Gaussian Beams

- Most lasers use cylindrically symmetric cavities
- Diffraction occurs at edge of circular cavity mirrors
- Creates Gaussian Spherical Waves
- Recall E field for Gaussian

$$u(x,y,R,t) = \frac{U_0}{R} exp\left(i\left[\omega t - Kr - \frac{\left(x^2 + y^2\right)}{2R} \right] \right)$$

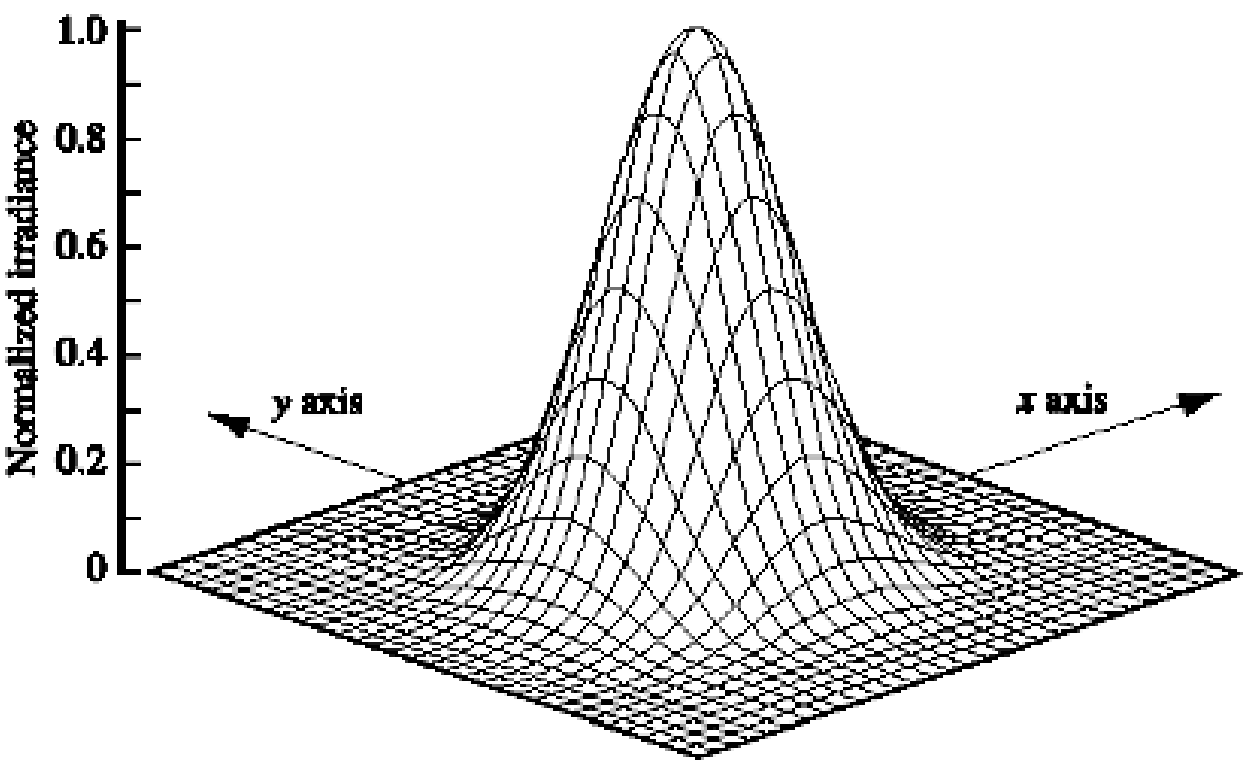
- R becomes the radius of curvature of the wave front
- These are really TEM_{00} mode emissions from laser
- Creates a Gaussian shaped beam intensity

$$I(r) = I_0 \, exp\left(\frac{-2r^2}{w^2} \right) = \frac{2P}{\pi w^2} exp\left(\frac{-2r^2}{w^2} \right)$$

Where P = total power in the beam

w = $1/e^2$ beam radius

- w changes with distance z along the beam ie. w(z)



Measurements of Spotsize

- For Gaussian beam important factor is the "spotsize"
- Beam spotsize is measured in 3 possible ways

(a) 1/e radius of beam

(b) $1/e^2$ radius = w(z) of the radiance (light intensity)
most common laser specification value
13% of peak power point
point where emag field is down by 1/e

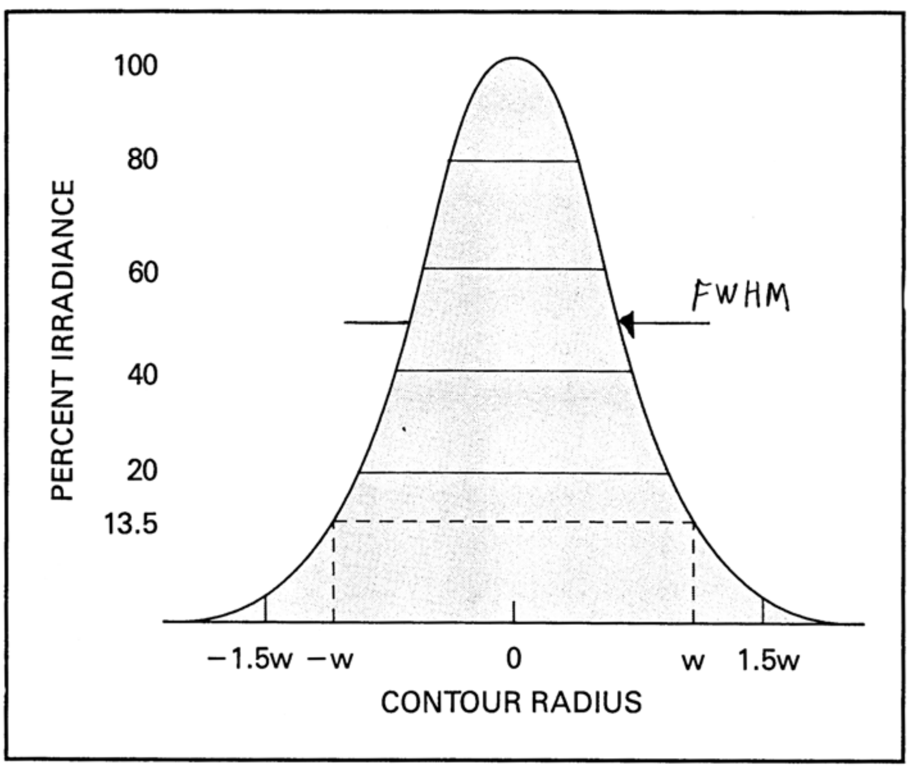
(c) Full Width Half Maximum (FWHM)
point where the laser power falls to half its initial value
good for many interactions with materials

- Useful relationships

$$FWHM = 1.177w = 1.177r_{1/e^2}$$

$$w = r_{1/e^2} = 0.849\ FWHM$$

$$FWHM = 1.665r_{1/e}$$



GAUSSIAN IRRADIANCE PROFILE for TEM_{00} mode, showing definitions of beam radius w.

Gaussian Beam Changes with Distance

- The Gaussian beam radius of curvature of E field with distance

$$R(z) = z\left[1 + \left(\frac{\pi w_0^2}{\lambda z}\right)^2\right]$$

- Gaussian spot size with distance

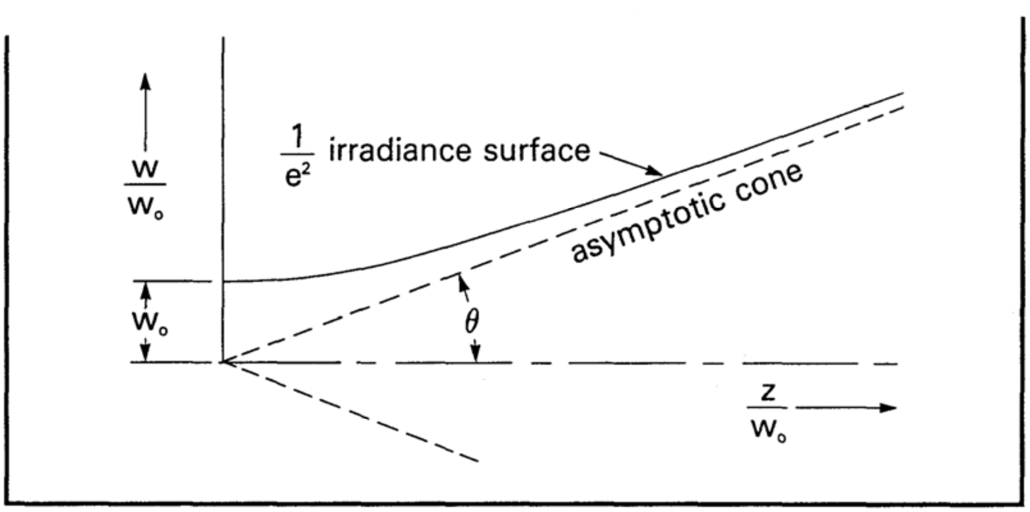
$$w(z) = w_0\left[1 + \left(\frac{\lambda z}{\pi w_0^2}\right)^2\right]^{1/2}$$

- Note: for lens systems lens diameter must be $3w_0$.= 99% of power
- Note: some books define w_0 as the full width rather than half width
- As z becomes large relative to the beam asymptotically approaches

$$w(z) \approx w_0\left(\frac{\lambda z}{\pi w_0^2}\right) = \frac{\lambda z}{\pi w_0}$$

- Asymptotically light cone angle (in radians) approaches

$$\theta \approx \frac{w(z)}{Z} = \frac{\lambda}{\pi w_0}$$



GROWTH IN 1/e² CONTOUR RADIUS with distance propagated away from Gaussian waist.

Rayleigh Range of Gaussian Beams

- Spread in beam is small when width increases < $\sqrt{2}$
- Called the Rayleigh Range z_R

$$z_R = \frac{\pi w_0^2}{\lambda}$$

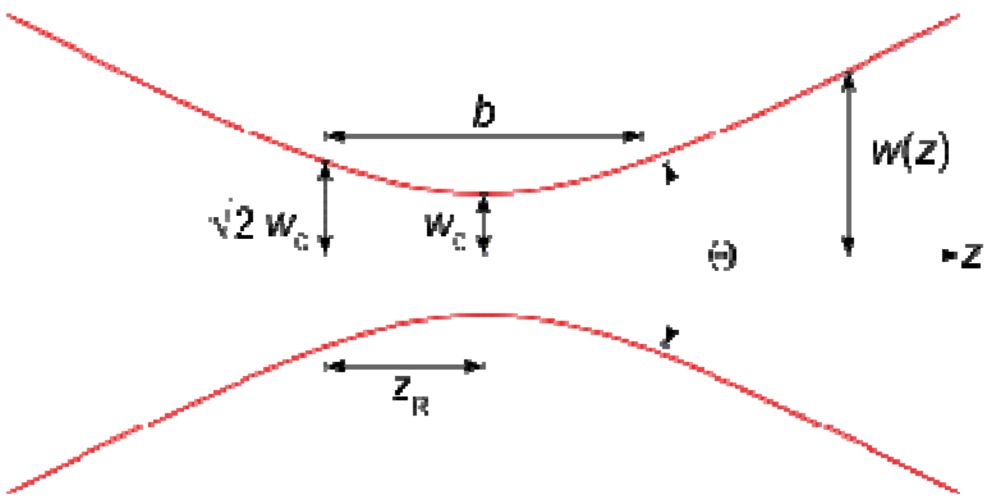
- Beam expands $\sqrt{2}$ for - z_R to $+z_R$ from a focused spot
- Can rewrite Gaussian formulas using z_R

$$R(z) = z\left[1 + \frac{z_R^2}{z^2}\right]$$

$$w(z) = w_0\left[1 + \frac{z^2}{z_R^2}\right]^{1/2}$$

- Thus w = 2 w_0 when z= 1.73 z_r and peak power = 25% of z=0
- Again for z >> z_R

$$w(z) \approx w_0 \frac{z}{z_R}$$



Beam Expanders

- Telescope beam expands changes both spotsize & Rayleigh Range
- Beam expanders are telescopes focused at infinity
- Kepler inverts the beam , Galilean does not
- For magnification **m** of side 2 relative side 1 then
  as before change of beam size is

$$w_{02} = m w_{01}$$

- Ralyeigh Range becomes

$$z_{R2} = \frac{\pi\, w_{02}^2}{\lambda} = m^2 z_{R1}$$

- where the magnification is

$$m = \frac{f_2}{f_1}$$

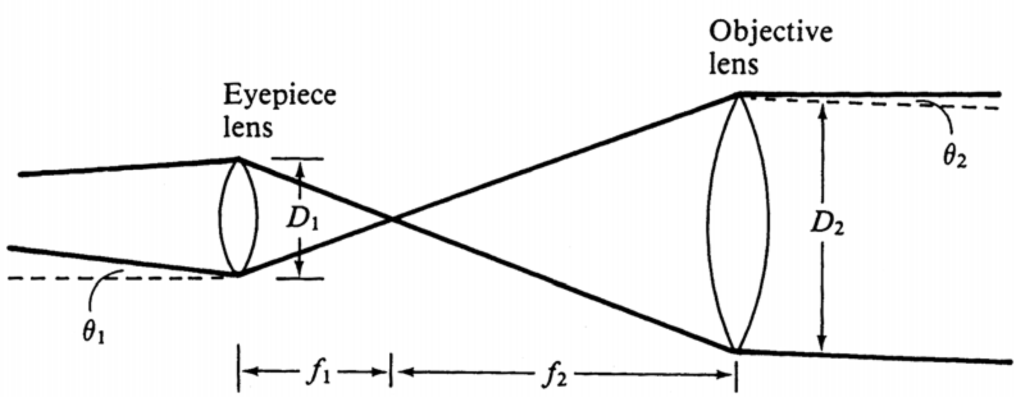


Fig. 3.7 Reduction in beam divergence using a reverse telescope arrangement. With eyepiece and objective lenses having diameters and focal lengths of D_1, f_1 and D_2, f_2 respectively, then the beam width is enlarged by the factor $D_2/D_1 = f_2/f_1$ and the divergence is decreased by the factor f_1/f_2.

Example of Beam Divergence

- eg HeNe 4 mW laser has 0.8 mm rated diameter.
  What is its z_R, spotsize at 1 m, 100 m and the expansion angle
- For HeNe wavelength λ = 632.8 nm
- Rayleigh Range is

$$z_R = \frac{\pi w_0^2}{\lambda} = \frac{\pi(0.0004)^2}{6.328x10^{-7}} = 0.794\ m$$

- At z = 1 metre

$$w(z) = w_0\left[1 + \frac{z^2}{z_R^2}\right]^{1/2} = 0.0004\left[1 + \frac{1^2}{0.794^2}\right]^{1/2} = 0.000643\ m = 0.643\ mm$$

- At z = 100 m >> z_R

$$\theta \approx \frac{w(z)}{z} = \frac{\lambda}{\pi w_0} = \frac{6.328x10^{-7}}{\pi 0.0004} = 5.04x10^{-4}\ Radians$$

$$w(z) \approx z\theta = 100(5.04x10^{-4}) = 0.0504\ m = 50.4\ mm$$

- What if beam was run through a beam expander of m = 10

$$w_{02} = mw_{01} = 10(0.0004) = 0.004\ m = 4\ mm$$

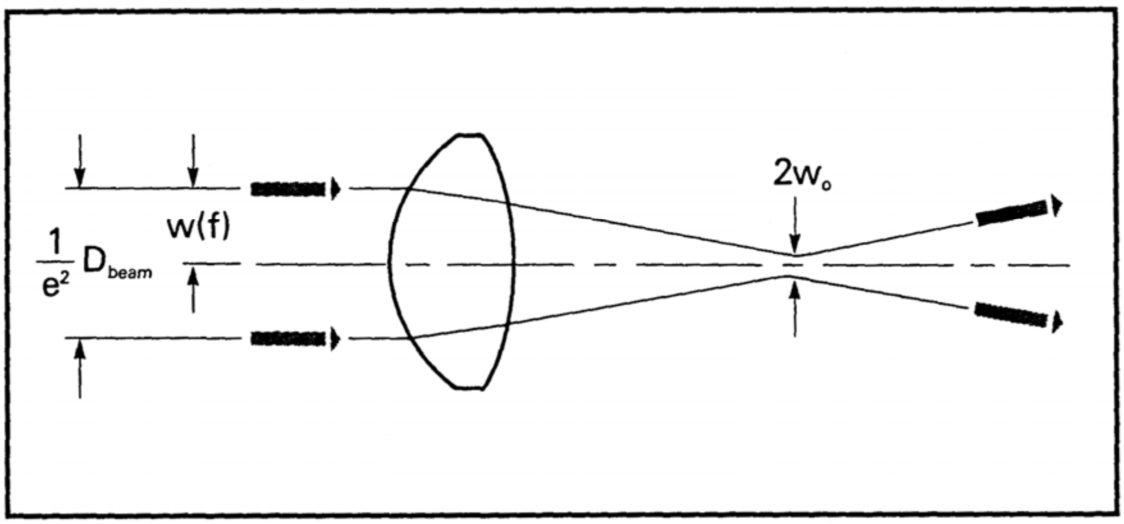
$$\theta_m = \frac{\theta}{m} = \frac{5.04x10^{-4}}{10} = 5.04x10^{-5}\ Radians$$

$$w(z) \approx z\theta = 100(5.04x10^{-5}) = 0.00504\ m = 5.04\ mm$$

- Hence get a smaller beam at 100 m by creating a larger beam first

Focused Laser Spot

- Lenses focus Gaussian Beam to a Waist
- Modification of Lens formulas for Gaussian Beams
- From S.A. Self "Focusing of Spherical Gaussian Beams"
  App. Optics, pg. 658. v. 22, 5, 1983
- Use the input beam waist distance as object distance s
  to primary principal point
- Output beam waist position as image distance s"
  to secondary principal point



CONCENTRATION OF LASER BEAM by a laser line focusing singlet. Size of the focal waist has been greatly exaggerated for illustrative purposes.

Gaussain Beam Lens Formulas

- Normal lens formula in regular and dimensionless form

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \quad or \quad \frac{1}{\left(\frac{s}{f}\right)}+\frac{1}{\left(\frac{s'}{f}\right)}=1$$

- This formula applies to both input and output objects
- Gaussian beam lens formula for input beams
  includes Rayleigh Range effect

$$\frac{1}{\left(s+\frac{z_R^2}{s-f}\right)}+\frac{1}{s'}=\frac{1}{f}$$

- in dimensionless form

$$\frac{1}{\left(\frac{s}{f}\right)+\frac{\left(\frac{z_R}{f}\right)^2}{\left(\frac{s}{f}-1\right)}}+\frac{1}{\left(\frac{s'}{f}\right)}=1$$

- in far field as z_R goes to 0
  (ie spot small compared to lens)
  this reduces to geometric optics equations

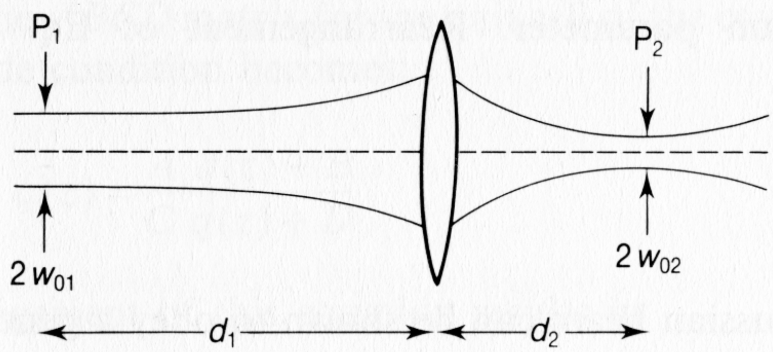
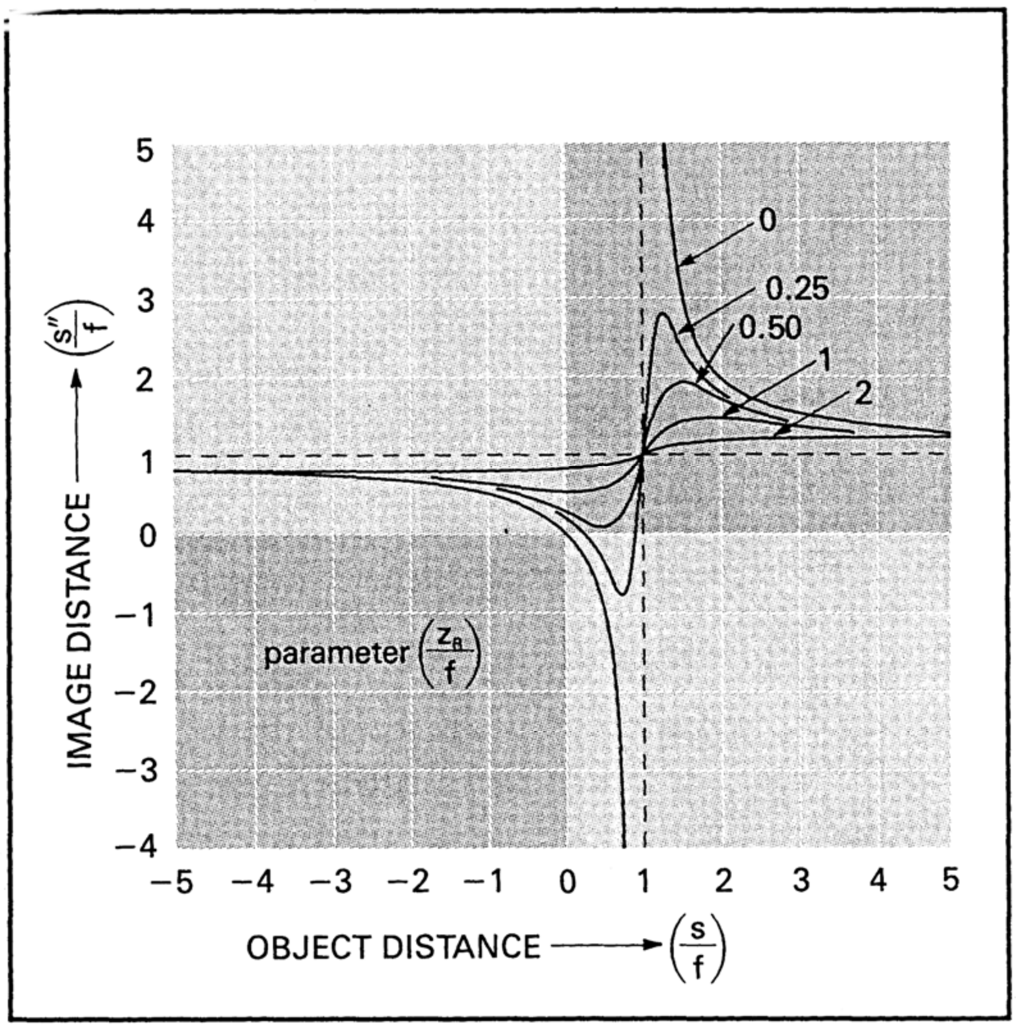


Figure 4.23 Focusing of a Gaussian beam by a convex lens.

Gaussain Beam Lens Behavior

- Plot shows 3 regions of interest for positive thin lens
- Real object and real image
- Real object and virtual image
- Virtual object and real image



PLOT OF THE LENS FORMULA for Gaussian beams showing normalized image distance vs normalized object distance, with normalized Rayleigh range of the input beam as the parameter.

Main Difference of Gaussian Beam Optics

- For Gaussian Beams there is a maximum and minimum image distance
- Maximum image not at s = f instead at

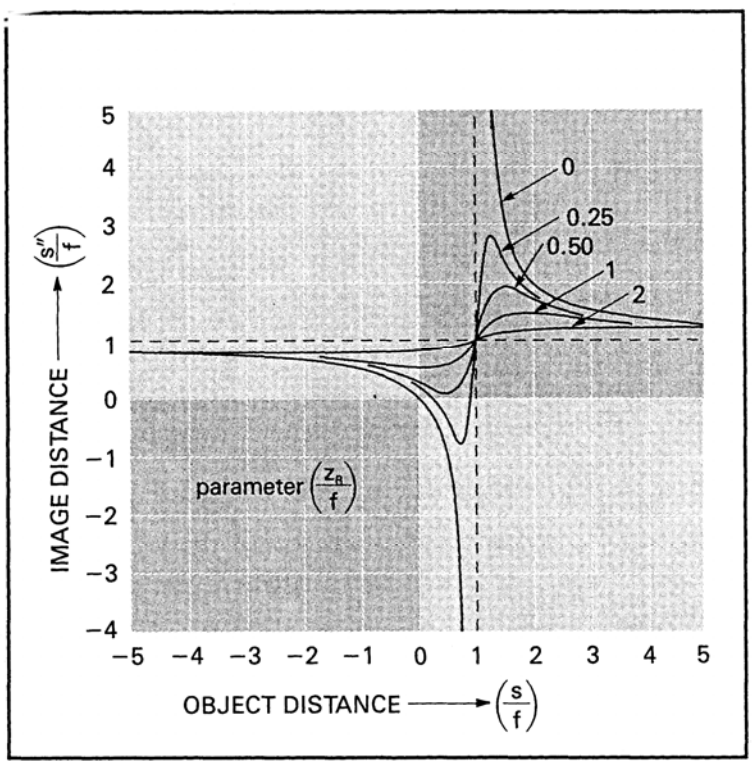
$$s = f + z_R$$

- There is a common point in Gaussian beam expression at

$$\frac{s}{f} = \frac{s''}{f} = 1$$

For positive lens when incident beam waist at front focus then emerging beam waist at back focus

- No minimum object-image separation for Gaussian
- Lens f appears to decrease as z_R/f increases from zero i.e. Gaussian focal shift



PLOT OF THE LENS FORMULA for Gaussian beams showing normalized image distance vs normalized object distance, with normalized Rayleigh range of the input beam as the parameter.

Magnification and Output Beams

- Calculate z_R and w_0, s and s" for each lens
- Magnification of beam

$$m = \frac{w_0''}{w_0} = \frac{1}{\left\{\left[1 - \left(\frac{s}{f}\right)\right]^2 + \left(\frac{z_R}{f}\right)^2\right\}^{1/2}}$$

- Again the Rayleigh range changes with output

$$z_R'' = m^2 z_R$$

- The Gaussian Beam lens formula is not symmetric
  From the output beam side

$$\frac{1}{s} + \frac{1}{\left[s'' + \frac{z_R''^2}{(s'' - f)}\right]} = \frac{1}{f}$$

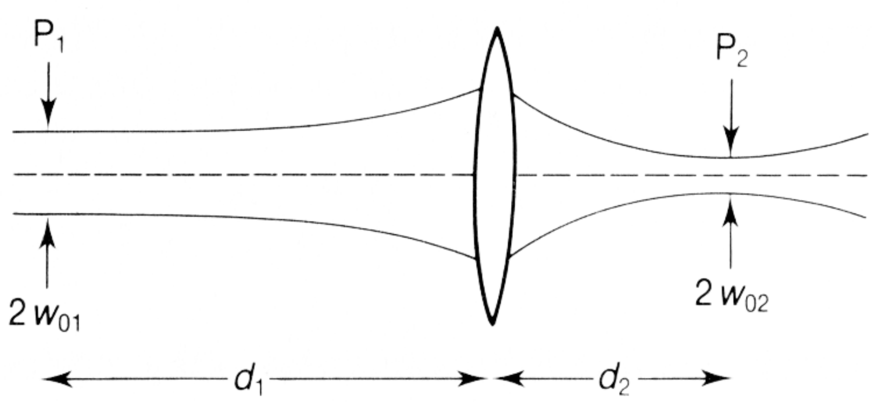


Figure 4.23 Focusing of a Gaussian beam by a convex lens.

Special Solution to Gaussian Beam

- Two cases of particular interests

Input Waist at First Principal Surface

- s = 0 condition, image distance and waist become

$$s'' = \frac{f}{1 + \left(\frac{f}{z_R}\right)^2}$$

$$w'' = \frac{\frac{\lambda f}{\pi w_0}}{\left[1 + \left(\frac{f}{z_R}\right)^2\right]^{1/2}}$$

Input Waist at First Focal Point

- s = f condition, image distance and waist become

$$s'' = f$$

$$w'' = \frac{\lambda f}{\pi w_0}$$

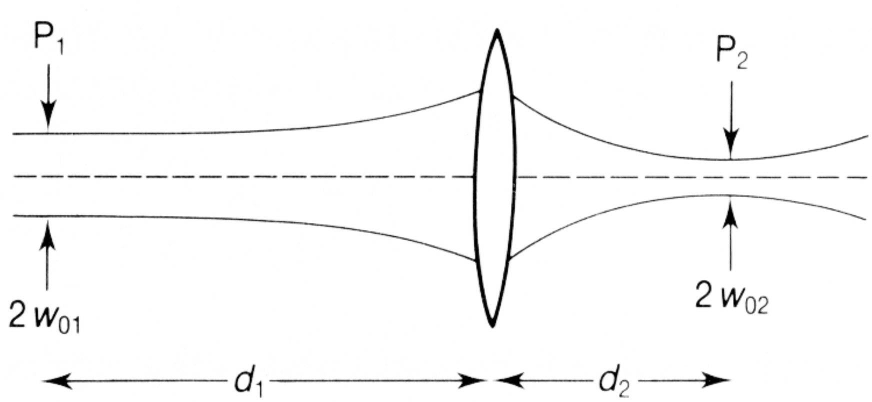


Figure 4.23 Focusing of a Gaussian beam by a convex lens.

Gaussian Spots and Cavity Stability

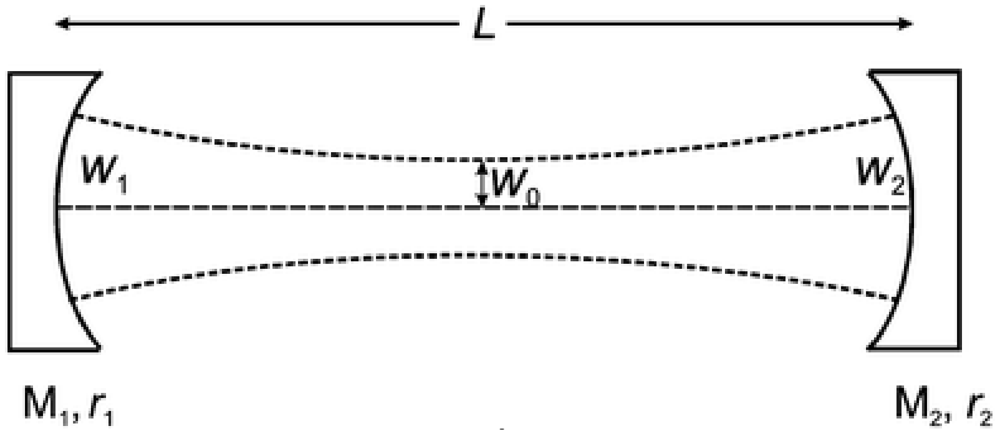
- In laser cavities waist position is controlled by mirrors
- Recall the cavity g factors for cavity stability

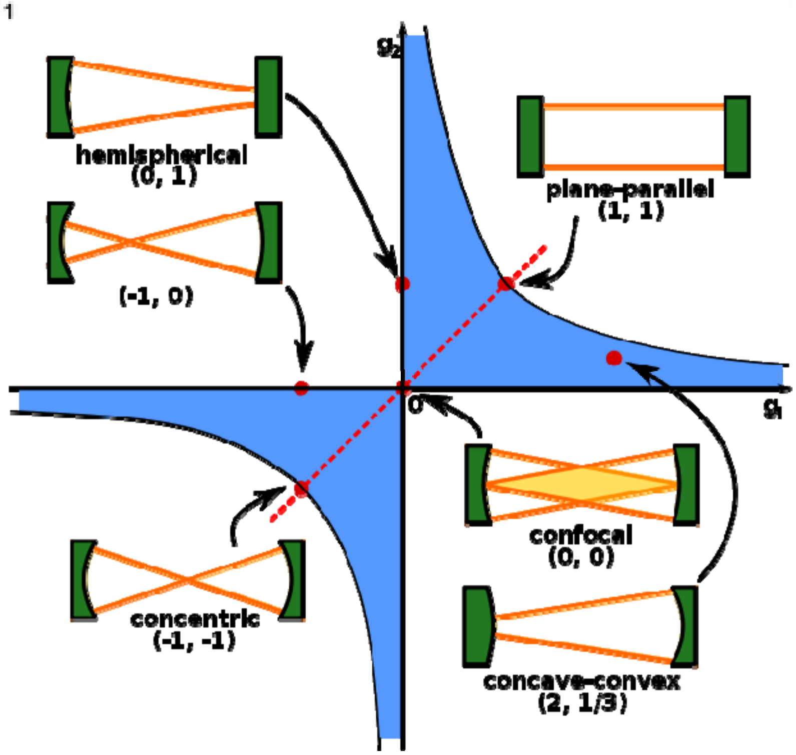
$$g_i = 1 - \frac{L}{r_i}$$

- Waist of cavity is given by

$$w_0 = \left(\frac{\lambda L}{\pi}\right)^{1/2} \left\{ \frac{[g_1 g_2 (1 - g_1 g_2)]}{[g_1 + g_2 - 2 g_1 g_2]^2} \right\}^{1/4}$$

where i=1=back mirror, i=2= front





Gaussian Waist within a Cavity

- Waist location relative to output mirror for cavity length L is

$$z_2 = \frac{g_1(1-g_2)L}{g_1+g_2-2g_1g_2}$$

$$w_1 = \left(\frac{\lambda L}{\pi}\right)^{1/2}\left\{\frac{g_2}{g_1[1-g_1g_2]}\right\}^{1/4} \qquad w_2 = \left(\frac{\lambda L}{\pi}\right)^{1/2}\left\{\frac{g_1}{g_2[1-g_1g_2]}\right\}^{1/4}$$

- If $g_1 = g_2 = g=0$ (i.e. r = L) waist becomes

$$w_0 = \left(\frac{\lambda L}{2\pi}\right)^{1/2} \qquad z_2 = 0.5$$

- If $g_1 =0$, $g_2 = 1$ (curved back, plane front)
  waist is located $z_2 = 0$ at the output mirror
  (common case for HeNe and many gas lasers)
- If $g_1 = g_2 = 1$ (i.e. plane mirrors) there is no waist

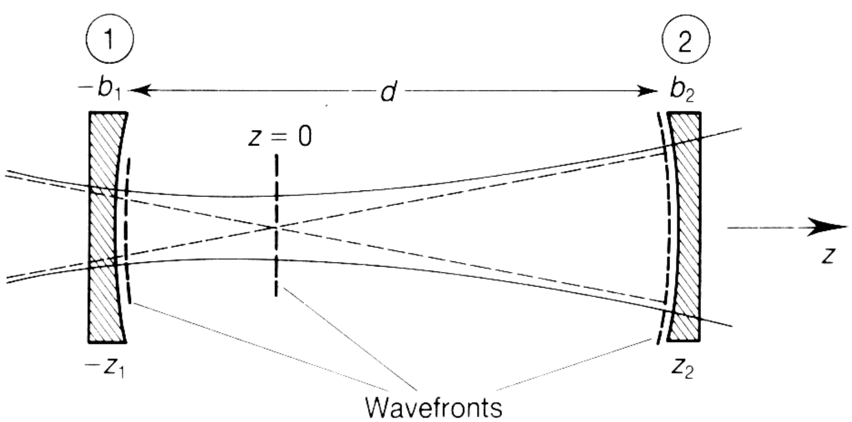


Figure 4.15 Schematic diagram of an open resonator showing the wavefront curvatures matching the mirror surfaces at the two concave mirrors. The waist occurs somewhere between them.

General Laser Types

- Solid State Laser (solid rods): eg ruby
- Gas and Ion lasers: eg He-Ne, Argon
- Dye Lasers
- Semiconductor Laser: GaAs laser diode
- Chemical Lasers
- Free Electron Lasers

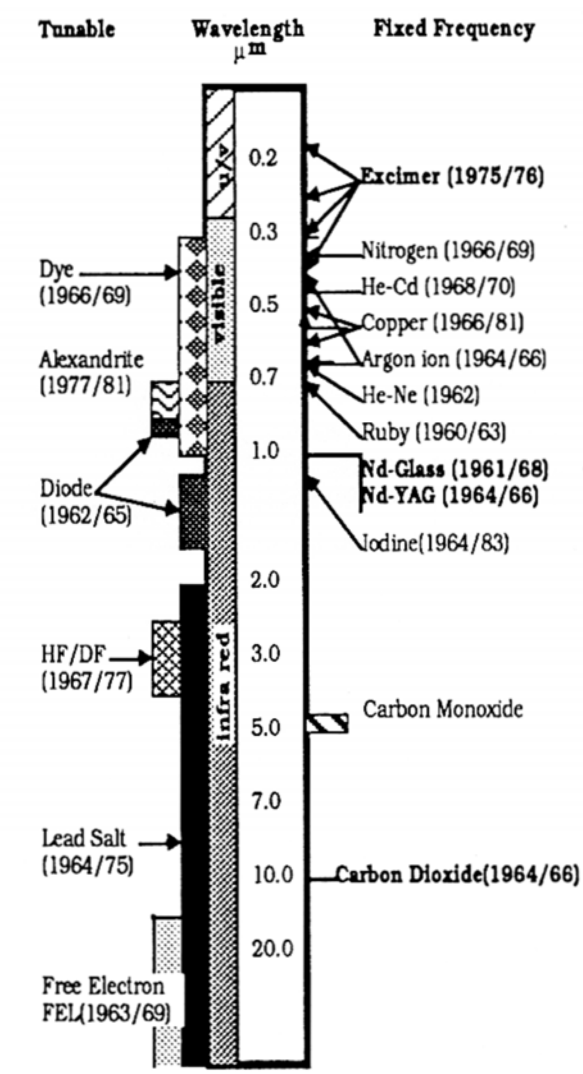
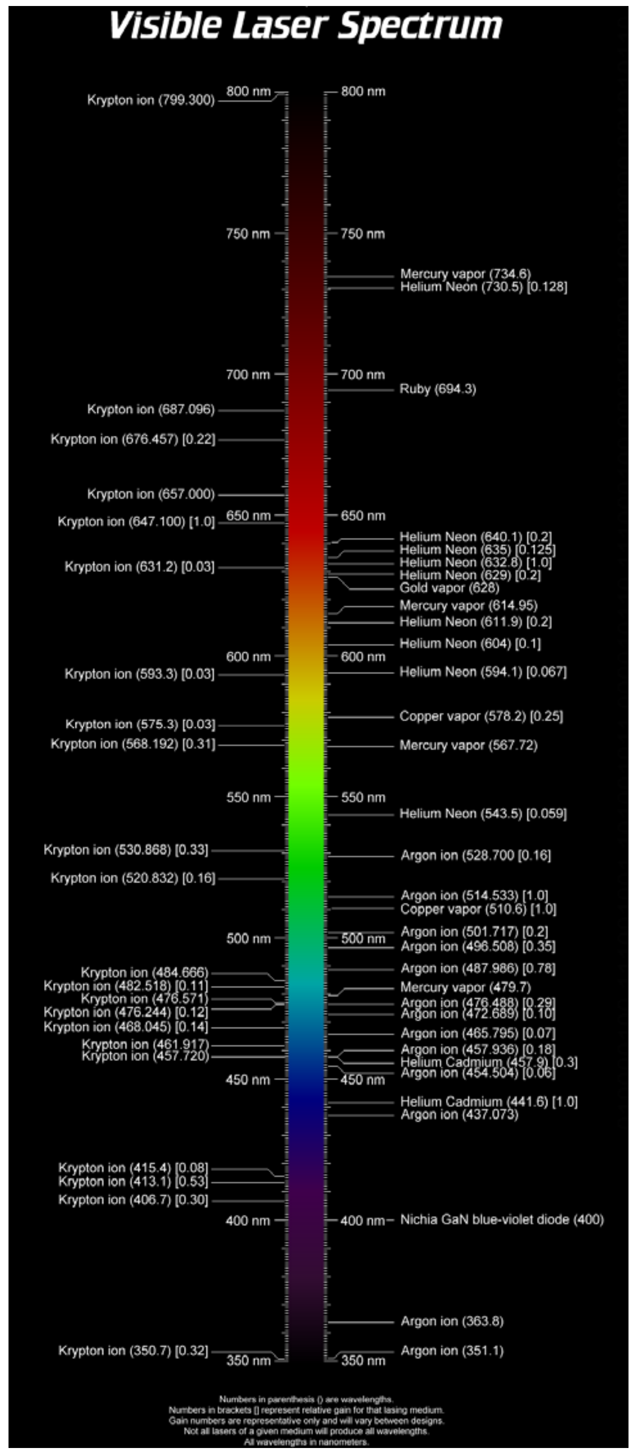


Fig. 2. Range of wavelengths for current commercial lasers. First date is date of discovery, the second is of commerciali-sation (4).



Solid State Lasers

- Was first type of laser (Ruby 1960)
- Uses a solid matrix or crystal carrier eg Glass or Sapphire
- Doped with ~1%-0.001% transition metal or rear earth ions
- eg Chromium (Cr) or Neodynmium (Nd)
- Mirrors at cavity ends (either on the rod or separate)
- Typically pumped with light
- Most common a Flash lamp: produces high power in ~1/1000 s
- Newer ones pumped by laser diodes (more efficient)
- Light adsorbed by doped ion, emitted as laser light
- Mostly operates in pulsed mode (newer CW)

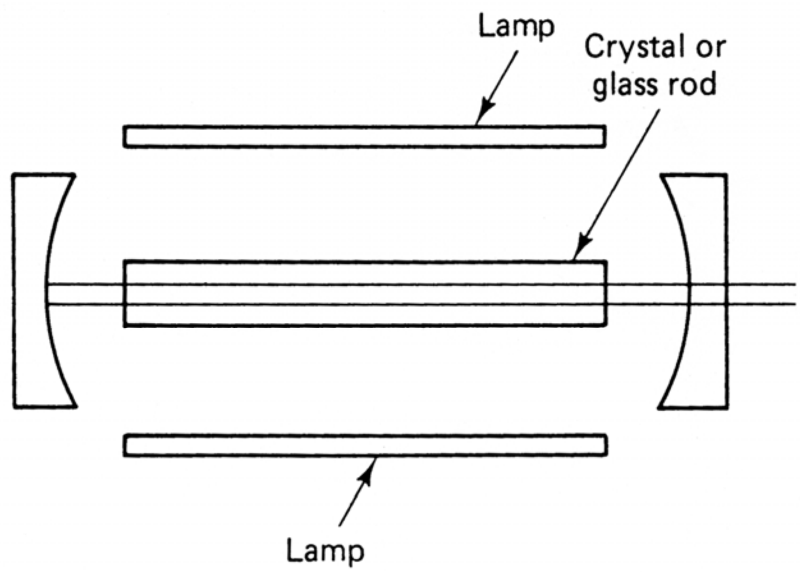
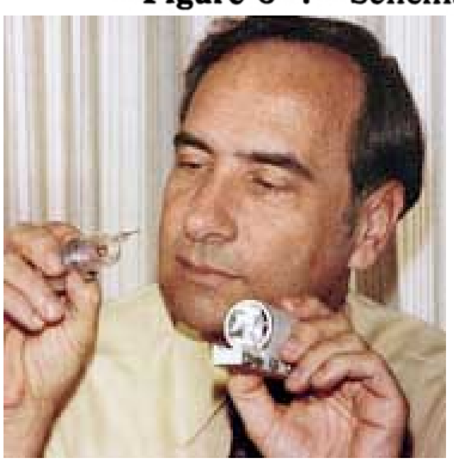
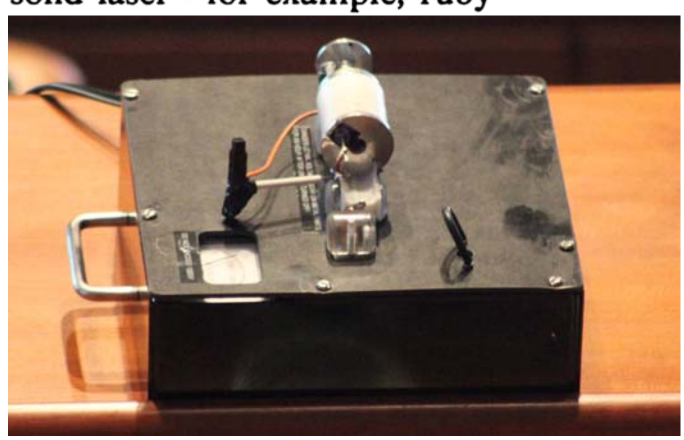


Figure 8–4 Schematic of solid laser—for example, ruby



Ted Maimin & laser



Original Ruby Laser

Flash Lamp Pumping

- Use low pressure flash tubes (like electronic flash)
- Xenon or Krypton gas at a few torr (mm of mercury pressure)
- Electrodes at each end of tube
- Charge a capacitor bank:
  50 - 2000 μF, 1-4 kV
- High Voltage pulse applied to tube
- Ionizes part of gas
- Makes tube conductive
- Capacitor discharges through tube
- Few millisec. pulse
- Inductor slows down discharge

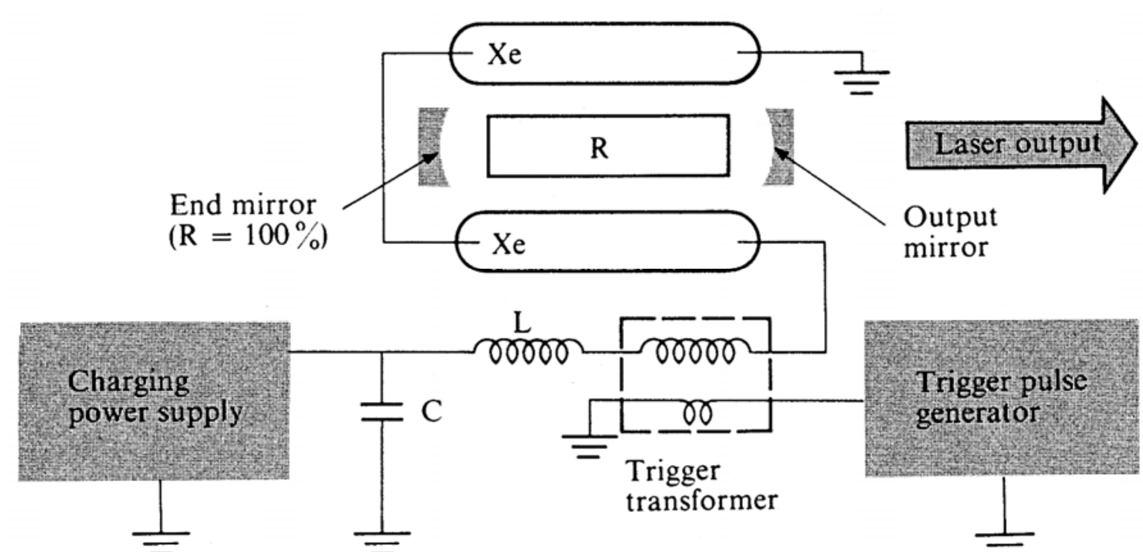
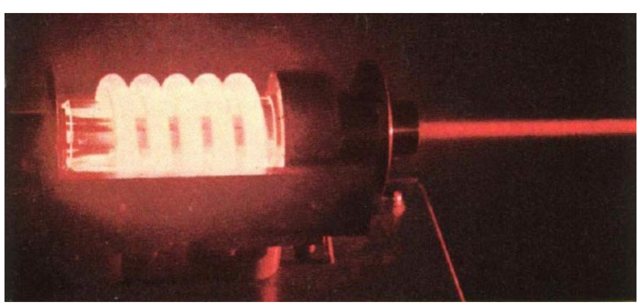


Fig. 6.16 Schematic of a simple flashlamp-pumped laser. The trigger pulse generator and transformer provide a high-voltage pulse sufficient to cause the xenon gas in the lamps (Xe) to discharge. The ionized gas provides a low-resistance discharge path to the storage capacitor (C). The inductor (L) shapes the current pulse, maintaining the discharge. The discharge of the lamps optically pumps the laser rod (R).



Light Source Geometry

- Earlier spiral lamp: inefficient but easy
- Now use reflectors to even out light distribution
- For CW operation use steady light sources
  Tungsten Halogen or Mercury Vapour
- Use air or water cooling on flash lamps

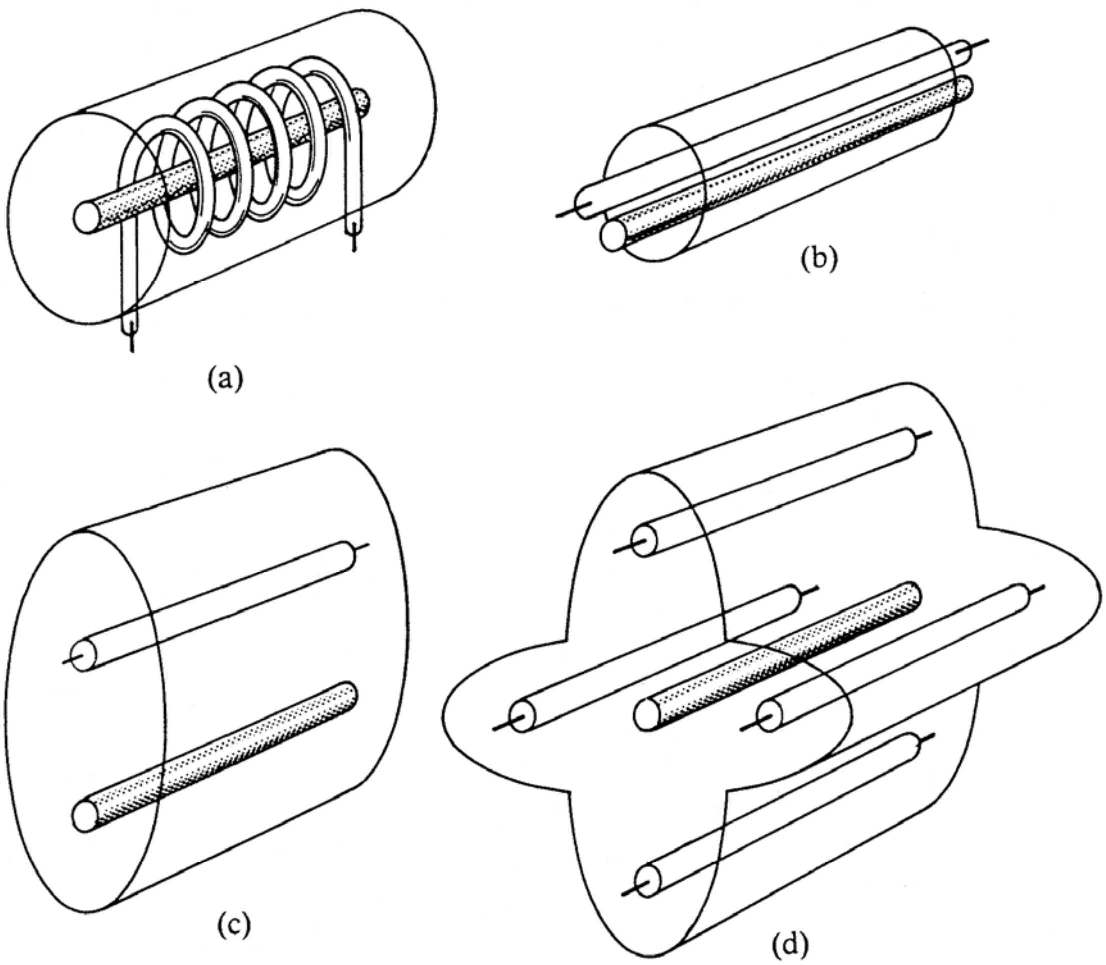


Fig. 2.4 Some of the more common flashtube geometries used for optical pumping: (a) a helical flashtube round the laser rod; (b) close coupling between flashtube and rod; (c) flashtube and rod along the two foci of an elliptical cavity and (d) a multi-elliptical cavity.

Ruby Laser

- First laser built used Ruby rods: Maiman 1960
- Crystal is Aluminium Oxide Al_2O_3: Sapphire
- 0.05% Cr_{3+}
- 3 level system: absorbs green/blue
- Emission at 694 nm (deep red)
- Pulsed operation

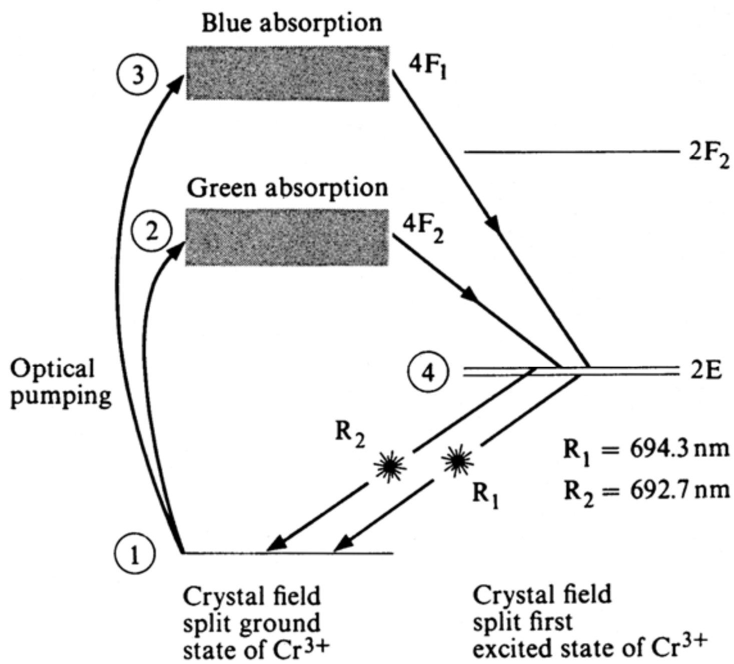
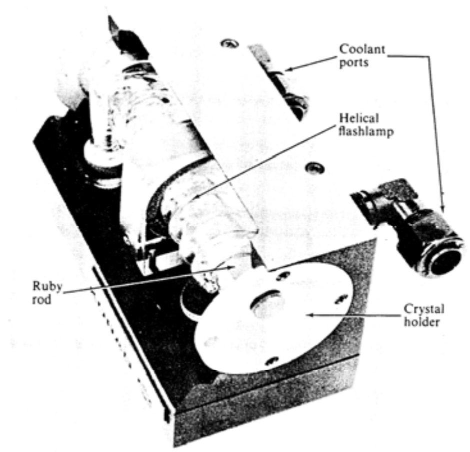


Fig. 6.14 Energy-level diagram for the ruby laser. The wavelengths of the two laser lines, R_1 and R_2, depend upon temperature. The values given here are typical.

Ruby Laser
694 nm

Ruby Laser Design

- Typically uses helix flash lamp
- Mirrors may be plated onto rod
- Mostly used for tattoo removal now
- Why Ruby laser was first – small rod inside of the helix flash
- Easy to get all light onto rod
- Ruby has strong meta-stable state (Maimin had measured it)
- Rod ends polished optically aligned – simple & stable
- Mirrors aluminum plated on rods end (now aligned)
- Front mirror has small hole to allow output (Makes R<1)
- Much easier to create stable cavity & pump than gas lasers



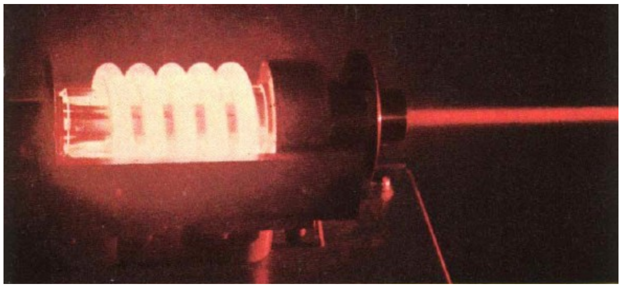
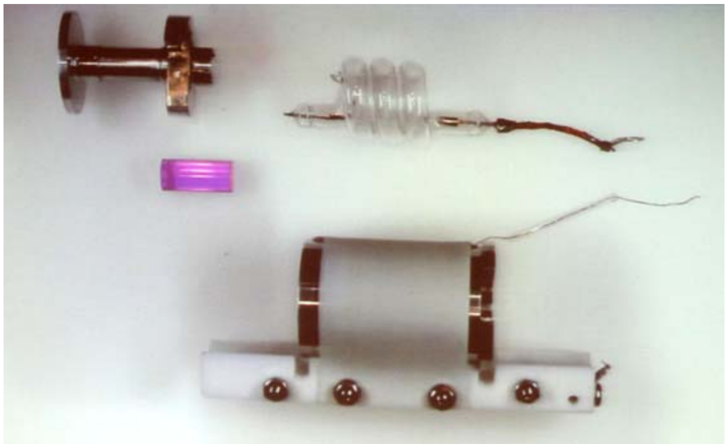
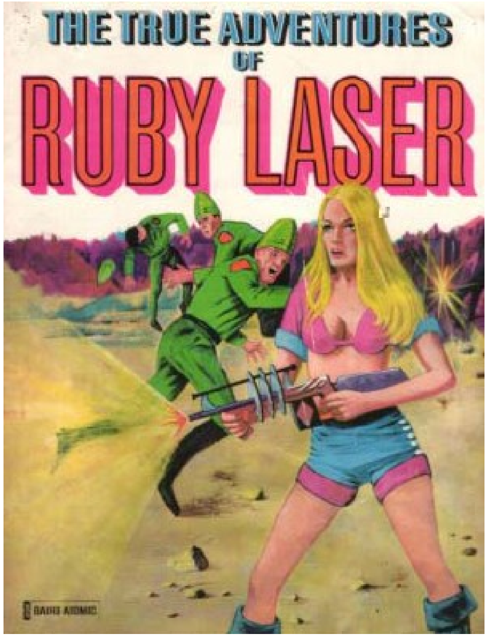


Fig. 6.15 Interior of a ruby laser head. (Courtesy of Korad, Division of Hadron Corporation.)



First Laser opened up



Transition Metal Impurity Ion Energy levels

- Solid states using transition metals in clear matrix
- For Ruby Chromium Cr_{3+} ion
- Atom has energy levels (shells)
  (orbit)(shell)$_{\text{(no. electrons)}}$
  $1s_2\ 2s_2\ 2p_6\ 3s_2\ 3p_6$
- In ions unfilled orbital electrons interact
- Inter-electron coulomb interaction split the energies
  (capital letter the L quantum)$_{\text{(spin quantum)}}$
- Ion then interacts with crystal field
  splits energy levels more

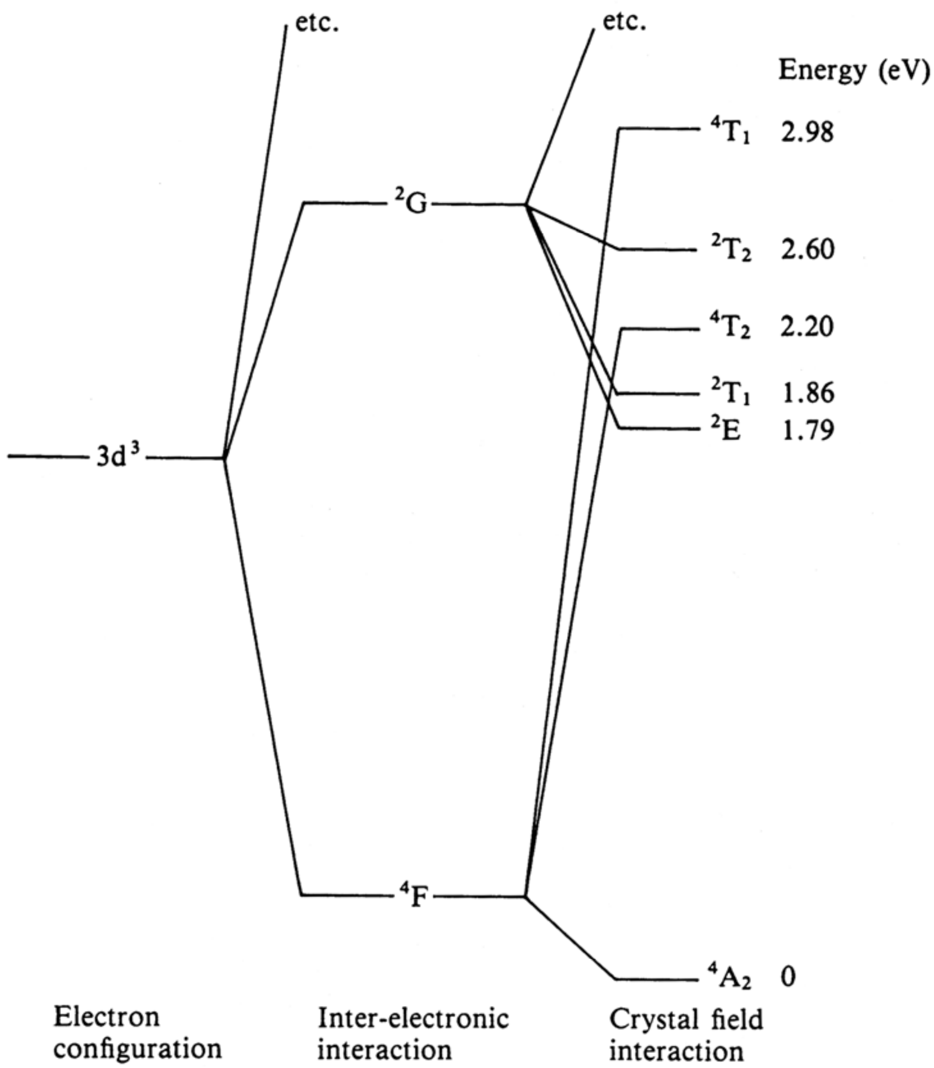


Fig. 2.1 The origins of the low-lying electronic energy levels in Cr^{3+}. Successive interactions split the original $3d^3$ electron configuration into an increasing number of energy levels.

Rare Earth Impurity Ion Energy levels

- Spin of electrons interacts with orbit
- Splits the inter-electronic levels

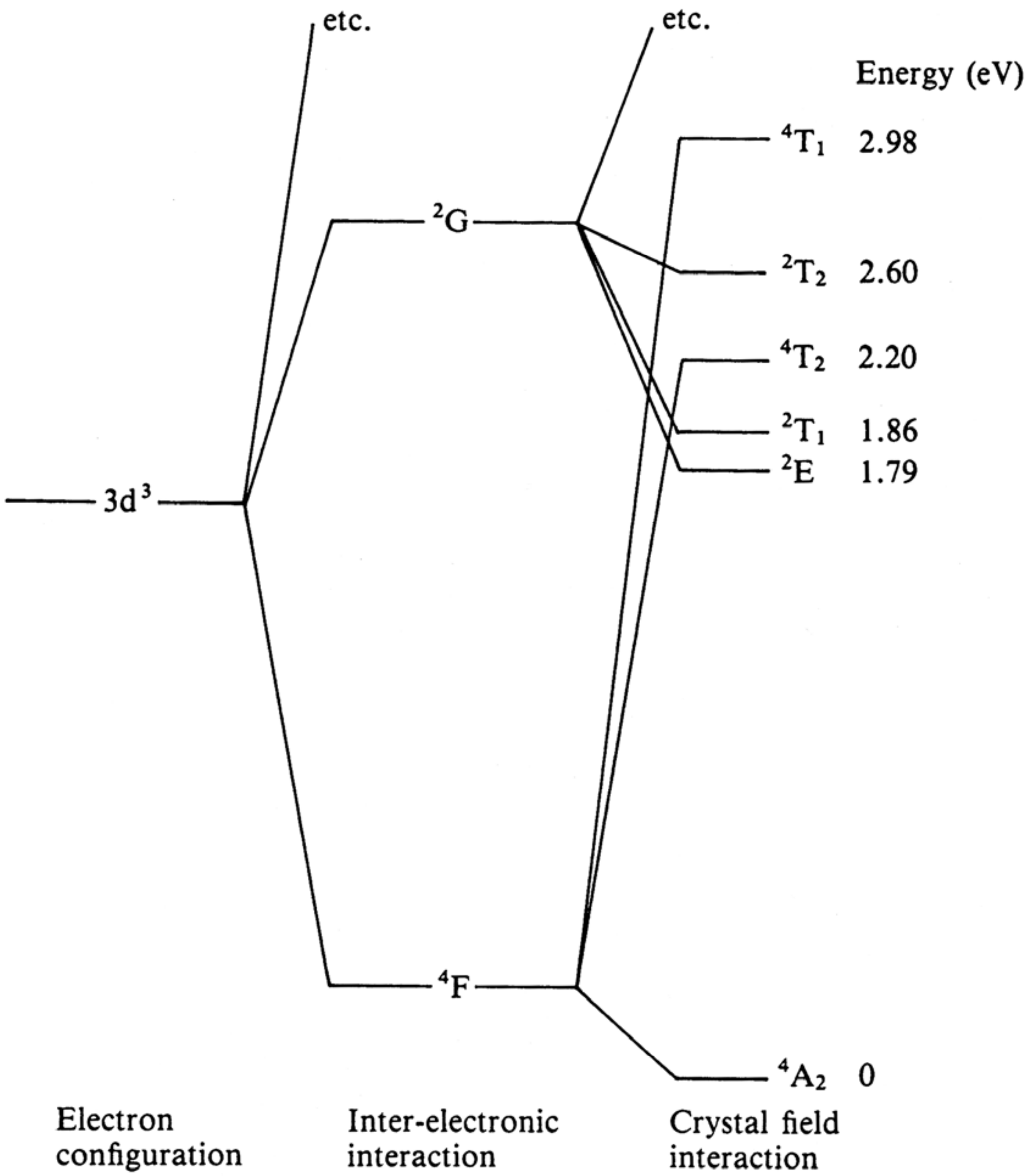


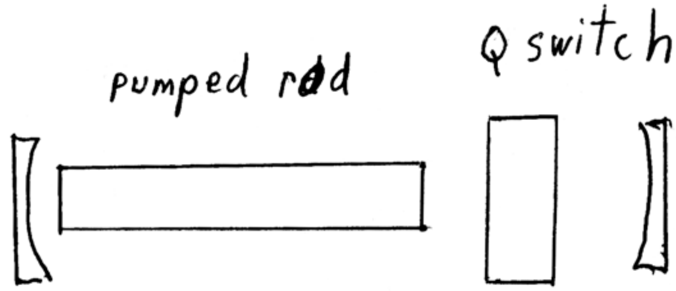
Fig. 2.1 The origins of the low-lying electronic energy levels in Cr^{3+}. Successive interactions split the original $3d^3$ electron configuration into an increasing number of energy levels.

Q Switch Pulsing

- Most solid states use Q switching to increase pulse power
- Block a cavity with controllable absorber or switch
- Acts like an optical switch
- During initial pumping flash pulse switch off
- Recall the Quality Factor of resonance circuits (eg RLC)

$$Q = \frac{2\pi \ energy\ stored}{energy\ lost\ \ per\ light\ \ pass}$$

- During initial pulse Q low
- Allows population inversion to increase without lasing
- No stimulated emission
- Then turn switch on
- Now sudden high stimulated emission
- Dump all energy into sudden pulse
- Get very high power level, but less energy
- Types of Q switches:
- Delayed triggered – set delay after pump pulse
- Electro-Mechanical, electro-optics, acousto-optic,
- Delay set by light in cavity - saturable dye



Q Switch Process During Laser Pulse

- Flash lamp rises to max then declines (~triangle pulse)
- Q switch makes cavity Q switch on after max pumping
- Low Q, so little spontaneous light
- Population inversion rises to saturation
- The Q switch creates cavity: population suddenly declines due to stimulated emission
- Laser pulse during high Q & above threshold conditions

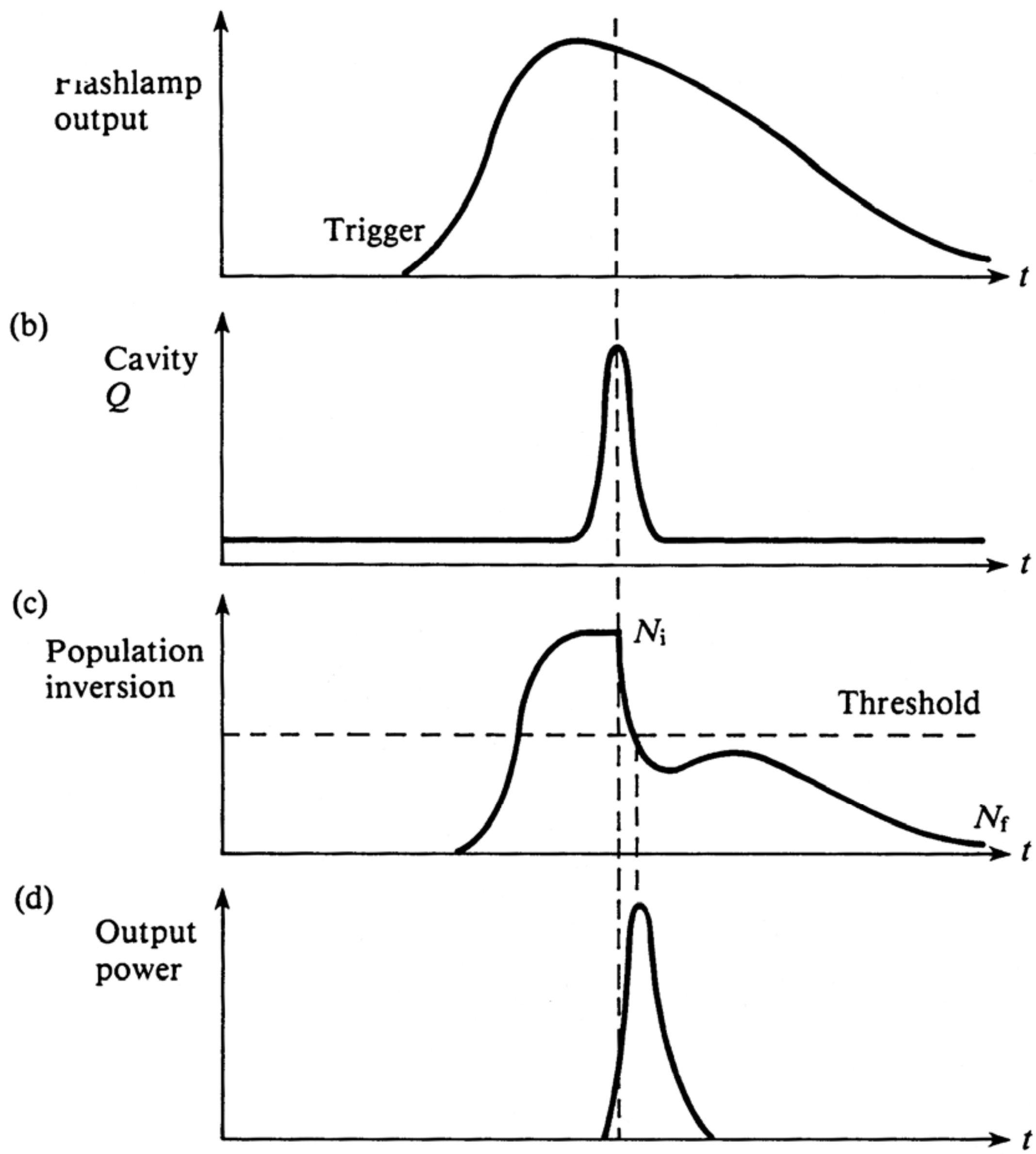


Fig. 3.11 Schematic representation of the variation of the parameters: (a) flashlamp output; (b) cavity *Q*; (c) population inversion; (d) output power as a function of time during the formation of a Q-switched laser pulse.

Energy Loss due to Mirrors & Q

- Q switching can be related to the cavity losses
- Consider two mirrors with reflectance R_1 and R_2
- Then the rate at which energy is lost is

$$\frac{E}{\tau_c} = \frac{(1 - R_1 R_2)E}{\tau_r}$$

where τ_c = photon lifetime
τ_r = round trip time = 2L/c
E = energy stored in the cavity

- Average number of photon round trips is the lifetime ratio

$$\frac{\tau_c}{\tau_r} = \frac{1}{(1 - R_1 R_2)}$$

Q Equations for Optical Cavity

- Rewrite energy equation in terms of photon lifetime τ_c
- First note the energy lost in the time of one light cycle $t_f = 1/f$

$$E_{lost\,/\,cycle} = \frac{Et_f}{\tau_c} = \frac{E}{f\tau_c}$$

where f = frequency

- Thus the cavity's Q is

$$Q = \frac{2\pi E}{E_{lost\,/\,cycle}} = \frac{2\pi E}{\left(\frac{E}{f\tau_c}\right)} = 2\pi f\tau_c$$

- Thus for a laser cavity:

$$Q = 2\pi\ f\tau_c = \frac{2\pi\ f\tau_r}{(1 - R_1R_2)} = \frac{4\pi\ fL}{c(1 - R_1R_2)} = \frac{4\pi\ L}{\lambda(1 - R_1R_2)}$$

- Q switch: go form high reflectivity to low reflectivity on one mirror
- Also Q is related to the bandwidth of the laser

(from resonance cavity circuits).

$$Q = \frac{f}{\Delta f}$$

- Thus lifetime relates to the bandwidth

$$\Delta f = \frac{1}{2\pi\tau_c}$$