What Makes a Laser

- Light Amplification by Stimulated Emission of Radiation

Main Requirements of the Laser

- Laser Gain Medium (provides the light amplification)
- Optical Resonator Cavity (greatly increase amplification)
  Also creates the confined beam
- Sufficient means of Excitation of Gain medium
  (called pumping) eg. light, current, chemical reaction
- Population Inversion in the Gain Medium due to pumping
  Creates the stimulated emission of radiation

Laser Types

- Two main types depending on time operation
- Continuous Wave (CW)
- Pulsed operation
- Pulsed is easier, CW more useful

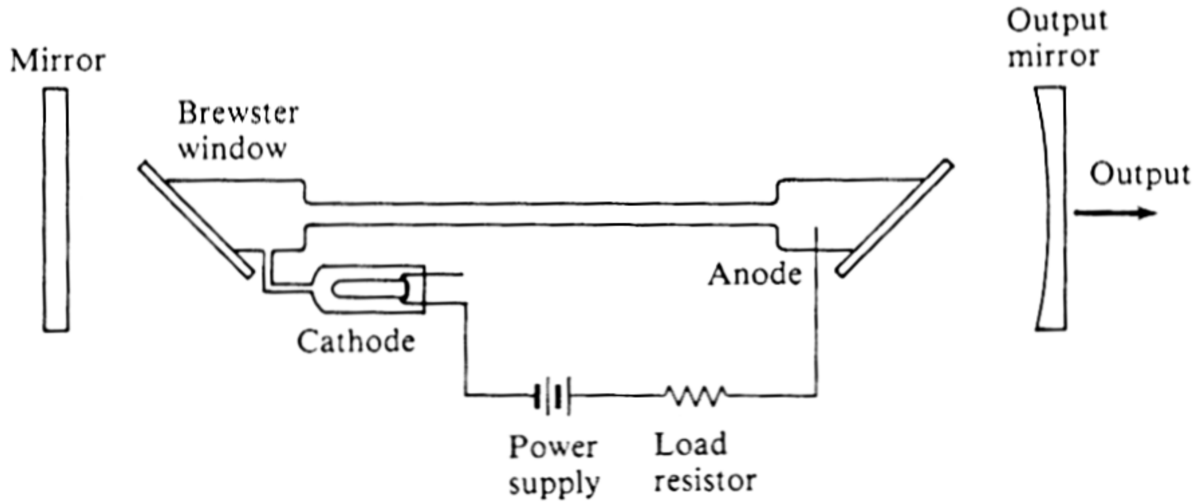
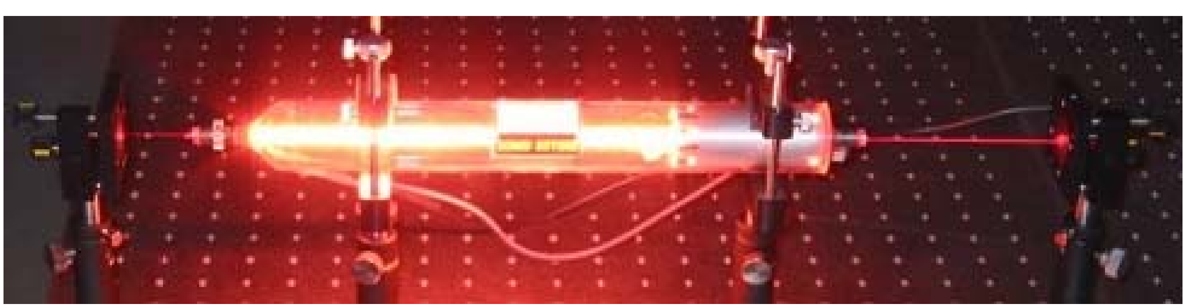


Fig. 2.25 Schematic construction of a low-power gas laser such as the helium–neon laser. The load resistor serves to limit the current once the discharge has been initiated.

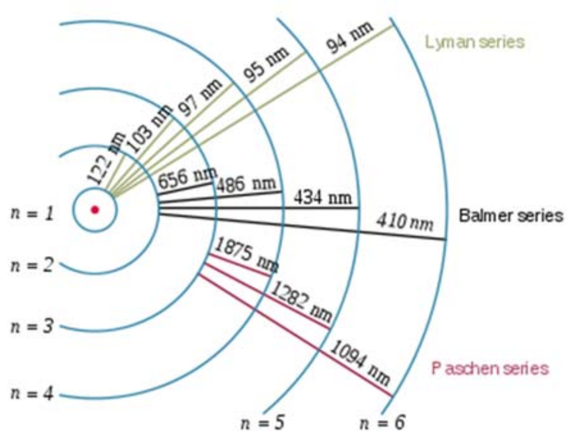


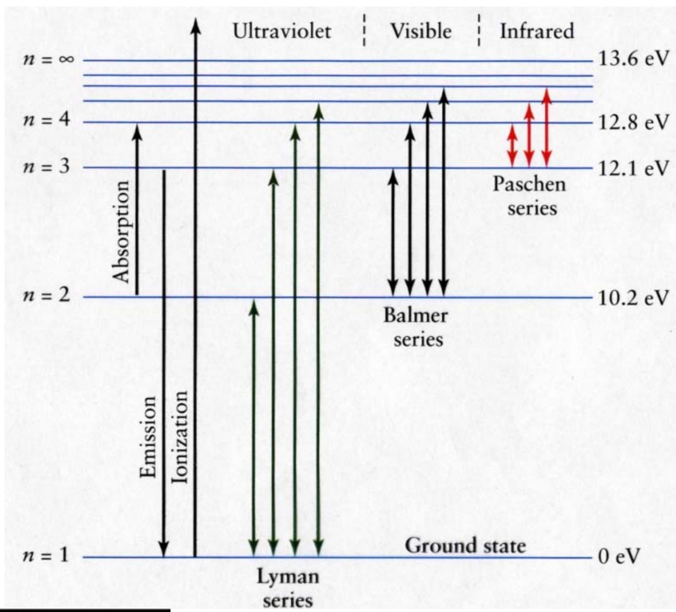
Quantum Energy Levels

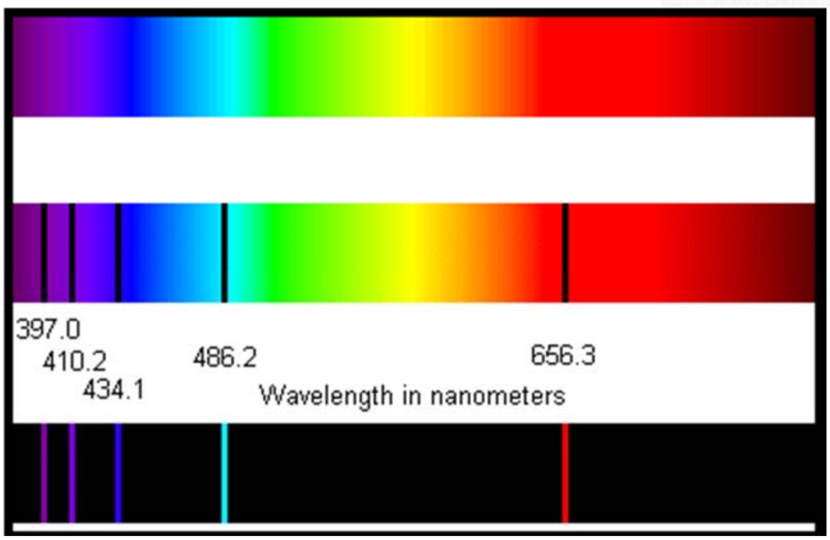
- Atoms and molecules all have quantized energy levels
- ie electrons/atoms gain or output energy at specific levels
- The exact level depends on characteristics of the atom/molecule
- Going from higher energy to lower is emission of light
- Eg in H atom going from n=3 (12.1 ev) to n=2 (10.2 ev) is 1.9ev
- Emitted wavelength is

$$\lambda = \frac{hc}{E_3 - E_2} = \frac{1.24x10^{-6}}{12.1 - 10.2} = 0.656x10^{-6}m = 656nm$$

- This is the hydrogen alpha line
- Lasers operation depends on the quantum energy levels







Regular Light Sources: Equilibrium Energy Populations

- Laser are quantum devices using characteristics of atomic levels
- Assume gas in thermal equilibrium at temperature T
- Some atoms in a Gas are in an excited state
- Quantization means discrete energy levels
- Initially N_i Atoms (atoms/m^3) at a given ith energy level E_i
- E_0 is the ground state (unexcited)
- Under steady state thermal most at lowest energy levels
- Fraction at a given energy follows a Boltzmann distribution

$$\frac{N_i}{N_0} = exp\left(- \frac{[E_i - E_0]}{KT} \right)$$

T = degrees K

K = Boltzman constant 1.38 x 10^{-23} J/K = 8.62 x 10^{-5} eV/K

- Boltzman distribution is what creates the Black body emission

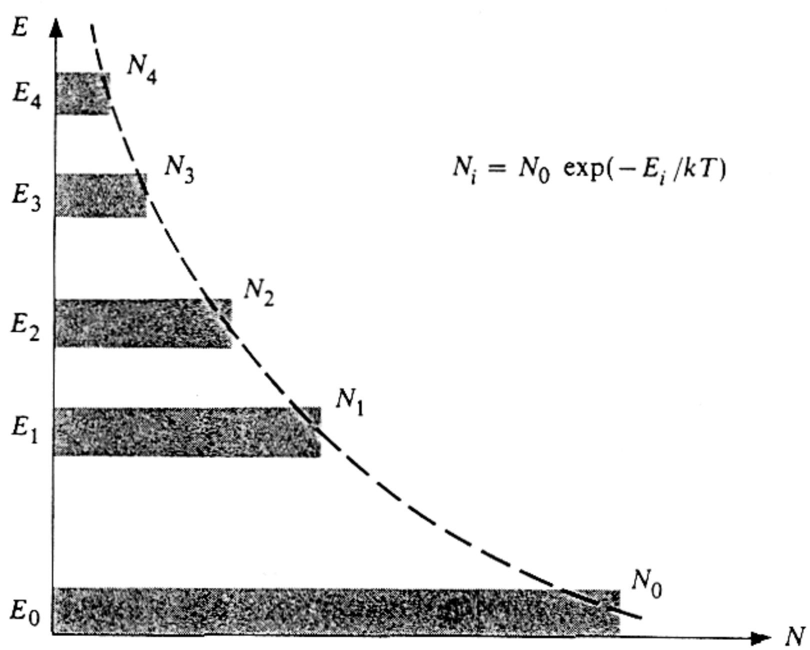


Fig. 3.3 Boltzmann distribution for several energy levels. Dashed line indicates the population of levels if the distribution of energy levels were continuous rather than discrete, as shown here.

Spontaneous and Stimulated Emission

- Consider 2 energy levels E_0 (ground state) and E_1 (excited state)
- Photon can cause **Stimulated Absorption** E_0 to E_1
- Excited state has some finite lifetime, τ_{10}
  (average time to change from state 1 to state 0)
- **Spontaneous Emission** of photon when transition E_1 to E_0 occurs
- Randomly emitted photons when change back to level 0
- Passing photon of same λ can cause **"Stimulated Emission"**
- Stimulated photon is emitted in phase with causal photon
- Einstein proposed this in 1916
- Emag field of passing photon causes transistion
- Stimulated emission the foundation of laser operation

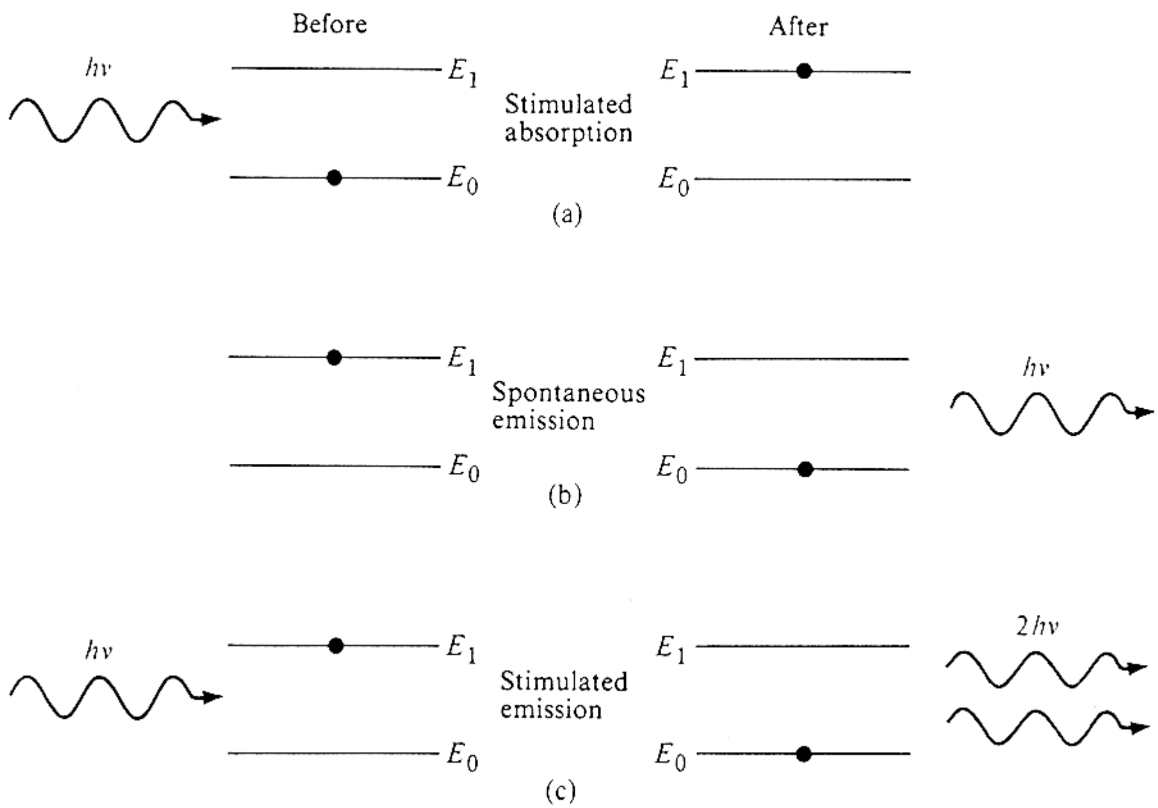


Fig. 3.1 Energy-state-transition diagram differentiating between stimulated absorption, spontaneous emission, and stimulated emission. A black dot indicates the state of the atom before and after the transition takes place. In the stimulated emission process, energy is added to the stimulating wave during the transition; in the absorption process, energy is extracted from the wave.

Einstein's Rate Equations

- Between energy levels 2 and 1 the rate of change from 2 to 1 is

$$\frac{dN_{21}}{dt} = -A_{21}N_2$$

- where A_{21} is the Einstein Coefficient (s^{-1})
- After long time energy follows a Boltzmann distribution

$$\frac{N_2}{N_1} = exp\left(- \frac{[E_2 - E_1]}{KT} \right)$$

- If $(E_2 - E_1) >> KT$ then over a long time

$$N_2(t) = N_2(0)\, exp(A_{21}t)$$

- Thus in terms of the lifetime of the level τ_{21} sec,

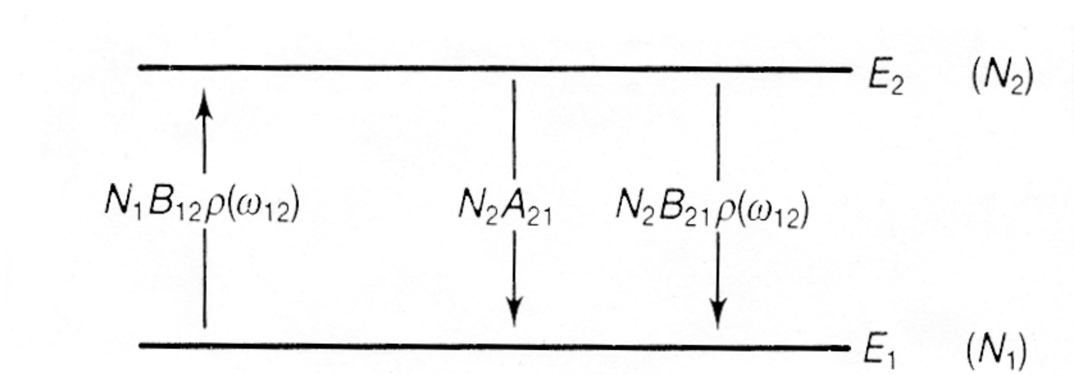
$$A_{21} = \frac{1}{\tau_{21}}$$

- illuminated by light of energy density $\rho = nh\nu$ (J/m^3) (n= number of photons/m^3) of frequency ν_{12} the absorption is
- At frequency ν_{12} the absorption is

$$\frac{dN_1}{dt} = N_1 B_{12} \rho(\nu_{12}) \quad emissions/m^3 s$$

- B_{12} is the Einstein absorption coefficient (from 1 to 2)
- Similarly stimulated emission rate (with B_{21}=B_{12}) is

$$\frac{dN_2}{dt} = N_2 B_{21} \rho(\nu_{21}) \quad emissions/m^3 s$$



Two level system: Population Inversion

- In thermal equilibrium lower level always greater population
- $N_1 >> N_2$
- Can suddenly inject energy into system - pumping
- Now not a equilibrium condition
- If pumped hard enough get "**Population Inversion**"
- Upper level greater than lower level: $N_2 >> N_1$
- Reason is the τ_{21} lifetime increases upper level population
- Population Inversion is the foundation of laser operation
  Creates the condition for high stimulated emission
- In practice difficult to get 2 level population inversion
- Best pumping with light gives is equal levels
- Reason is Einstein's rate equations

$$\frac{dN_2}{dt} = N_2 B_{21} \rho(\nu_{21}) = N_1 B_{12} \rho(\nu_{21}) = \frac{dN_1}{dt} \quad emissions / m^3 s$$

- Since $B_{21}=B_{12}$ then $N_1=N_2$ with light pumping
- Need more levels to get population inversion

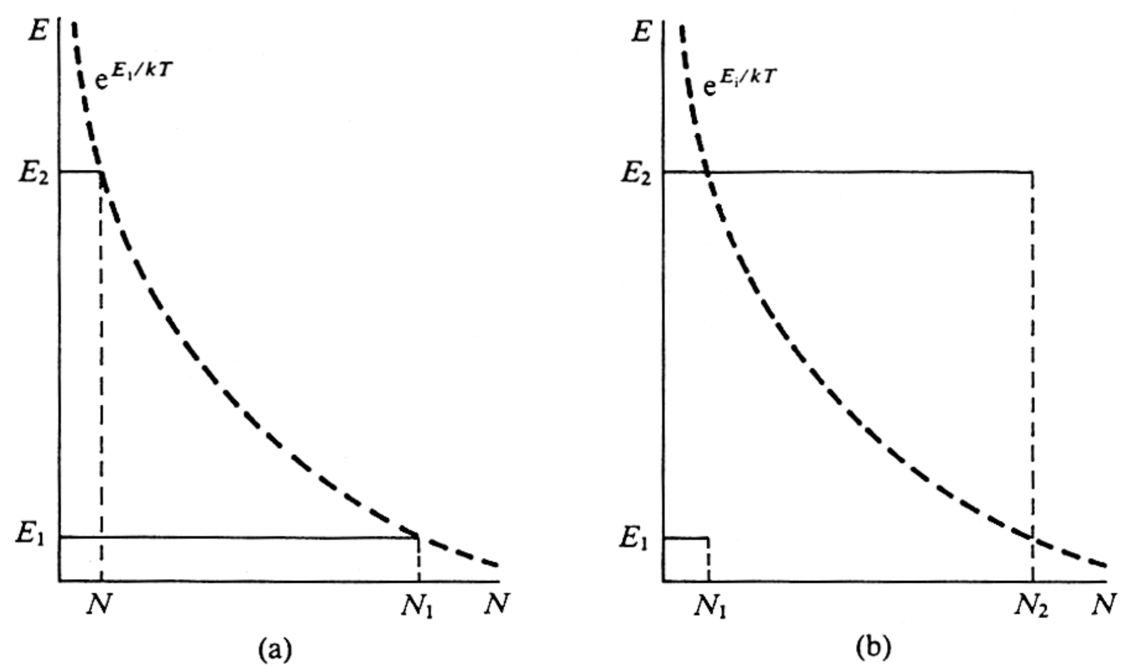


Fig. 1.11 Populations of a two-level energy system (a) in thermal equilibrium, (b) after a population inversion has been produced.

Three level systems

- Pump to E_0 level E_2, but require E_2 to have short lifetime
- Rapid decay to E_1
- E_1 must have very long lifetime: called **Metastable**
- Now population inversion readily obtained with enough pumping
- Always small amount of spontaneous emission (E_1 to E_0)
- Spontaneous create additional stimulated emission to E_0
- If population inversion: stimulated emission dominates: Lasing
- Common example Ruby (1st laser) and Nd:Yag laser
- Problem: E_0 often very full

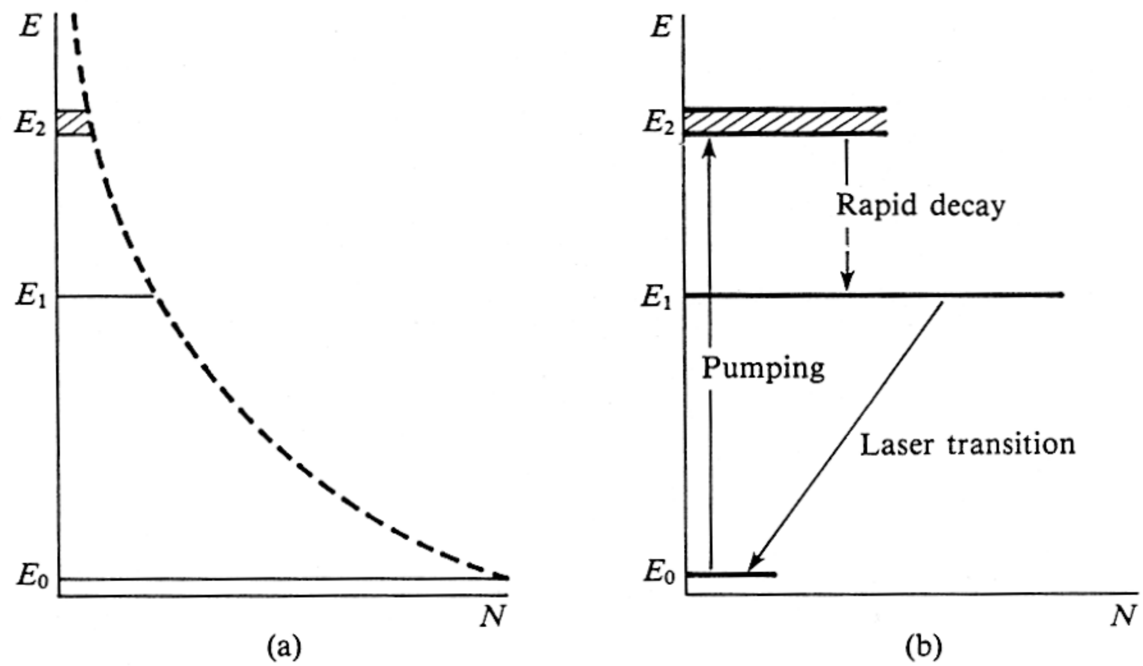
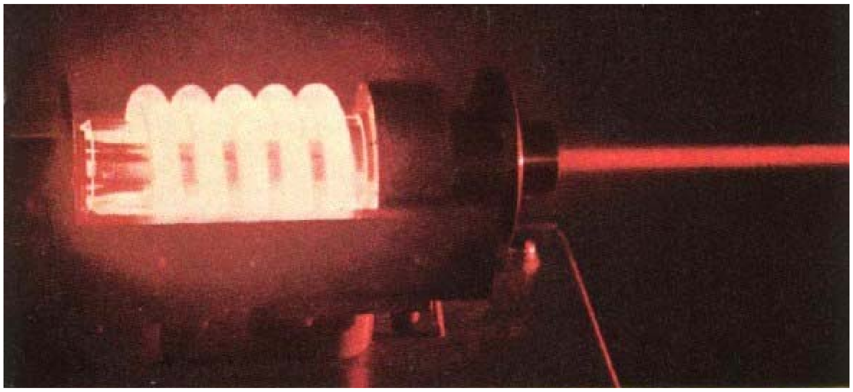


Fig. 1.12 Population of the energy levels by pumping in a three-level system: (a) Boltzmann distribution before pumping; (b) distribution after pumping and the transitions involved.



He-Ne Laser Energy levels

- He-Ne laser is example of 3 level laser
- Levels are created in both He and Ne atoms
- Fast decay from lower state (2p) to ground state
- First laser (Ruby) was a 3 level system

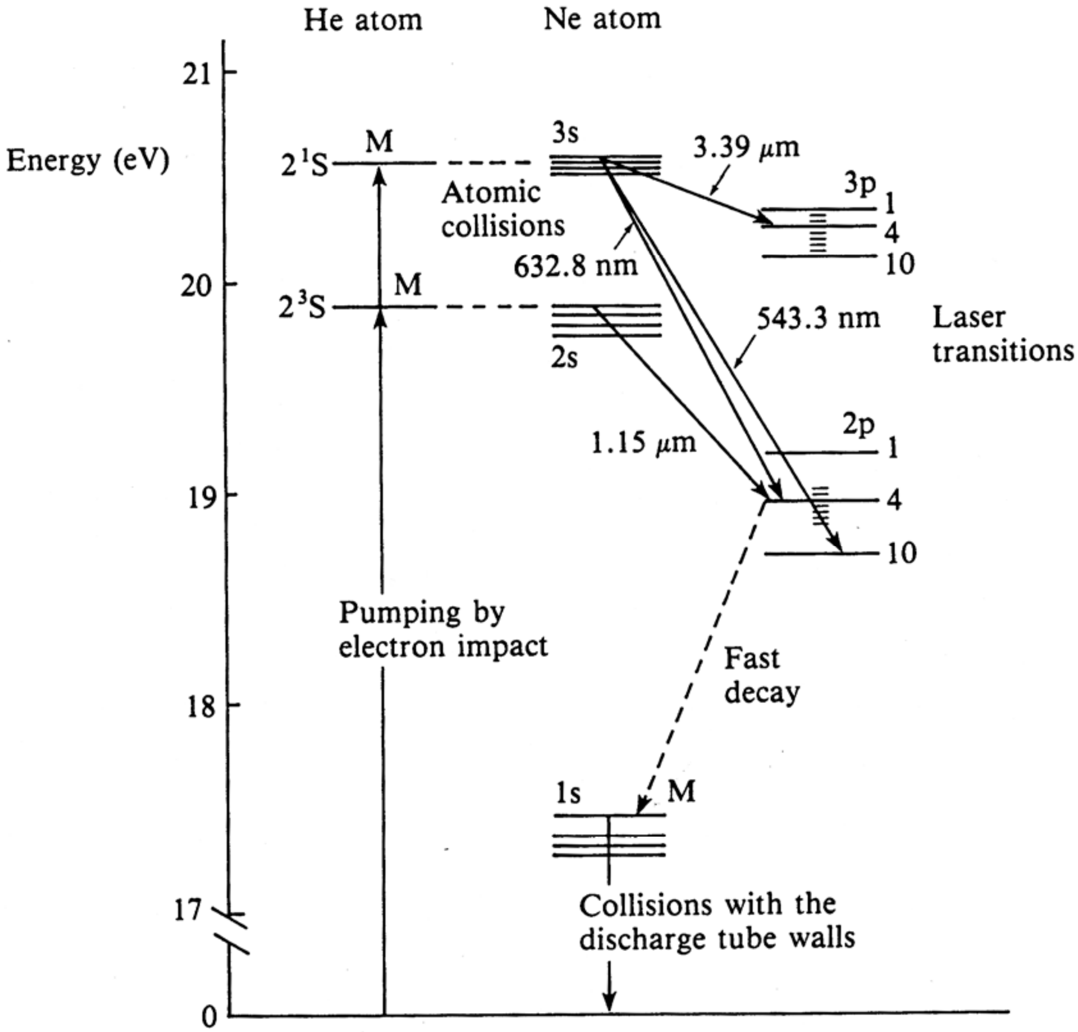
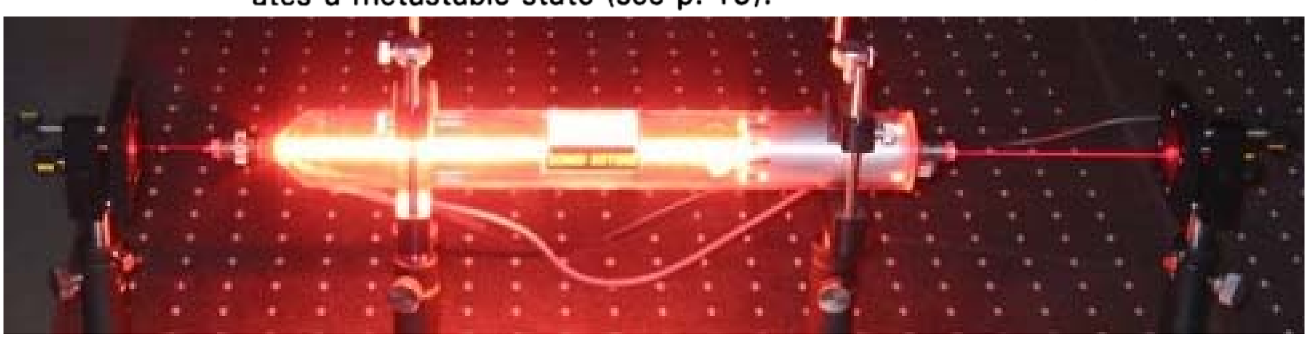


Fig. 2.26 Energy levels relevant to the operation of the HeNe laser. M indicates a metastable state (see p. 19).



Four Level Systems

- Pump to level E_3, but require E_3 to have short lifetime
- Rapid decay to E_2 so E_3 is always empty
- E_2 must have very long lifetime: metastable so fills up
- Also require E_1 short lifetime for decay to E_0
- Now always have E_1 empty relative to E_2
- In principal easier for population inversion but lowers efficiency
- Always small amount of spontaneous emission (E_2 to E_1)
- Spontaneous photons create additional stimulated emission to E_1
- If population inversion: stimulated emission dominates
- Problem: low efficiency: energy losses at E_3 to E_2 and E_1 to E_0
- Example Carbon Dioxide laser (very efficient)

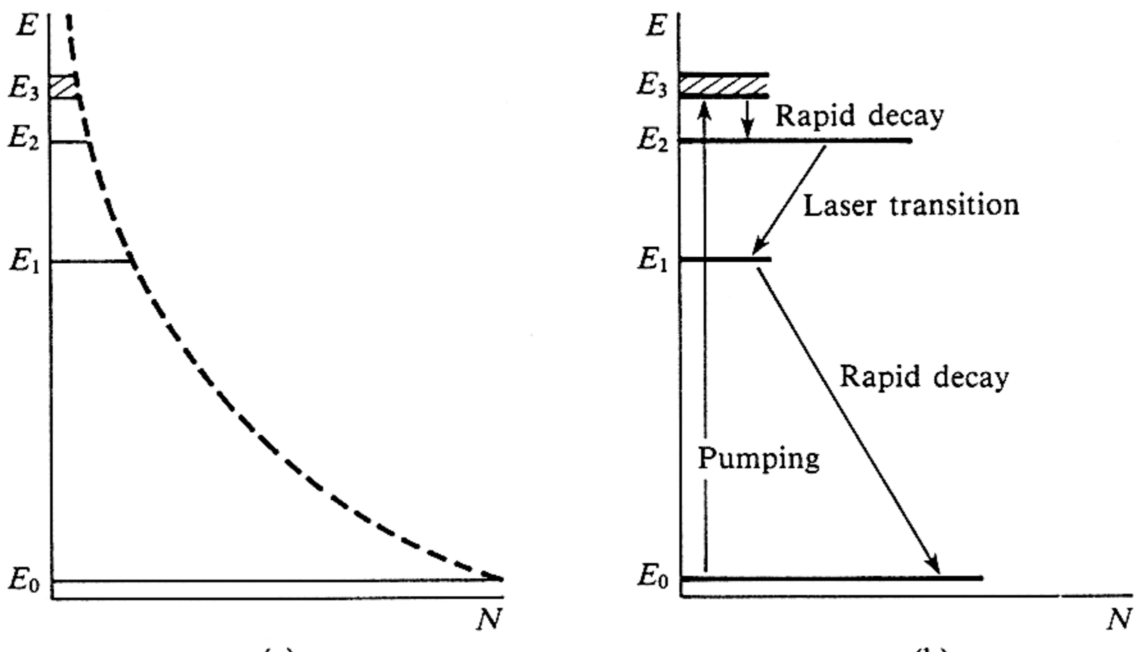
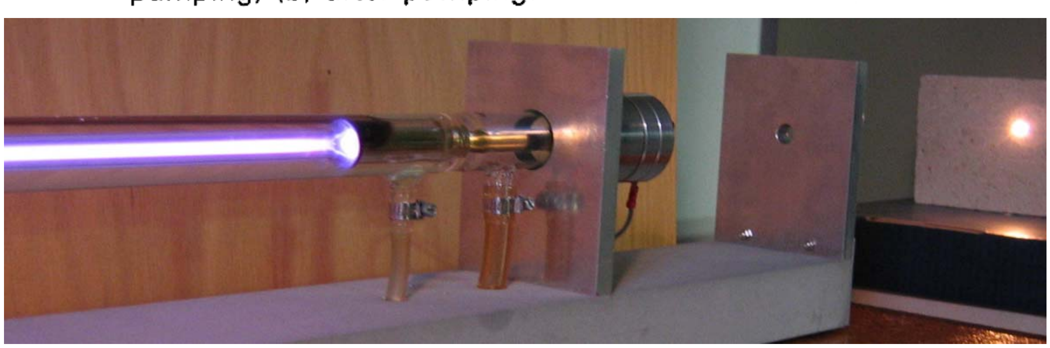


Fig. 1.13 Population of the energy levels in a four-level system: (a) before pumping; (b) after pumping.



Absorption in Homogeneous Mediums

- Monochromatic beam passing through absorbing medium homogeneous medium
- Change in light intensity I is

$$\Delta I = I(x + \Delta x) - I(x)$$

$$\Delta I = -\alpha \Delta x I(x)$$

where α = the absorption coefficient (cm^{-1})

- In differential form

$$\frac{dI(x)}{dx} = -\alpha I(x)$$

- This differential equation solves as

$$I(x) = I_0 \exp(-\alpha x)$$

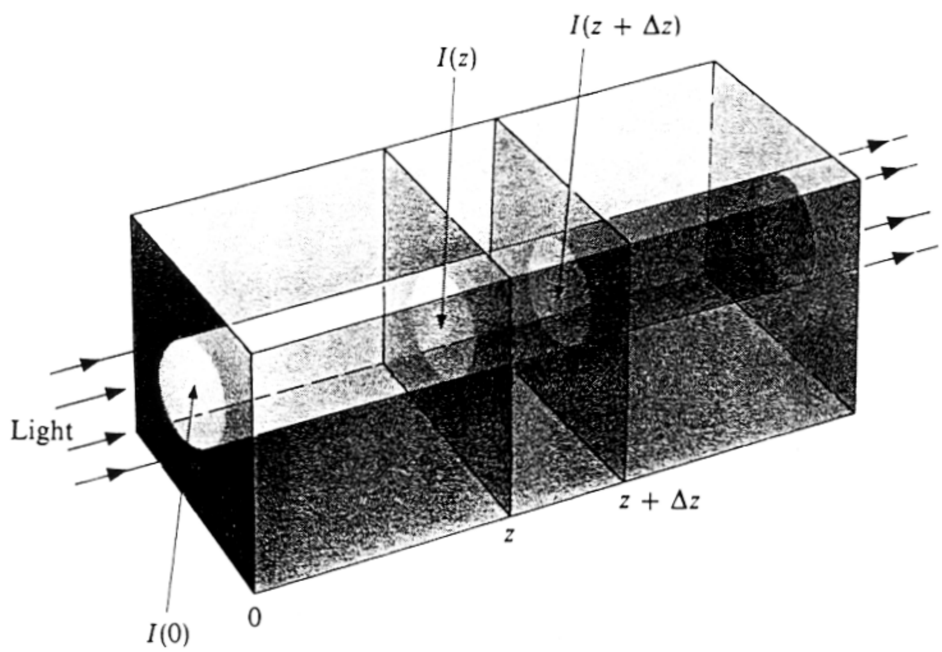


Fig. 3.2 Collimated beam of light traversing an absorbing gas. The change in the irradiance across a small slab of the gas is proportional to the irradiance at the slab and to the thickness of the slab, Δz.

Gain in Homogeneous Mediums

- If we have a population inversion increase I
- Stimulated emission adds to light: gain

$$I(x) = I_0 \exp(gx)$$

g = small signal gain coefficient (cm^{-1})

- In practice get both absorption and gain

$$I(x) = I_0 \exp([g - a]x)$$

- Gain is related directly to the population inversion

$$g = g_0(N_1 - N_0)$$

g_0 = a constant for a given system

- This seen in the Einstein B Coefficients
- Thus laser needs gain medium to amplify signal

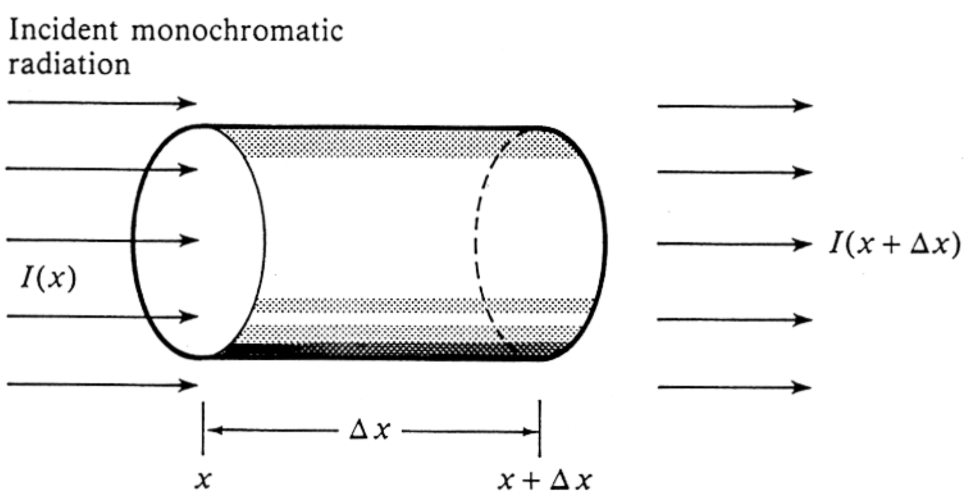


Fig. 1.10 Radiation passing through a volume element of length Δx and unit cross-sectional area of an absorbing medium.

Optical Resonator Cavity

- In laser want to confine light: have it bounce back and forth
- Then it will gain most energy from gain medium
- Need several passes to obtain maximum energy from gain medium
- Confine light between two mirrors (Resonator Cavity)
  Also called Fabry Perot Etalon
- Have mirror (M_1) at back end highly reflective
- Front end (M_2) not fully transparent
- Place pumped medium between two mirrors: in a resonator
- Needs very careful alignment of the mirrors (arc seconds)
- Only small error and cavity will not resonate
- Curved mirror will focus beam approximately at radius
- However is the resonator stable?
- Stability given by g parameters: g_1 back mirror, g_2 front mirror:

$$g_i = 1 - \frac{L}{r_i}$$

- For two mirrors resonator stable if

$$0 < g_1 g_2 < 1$$

- Unstable if

$$g_1 g_2 < 0 \qquad g_1 g_2 > 1$$

- At the boundary ($g_1 g_2 = 0$ or 1) marginally stable

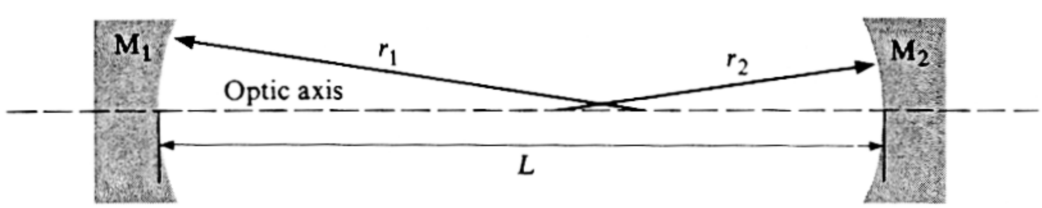


Fig. 3.8 Optical resonator consisting of two spherical mirrors. Radii are defined as having positive values if the mirrors are concave.

Stability of Different Resonators

- If plot g_1 vs g_2 and $0 < g_1 g_2 < 1$ then get a stability plot
- Now convert the g's also into the mirror shapes

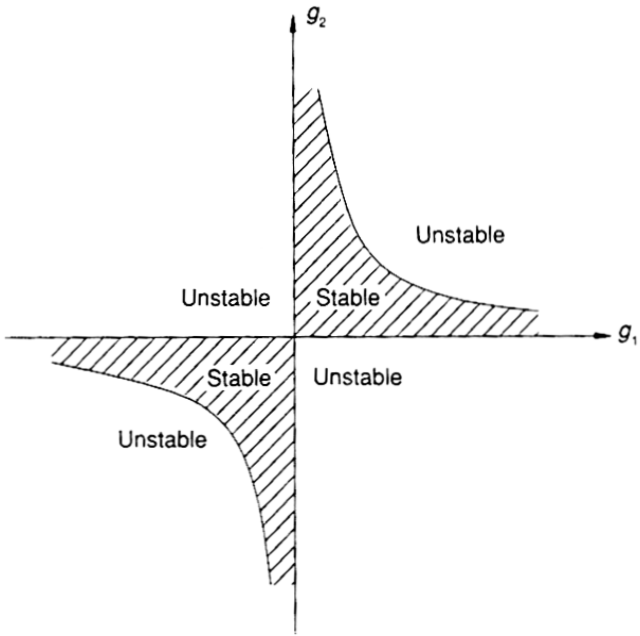


Fig. 1.9 Fox and Li stability diagram.

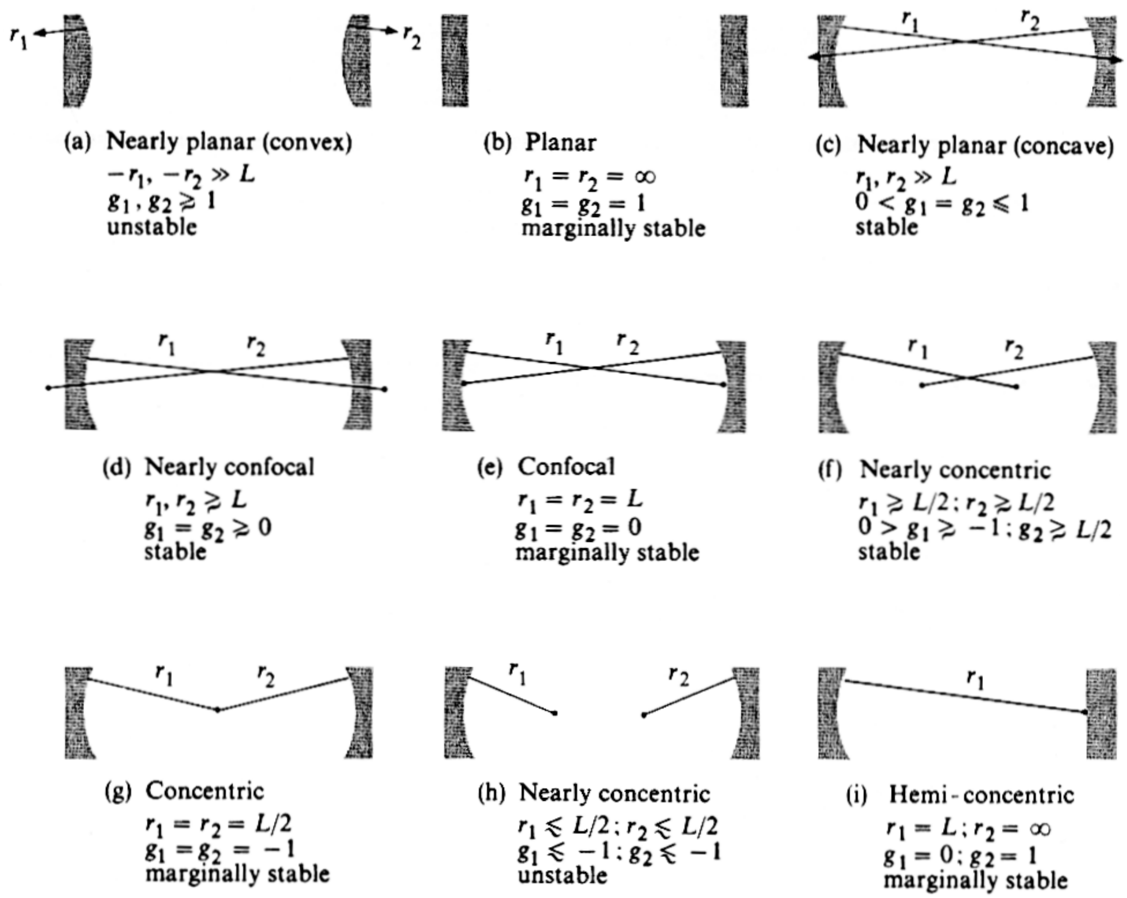


Fig. 3.9 Laser cavity mirror configurations. Stability for each of these configurations is indicated.

Polarization and Lasers

- Lasers often need output windows on gain medium in cavity
- Output windows often produce polarized light
- Normal windows lose light to reflection
- Least loss for windows if light hits glass at **Brewster Angle**
- Perpendicular polarization reflected
- Parallel polarization transmitted with no loss (laser more efficient)
- Called a Brewster Window & the Brewster Angle recall is

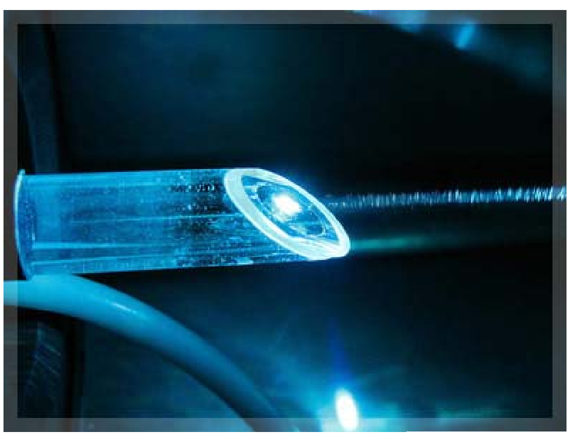
$$\theta_b = tan^{-1}\left(\frac{n_2}{n_1}\right)$$

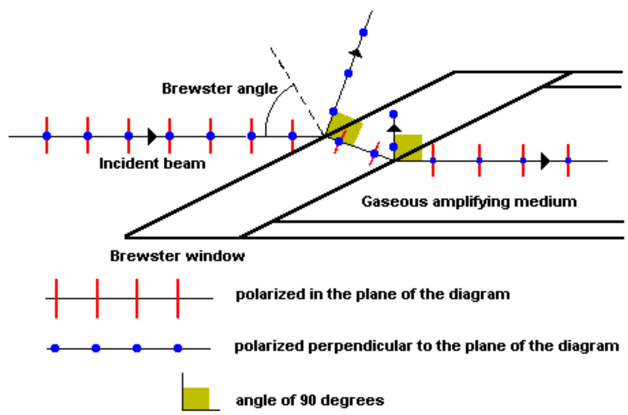
n_1 = index of refraction of air

n_2 = index of refraction of window

- Example: What is Brewster for Glass of $n_2 = 1.5$

$$\theta_b = tan^{-1}\left(\frac{n_2}{n_1}\right) = tan^{-1}\left(\frac{1.5}{1}\right) = 56.6^o$$





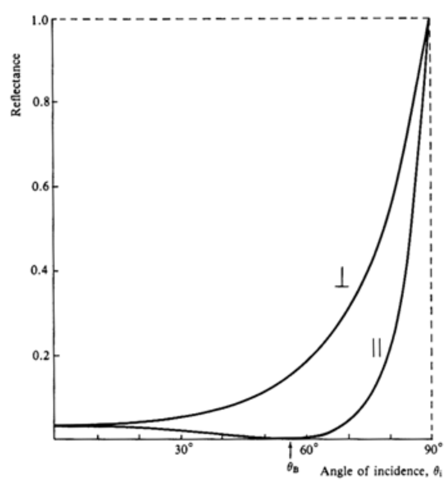


Fig. 2.5 Reflectance as a function of the angle of incidence for light polarized parallel (∥) and perpendicular (⊥) to the plane of incidence.

Gain Medium & Laser Threshold

- With good Gain Medium in optical cavity can get lasing but only if gain medium is excited enough
- Low pumping levels: mostly spontaneous emission
- At some pumping get population inversion in gain medium
- Beyond inversion get **Threshold** pumping for lasing set by the losses in cavity
- Very sensitive to laser cavity condition eg slight misalignment of mirrors threshold rises significantly
- At threshold gain in one pass = losses in cavity

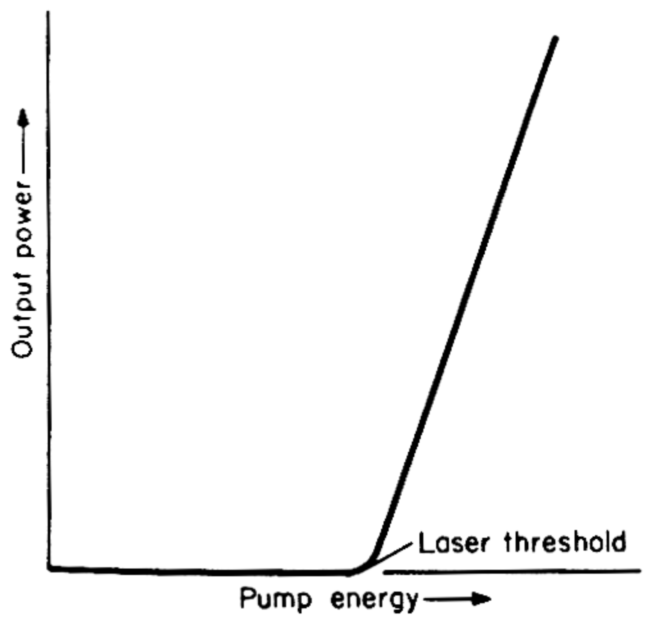


Figure 3.5 Laser threshold phenomenon—a laser does not generate significant optical output until the pump energy passes a threshold. At higher pump energies, the output power increases rapidly. In practice, each laser has limits on output, and eventually the output-input curve bends over.

Round Trip Power Gain

- Within medium light intensity I gained in one pass

$$I(L) = I_0 \exp[(g - \alpha)L]$$

where g = small signal gain coefficient
α = the absorption coefficient
L = length of cavity

- Thus calculate **Round Trip Power** Gain G_r
- Each mirror has a reflectivity R_i
R=1 for perfect reflection off a mirror

$$G_r = \frac{I(2L)}{I(0)} = R_1 R_2 \exp[(g - \alpha)2L]$$

- At threshold $G_r = 1$
- Thus threshold gain required for lasing is

$$g_{th} = \alpha + \frac{1}{2L} \ln\left(\frac{1}{R_1 R_2}\right)$$

- Some of the loss = laser emission

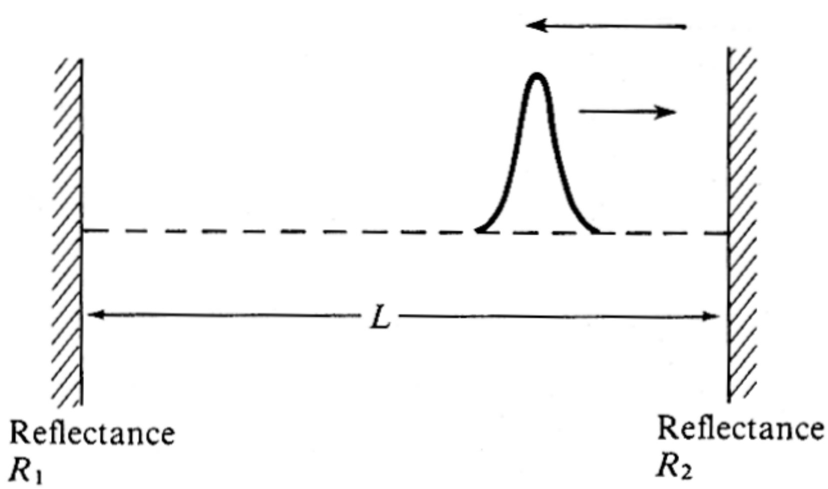


Fig. 3.3 A bunch of photons bouncing back and forth between the mirrors in a laser cavity.

Transition Line Shape

- Distribution of energy levels creates width of emission
- Usually Gaussian shape
- Width broadened by many mechanisms
- Most more important in gases
- Doppler Broadening
  (movement of molecules creates Doppler shift in wavelength)
- Collision Broadening (atomic/molecular collisions)
- Radiative Lifetime Broadening
  (From the finite lifetime of transitions)

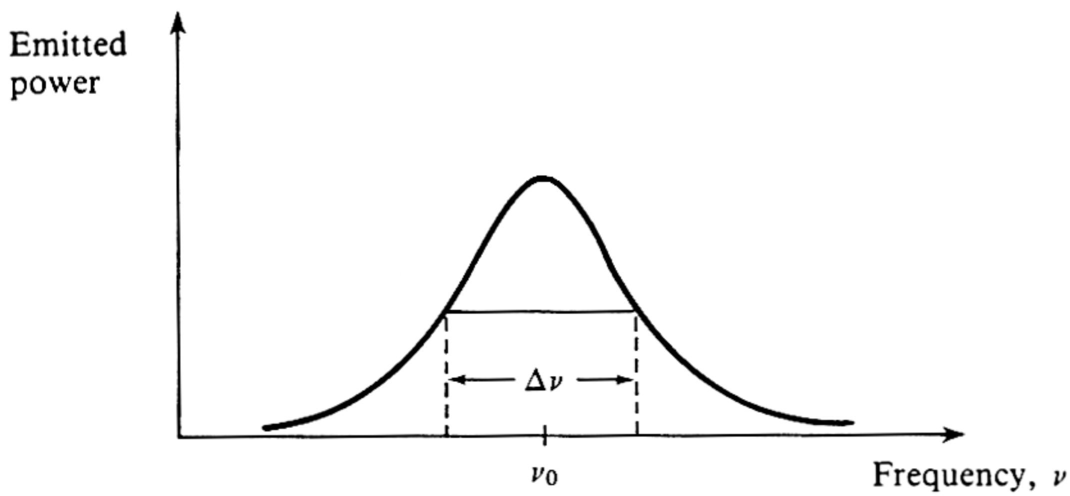


Fig. 1.16 Frequency distribution of the radiation emitted from a group of atoms following transitions from energy levels E_2 to E_1. The precise shape of this distribution, that is the lineshape function $g(\nu)$, depends on the dominant spectral broadening mechanisms.

Axial Modes of Laser

- Proper phase relationship only signal in phase in cavity
- Thus cavity must be integer number of half wavelengths

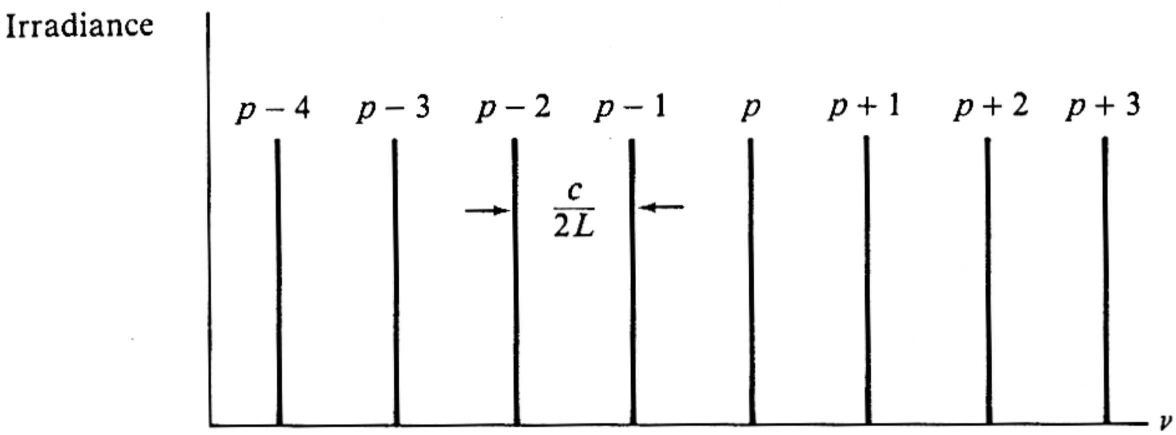
$$L = \frac{p\lambda}{2}$$

where p = integer

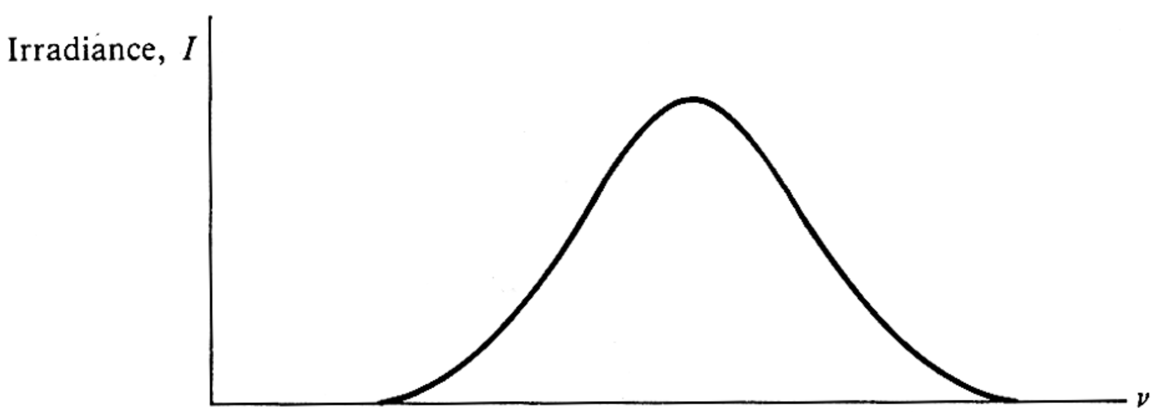
- Results in frequency separation of peaks

$$\Delta\nu = \frac{c}{2L}$$

- Emission from the atomic transitions is a wider Gaussian
- Result is axial modes imposed on Gaussian Transition spread



(a)



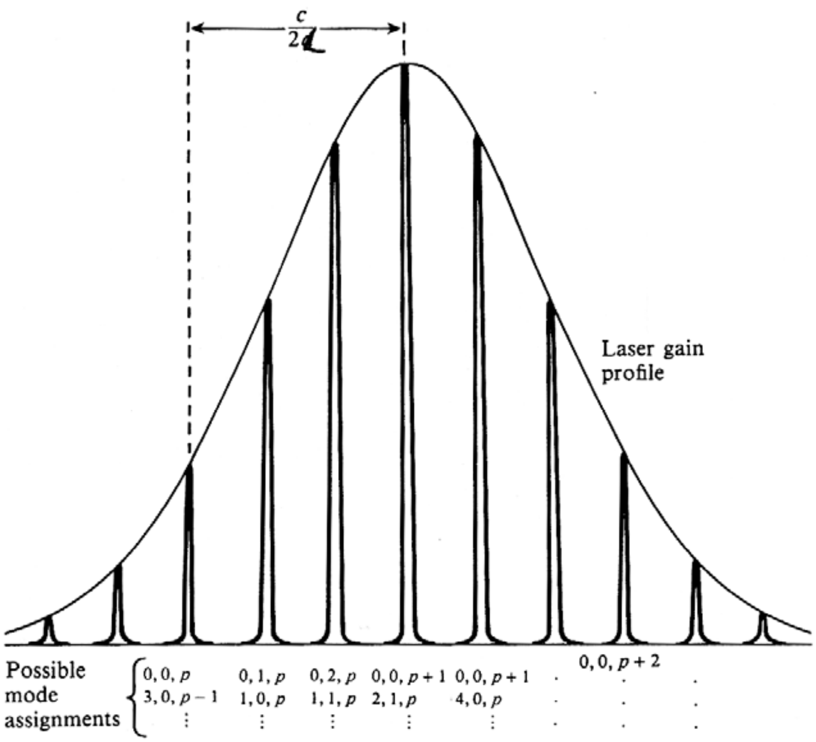
(b)

Axial modes within Transition Gaussian

- Each axial mode is Gaussian shape narrower than transition peak
- eg for L=1 m Argon laser at 514 nm

$$\Delta\nu = \frac{c}{2L} = \frac{3.00x10^{8}}{2} = 150\ MHz \qquad \nu = \frac{c}{\lambda} = \frac{3.00x10^{8}}{5.14x10^{-7}} = 5.83x10^{14}\ Hz$$

- Thus emission much smaller than 0.0026% of frequency
  Much narrower than other light sources.



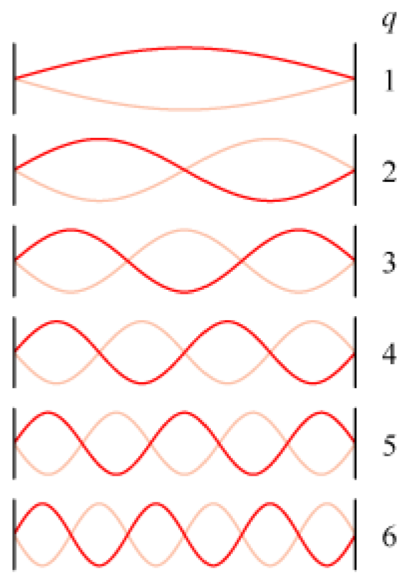


Fig. 3.2 The mode frequency spectrum for a cavity (where $r_1 = r_2 = 2L$) superimposed on the laser gain profile. Note that many modes have the same frequency (that is they are 'degenerate').

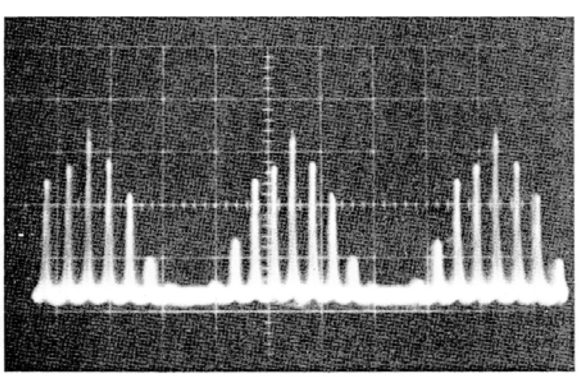
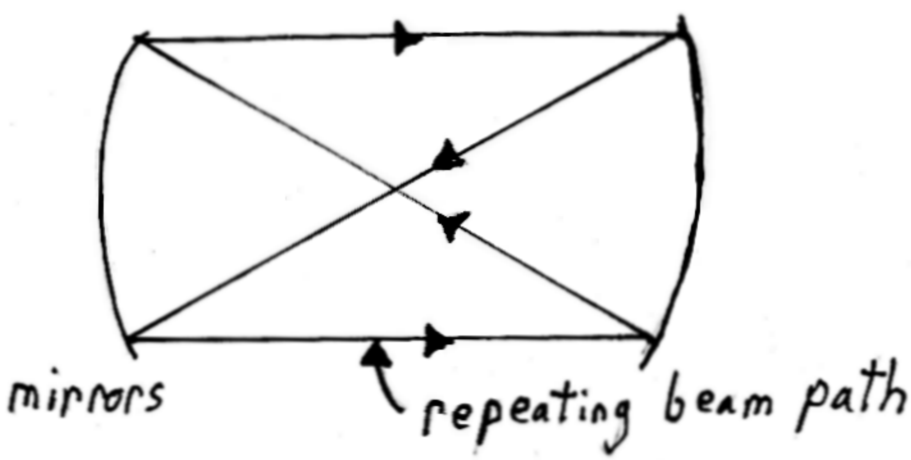


Fig. 1.18 Axial modes formed in a HeNe laser — the mode pattern is repeated (here three times) as the optical frequency analyzer scans through the gain curve of the laser (Photograph courtesy Dr. I. D. Latimer, School of Physics, Newcastle upon Tyne Polytechnic)

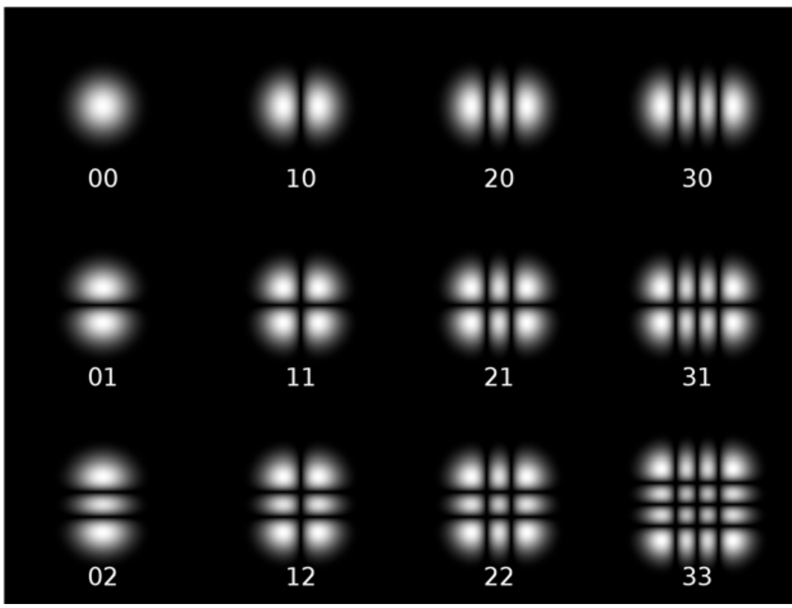
Transverse Modes

- Name comes from microwave cavities calculations
- Idea: light (EMag waves) bounce off the mirrors
- Want a path that always repeats
- Some waves travel off axis, but within cavity
- Result is Phase changes in different repeating paths
- These can change shape of output
- When phase is 180^o out get local minimums (nulls)
- Result is output beam shape varies with the allowed modes
- Reduce these by narrowing the beam
- Called Transverse ElectroMagnetic, TEM
- Different modes for rectangular and cylindrical geometries
- Generate solutions to EMag equations
- If Brewster window get Rectangular geometries
- Cylindrical if no Brewster

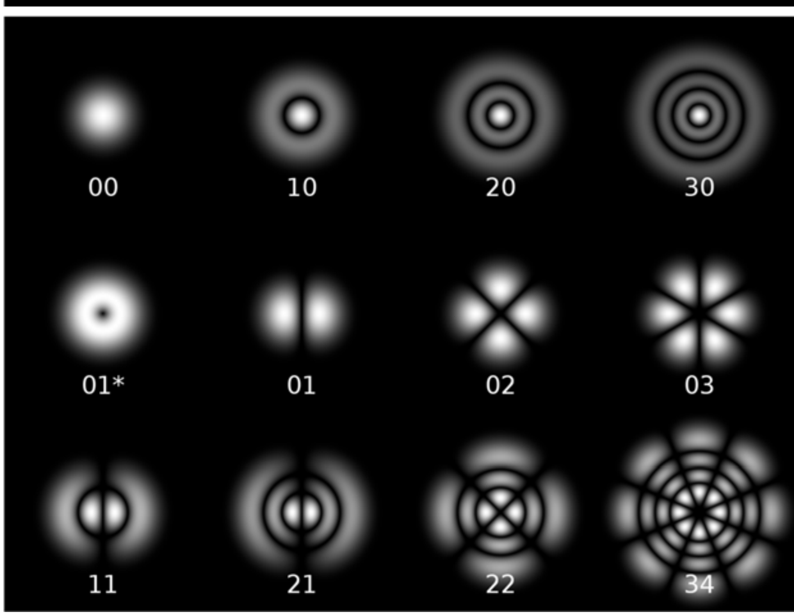


Transverse Modes

- Transverse ElectroMagnetic, TEM depend on cavity type
- In either geometry two possible phase reversal orientations
- Rectangular horizontal (q) and vertical (r) - show these as TEM_{qr}
- q and r give number of null's in output beam
- Horizontal (q) gives number of vertical running nulls
- Vertical (r) gives number of horizontal running nulls
- Thus TEM_{12} has 2 vertical, 1 horizontal nulls
- In Cylindrical q=radial, r=angular
- Special mode TEM_{01*} or donut mode
- Comes from rapid switching from TEM_{01} to TEM_{10}



Rectangular TEM



Cylindrical TEM

Einstein Coefficients and Lasers

- Recall the Einstein Emission/Absorption Coefficients
- Consider again a 2 level system being pumped by light
- A_{21} is the Einstein spontaneous emission Coefficient
- B_{12} is the Einstein spontaneous absorption coefficient
- B_{21} is the Einstein stimulated emission coefficient
- At equilibrium the absorption from pumping equals the spontaneous and stimulated emission.

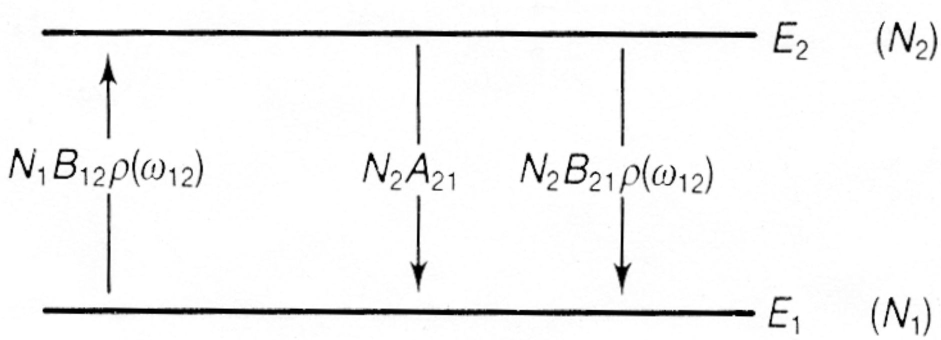
$$N_1 \rho B_{12} = N_2 A_{21} + N_2 \rho B_{21}$$

- Now recall the Boltzman distribution

$$\frac{N_1}{N_0} = exp\left(\frac{[E_1 - E_0]}{KT}\right) = exp\left(\frac{h\nu}{KT}\right)$$

- ν = the frequency of the light
- $h\nu$ = energy in photon
- Thus

$$\rho B_{12}\, exp\left(\frac{h\nu}{KT}\right) = A_{21} + \rho B_{21}$$



Einstein Coefficients relationships

- Solving for the emitted photon energy density

$$\rho = \frac{A_{21}}{B_{12}\, exp\left(\frac{h\nu}{KT}\right) - B_{21}}$$

- From Planck's law the photons emitted at a given temperature are:

$$\rho = \frac{8\pi h\nu^3}{c^3\left[exp\left(\frac{h\nu}{KT}\right) - 1\right]}$$

- From these two can solve noting $B_{12} = B_{21}$

$$A_{21} = \frac{8\pi h\nu^3}{c^3} B_{12} = \frac{8\pi h}{\lambda^3} B_{12}$$

- Note absorption to emission increases rapidly with wavelength

Under Lasing Conditions

- Spontaneous emission is nil so total emission/unit area is

$$\frac{dI}{dx} = (N_2 - N_1) B_{21} \rho h \nu$$

- For a linear beam I = ρ c (energy density times speed) thus

$$\frac{dI}{dx} = \frac{(N_2 - N_1) B_{21} h \nu I}{c}$$

- Thus the gain is

$$g = \frac{(N_2 - N_1) B_{21} h \nu}{c}$$

- Or substituting for the spontaneous coefficient

$$g = \frac{(N_2 - N_1) A_{21} c^2}{8\pi \nu^2} = \frac{(N_2 - N_1) \lambda^2}{8\pi \tau_{21}}$$

- Three important implications because we need $g > g_{th}$ to lase

(1) For gain $N_2 > N_1$ (i.e. population inversion)

(2) Shorter wavelength the lower the gain
Hence much harder to make UV than IR lasers

(3) Want short lifetime τ_{21} for higher gain

- But want metastable long τ_{21} to get population inversion!
- Hence tradeoff: τ_{21} must be short enough for threshold g but long enough for population inversion
- More difficult to achieve for CW (continuous) lasers