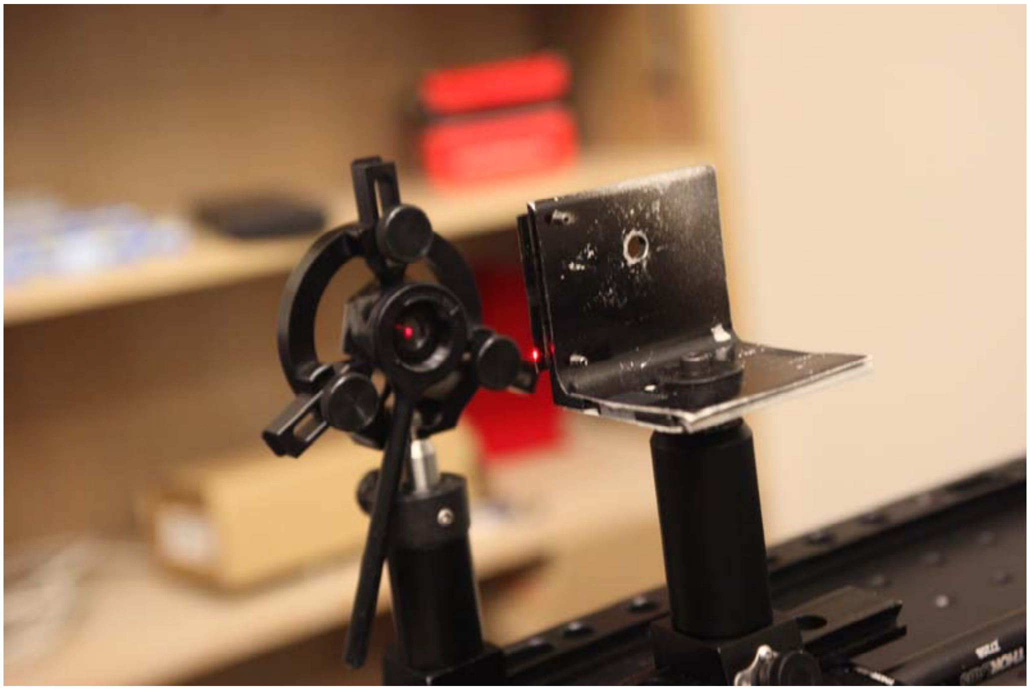
Lab 3: measurement of Laser Gaussian Beam Profile
v5.1 (Dec. 11, 2013)

- Lab 3: basic experience working with laser

(1) To create a beam expander for the Argon laser

(2) To measure the spot size and profile of the Argon laser
Measure before and after the beam expansion

- Do this by moving a knife edge through the beam
- Have a computer controlled knife that moves through beam



Knife Edge measurement of Gaussian Beam

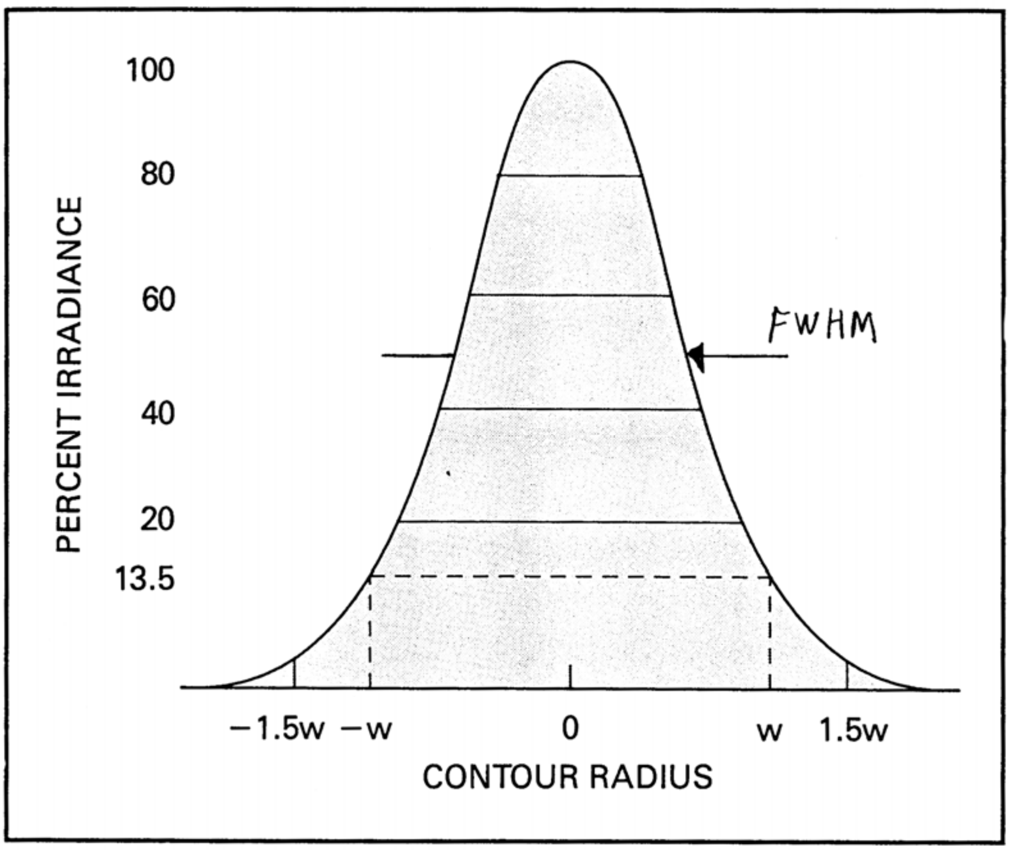
- Consider a Gaussian shaped beam

$$I(r) = I_0 \exp\left(\frac{-2r^2}{w^2}\right) = \frac{2P}{\pi w^2} \exp\left(\frac{-2r^2}{w^2}\right)$$

Where P = total power in the beam

w = $1/e^2$ beam radius at point w(z)

- This is in cylindrical coordinates
- r is the radius of the central area



GAUSSIAN IRRADIANCE PROFILE for TEM_{00} mode, showing definitions of beam radius w.

Knife Edge and Gaussian

- Straight knife edge cutting into a Gaussian shaped beam
- Measure the total power seen when knife move in x direction
- Must convert to Cartesian coordinates & integrate
- Assume -∞ is when the knife fully below the beam

$$I(x) = \frac{2P}{\pi w^2} \int_{-\infty}^{x} \exp\left(\frac{-x^2}{w^2}\right) dx \int_{-\infty}^{\infty} \exp\left(\frac{-y^2}{w^2}\right) dy$$

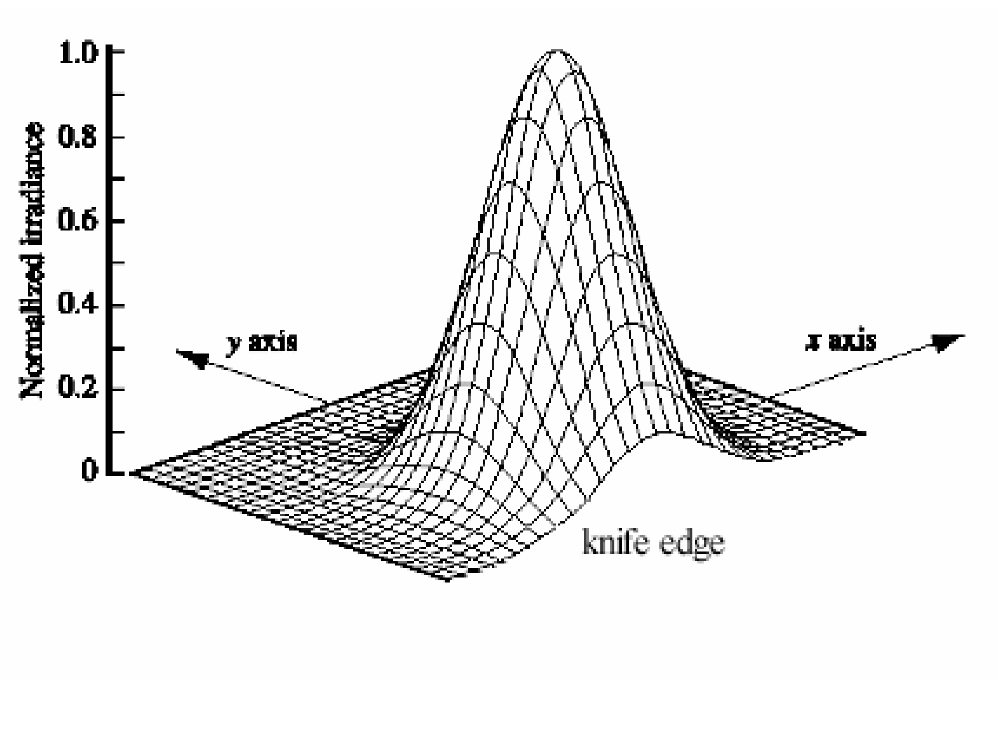
Where P is the total power of the beam

I(x) is the intensity measured at position x

- In x direction the beam is cut: Integrate from x to -∞
- In y direction get full beam: integrate from -∞ to +∞
- To solve this use the error function or integral of the normal

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_{0}^{x} e^{-s^2} ds$$

- Two ways of fitting this:
- Fit the power measured: that is the integral
- Fit the derivative



Fitting the power measured

- The power measured at the detector is the integral
- x_0 = centre of beam
- Then the power measured is given by for x >x_0

$$I(x) = \frac{P}{2} erf\left(\frac{(x - x_0)}{w}\right)$$

For x<x_0

$$I(x) = \frac{P}{2}\left[1 + erf\left(\frac{(-x + x_0)}{w}\right)\right]$$

- Must also assume some background light level B
- In Excel use the Normdist (Normal distribution function)
- This is slightly different from erf function
- Fit with the excel function of the following formula

I(x)=P*normdist(-x,-x_0,w/1.414,1)+B

Where x is the position (starting with x below x_0)

x_0 is the fitted centre point of the beam

w = 1/e^2 size of beam you fit

1 is to make it the integration of the normal distribution

B is the background or offset level

- Set up a spreadsheet with initial estimates of each parameter
- Have columns with x, I(x_n), fitted I(x_n),
- I_{fit}(x) -I(x) = error (called the error of fit or residual)
- $error^2$ =(I_{fit}(x)-I(x))^2 (called the residual squared in statistics)
- Set a column to sum the $error^2$ (sum of the squares)

Running the Fit

- Want to minimize the sum of the squares
- In stats were shown this gives the best statistical fit to data
- Use the Excel solver function to do this
- Set solver to minimize the sum of squares cell
- Then use solver under tools tab to fit
- Set to minimizing sum of squares cell
- Use sum of squares as fit as minimization,
- Set P, x_0, w and B as variables to be changed for fit
- Solver need initial estimates – these important for getting fit
- Getting good initial values for the fit
- P - use the measured I before the knife edge cuts (start of data)
- x_0 – use x for P/2 point from the data (nearly right)
- B – use background light level
- w (spot size) is the difficult one to estimate and the hardest to get.
- See the difference discussion next page to estimate
- Plots help evaluate the fit
- Plot I(x) vs x for both data and fit
- Suggest put both on the same plot so you tell how good a fit
- Useful to plot the errors against position (called residuals)
- Thus plot residuals $I_{fit}(x)-I(x)$ vs x
- Ideal fit residuals should be small
- Residuals should be on both sides of 0 line (ie + and -)
- See sample excel layout in appendix B
- See Appendix A for running solver

Fit the difference of Power Measured

- The derivative
- Take a derivative of the measurements
- Best if take a simple derivative

$$\frac{dI_j}{dx} = \frac{\left[I\left(x_{j+1}\right) - I\left(x_j\right)\right]}{\left[x_{j+1} - x_j\right]}$$

- Plot dI/dx vs x for your data
- Then the plot is a Gaussian shape with the formula:

$$\frac{dI(x)}{dx} = \frac{P}{w\sqrt{\pi}} \exp\left(- \frac{\left[x - x_0\right]^2}{w^2} \right)$$

- Note need to be careful with the derivatives units you use
- Suggest you plot the derivative but not fit it
- Derivatives are very prone to errors (small errors magnified)
- Plotting shows if the curve shape you are getting
- Check does it really look like a Gaussian
- This is best way to estimate w
- Take the plot and find half the peak dI/dx value
- Width of curve at half point is the FWHM of laser beam
- Then convert FWHM to w ($1/e^2$)
- FWHM*0.849= w
- Gives a good estimate of w for curve fit
- See the plots in Appendix B

Appendix 1: Solver in excel

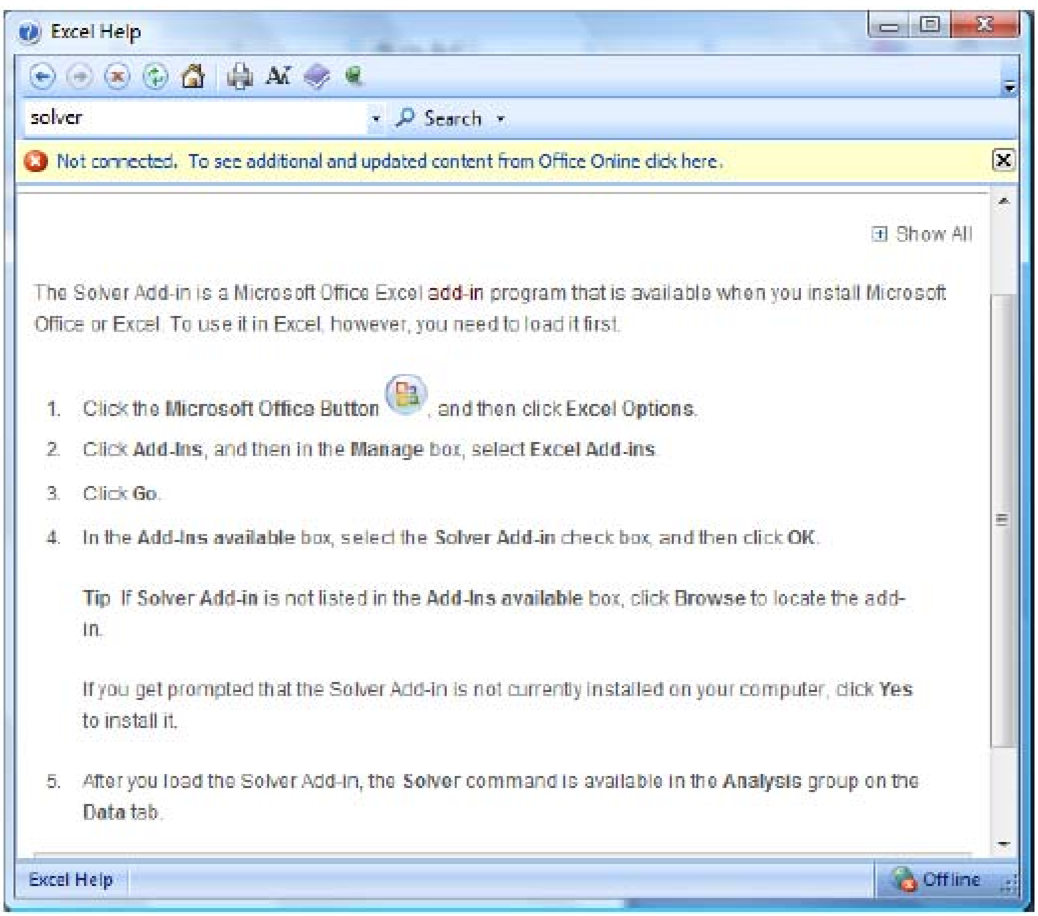
Adding solver to excel

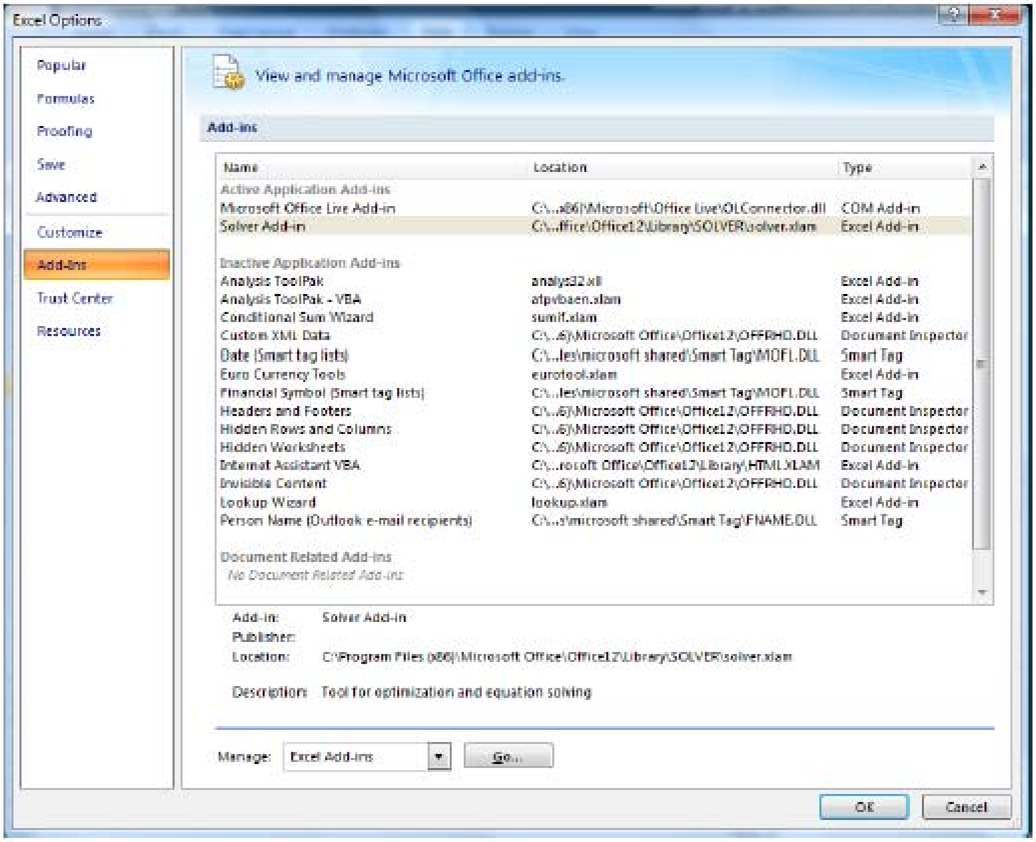
For the lab 3 you will probably want to use the excel solver add on in excel (matlab is not friendly for this)

Students who are using MS office 2007/2010 may find that the solver was not loaded into the excel and does not appear in the data menu. In earlier versions a pull down tab showed it still needed to be installed but 2007 (and 2010) does not. Check your data tab ribbon – will show solver on the furthest right if installed.

To get the solver:

The instructions are hard to find in help also. Attached is a screen shot of the help instructions on MS web site and the add-ins list window showing where the solver is once installed.




Excel Options
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Formulas
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Trust Center
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COM Add-in
Solver Add-in
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Excel Add-in
Inactive Application Add-ins
Analysis ToolPak
analys32.xll
Excel Add-in
Analysis ToolPak - VBA
atpvbaen.xlam
Excel Add-in
Conditional Sum Wizard
sumif.xlam
Excel Add-in
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Document Inspector
Date (Smart tag lists)
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Smart Tag
Euro Currency Tools
eurotool.xlam
Excel Add-in
Financial Symbol (Smart tag lists)
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Smart Tag
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Document Inspector
Hidden Rows and Columns
C:\...6)\Microsoft Office\Office12\OFFRHD.DLL
Document Inspector
Hidden Worksheets
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Document Inspector
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Excel Add-in
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Document Inspector
Lookup Wizard
lookup.xlam
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Person Name (Outlook e-mail recipients)
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Document Related Add-ins
No Document Related Add-ins
Add-in: Solver Add-in
Publisher:
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Manage: Excel Add-ins
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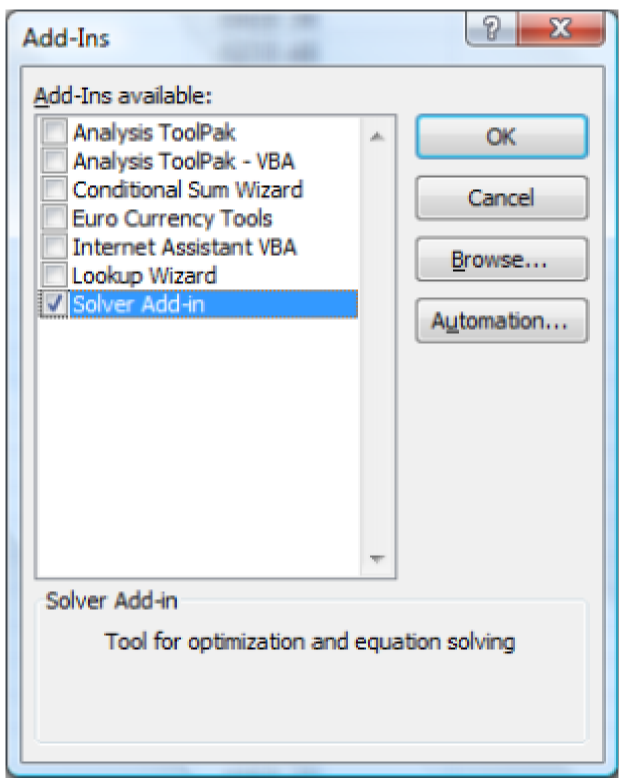
Add-Ins
Add-Ins available:
Analysis ToolPak
Analysis ToolPak - VBA
Conditional Sum Wizard
Euro Currency Tools
Internet Assistant VBA
Lookup Wizard
Solver Add-in
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Solver Add-in
Tool for optimization and equation solving

Using Solver:
Excel help is not good in describing this
Here are links to several good sites that give nice examples of how to use the solver for a problem where you are adjusting several variables to minimize one parameter (sum of squares in the lab)
General instructions
http://chandoo.org/wp/2011/05/11/using-solver-to-assign-item/

http://archives.math.utk.edu/ICTCM/VOL23/C006/paper.pdf

Using solver in nonlinear fits (as in this lab)
http://web.chem.ucsb.edu/~laverman/Chem116/PDF116CL/Solver.pdf

Solver is useful for your work in other labs

Appendix B

Printout of excel sheet for laser profile fitting.

Nd-YAG Glenn

first round: (this is used to roughly profile the laser beam and decide on the resolution for the z-axis needed)

2150	91.7
2225	84.6
2300	70.2
2375	47.5
2450	23.1
2525	8.01
2600	2.08
2675	0.473
2750	0.235
2825	0.152
2900	0.075
2975	0.039
3050	0.015

without laser: 0.032 mW

				Z0	W/1.414	B	P	W
second round:			fitted	2370.972	111.4439	0.067357	94.3335	157.5817
step	I(z)	diff		I(z)fitted				
2150	93.4	2.4		92.16568	1.523558			
2175	91	3		90.69044	0.095829			
2200	88	3.4		88.50537	0.255396			
2225	84.6	4.1		85.42711	0.684115			
2250	80.5	5.1		81.30253	0.644056			
2275	75.4	5.4		76.0461	0.41744			
2300	70	7.5		69.67464	0.105861			
2325	62.5	7.7		62.32911	0.029203			
2350	54.8	8.3		54.27452	0.276125			
2375	46.5	8.4		45.87412	0.391724			
2400	38.1	8.4		37.54127	0.312174			
2425	29.7	7.5		29.67945	0.000422			
2450	22.2	6		22.62458	0.180267			
2475	16.2	4.9		16.60326	0.162615			
2500	11.3	3.63		11.71526	0.172441			
2525	7.67	2.69		7.941225	0.073563			
2550	4.98	1.82		5.169711	0.03599			
2575	3.16	1.18		3.233894	0.00546			
2600	1.98	0.84		1.947875	0.001032			
2625	1.14	0.443		1.135296	2.21E-05			
2650	0.697	0.237		0.646959	0.002504			
2675	0.46	0.125		0.367827	0.008496			
2700	0.335	0.065		0.216076	0.014143			
2725	0.27	0.038		0.137608	0.017528			
2750	0.232	0.029		0.099017	0.017684			
2775	0.203	0.026		0.080966	0.014892			
2800	0.177	0.027		0.072935	0.01083			
2825	0.15	0.028		0.069536	0.006474			
2850	0.122	0.0257		0.068169	0.002898			
2875	0.0963	0.0223		0.067645	0.000821			
2900	0.074	0.019		0.067454	4.28E-05			
2925	0.055	0.0145		0.067388	0.000153			
2950	0.0405	0.0103		0.067367	0.000722			
2975	0.0302	0.0073		0.06736	0.001381			
3000	0.0229	0.0047		0.067358	0.001977			
3025	0.0182	0.0182		0.067357	0.002416			
				sum sq	5.470255