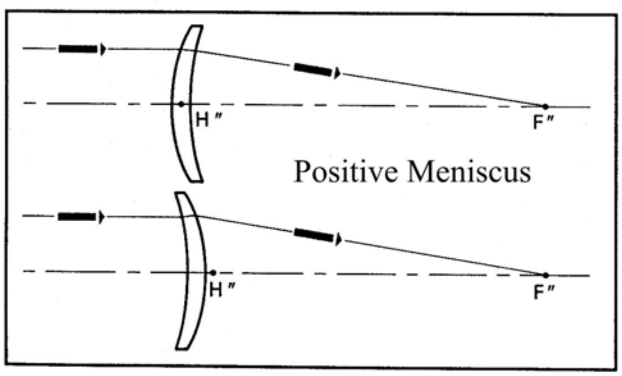
- Object and image distances are measured from the Principal Points
- Secondary principal point H” location depends on the lens shape
- H” also depends on a thin lens orientation
- Note if you reverse a lens it often does not focus at the same point
- Only symmetric lens shapes have symmetric principal & focal pt.
- Need to look at these lens specifications for principal points
- Note: Plano lenses when plane side toward source H” at vertex
- Thick lenses have separate Principal points

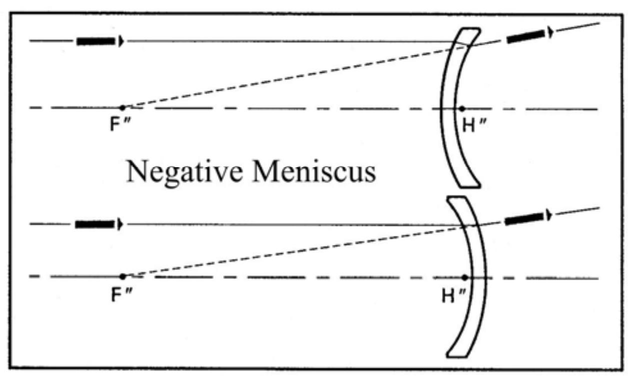






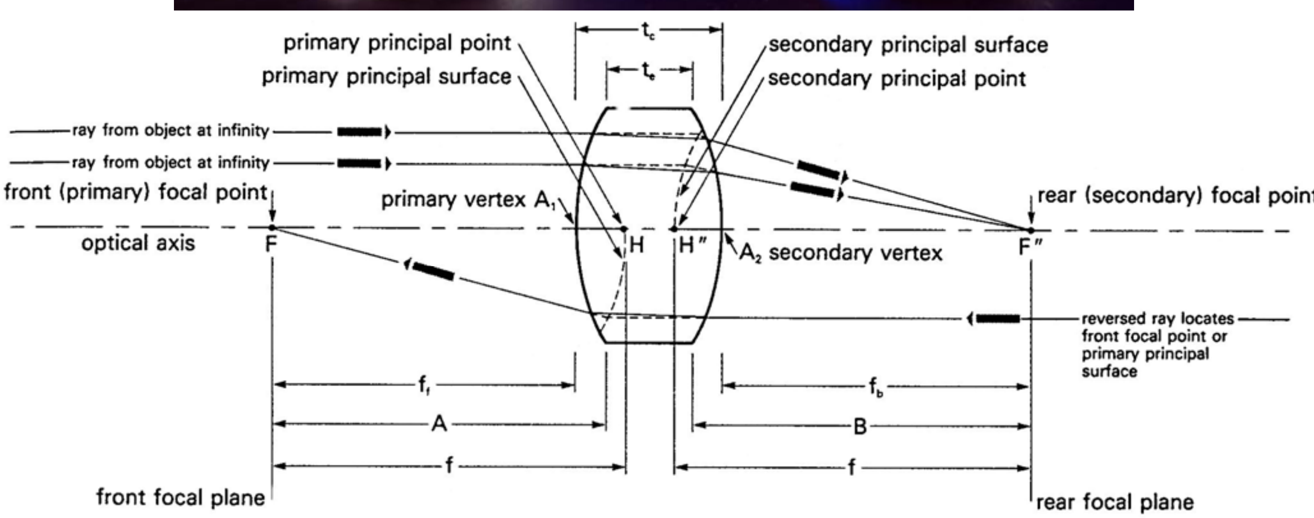






Thin and Thick Lens & Principal Surface

- Thin lens formulas work were radius of curvature is small
- Radius of curvature r >> than diameter ϕ
- Thus high F# & low light gathering power
- When lens has radius of curvature 2 or 3 times diameter then thick
- Now formulas must change.
- For biconvex/concave the thickness of centre becomes important
- Also now go from principal pt to principal surface
- Principal surface is line in lens where light appears to refract from

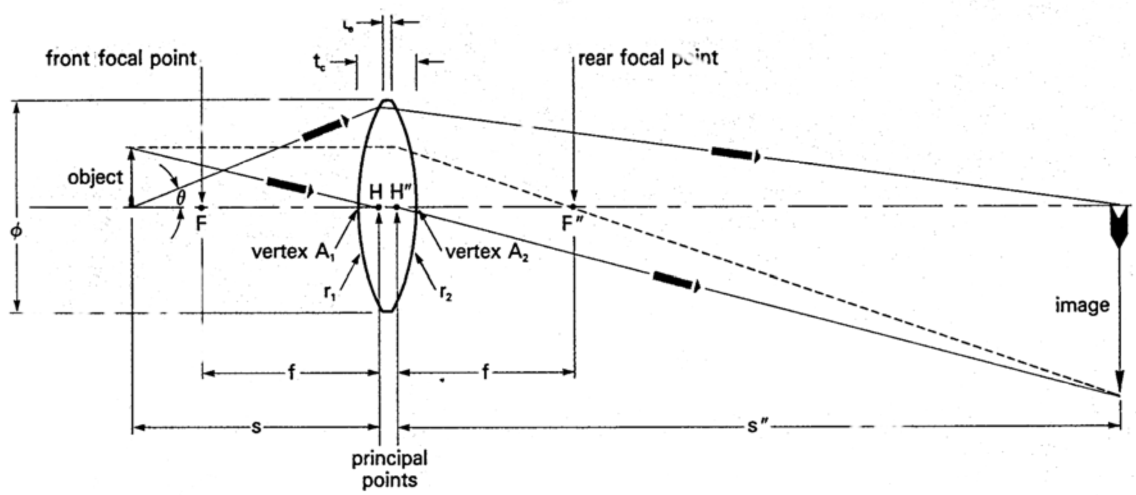


Thick Lens Formula

- Lens thickness t_c (between vertex at the optical axis i.e. centre)
- Now lens formula much more complicated
- Distances measured relative to the principal points
  H" for light coming from the front (left)
  H for light coming from the back of lens (right)

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \frac{(n-1)^2}{n}\left[\frac{t_c}{r_1 r_2}\right]$$

- Note simple lens formula assumes $t_c = 0$ which is never true
- But if f is large then r's large and t_c is small so good approximation
- Note plano-convex $r_2 = \infty$ and $f_{thin} = f_{thick}$ but principal point changes



Note location of object and image relative to front and rear focal points.

ϕ = Lens diameter

r_1 = Radius of curvature of 1st surface (positive if center of curvature is to right)

r_2 = Radius of curvature of 2nd surface (negative if center of curvature is to left)

r_2 = $-r_1$ for symmetric lens

m = s"/s = magnification = conjugate ratio, said to be infinite if either s" or s is infinite

θ = Arctan (ϕ/2s)

s = Object distance, positive for object (whether real or virtual) to the left of principal point H

s" = Image distance (s and s" are collectively called conjugate distances, with object and image in conjugate planes), positive for image (whether real or virtual) to the right of the principal point H"

t_c = Center thickness

t_e = Edge thickness

f = Effective focal length (EFL), may be positive (as shown or negative. f represents both FH and H"F", assuming lens to be surrounded by medium of index 1.0

Very Thick Lenses

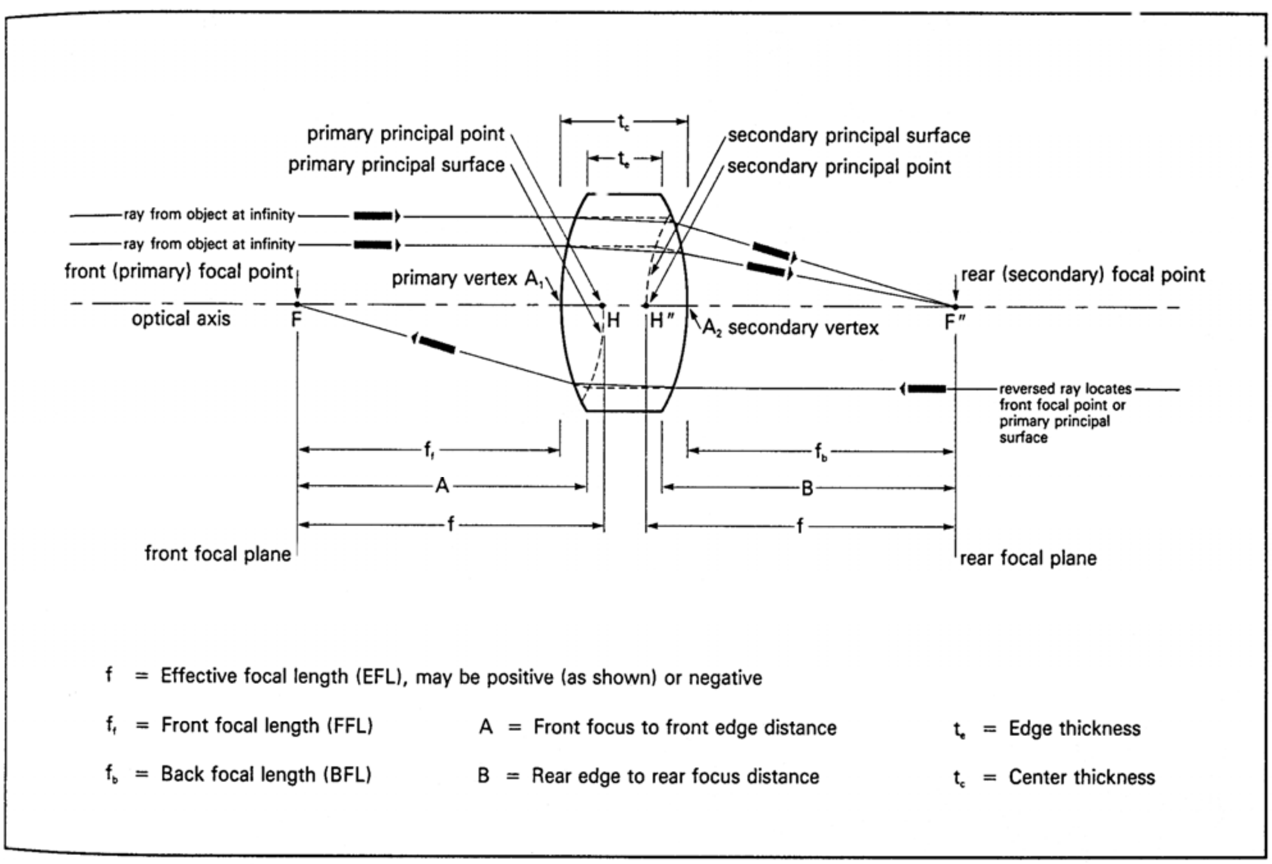
- Now primary and secondary principal points very different
- A_1 = front vertex (optical axis intercept of front surface)
- H = primary (front) principal point
- A_2 = back vertex (optical axis intercept of back surface)
- H" = secondary (back) principal point
- t_c = centre thickness: separation between vertex at optic axis
- Relative to the front surface the primary principal point is

$$A_1 - H = f t_c \left(\frac{n-1}{r_2} \right)$$

- Relative to the back surface the secondary principal point is

$$A_2 - H'' = f t_c \left(\frac{n-1}{r_1} \right)$$

- f_{efl} effective focal length (EFL): usually different for front and back



FRONT AND BACK FOCAL LENGTHS of a lens having spherical surfaces and surrounded by air. Under these conditions, distances labeled f are equal whether or not the lens is symmetric, but distances f_f and f_b are equal *only* if the lens is symmetric. In the paraxial limit (see text), the curvature of the principal surfaces may be neglected.

Combining Lenses

- Can combine several lenses to give single lens with same effect
- Create a Combination Effective Focal Length f_e
- If many thin lenses in contact (ie no space between them) then

$$\frac{1}{f_e} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} \cdots$$

- Two lenses f_1 and f_2 separated by distance d
- To completely replace two lens for all calculations
- New image distance for object at infinity (eg laser beam)

$$\frac{1}{f_e} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \quad or \quad f_e = \frac{f_1 f_2}{f_1 + f_2 - d}$$

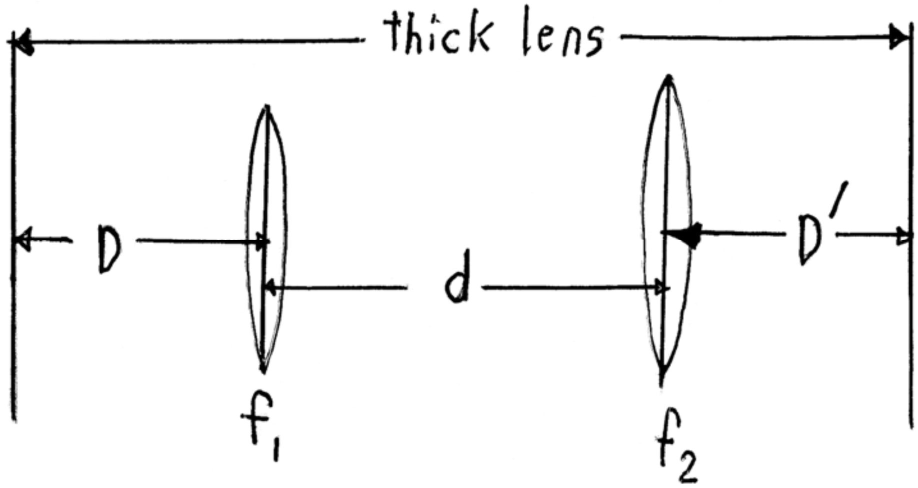
- Distance from first lens primary principal point
  to combined lens primary principal point

$$D = -\frac{d f_e}{f_2}$$

- Distance from second lens secondary principal point
  to combined lens secondary principal point

$$D' = -\frac{d f_e}{f_1}$$

- Combined "thick lens" extends from D to D'



Combining Two Lens Elements

- Combined object distance s_e

$$s_e = s_1 - D$$

- Combined image distance s'_e

$$s'_e = s'_2 - D'$$

- NOTE: Combined object/image distance may change sign
- The thick lens follows the standard formula

$$\frac{1}{s_e} + \frac{1}{s'_e} == \frac{1}{f_e}$$

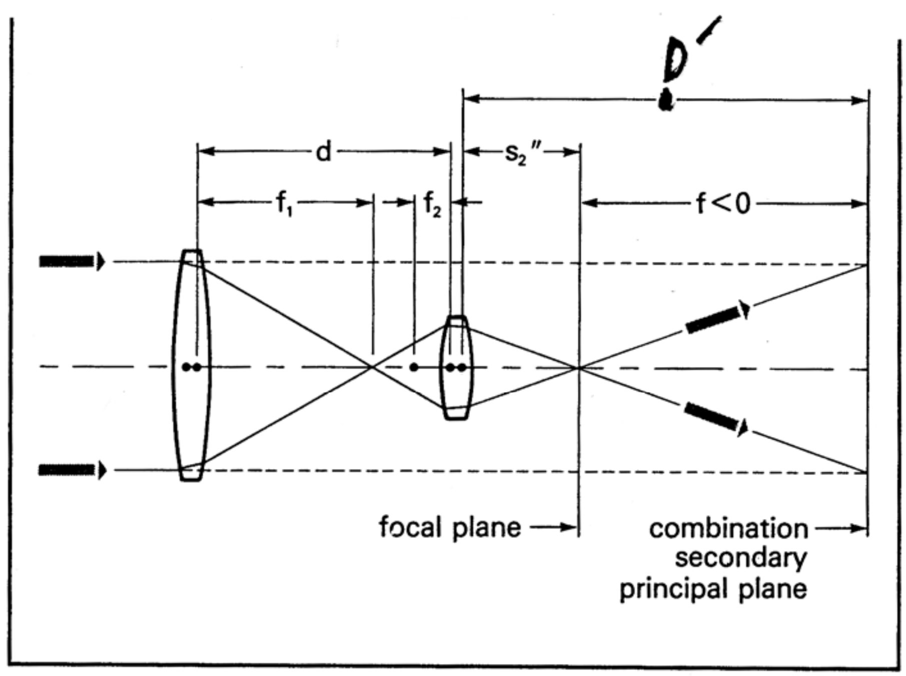
- Combined magnification

$$m_e = -\frac{s'_e}{s_e}$$

- Secondary focus distance relative to 2nd lens vertex is:

$$f = f_e + D$$

- Note some devices (e.g. telescopes) cannot use these formulas
- Reason telescopes require $f_1 + f_2 - d = 0$



PAIR OF POSITIVE LENSES separated by distance d greater than $f_1 + f_2$.

Matrix Methods in Optics(Hecht 6.2.1)

- Alternative Matrix methods
- Both matrix & CAD are based on Ray Tracing concepts
- Solve the optical system by tracing many optical rays
- In free space a ray has position and angle of direction
  y_1 is radial distance from optical axis
  V_1 is the angle (in radians) of the ray
- Now assume you want to a Translation:
  find the position at a distance t further on
- Then the basic Ray equations are in free space
  making the parallex assumption

$$y_2 = y_1 + V_1 t$$

$$V_2 = V_1$$

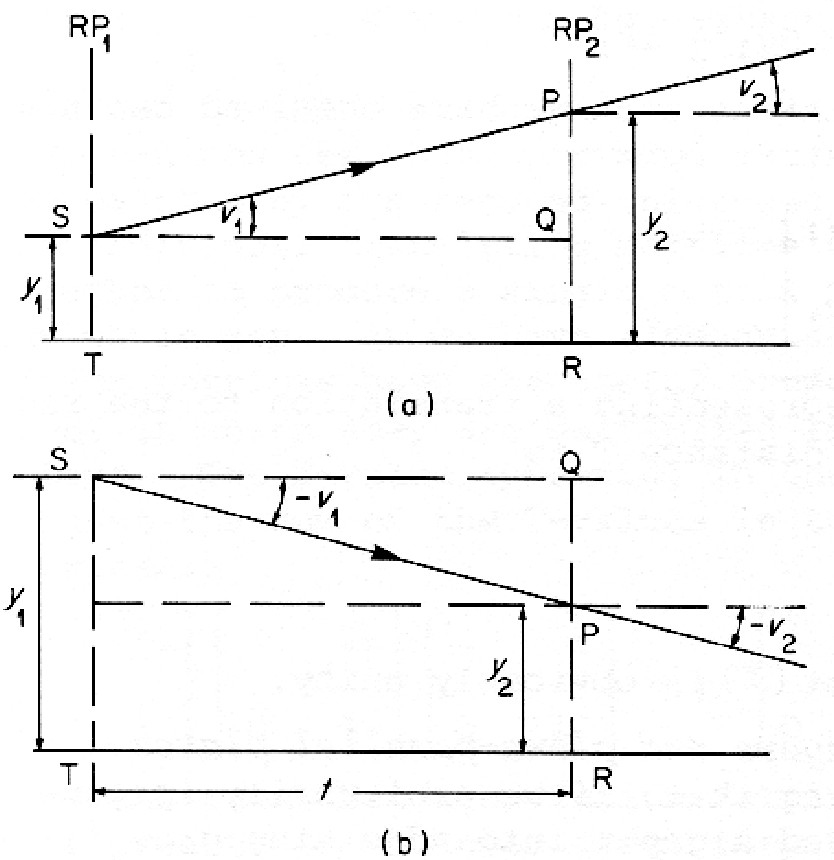


Figure II.2

Matrix Method: Translation Matrix

- Can define a matrix method to obtain the result for any optical process
- Consider a simple translation distance t
- Then the Translation Matrix (or *T* matrix)

$$\begin{bmatrix} y_2 \\ V_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} y_1 \\ V_1 \end{bmatrix} = \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ V_1 \end{bmatrix}$$

- The reverse direction uses the inverse matrix

$$\begin{bmatrix} y_1 \\ V_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} \begin{bmatrix} y_2 \\ V_2 \end{bmatrix} = \begin{bmatrix} D & -B \\ -C & A \end{bmatrix} \begin{bmatrix} y_2 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1 & -t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ V_1 \end{bmatrix}$$

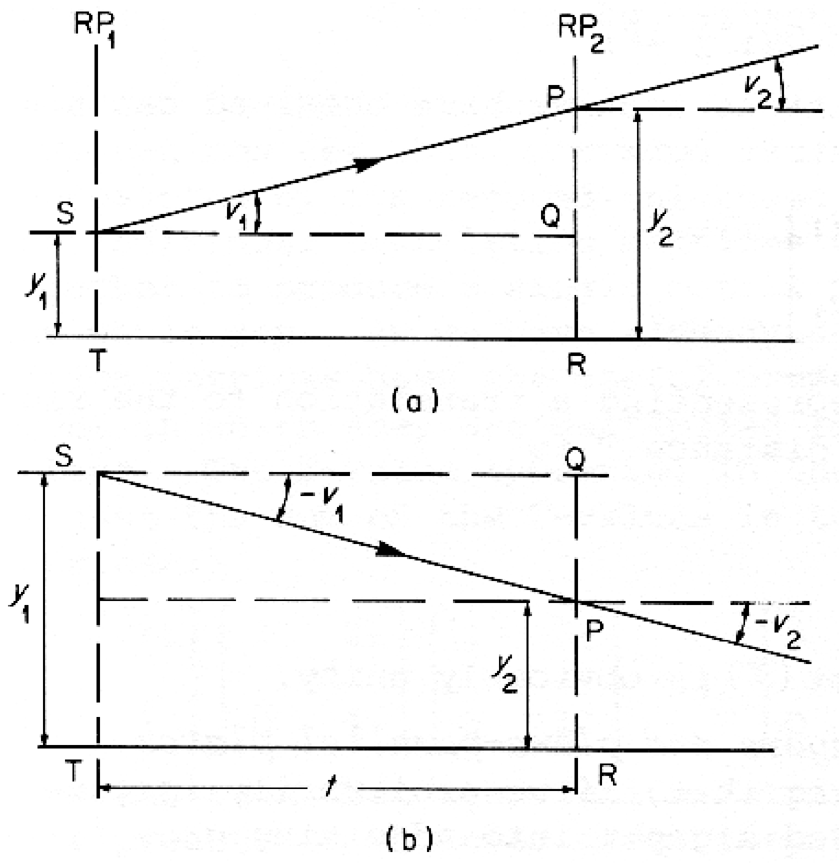
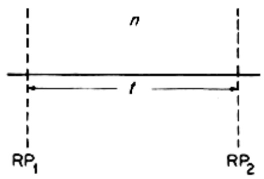
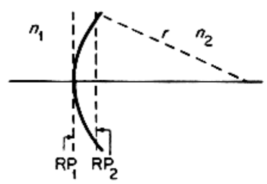
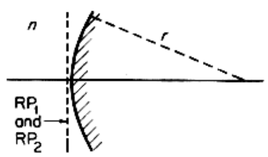
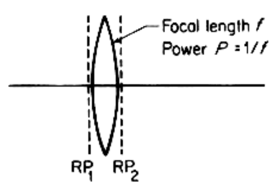


Figure II.2

General Matrix for Optical Devices

- Optical surfaces however will change angle or location
- Example a lens will keep same location but different angle
- Reference for more lens matrices & operations
  A. Gerrard & J.M. Burch,
  "Introduction to Matrix Methods in Optics", Dover 1994
- Matrix methods equal Ray Trace Programs for simple calculations

Table 1

Number	Description	Optical Diagram	Ray-transfer matrix
1	Translation ($\mathcal{T}$-matrix)		$\begin{bmatrix}1 & t/n\\ 0 & 1\end{bmatrix} = \begin{bmatrix}1 & T\\ 0 & 1\end{bmatrix}$
2	Refraction at single surface ($\mathcal{R}$-matrix)		$\begin{bmatrix}1 & 0\\ \frac{-(n_2 - n_1)}{r} & 1\end{bmatrix} = \begin{bmatrix}1 & 0\\ -P & 1\end{bmatrix}$
3	Reflection at single surface (for convention see section II.11)		$\begin{bmatrix}1 & 0\\ \frac{2n}{r} & 1\end{bmatrix} = \begin{bmatrix}1 & 0\\ -P & 1\end{bmatrix}$
4	Thin lens in air (focal length f, power P)		$\begin{bmatrix}1 & 0\\ -(n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) & 1\end{bmatrix} = \begin{bmatrix}1 & 0\\ -1/f & 1\end{bmatrix}$

General Optical Matrix Operations

- Place Matrix on the left for operation on the right
- Can solve or calculate a single matrix for the system

$$\begin{bmatrix} y_2 \\ V_2 \end{bmatrix} = \begin{bmatrix} M_{image} \end{bmatrix} \begin{bmatrix} M_{lens} \end{bmatrix} \begin{bmatrix} M_{object} \end{bmatrix} \begin{bmatrix} y_1 \\ V_1 \end{bmatrix}$$

$$\begin{bmatrix} y_2 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1 & s' \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} \begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ V_1 \end{bmatrix}$$

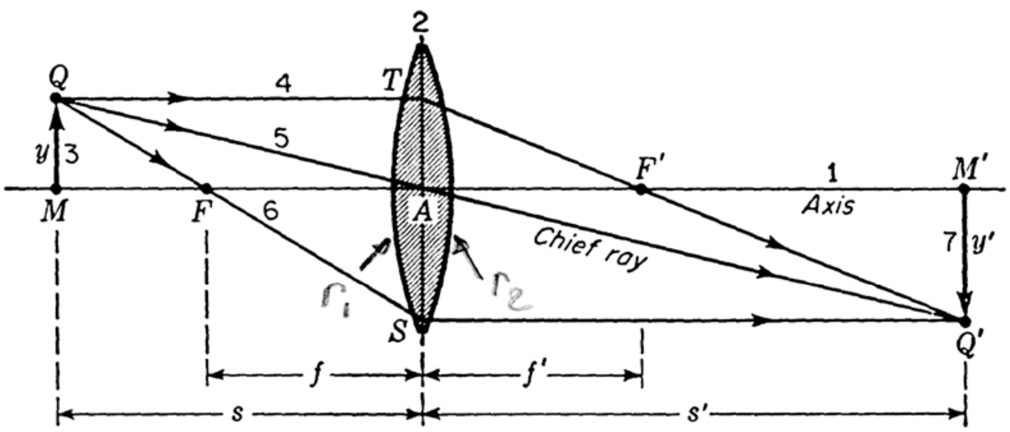


FIGURE 4D
The parallel-ray method for graphically locating the image formed by a thin lens.

Solving for image with Optical Matrix Operations

- For any lens system can create an equivalent matrix
- Combine the lens (mirror) and spacing between them
- Create a single matrix

$$[M_n]\cdots[M_2][M_1] = [M_{system}] = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

- Now add the object and image distance translation matrices

$$\begin{bmatrix} A_s & B_s \\ C_s & D_s \end{bmatrix} = [M_{image}][M_{lens}][M_{object}]$$

$$\begin{bmatrix} A_s & B_s \\ C_s & D_s \end{bmatrix} = \begin{bmatrix} 1 & s' \\ 0 & 1 \end{bmatrix}\begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} A_s & B_s \\ C_s & D_s \end{bmatrix} = \begin{bmatrix} A + s'C & As + B + s'(Cs + D) \\ C & Cs + D \end{bmatrix}$$

- Image distance s' is found by solving for $B_s=0$
- Image magnification is

$$m_s = \frac{1}{D_s}$$

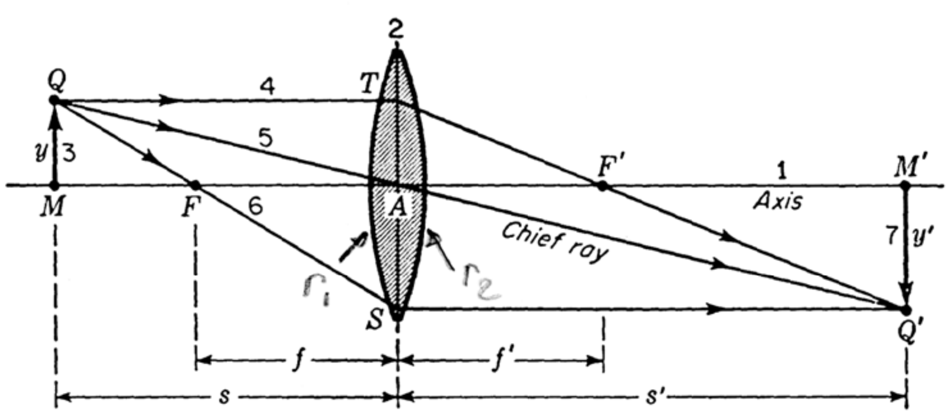


FIGURE 4D
The parallel-ray method for graphically locating the image formed by a thin lens.

Example Solving for the Optical Matrix

- Two lens system: solve for image position and size
- Biconvex lens f_1=8 cm located 24 cm from 3 cm tall object
- Second lens biconcave f_2= -12 cm located d=6 cm from first lens
- Then the matrix solution is

$$\begin{bmatrix} A_s & B_s \\ C_s & D_s \end{bmatrix} = \begin{bmatrix} 1 & X \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{12} & 1 \end{bmatrix} \begin{bmatrix} 1 & 6 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{8} & 1 \end{bmatrix} \begin{bmatrix} 1 & 24 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} A_s & B_s \\ C_s & D_s \end{bmatrix} = \begin{bmatrix} 1 & X \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 6 \\ \frac{1}{12} & \frac{3}{2} \end{bmatrix} \begin{bmatrix} 1 & 24 \\ -\frac{1}{8} & -2 \end{bmatrix}$$

$$\begin{bmatrix} A_s & B_s \\ C_s & D_s \end{bmatrix} = \begin{bmatrix} 1 & X \\ 0 & 1 \end{bmatrix} \begin{bmatrix} A_{s1} & B_{s1} \\ C_{s1} & D_{s1} \end{bmatrix} = \begin{bmatrix} 1 & X \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0.25 & 12 \\ -0.1042 & -1 \end{bmatrix}$$

- Solving for the image position using the s1 matrix & X matrix:

$$B_s = B_{s1} + XD_{s1} = 0 \quad or \quad X = \frac{-B_{s1}}{D_{s1}} = \frac{-12}{-1} = 12\ cm$$

- Then the magnification is

$$m = \frac{1}{D_s} = \frac{1}{D_{s1}} = \frac{1}{-1} = -1$$

- Thus the object is at 12 cm from 2nd lens, -3 cm high

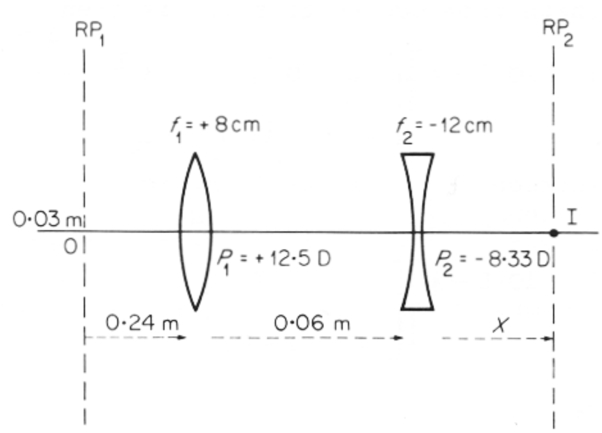


Figure II.14

Matrix Method and Spread Sheets

- Easy to use matrix method in Excel or matlab or maple
- Use mmult array function in excel
- Select array output cells (eg. matrix) and enter =mmult(
- Select space 1 cells then comma
- Select lens 1 cells (eg =mmult(G5:H6,I5:J6))
- Then do control+shift+enter (very important)
- Here is example from previous page

E460 example lesson 6			Distances in cm						
Lens Matrix		Lens 2		Matrx 1		Space 1		Lens 1	
		f2	-12			d	6	f1	8
0.25	6	1	0	0.25	6	1	6	1	0
-0.104167	1.5	0.083333	1	-0.125	1	0	1	-0.125	1
second focal length		-1/C	9.6						
second focal point		-A/C	2.4						
Image		System Matrix S1		Lens Matrix		Object			
						d	24		
1	X	0.25	12	0.25	6	1	24		
0	1	-0.104167	-1	-0.104167	1.5	0	1		
Object size		y	3						
image distance		=-Bs1/Ds1	12						
Magnification		=1/Ds1	-1						
Object size		=y/Ds1	-3						

Optical Matrix Equivalent Lens

- For any lens system can create an equivalent matrix & lens
- Combine all the matrices for the lens and spaces
- The for the combined matrix

$$[M_n]\cdots[M_2][M_1]=[M_{system}]=\begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

- Table shows how to calculate focal, nodal and principal points

where RP_1 = first lens left vertex

RP_2 = last lens right most vertex

n_1=index of refraction before 1st lens

n_2=index of refraction after last lens

System parameter described	Measured From	To	Function of matrix elements	Special case $n_1 = n_2 = 1$
First focal point	RP_1	F_1	n_1D/C	D/C
First focal length	F_1	H_1	$-n_1/C$	$-1/C$
First principal point	RP_1	H_1	$n_1(D-1)/C$	$(D-1)/C$
First nodal point	RP_1	L_1	$(Dn_1-n_2)/C$	$(D-1)/C$
Second focal point	RP_2	F_2	$-n_2A/C$	$-A/C$
Second focal length	H_2	F_2	$-n_2/C$	$-1/C$
Second principal point	RP_2	H_2	$n_2(1-A)/C$	$(1-A)/C$
Second nodal point	RP_2	L_2	$(n_1-An_2)/C$	$(1-A)/C$

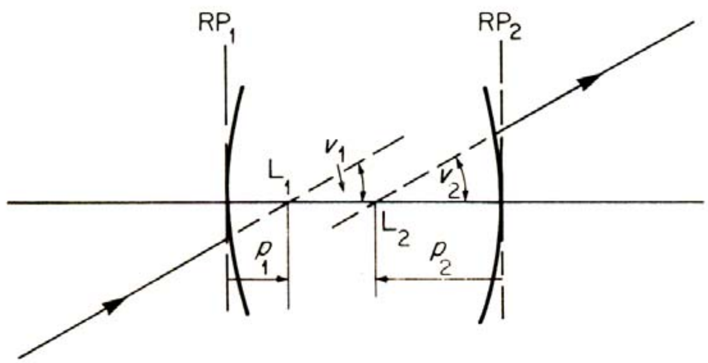


Figure II.17c

Example Combined Optical Matrix

- Using Two lens system from before
- Biconvex lens f_1=8 cm
- Second lens biconcave f_2= -12 cm located 6 cm from f_1
- Then the system matrix is

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{12} & 1 \end{bmatrix} \begin{bmatrix} 1 & 6 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{8} & 1 \end{bmatrix} = \begin{bmatrix} 0.25 & 6 \\ -0.1042 & 1.5 \end{bmatrix}$$

- Second focal length (relative to H_2) is

$$f_{s2} = -\frac{1}{C} = -\frac{1}{-0.1042} = 9.766\ cm$$

- Second focal point, relative to RP_2 (second vertex)

$$f_{rP2} = -\frac{A}{C} = -\frac{0.25}{-0.1042} = 2.400\ cm$$

- Second principal point, relative to RP_2 (second vertex)

$$H_{s2} = \frac{1-A}{C} = \frac{1-0.25}{-0.1024} = -7.198\ cm$$

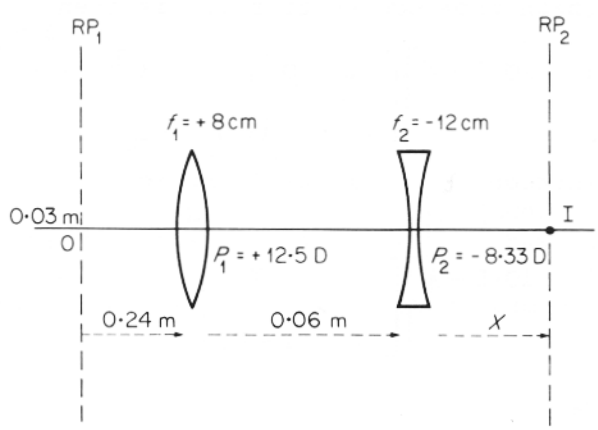


Figure II.14

Numerical Aperture (NA)

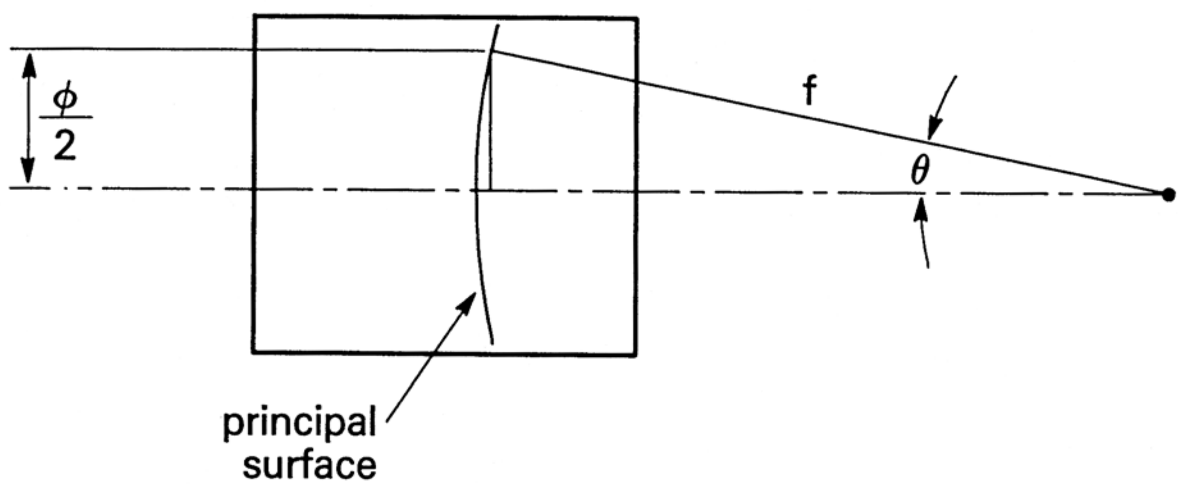
- NA is the sine of the angle the largest ray a parallel beam makes when focused

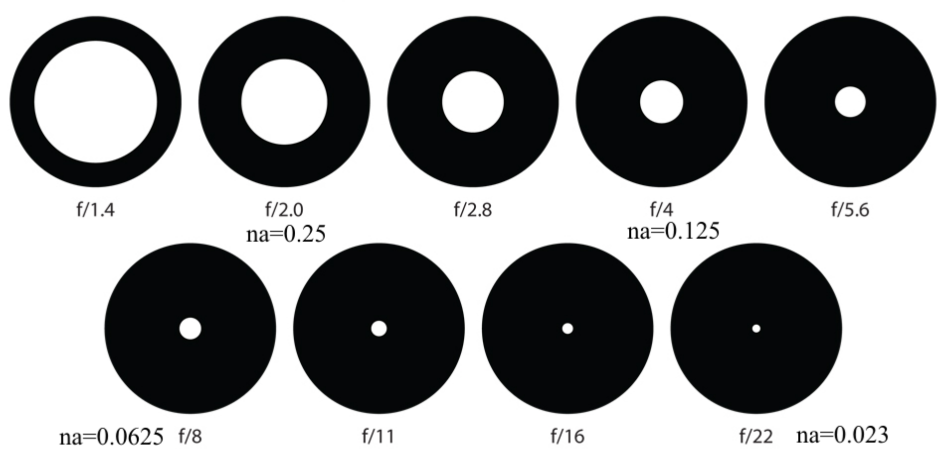
$$NA = sin(\theta) = \frac{\phi}{2f}$$

where θ = angle of the focused beam
ϕ = diameter of the lens

- NA <1 are common
- High NA lenses are faster lenses
- NA is related to the F#

$$F\# = \frac{1}{2NA}$$





Human Eye

- Human eye is a simple single lens system
- Cornea: outer surface protection
- Aqueous humor is water like liquid behind cornea
- Iris: control light
- Crystalline lens provide focus
- Retina: where image is focused
- Note images are inverted
- Brain's programming inverts the image

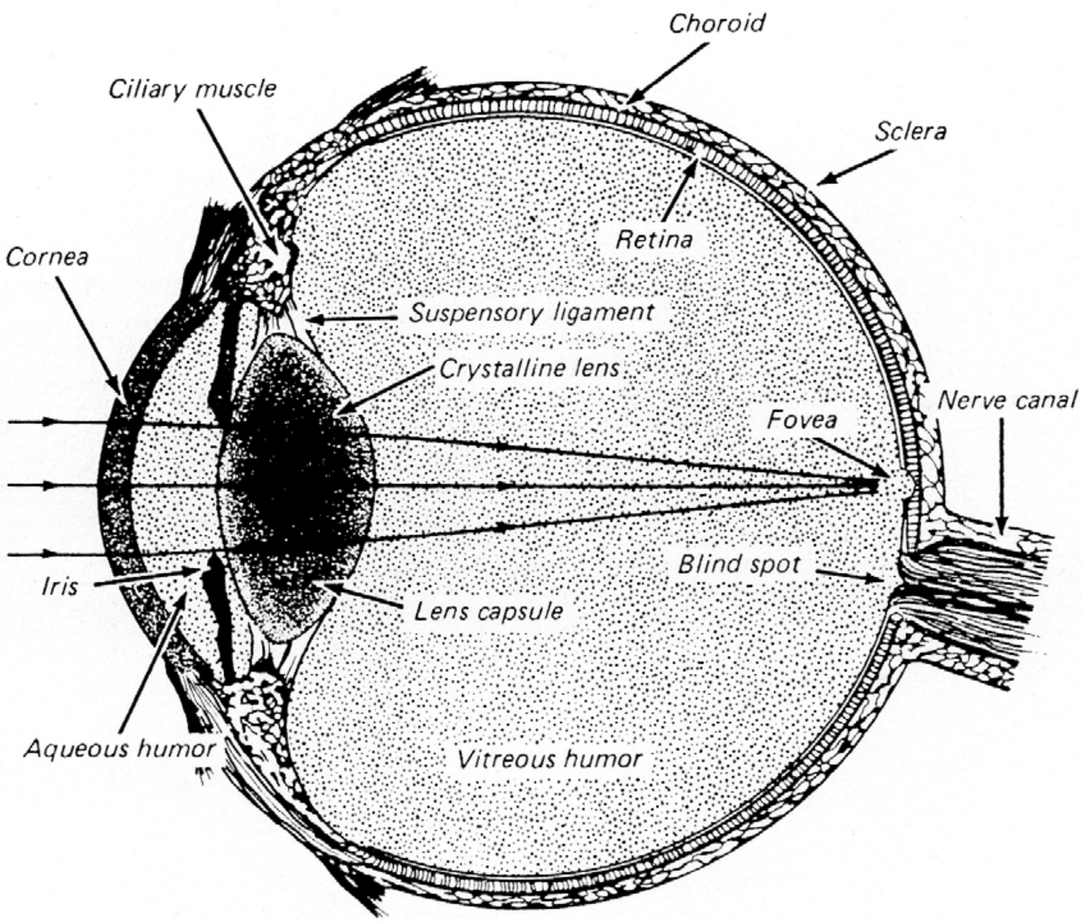


FIGURE 10A
A cross-sectional diagram of a human eye, showing the principal optical components and the retina.

Human Eye Distance

- Crystalline lens to retina distance 24.4 mm
- Eye focuses object up to 25 cm from it
- Called the near point or $D_v = 25$ cm
- Eye muscles to change focal length of lens over $2.22<f<2.44$ cm
- Near sighted: retina to lens distance too long, focused in front
- Infinity object focused in front of retina: out of focus at it
- When bring objects closer focus moves to retina
- Near sighted people can see objects with $D_v < 25$ cm
- Far sighted: eye is too short, focuses behind retina, $D_v > 25$ cm

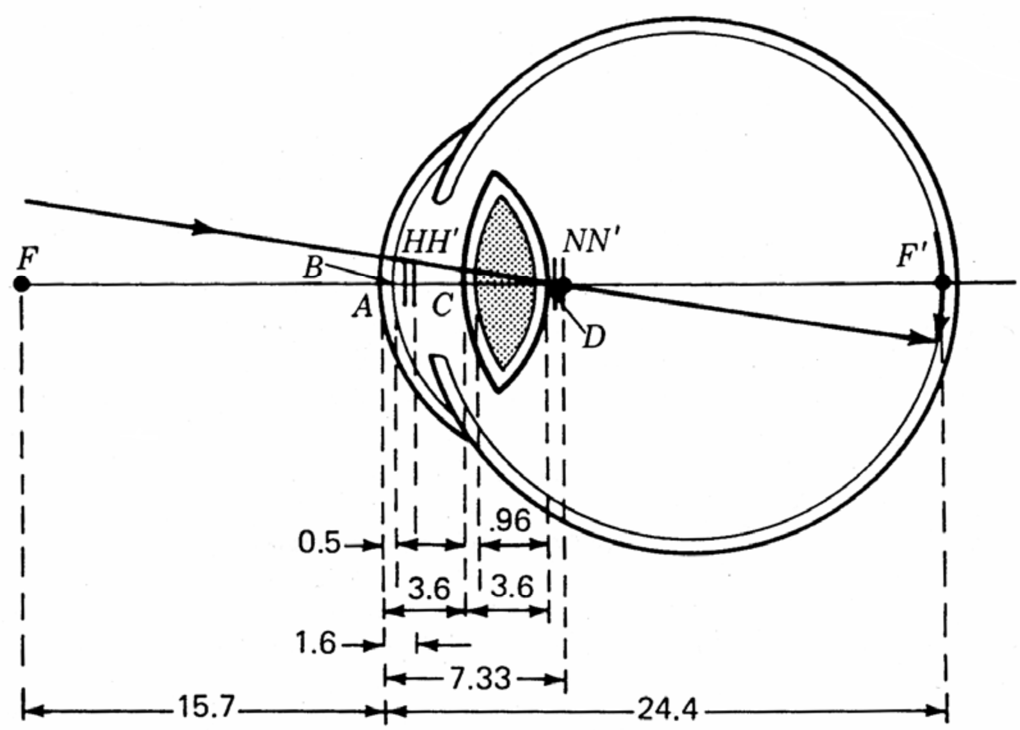


FIGURE 10B
Schematic eye as developed by Gullstrand, showing the real and inverted image on the retina (dimensions are in millimeters).

Magnification of Lens

- Lateral change in distance equals change in image size
- Measures change in apparent image size

$$m = M = \frac{y'}{y} = -\frac{s'}{s}$$

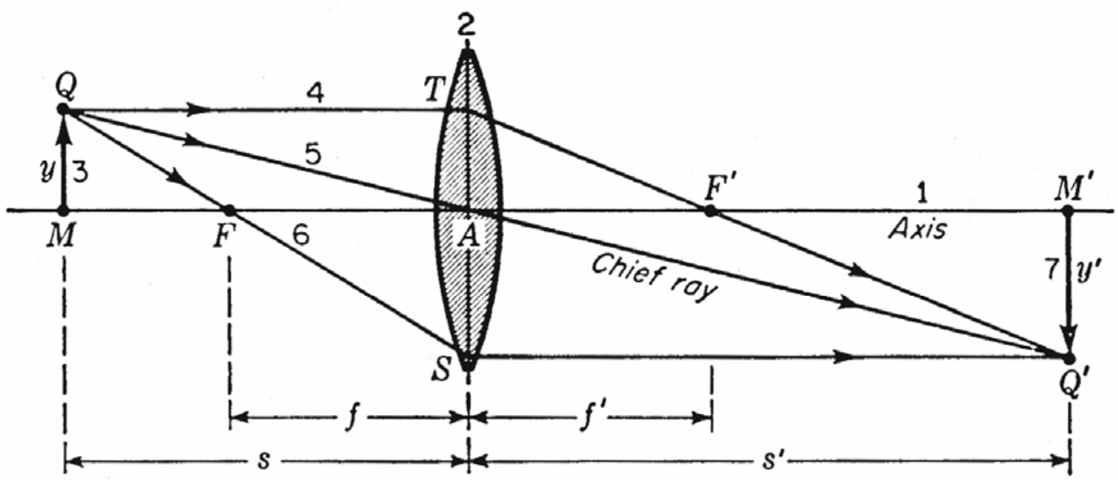


FIGURE 4D
The parallel-ray method for graphically locating the image formed by a thin lens.

Magnification with Index Change

- Many different ways of measuring magnification
- With curved index of refraction surface
  measure apparent change in distance to image
- Called Lateral Magnification

$$m = -\frac{s' - r}{s + r}$$

- m is + if image virtual, - if real

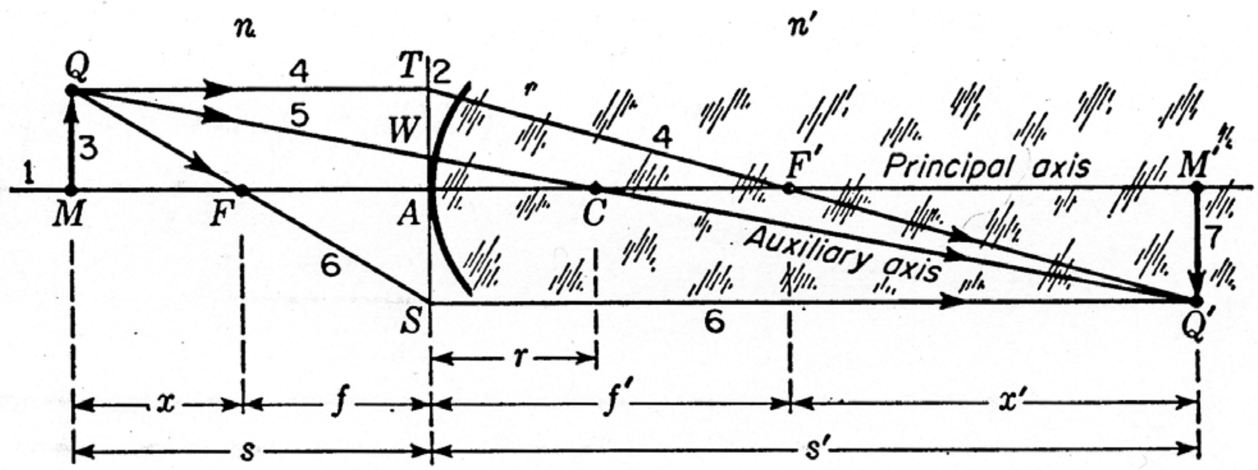


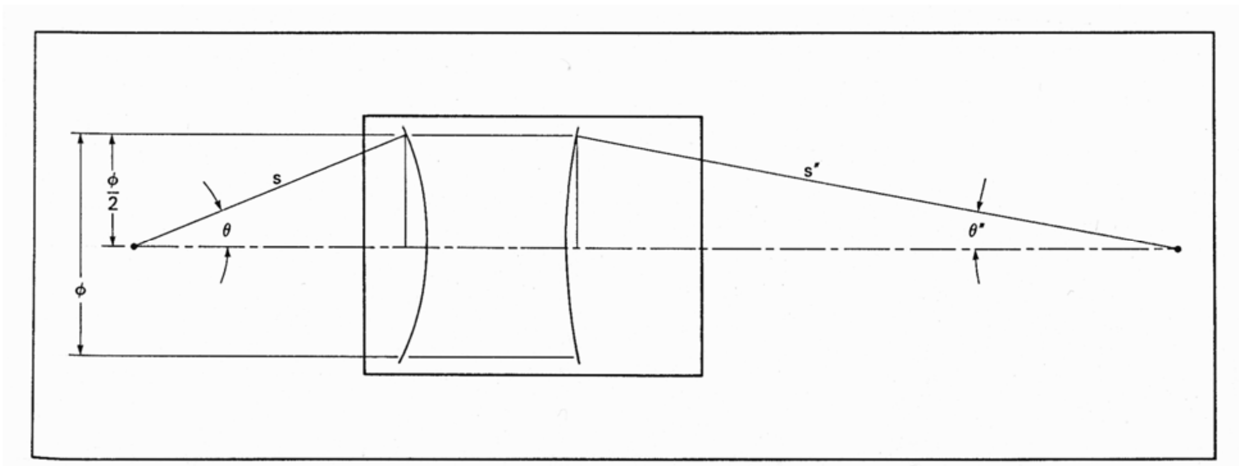
FIGURE 3F
Parallel-ray method for graphically locating the image formed by a single spherical surface.

Angular Magnification

- For the eye look at angular magnification

$$m = M = \frac{\theta'}{\theta}$$

- Represents the change in apparent angular size



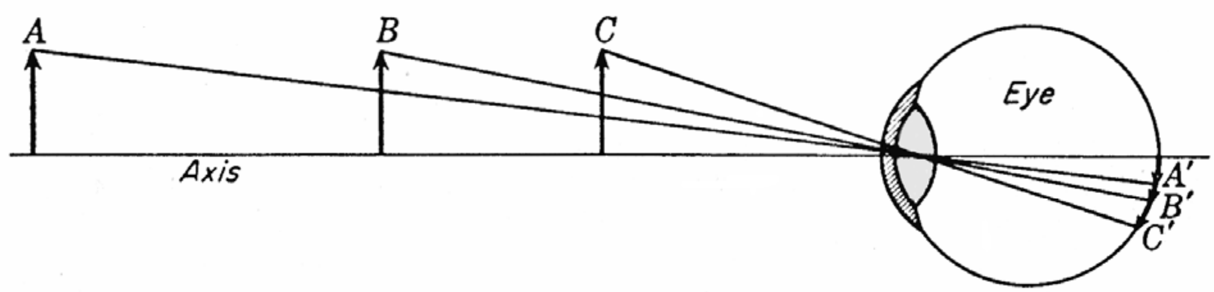


FIGURE 10H
The angle subtended by the object determines the size of the retinal image.

Simple Magnifying Glass

- Human eye focuses near point or D_v = 25 cm
- Magnification of object:
  ratio of angles at eye between unaided and lens
- Angle of Object with lens

$$tan(\theta) = \frac{y}{D_v} = \frac{y}{25} \approx \theta$$

- For maximum magnification place object at lens f (in cm)

$$\theta' = \frac{y}{f}$$

- Thus magnification is (where *f* in cm)

$$m = \frac{\theta'}{\theta} = \frac{25}{f}$$

- e.g. What is the magnification of a lens f = 1 inch = 2.5 cm

$$m = \frac{\theta'}{\theta} = \frac{25}{f} = \frac{25}{2.5} = 10$$

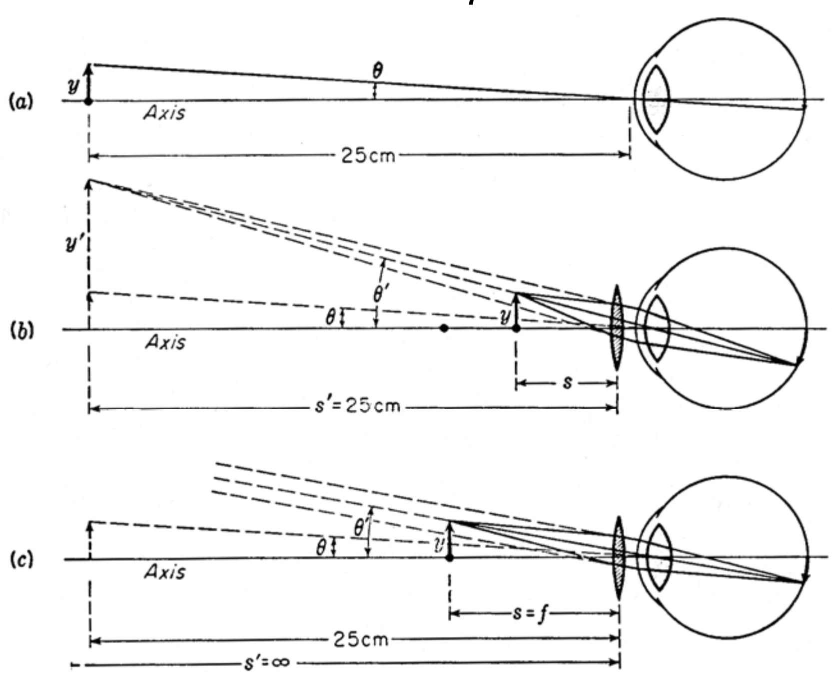


FIGURE 10I
The angle subtended by (*a*) an object at the near point to the naked eye, (*b*) the virtual image of an object inside the focal point, (*c*) the virtual image of an object at the focal point.

Power of a Lens or Surface

- Power: measures the ability to create converging/diverging light by a lens
- Measured in Diopters (D) or 1/m
- For a simple curved surface

$$P = \frac{n' - n}{r}$$

- For a thin lens

$$P = \frac{1}{f}$$

- Converging lens have + D, diverging - D
- eg f = 50 cm, D = +2 D
  f = -20 cm, D = -5 D
- Recall that for multiple lens touching

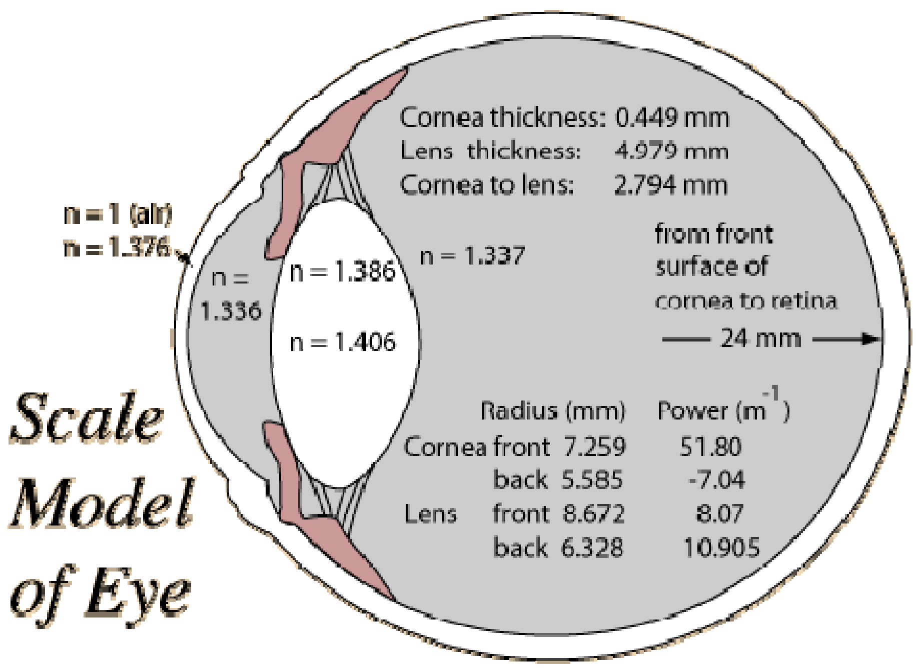
$$\frac{1}{f_e} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} \cdots$$

- Hence power in Diopters is additive

$$D = D_1 + D_2 \cdots$$

Human Eye: A two Lens System

- Eye is often treated as single simple lens
- Actually is a two lens system
- Cornea with n=1.376 makes main correction
- Aqueous humor is nearly water index
- Lens n=1.406 relative to aqueous humor Δn causes change
- Eye muscles shape the lens and adjusts focus
- Cornea gives 44.8 D of correction
- Lens gives ~18.9 D of correction
- Cannot see in water because water index 1.33 near cornea
- Thus cornea correction is not there.



Eyeglasses (Hecht 5.7.2)

- Use Diopters in glasses
- Farsighted, Hypermetopia: focus light behind retina
  Use convex lens, +D to correct
- Nearsighted, Myopia: focus in front of retina
  use concave lens, -D to correct
- Normal human eye power is ~58.6 D
- Nearsighted glasses create a reverse Galilean telescope
- Makes objects look smaller.

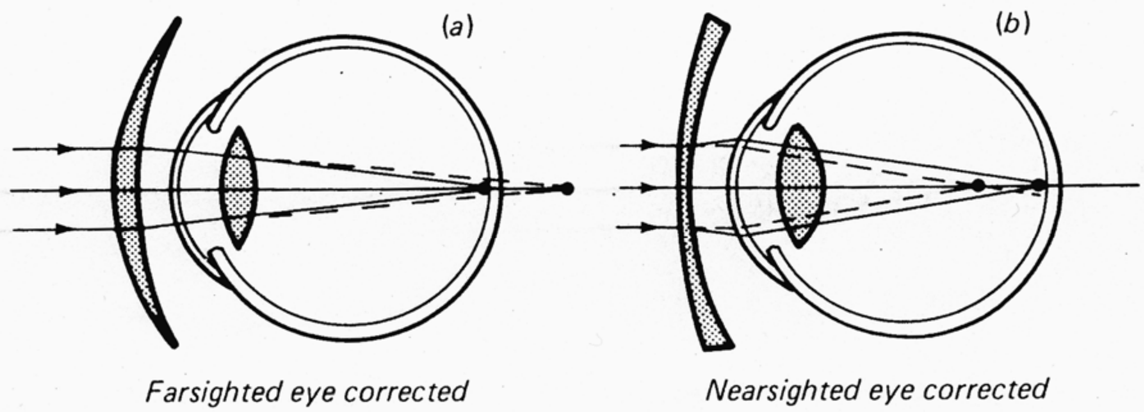
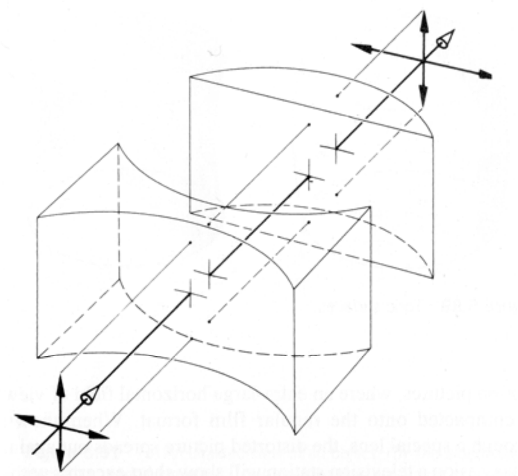
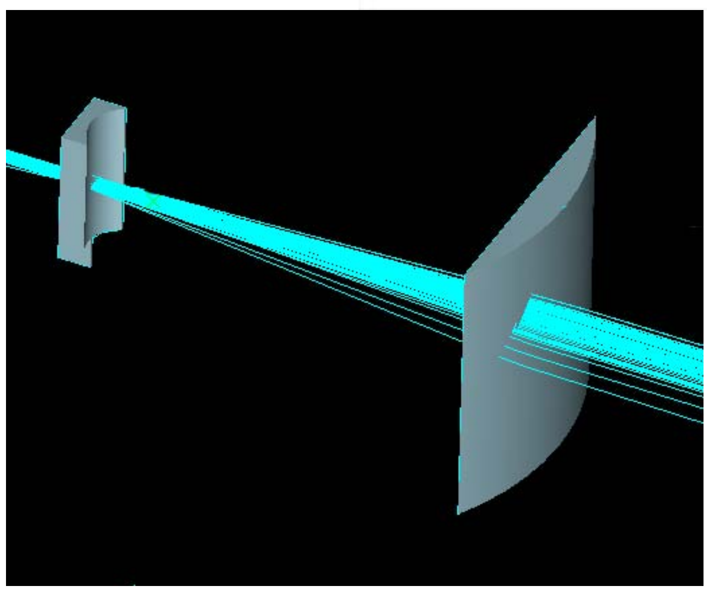


FIGURE 10L
Typical eye defects can be corrected by spectacle lenses.

Anamorphic Lenses

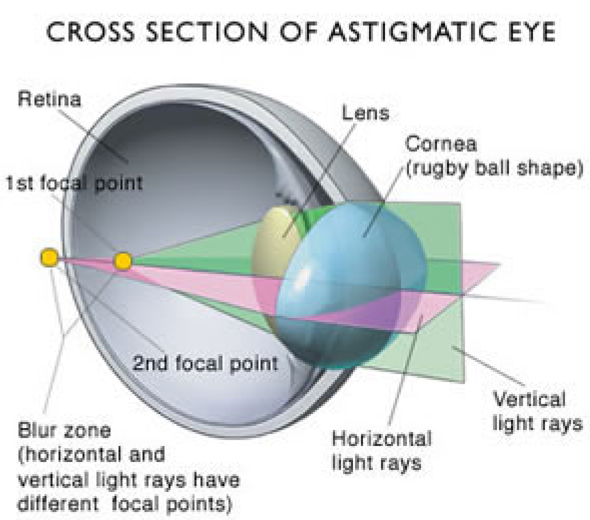
- Lenses & Mirrors do not need to be cylindrically symmetric
- Anamorphic Lenses have different characteristics in each axis
- Sphero-cylinderical most common
- One axis (eg vertical): cylindrical curve just like regular lens
- Other axis (e.g. horizontal): has no curve
- Result light is focused in horizontal axis but not vertical
- Often used to create a line of light





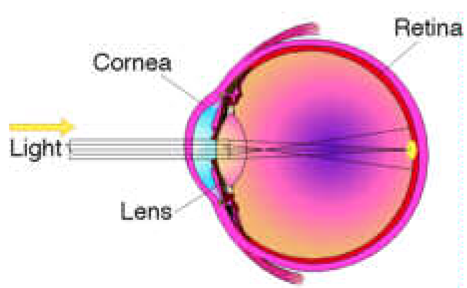
Astigmatism

Astigmatism means light is focused in on axis not other
Cylinderical lens cause as Astigmatism: focus in one plan
In eyes astigmatism caused by shape of eye (& lens)
Image is compressed in one axis and out of focus
Typically measure D in both axis
Rotation of astigmatism axis is measured
Then make lens slightly cylindrical
i.e. perpendicular to axis may have higher D in one than other
eg. eyeglass astigmatism prescription gives +D and axis angle
+D is difference between the two axis.

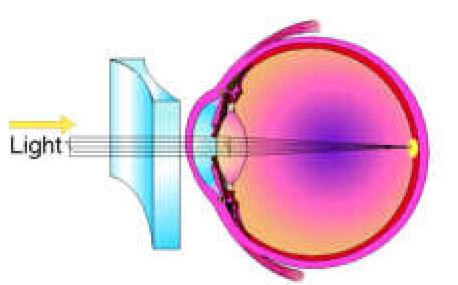


uncorrected

corrected



Horizontal line out of focus



Astigmatism corrected by lens

Ray Tracing (Hecht 6.2)

- For more complicated systems use CAD tools
- Both are based on Ray Tracing concepts
- Solve the optical system by tracing many optical rays
- Use exact surface positions & surface
- Do not make parallex assumption – use Snell's law
- Eg.of programs Z max, Code 5, OSLO
- Optical Ray Tracer is a free (modest complexity) java program https://arachnoid.com/OpticalRayTracer/