JPEG Image Compression

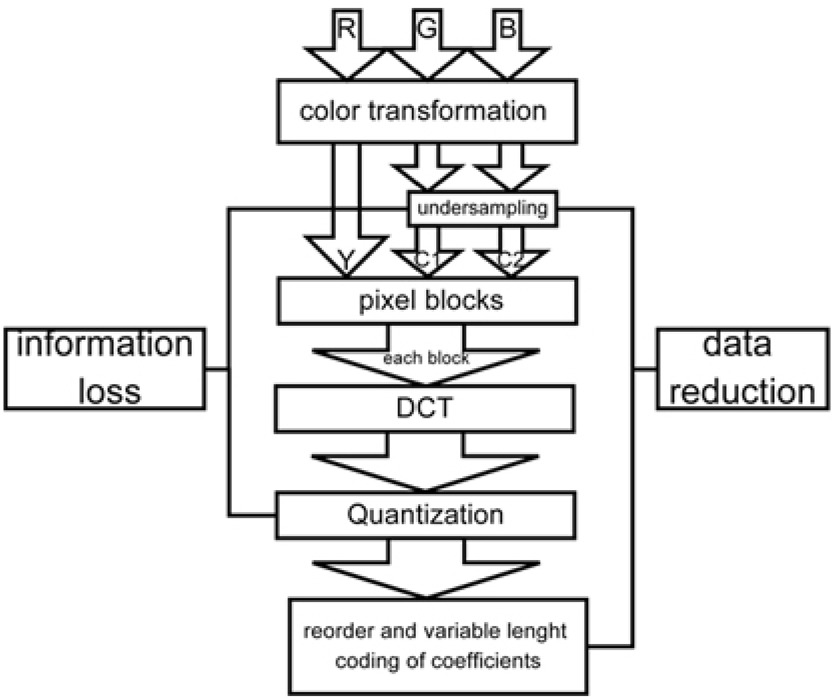
- JPEG is the standard lossy image compression
- Based on Discrete Cosine Transform (DCT)
- Comes from the Joint Photographic Experts Group
- Formed in1986 and issued format in 1992
- Consider an image of 73,242 pixels with 24 bit color
- Requires 219,726 bytes

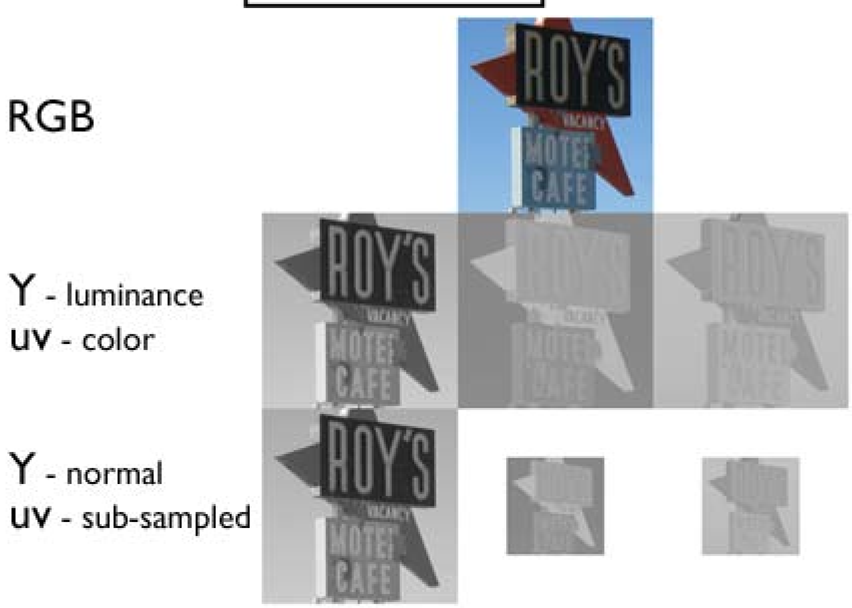
Image	Quality	Size	Ratio	
	Highest Q=100	81,447	2.7:1	Extremely minor artifacts
	High Quality Q=50	14,679	15:1	Initial signs of subimage artifacts

	Medium Q	9,407	23:1	Stronger artifacts; loss of high frequency information
	Low	4,787	46:1	Severe high frequency loss leads to obvious artifacts on subimage boundaries ("macroblocking")
	Lowest	1,523	144:1	Extreme loss of color and detail; the leaves are nearly unrecognizable

JPEG How it works

- Begin with a color translation
- RGB goes to $Y'C_BC_R$
- Luma and two Chroma colors
- Y is brightness
- C_B is B-Y
- C_R is R-Y
- Downsample or Chroma Subsampling
- Chroma data resolutions reduced by 2 or 3
- Eye is less sensitive to fine color details than to brightness

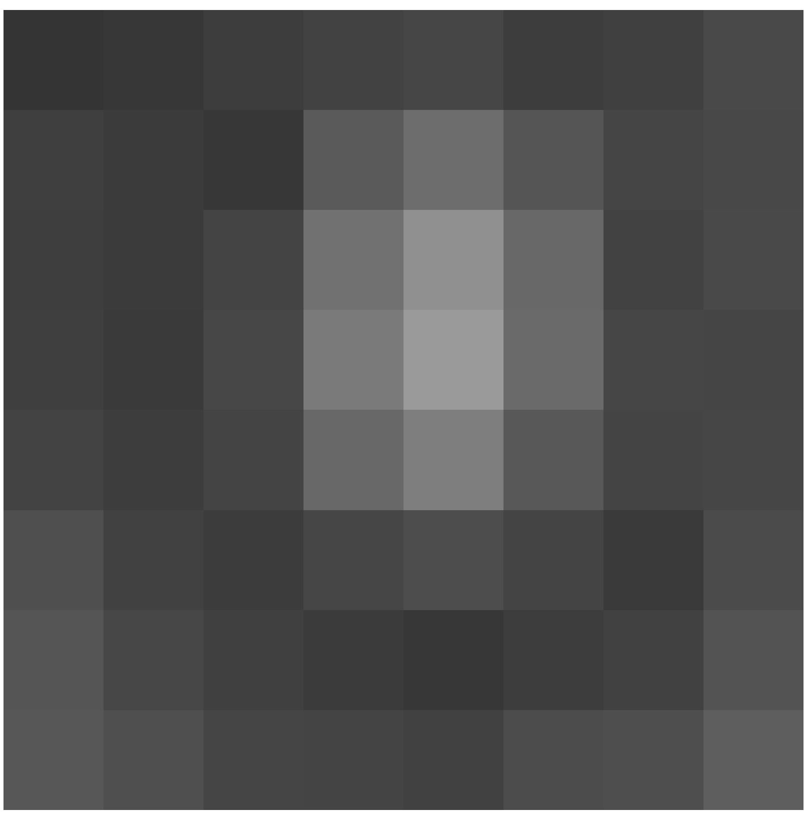




Block splitting

- Each channel broken into 8x8 blocks no subsampling
- Or 16x8 most common at medium compression
- Or 16x16
- Must fill in remaining areas of incomplete blocks
- This gives the values

$$\begin{bmatrix} 52 & 55 & 61 & 66 & 70 & 61 & 64 & 73 \\ 63 & 59 & 55 & 90 & 109 & 85 & 69 & 72 \\ 62 & 59 & 68 & 113 & 144 & 104 & 66 & 73 \\ 63 & 58 & 71 & 122 & 154 & 106 & 70 & 69 \\ 67 & 61 & 68 & 104 & 126 & 88 & 68 & 70 \\ 79 & 65 & 60 & 70 & 77 & 68 & 58 & 75 \\ 85 & 71 & 64 & 59 & 55 & 61 & 65 & 83 \\ 87 & 79 & 69 & 68 & 65 & 76 & 78 & 94 \end{bmatrix}.$$



DCT - centering

- Center the data about 0
- Range is now -128 to 127
- Middle is zero

$$
g = \overset{\displaystyle x}{\overrightarrow{}}
\begin{bmatrix}
-76 & -73 & -67 & -62 & -58 & -67 & -64 & -55 \\
-65 & -69 & -73 & -38 & -19 & -43 & -59 & -56 \\
-66 & -69 & -60 & -15 & 16 & -24 & -62 & -55 \\
-65 & -70 & -57 & -6 & 26 & -22 & -58 & -59 \\
-61 & -67 & -60 & -24 & -2 & -40 & -60 & -58 \\
-49 & -63 & -68 & -58 & -51 & -60 & -70 & -53 \\
-43 & -57 & -64 & -69 & -73 & -67 & -63 & -45 \\
-41 & -49 & -59 & -60 & -63 & -52 & -50 & -34
\end{bmatrix}
\Big\downarrow y.
$$

Discrete cosine transform formula

- Apply as 2D DCT using the formula
- Creates a new matrix
- Top left (largest) is the DC coefficient constant component
- Gives basic hue for the block
- Remaining 63 are AC coefficients

$$G_{u,v} = \frac{1}{4}\alpha(u)\alpha(v)\sum_{x=0}^{7}\sum_{y=0}^{7} g_{x,y} \cos\left[\frac{(2x+1)u\pi}{16}\right] \cos\left[\frac{(2y+1)v\pi}{16}\right]$$

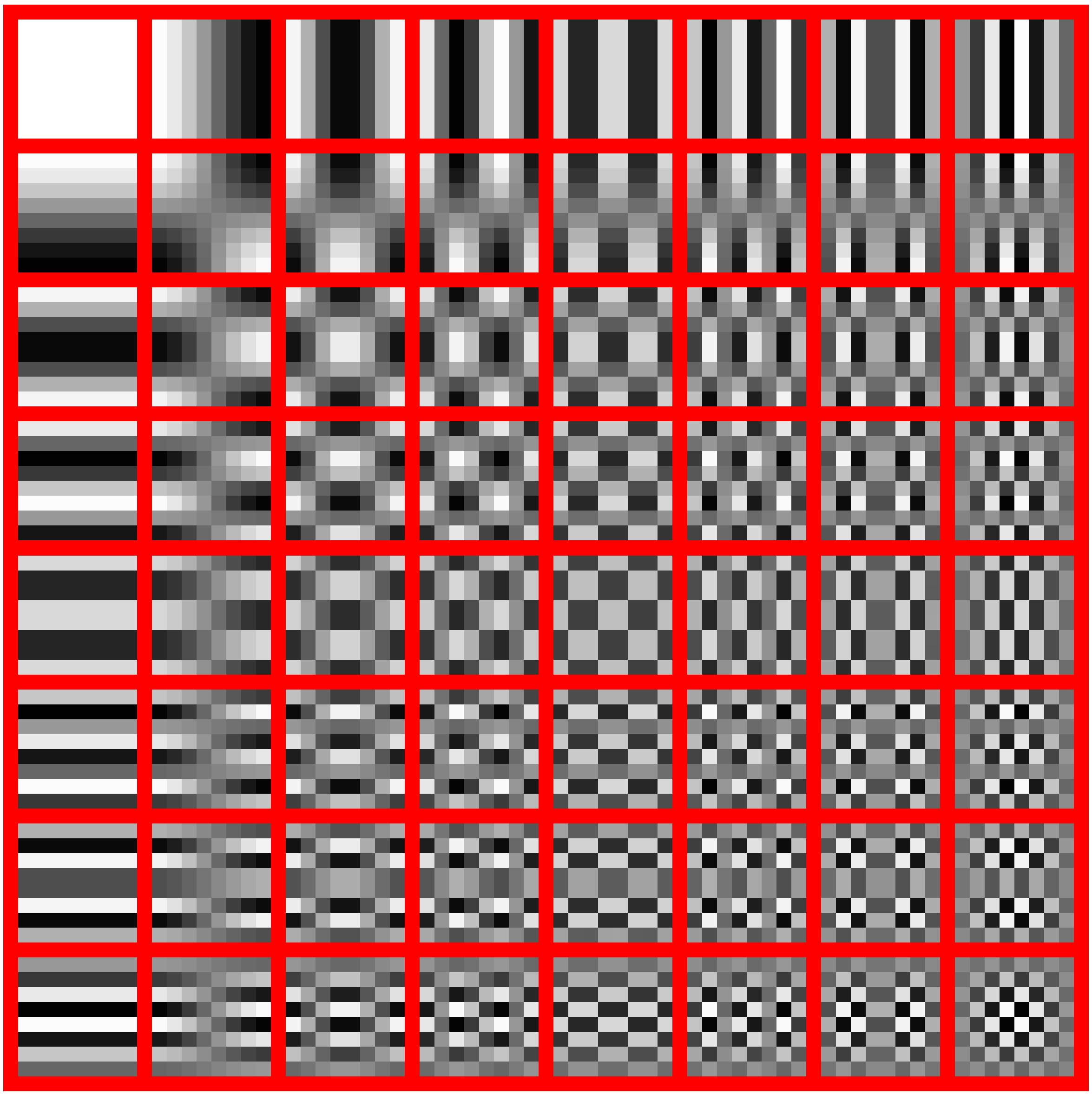
where

- u is the horizontal spatial frequency, for the integers $0 \leq u < 8$.
- v is the vertical spatial frequency, for the integers $0 \leq v < 8$.
- $\alpha(u) = \begin{cases} \frac{1}{\sqrt{2}}, & \text{if } u = 0 \\ 1, & \text{otherwise} \end{cases}$ is a normalizing scale factor to make the transformation orthonormal
- $g_{x,y}$ is the pixel value at coordinates (x, y)
- $G_{u,v}$ is the DCT coefficient at coordinates (u, v).

$$G = \overset{\xrightarrow{u}}{\begin{bmatrix} -415.38 & -30.19 & -61.20 & 27.24 & 56.12 & -20.10 & -2.39 & 0.46 \\ 4.47 & -21.86 & -60.76 & 10.25 & 13.15 & -7.09 & -8.54 & 4.88 \\ -46.83 & 7.37 & 77.13 & -24.56 & -28.91 & 9.93 & 5.42 & -5.65 \\ -48.53 & 12.07 & 34.10 & -14.76 & -10.24 & 6.30 & 1.83 & 1.95 \\ 12.12 & -6.55 & -13.20 & -3.95 & -1.87 & 1.75 & -2.79 & 3.14 \\ -7.73 & 2.91 & 2.38 & -5.94 & -2.38 & 0.94 & 4.30 & 1.85 \\ -1.03 & 0.18 & 0.42 & -2.42 & -0.88 & -3.02 & 4.12 & -0.66 \\ -0.17 & 0.14 & -1.07 & -4.19 & -1.17 & -0.10 & 0.50 & 1.68 \end{bmatrix}} \Bigg\downarrow v.$$

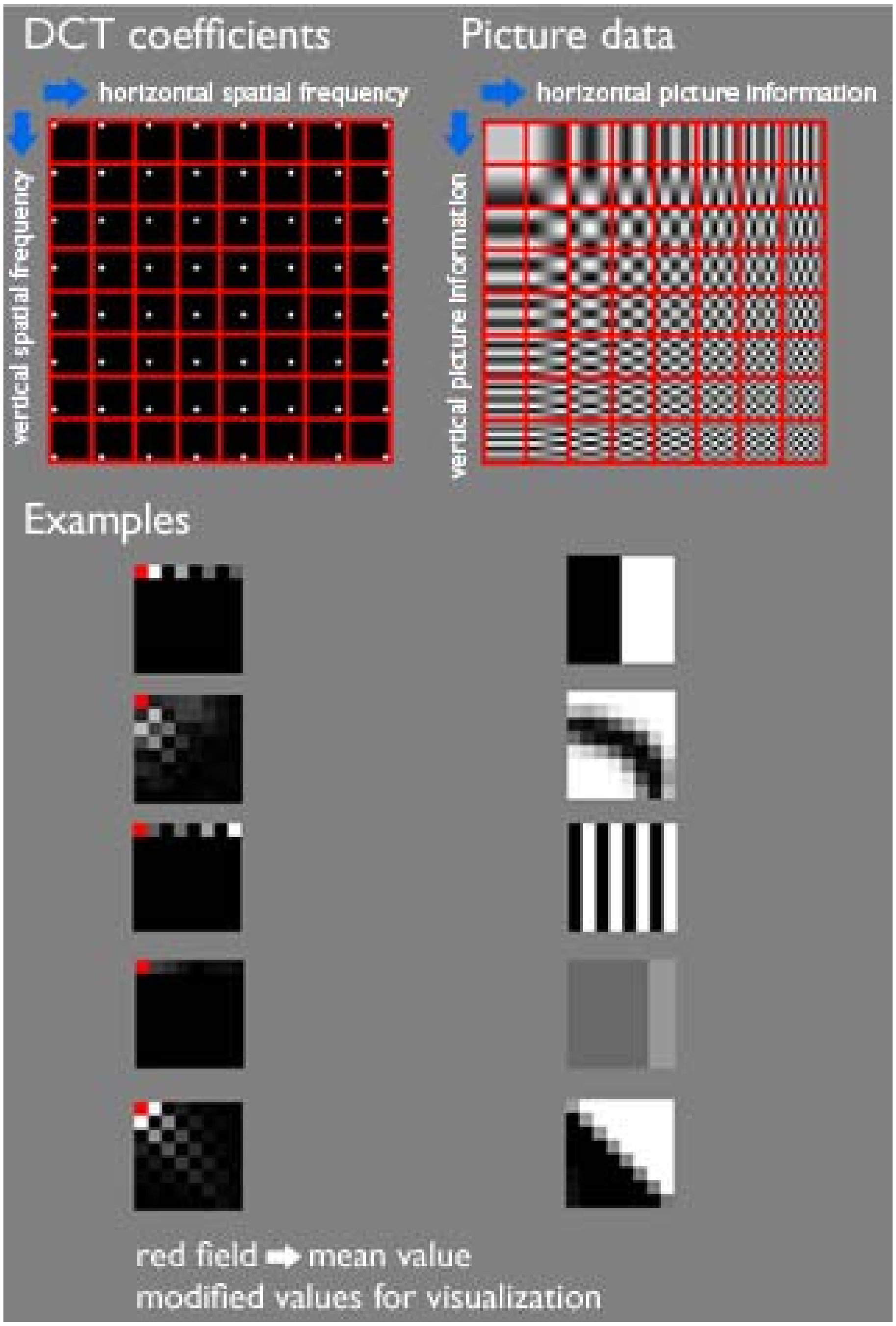
Discrete cosine transform

- The DCT transforms an 8×8 block of input values
- to a linear combination of these 64 patterns.
- The patterns are referred to as the 2D DCT basis functions,
- the output values are referred to as transform coefficients.
- The horizontal index is u {\displaystyle u} u and
- the vertical index is v {\displaystyle v} v.



DCT example

- How DCT is expressed



Quantization

- human eye is good at seeing small differences in brightness over a relatively large area
- not so good at distinguishing the exact strength of a high frequency brightness variation
- allows one to greatly reduce the amount of information in the high frequency components
- This rounding operation is the only lossy operation in the whole process (other than chroma subsampling)
- Results in many higher frequency components are rounded to zero
- Quantization matrix controls the compression ratio
- Large values high compression,

$$Q = \begin{bmatrix} 16 & 11 & 10 & 16 & 24 & 40 & 51 & 61 \\ 12 & 12 & 14 & 19 & 26 & 58 & 60 & 55 \\ 14 & 13 & 16 & 24 & 40 & 57 & 69 & 56 \\ 14 & 17 & 22 & 29 & 51 & 87 & 80 & 62 \\ 18 & 22 & 37 & 56 & 68 & 109 & 103 & 77 \\ 24 & 35 & 55 & 64 & 81 & 104 & 113 & 92 \\ 49 & 64 & 78 & 87 & 103 & 121 & 120 & 101 \\ 72 & 92 & 95 & 98 & 112 & 100 & 103 & 99 \end{bmatrix}.$$

The quantized DCT coefficients are computed with

$$B_{j,k} = \text{round}\left(\frac{G_{j,k}}{Q_{j,k}}\right) \text{ for } j = 0, 1, 2, \ldots, 7; k = 0, 1, 2, \ldots, 7$$

where G is the unquantized DCT coefficients; Q is the quantization matrix above; and B is the quantized DCT coefficients.

Quantization compression

- Quantization example

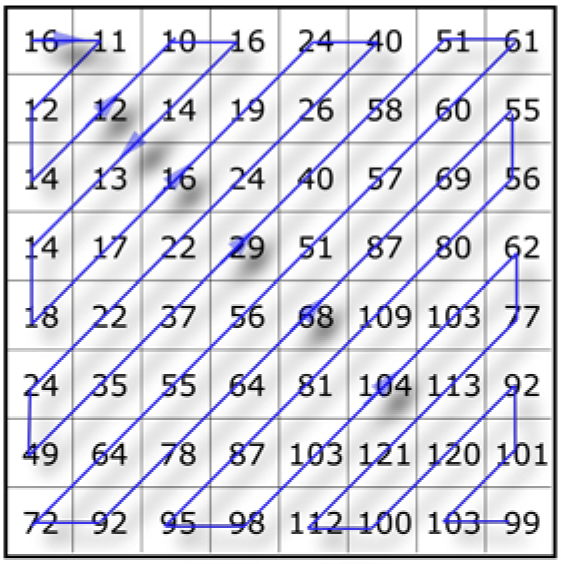
$$B = \begin{bmatrix} -26 & -3 & -6 & 2 & 2 & -1 & 0 & 0 \\ 0 & -2 & -4 & 1 & 1 & 0 & 0 & 0 \\ -3 & 1 & 5 & -1 & -1 & 0 & 0 & 0 \\ -3 & 1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

For example, using −415 (the DC coefficient) and rounding to the nearest integer

$$\mathrm{round}\left(\frac{-415.37}{16}\right) = \mathrm{round}\left(-25.96\right) = -26.$$

Reorder and variable length encoding

- At high compression more coef reduced to zero
- Data is reordered to reflect this
- After reordering, very likely, that some values at the beginning
- and then a lot of 0's.
- Instead of storing "0 0 0 0 0 0 0 0 0 0", we can easily store "10x 0".



16	11	10	16	24	40	51	61
12	12	14	19	26	58	60	55
14	13	16	24	40	57	69	56
14	17	22	29	51	87	80	62
18	22	37	56	68	109	103	77
24	35	55	64	81	104	113	92
49	64	78	87	103	121	120	101
72	92	95	98	112	100	103	99