Why Did Progress Take So Long?

Stagnation and Innovation Before Agriculture

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Economic progress is almost entirely a phenomenon of the last 10,000 years. Before that, evidence from paleodemography reveals prolonged periods of stagnation, although there were occasional bursts of technical innovation. We develop a model in which knowledge is subject to mutation and selection. In a static environment, long run stagnation is the norm. But in a dynamic setting, climate shocks can induce experimentation with latent resources, leading to punctuated equilibria with greater technical capabilities and higher population densities at successive plateaus. Sustained progress only became possible with the end of the last Ice Age and the transition to agriculture.

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1. Introduction

For most of the 200,000 years during which anatomically modern humans have existed, little progress can be discerned. There were a few bursts of innovation at earlier dates, but only in the last 13,000 years do we see precursors of modern society such as agriculture, draft animals, metallurgy, writing, and cities. Why did progress take so long?

Even if we go back just 40,000 years on the grounds that modern human brains, as opposed to bodies, took longer to evolve (Klein, 2001; Ervard et al., 2002), the puzzle remains: why was there minimal progress over tens of millennia? A parallel puzzle exists for contemporary foraging societies that continued to use stone-age techniques until they encountered the modern world (Johnson and Earle, 2000).

Economists have typically viewed such questions through the lenses of growth theory (Kremer, 1993; DeLong, 1998; Becker, Glaeser, and Murphy, 1999; Galor and Weil, 2000; Jones, 2001; Galor, 2005; Olsson and Hibbs, 2005). These authors note that world population growth before agriculture was extraordinarily slow compared to growth rates afterward. They also suggest reasons why low total population or population density might limit the rate of technological innovation, which in turn can explain slow population growth. We agree with these general points, but some further facts about foraging societies do not fit as comfortably into current models of long run growth.

Climate. According to archaeologists, climate conditions are a crucial factor in explaining prehistoric population. Climate amelioration was almost certainly necessary for the large-scale migration of humans from Africa to Asia and then onward to Europe, Australia, and the Americas. Climate was also a major determinant of population density at the regional and local levels. Until about 11,500 years ago, climate shocks were large and frequent, and had massive effects on natural resource availability in many parts of the world. We argue elsewhere that agriculture itself was a response to climate change in southwest Asia, and probably other regions as well (Dow, Reed, and Olewiler, 2007).

The natural environment is equally important for contemporary foragers. Baker (2007) has studied the technological sophistication and population density of indigenous cultures using the Standard Cross Cultural Sample (SCCS), which contains information on 186 societies. For hunter-gatherers, the variables that predict population density are habitat diversity, relatively high rainfall, flatter landscape, high primary production, and rainfall variability. All are determined by climate, geography, or ecological conditions. These exogenous variables are often omitted from long run growth models, but play a central role in the model we develop here.

Population. Economists who have written on the subject of prehistory tend to cite world population estimates obtained by identifying the inhabited regions of the world at various dates and multiplying these areas by modern hunter-gatherer population densities (e.g. Deevey, 1960; McEvedy and Jones, 1978). This yields a very small positive growth rate simply because humans slowly colonized new continents over time. Such estimates cannot be used to infer technological progress, because migration opportunities may have arisen mainly or entirely through exogenous climate changes. To make a convincing case for technical progress, one would need to show that population density increased within a fixed geographic region with fixed natural resources, and that the rise in density was not due solely to migration from adjacent regions.

Data that can be used to make such assessments are increasingly available from paleodemography. We will say more about this in section 2, but in general there is little evidence for exponential population growth (even at very low rates) within well-defined geographic regions prior to agriculture. At least as often, one finds population densities that remain roughly static for several millennia or fluctuate in ways that can be attributed with considerable confidence to exogenous climate events.

Technology. Direct archaeological evidence reveals a number of technological innovations beginning about 45 KYA (forty five thousand years ago). This history will be surveyed in section 2. These innovations were episodic and did not lead to sustained growth. Existing long run growth models, on the other hand, predict continuous (albeit slow) technical progress that is causally linked with continuous population growth.

We suggest the following way of thinking about these issues. In principle nature provides a virtually unlimited array of resources that could be exploited if a society had access to suitable technology. In a static environment, foragers typically become highly competent at exploiting some subset of these resources, but they face long run stagnation because (a) there is an upper bound on productivity for each resource; (b) latent resources remain unexploited due to the limitations of existing knowledge; and (c) knowledge does not improve for resources that are never used.

To escape from such a trap, a foraging society must be exposed to shocks from nature. For example, an improved climate tends to increase population in the long run. If this scale effect is big enough, it may become attractive to exploit latent resources. Once this occurs, cultural evolution generates improvements in the techniques used to harvest the new resources. As long as knowledge gains are irreversible, a series of positive and negative shocks can create a ratchet effect in technological capabilities, leading to higher population density for any given environment.

We define 'progress' to mean the increased capacity of a human population to obtain food in a given geographic region with given natural resources. We make the Malthusian assumption that the productivity gains from new techniques are absorbed through population growth in the long run. Therefore, on an archaeological time scale population density is a suitable measure of progress. However, population density can rise either because natural resources improved (holding technology constant) or because technology improved (holding resources constant). Our definition of progress excludes the former effect and focuses on the latter.

Unlike existing models of long run growth, our theory can explain the lengthy periods of economic stagnation observed in pre-agricultural societies, both in the distant past and among surviving hunter-gatherers. Moreover, our theory can account for two major episodes of 'punctuated' growth known from archaeology: the transition from the Middle to the Upper Paleolithic (starting around 45 KYA), and the transition from the Upper Paleolithic to the Mesolithic (starting around 15 KYA). According to our theory, in each case the causal sequence runs from climate improvement to population growth, then to greater dietary breadth, and finally to technological progress. These predictions agree with the sequence of events in the archaeological record.

Another contribution involves our model of technological innovation. We treat the techniques used to harvest food resources as finite binary strings, which are modified across generations through a process of cultural mutation and selection. This approach has several advantages. First, it is consistent with the fact that forager technology demands substantial human capital investment in children (Robson and Kaplan, 2006). Second, it captures the idea that foragers face an upper bound on labor productivity for any given resource, and can only achieve long run technical progress by broadening the spectrum of resources they exploit. Finally, it enables us to derive an explicit functional form for the probability of technical progress, which depends on (a) the labor time used to harvest a specific resource (a function of total population and the natural environment); (b) the current productivity level for the resource in question; (c) the rate of population growth; and (d) the mutation rate in the transmission of technical knowledge from adults to children. Existing models generally include these causal channels (if at all) through ad hoc specifications, rather than deriving them from consistent theoretical foundations.

Section 2 summarizes the archaeological data that constrain our analysis. Section 3 models technical change, section 4 defines equilibrium concepts, and section 5 studies responses to climate shocks (proofs of all propositions are provided in the appendix). In section 6 we suggest explanations for the Upper Paleolithic and Mesolithic transitions, as well as the geographic distribution of technological innovation and the punctuated nature of progress prior to the present Holocene climate regime. In section 7 we integrate our analysis into the existing literature on long run growth.

2. The Archaeological Record

Data from ice cores and other sources reveal a series of glacial periods (Ice Ages) lasting about 100,000 years on average, interrupted by milder interglacial periods lasting about 10,000 years on average (Cronin, 1999; McManus, 2004; Wolff et al., 2004). Glacial periods have low atmospheric CO_2, cold and dry climate, high variability in weather, and low sea levels, while interglacial periods have the opposite conditions. Combining climate data with archaeological evidence yields the following chronology (archaeological data are from Fagan, 2006, except where noted).

Fully modern human skeletons date from around 200 KYA in tropical Africa (McDougall, Brown, and Fleagle, 2005). Glacial and desert conditions confined humans to Africa at that time. An interglacial period occurred from about 126 to 116 KYA, after which glacial conditions gradually returned.

The earliest evidence for migration out of Africa into southwest Asia dates from around 100 KYA. Initial population levels were very low, perhaps as few as 50 migrants. Technology was mainly based on primitive flake tools, although there is evidence of bone tools and jewelry in South Africa by around 77 KYA (Campell et al., 2006: 397). Other migrations from Africa to Asia involving larger populations probably occurred after 70 KYA. During 78 to 64 KYA, glacial conditions intensified. Modern humans moved along the southern coast of Asia as far as Indonesia at about this time.

From 64 to 32 KYA there was a warming phase, although temperatures remained cold compared to the previous and present interglacials. By 40 KYA, modern humans had arrived in western Europe. Settlement in Australia is well documented after 36 to 38 KYA, where colonization required crossing at least 55 miles of open ocean. The current evidence implies that movement into China did not occur until 35 KYA. Migration into the Americas is usually dated at 13.5 KYA by land, with a minority view that settlement occurred earlier using watercraft.

Sometime before 40 KYA, a complex system of blade tools began to spread (see Fagan, 2006: 130). This involved more efficient use of flint and similar materials, and allowed the production of several specialized implements such as awls, burins, knives, and scrapers. In turn, these tools had the sharp edges needed to carve antler, bone, ivory, and other materials. This technical transition defines the boundary between the Middle and Upper Paleolithic in Europe and western Asia (see section 6 for further discussion).

Between 40 and 30 KYA in western Europe, there is evidence for the production of intricate personal adornments in the form of beads and pendants. Painted images of animals, signs, and anthropomorphic figures begin to appear on cave walls around 30 KYA. Artwork is worldwide by 25 KYA and common by 18 KYA. Intense glacial conditions again prevailed from 32 to 15 KYA, with the last glacial maximum (LGM) occurring about 21 KYA. The most sophisticated stone tools date to around 20 KYA.

The Mesolithic period after the LGM was warm and wet, with sedentary villages appearing in southwest Asia from 15 to 13 KYA. Semi-glacial conditions returned from 13 to 11.6 KYA (an event called the Younger Dryas). In this period, agriculture began in southwest Asia and probably also in China (see Dow, Reed, and Olewiler, 2007).

The evidence from paleodemography, taken as a whole, indicates that prior to agriculture (a) climate was clearly the main driving force behind population changes; and (b) technological progress had at most only a faint effect on population. Relevant studies include Gamble et al. (2005) on western Europe during the period 25 – 10 KYA; Shennan and Edinborough (2007) on Germany, Denmark, and Poland during the period 9 – 4 KYA; Rick (1987) on Peru during the period 13 – 3 KYA; Lourandos and David (2002) on Australia from 35 KYA until European contact; and Holdaway and Porch (1995) on

Tasmania during 35 – 10 KYA. Several of these studies find lengthy periods of static population and some find long swings or cycles that are clearly related to climate change. Rick (1987) cannot reject slow exponential growth for coastal Peru but there are potential biases in the data (rising sea levels may have removed early sites from the sample, and unobserved agriculture may have occurred late in the period).

Two major improvements in climate appear to have induced population growth, a broader diet, and technological progress (in that order). The first is associated with the boundary between the Middle and Upper Paleolithic. Stiner et al. (2000) infer an upward jump in the population of southwest Asia sometime between the late Middle Paleolithic (60-48 KYA) and the early Upper Paleolithic (about 44 KYA). This 'demographic pulse' coincides with a warming phase in the prevailing glacial conditions. The second led to Mesolithic society in southwest Asia after the LGM, with large increases in population density (Bar-Yosef, 2002a, 2002b). We discuss both of these transitions in section 6.

Although prehistoric foragers may have had adequate leisure time (Kelly, 1995: 14-23) and ample nutrition (Clark, 2006), there is very little information on changes in living standards in prehistory. Most archaeologists agree that in the millennia leading up to agriculture there was neither a systematic deterioration in nutrition or health, nor any pronounced gain (Cohen and Armelagos, 1984; Cohen, 1989). Of course, productivity improvements could have been taken in the form of increased leisure, which would be difficult to detect. Still, due to the power of compound interest even very small growth rates in population density or technological capabilities would have led to an observable transformation in pre-agricultural economies over 190,000 years, or 10,000 years, or as little as 1000 years (Richerson, Boyd, and Bettinger, 2001). There is no indication that any such dramatic transformation took place. Thus it is reasonable to conclude that before agriculture, equilibrium growth rates were centered on zero.

3. Technological Evolution in a Static Environment

We consider a foraging society that has access to an array of natural resources indexed by r = 1 . . R. Each resource can be converted into food (measured in some homogeneous units such as calories) according to a production function

$$(1) \qquad F_r(a_r, k_r, n_r) = a_r g_r(k_r) f_r(n_r)$$

where a_r is the abundance of resource r (regarded as a flow provided by nature in a given time period); k_r is the technique used to harvest it; and n_r is the labor used for harvesting. We will refer to $a = (a_1 \ldots a_R) > 0$ as 'climate'. The functions f_r are twice continuously differentiable and satisfy $f_r(0) = 0$; $0 < f_r'(n_r) < \infty$ for all $n_r \geq 0$ with $f_r'(n_r) \to 0$ as $n_r \to \infty$; and $f_r''(n_r) < 0$ for all $n_r \geq 0$.

Techniques are modeled as binary strings of uniform length Q so $k_r = (k_{r1} \ldots k_{rQ}) \in \{0, 1\}^Q$. Let k_r^* be the best method of converting resource r into food: $0 < g_r(k_r) < g_r(k_r^*)$ for all $k_r \neq k_r^*$. The function $g_r(k_r)$ is increasing in the number of digits of k_r that match k_r^*. It does not matter which digits are matched.

Q is assumed to be large enough that an exhaustive search for the ideal strings is infeasible. Instead, each generation inherits a repertoire of techniques from its parental generation. The repertoire available to the adults of generation t is $K^t = \{k_1^t \ldots k_R^t\}$. This set summarizes the state of technological knowledge.

The repertoire K^{t+1} for the next generation is derived as follows. Let there be N^t adults in period t. Each is endowed with one unit of labor time. The society's labor allocation is $n^t = (n_1^t \,.\,.\, n_R^t)$ where $\sum_r n_r^t = N^t$. All adults allocate time in the same way, so each spends n_r^t/N^t time units on resource r.

A typical child in period t has X opportunities to watch parents or other adults harvest resources, where X is a large number. Xn_r^t/N^t of these observations involve resource r (there are no observations for latent resources with $n_r^t = 0$). Each time a child i sees the exploitation of resource r, the string $k_r^t = (k_{r1}^t \,.\,.\, k_{rQ}^t)$ is copied. For each of the Q positions on the string there is a probability p that an error is made in copying the current digit, and with probability 1-p the digit is copied accurately. Mutations are independent across loci, observations, and agents. The copy is k_{rix}^t where x indexes observations.

Whenever a copy of k_r^t is made, child i uses a negligible amount of labor $\varepsilon > 0$ to determine the marginal product $a_r^t g_r(k_{rix}^t) f_r'(0)$. If the new copy achieves a higher value than the best previous copy, the child retains the new copy and discards any earlier copy. Otherwise, the best previous copy is retained and the new copy is discarded. At the end of this learning process, child i has a best string for resource r, which we denote by k_{ri}^{t+1}.

The number of children who survive to adulthood is N^{t+1}. When the parents from period t die and their children become adults, these new adults compare their strings for resource r and select (one of) the best available. The result for resource r is k_r^{t+1} = argmax $\{g_r(k_{ri}^{t+1})$ for $i = 1 \,.\,.\, N^{t+1}\}$.

This process gives updated strings for r such that $n_r^t > 0$. In most of the paper, we assume passive updating for latent resources: that is, $n_r^t = 0$ implies $k_r^{t+1} = k_r^t$. We discuss alternative productivity assumptions for latent resources at the end of section 5. The full updated repertoire $K^{t+1} = (k_1^{t+1} \, . \, . \, k_R^{t+1})$ is freely available to the adults in generation t+1.

Let q_r^t be the number of correct digits in the string $k_r^t \in K^t$. To model technical progress for active resources, we need to know the probability distribution over q_r^{t+1}.

<u>Proposition 1</u>. Assume $n_r^t > 0$ and define $\rho^t \equiv N^{t+1}/N^t$. Let the number of observations per child (X) approach infinity while the mutation probability (p) approaches zero such that $\lambda \equiv Xp > 0$ remains constant. In the limit, $\text{Prob}(q_r^{t+1} = q_r^t + 1) = 1 - \exp[-\lambda n_r^t \rho^t (Q-q_r^t)]$ and $\text{Prob}(q_r^{t+1} = q_r^t) = \exp[-\lambda n_r^t \rho^t (Q-q_r^t)]$. All other transition probabilities for resource r go to zero in the limit.

Proposition 1 shows that technical evolution for an active resource is directional. Regress requires every observation of k_r^t to have a negative mutation and the probability of this is zero in the limit. But progress only requires that at least one observation have a positive mutation and the probability of this does not vanish. The probability of progress is an increasing function of the mutation rate, the labor input for the resource, the population growth rate, and the distance from the ideal string. These results do not require a large population of agents; it is sufficient for each child to have many opportunities to observe adult behavior. Proposition 1 implies that if the transition probabilities are continuous at $n_r^t = 0$, the strings for latent resources remain constant.

4. <u>Equilibrium in a Static Environment</u>

Let $A_r(a_r, k_r) \equiv a_r g_r(k_r)$ be the productivity of resource r when its abundance is a_r and the string k_r is used. We will indicate the vector of productivities by A(a, K), where one or both arguments may be dropped when climate or technology is fixed. If the ideal string k_r^* is available, this is indicated by A_r^*. The repertoire consisting entirely of ideal strings is denoted by K* and the corresponding productivity vector is A*.

In each period the adult labor supply N is allocated across resources to maximize total food, which is shared equally. For given productivities $A > 0$ and population $N \geq 0$, a <u>short run equilibrium</u> (SRE) is a labor allocation $n(A, N) = [n_1(A, N) \,..\, n_R(A, N)]$ that achieves

(2) $H(A, N) \equiv \max \sum_{r=1..R} A_r f_r(n_r)$ subject to $n_r \geq 0$ for all r and $\sum_{r=1..R} n_r = N$.

<u>Proposition 2</u>. The solution in (2) is unique and continuous in (A, N). Moreover:

(a) <u>Scale effect</u>. Fix $A > 0$ and suppose $0 < N' < N''$. If $0 < n_r(A, N')$ then $n_r(A, N') < n_r(A, N'')$.

(b) <u>Substitution effect</u>. Fix $N > 0$ and suppose $0 < A_r' < A_r''$ with $0 < A_s' = A_s''$ for all $s \neq r$. Also suppose $0 < n_r(A', N)$ and $0 < n_s(A', N)$ for some $s \neq r$. Then $n_r(A', N) < n_r(A'', N)$ and $n_s(A', N) > n_s(A'', N)$.

(c) $H(A, N)$ is strictly concave in N and $y(A, N) \equiv H(A, N)/N$ is decreasing in N.

(d) $\lim_{N\to\infty} H(A, N)/N = 0$.

(e) $\lim_{N\to 0} H(A, N)/N = H_N(A, 0) = \max \{A_r f_r'(0)\}$ where $H_N(A, N)$ is the derivative with respect to N.

We turn now to the determination of population N. Every adult has an identical demand $\rho(y)$ for surviving children, where y is the adult's food income. We assume $\rho(0) = 0$ and $\rho(\infty) > 1$, where ρ is continuous and increasing. Surviving children are thus a normal good. This is consistent with the idea that surviving children are an argument in the adult's (direct) utility function, or with the Darwinian perspective that $\rho(y)$ is in fact the adult's (indirect) utility function. In either case there is a unique income $y^* > 0$ such that $\rho(y^*) = 1$. Integer problems involving the number of children are ignored.

The population evolves according to

$$(3) \qquad N^{t+1} = \rho[y(A^t, N^t)]N^t$$

where $y(A, N)$ is defined in Proposition 2. We restrict population dynamics as follows.

<u>Monotone population adjustment</u> (MPA). Suppose there is a population $N^* > 0$ such that $H(A, N^*)/N^* = y^*$. Keep A constant over time. If $H(A, N^0)/N^0 > y^*$ then $N^{t+1} > N^t$ for all $t \geq 0$. If $H(A, N^0)/N^0 < y^*$ then $N^{t+1} < N^t$ for all $t \geq 0$. In either case $\lim_{t\to\infty} N^t = N^*$.

MPA rules out oscillations around N^*. It holds when the direct positive effect of N^t on N^{t+1} in (3) outweighs the indirect negative effect of N^t through $y(A^t, N^t)$.

$N = 0$ is always a steady state in (3). From parts (c)-(e) of Proposition 2 there is a (unique) non-trivial steady state $N(A) > 0$ such that $y[A, N(A)] \equiv y^*$ if and only if some resource has $A_r f_r'(0) > y^*$. When such a steady state exists, population converges to it from any $N > 0$ due to MPA. In this case the steady state $N = 0$ is unstable and will be ignored. When $A_r f_r'(0) \leq y^*$ for all r, $N = 0$ is the only steady state and it is stable.

For a given productivity vector A, a <u>long run equilibrium</u> (LRE) is the population $N(A)$ defined by

$$(4) \qquad \begin{aligned} &y[A, N(A)] \equiv y^* && \text{when } \max \{A_r f_r'(0)\} > y^* \quad \text{or} \\ &N(A) \equiv 0 && \text{when } \max \{A_r f_r'(0)\} \leq y^*, \end{aligned}$$

along with the associated SRE labor allocation n[A, N(A)] from (2).

In every non-null LRE, per capita food production is y*. As the vector A varies with climate or technology, the long run population N(A) will generally vary but the long run standard of living will not. This is the Malthusian aspect of the model: an improved climate or technological progress can raise food consumption per person in the short run, but it yields a larger population with unchanged food per person in the long run.

We use the term 'very long run equilibrium' (VLRE) for a situation in which the LRE requirements are satisfied and in addition, the repertoire K is transmitted to the next generation with probability one. Proposition 1 showed that for an active resource ($n_r > 0$) there is a positive probability of progress whenever $k_r \neq k_r^*$. This implies that in a VLRE the repertoire K must include the ideal string for every active resource. Strings for latent resources ($n_r = 0$) must be compatible with corner solutions for these resources in SRE.

Formally, a very long run equilibrium (VLRE) for a fixed climate vector $a > 0$ is an array (K, N, n) such that

(a) $k_r = k_r^*$ for all r such that $n_r > 0$;

(b) N = N[A(a, K)] is derived from (4); and

(c) n = n[A(a, K), N] is derived from (2).

We say that (K, N, n) is a null VLRE if N = 0.

To characterize the set of VLREs, we require some additional terminology and notation. Let $S \subseteq \{1 \, .. \, R\}$ be a non-empty set of resources. A VLRE is said to be of type S if $k_r = k_r^*$ for $r \in S$ and $k_r \neq k_r^*$ for $r \notin S$. Let k_r^{min} be the string with the lowest productivity for resource r. Define the repertoire K^S by setting $k_r^S = k_r^*$ for $r \in S$ and k_r^S

$= k_r^{min}$ for $r \notin S$. Let A^S be the associated productivity vector, let $N^S = N(A^S)$ be the LRE population level for A^S, and let $n^S = n(A^S, N^S)$ be the SRE labor allocation for (A^S, N^S).

Proposition 3. The array (K^S, N^S, n^S) is a non-null VLRE if

(a) $A_r^* f_r'(0) > y^*$ for at least one $r \in S$ and

(b) $H_N(A^S, N^S) \geq A_r(k_r^{min}) f_r'(0)$ for all $r \notin S$.

Every other non-null VLRE of type S has the same population $N^S > 0$ and the same labor allocation n^S. If either (a) or (b) fails to hold, every VLRE of type S is null.

Corollary. Let K^* be the ideal repertoire with $N^* = N(A^*)$ and $n^* = n(A^*, N^*)$. If max $\{A_r^* f_r'(0)\} > y^*$ then (K^*, N^*, n^*) is a non-null VLRE. If max $\{A_r^* f_r'(0)\} \leq y^*$ then every VLRE is null.

A resource that satisfies $A_r^* f_r'(0) > y^*$ as in condition (a) of Proposition 3 will be called a staple. Such resources can support a positive population even in the absence of any other resource. Every non-null VLRE must have at least one staple in the set S and every staple in S must be active. Resources with $A_r^* f_r'(0) \leq y^*$ are called supplements. The set S may include one or more supplements but these resources need not be active.

Every resource $r \notin S$ is latent, whether it is a staple or a supplement. Condition (b) in Proposition 3 requires that for each of these resources, there is a harvesting method so unproductive that the resource is unexploited. If highly unproductive techniques exist for many different resources, then in general many VLREs will exist. All of the VLREs of type S are essentially identical: they support the same population and involve the same allocation of labor. The only distinctions among them involve the techniques for $r \notin S$, which must be sufficiently unproductive but are otherwise indeterminate.

The corollary provides a simple existence test. Whenever at least one staple exists, there is a non-null VLRE of the form (K*, N*, n*). We call this the maximal VLRE because no other equilibrium supports a larger population, and any VLRE with fewer active resources must have a smaller population. VLREs with populations below the maximum level will be called stagnation traps.

5. Climate Change and Technical Progress

We are now ready to address a central theoretical question: what environmental conditions are most conducive to technical progress? In particular, can climate change help a society escape from a stagnation trap?

As explained in section 1, we define progress to mean changes in technique that enable a society to support a larger population with a given climate. This requires us to separate population growth due to technological change (holding climate constant) from population growth due to climate change (holding technology constant). We solve this problem by the following thought experiment: let climate jump from an initial state a^0 to a new state a', and then back to a^0. If population in the final equilibrium is higher than in the initial one, this can only be due to technological progress along the adjustment path.

The first task is to show that the system converges to a VLRE from any initial state. This is done in Proposition 4. The second task is to study the impact of climate shocks on technology, population, and labor allocation. Proposition 5 provides such an analysis for neutral shocks that affect all resources in the same proportion. We show that positive shocks can stimulate progress while negative shocks cannot. We then briefly discuss biased shocks in a setting with two resources. Finally, we discuss the possibility of regress due to the loss of strings when active resources shut down.

Before defining convergence to a VLRE, we need to spell out how techniques and population are updated over time for a fixed climate vector $a = (a_1 \ldots a_R) > 0$. Let (K^t, N^t) be the state in period t. We obtain (K^{t+1}, N^{t+1}) as follows.

(a) K^t determines the productivities $A^t = [a_1 g_1(k_1^t) \ldots a_R g_R(k_R^t)]$.

(b) A^t and N^t determine the SRE labor allocation n^t as in (2).

(c) $H(A^t, N^t)$ determines N^{t+1} as in (3).

(d) K^t, n^t, and N^{t+1} determine the probability distribution over K^{t+1} as in Proposition 1. This yields a new state (K^{t+1}, N^{t+1}).

Proposition 1 is expressed using q_r^t (the number of digits in k_r^t that match the ideal string k_r^*), but it generates a probability distribution over k_r^{t+1} for given k_r^t because there is an equal probability of mutation at every locus of k_r^t that does not yet match k_r^*. We assume existing strings for latent resources are retained with probability one.

<u>Proposition 4</u>. Fix the climate $a > 0$ and consider any initial state (K^0, N^0) with $N^0 > 0$.

(a) Each sample path $\{K^t, N^t\}$ has some finite $T \geq 0$ and K' such that $K^t = K'$ for all $t \geq T$. We call K' the <u>terminal repertoire</u> for the sample path and $A' = A(K')$ the <u>terminal productivity vector</u>. For a given sample path, $\{N^t\} \to N' = N(A')$ and $\{n^t\} \to n' = n(A', N')$. Accordingly, we say that K' <u>generates</u> (K', N', n').

(b) With probability one, the terminal array (K', N', n') is a VLRE.

(c) If $N^0 < N[A(K^0)]$ then $\{N^t\}$ is increasing. If $N^0 = N[A(K^0)]$ then $\{N^t\}$ is non-decreasing. If $N^0 > N[A(K^0)]$ then $\{N^t\}$ may decrease for all $t \geq 0$, or it may decrease until some $T > 0$ and become non-decreasing for all $t \geq T$.

Because regress is impossible for active resources and strings are conserved for latent resources, $\{A^t\}$ is non-decreasing and the system cannot converge to any VLRE whose productivity vector is dominated by A^0. However, it is difficult to say much about the probability of converging to a specific VLRE because this depends in a complex way on how mutations affect productivities, which affect population dynamics, which then feed back into the mutation probabilities as described in Proposition 1.

Fortunately, more can be said about responses to neutral climate shocks, which do not alter relative resource abundances.

Proposition 5 (neutral shocks). Let (K^0, N^0, n^0) be a non-null VLRE for the climate $a^0 > 0$. Define $A^0 = A(a^0, K^0)$ and consider a permanent climate change $a' = \theta a^0$ where $\theta > 0$.

(a) Negative shocks. Suppose $\theta \in (0, 1)$. The system converges to the new VLRE (K', N', n') with $K' = K^0$; $N' = N(\theta A^0) < N^0$; and $n' = n[\theta A^0, N(\theta A^0)]$. The set of active resources in n' is a subset of the active resources in n^0. If $N' > 0$ and the climate returns permanently to a^0 starting from (K', N', n'), the system converges to the original VLRE (K^0, N^0, n^0).

(b) Positive shocks. Suppose $\theta > 1$. Due to Proposition 4(b), with probability one the system converges to a VLRE (K', N', n'). This new VLRE has $K' \neq K^0$ iff

(*) $n_r[\theta A^0, N(\theta A^0)] > 0$ for some r such that $k_r^0 \neq k_r^*$.

The new population satisfies $N' \geq N(\theta A^0) > N^0$, where $N' > N(\theta A^0)$ iff (*) holds. If a^0 is permanently restored starting from (K', N', n') and (*) does not hold, the system converges to the original VLRE (K^0, N^0, n^0). If a^0 is permanently restored starting from (K', N', n') and (*) does hold, the system converges to the VLRE (K'', N'', n'') where $K'' = K' \neq K^0$ and $N' > N'' \geq N^0$. The last inequality is strict iff $n[A(a^0, K''), N^0] \neq n^0$. In this case, some resource with $n_r^0 = 0$ has $n_r'' > 0$.

Proposition 5(a) shows that a neutral negative shock cannot stimulate technical progress because it cannot change the repertoire K. Population drops, and some of the previously active resources may become latent (the diet may narrow). Reversing the shock returns the system to the original VLRE, provided that the society has not become extinct in the meantime, and the initial population is restored. This reflects the absence of technical change in response to the climate fluctuation.

Proposition 5(b) shows that a neutral positive shock can stimulate permanent progress. A necessary and sufficient condition for this outcome is that the shock must lead to exploitation of a latent resource whose technique can be improved. This cannot occur in the short run through substitution effects because relative resource abundances are unchanged. Instead, the key channel is a scale effect involving population.

Without technical change, the improved climate would lead to a larger population $N(\theta A^0)$ in the long run. This population growth could make one or more latent resources active. If so, the technical repertoire improves through learning by doing, and population expands beyond the level $N(\theta A^0)$ induced by climate change alone. But if the scale effect is too small to activate a latent resource, technology remains static and population grows only to the extent that climate permits.

If the climate returns to its original state a^0 and technology has not improved in the meantime ($K' = K^0$), clearly population must return to its original level N^0. The same is true if technology improves as a result of the climate shock, but not by enough to alter the set of active resources used in the original climate a^0. For progress to become visible in the population level ($N'' > N^0$) after climate reverts back to a^0, the string for at least one previously latent resource must improve to a point where the old labor allocation n^0 is no longer optimal at the old population level N^0. In this case, at least one new resource will be used after the climate returns to a^0. Simultaneously, some resources initially active at a^0 may be abandoned due to substitution effects.

Provided that strings for latent resources are conserved, the new technical plateau will be permanent. Proposition 5(a) shows that subsequent negative shocks cannot force a technological retreat. The result is a ratchet effect in which knowledge can gradually improve. But unlike conventional growth models, our framework predicts a 'punctuated' process where occasional productivity gains stimulated by positive climate shocks can be separated by long periods of stagnation during which technology does not change and the average population growth rate is zero. During these intervals population will fluctuate in response to climate, and individual resources may go in and out of use as a result, but there is no lasting improvement in technological capabilities.

These conclusions all hinge on two key assumptions: first that climate shocks are neutral, and second that existing strings for latent resources are conserved. In the rest of this section we consider the consequences of relaxing each assumption.

Shocks biased toward or against particular resources create short run substitution effects that can activate latent resources even before population has time to adjust. It will be convenient to discuss these effects in the context of two resources (R = 2). We start from an initial VLRE (K^0, N^0, n^0) associated with climate a^0 in which resource 1 is active and resource 2 is latent. Because a negative shock to a latent resource cannot affect the system, this case is ignored.

<u>A positive shock to the latent resource</u>. There is an immediate substitution effect away from resource 1 toward resource 2 with N^0 constant. If the shock is large enough, $n_2^0 > 0$ will occur (otherwise it is irrelevant). In the long run population grows and the productivity A_2 rises. For both reasons, n_2^t increases. With probability one, the ideal string k_2* is eventually identified and the ideal repertoire K* is achieved.

<u>A positive shock to the active resource</u>. An immediate substitution effect keeps resource 1 active and reinforces the latency of resource 2. However, population grows in the long run and this scale effect may eventually outweigh the substitution effect to give $n_2^T > 0$ at some T > 0. If resource 2 ever becomes active it remains so, and the analysis from that point on is the same as before.

<u>A negative shock to the active resource</u>. An immediate substitution effect favors resource 2 at the expense of resource 1. If the shock is large enough, $n_2^0 > 0$ may occur. There are then two possibilities: (a) $n_2^t > 0$ for all $t \geq 0$; or (b) there is a T > 0 such that $n_2^t = 0$ for all $t \geq T$. Case (a) occurs when technical progress for resource 2 is rapid enough to outweigh the population decline resulting from the negative shock. In this scenario, population could eventually begin to grow and the ideal repertoire K* could be achieved. Case (b) occurs when technical progress for resource 2 is too slow, so that this resource

eventually shuts down due to the declining population. This aborts further progress and leads to a new VLRE in which resource 2 is again latent.

These conclusions and those for neutral shocks in Proposition 5 are all tilted in favor of progress by our assumption that strings for latent resources are conserved with certainty. This assumption is appealing both on grounds of tractability and because it is the limiting case of Proposition 1. However, it is worth considering other possibilities. One might suppose, for example, that there is a positive probability of deterioration in a string if a technique must be passed down through oral traditions, in the absence of any practical experience with the resource in question.

An extreme way to introduce regress would be to assume that when a resource shuts down, the associated string drops out of the repertoire entirely. Then the only way to revive the resource would be to borrow a string already in use for an active resource. This string would gradually be adapted to the new resource as in Proposition 1. Within the two-resource framework, it can be shown that in this case negative shocks may lead to regress and the population may not fully recover when such shocks are reversed.

Overall, the analysis of this section identifies three key prerequisites for sustained progress. First, climate shocks must be large enough to trigger experimentation with new resources. Second, they should permit the preservation of existing knowledge. This is more likely when shocks are roughly neutral across individual resources, because strong substitution effects that flip the system from one corner solution to another interrupt the process of 'remembering by doing'. Finally, a climate improvement must persist long enough for new techniques to evolve. When a positive shock is too brief, there is little opportunity for progress and the previous equilibrium is likely to be restored.

It is also helpful if a society allocates a large amount of labor to new resources, without abandoning previous resources, and if population grows rapidly in response to a positive shock. As shown in section 3, these conditions tend to accelerate productivity gains during intervals of favorable climate, which makes continued exploitation of the new resources more likely after climate reverts to its previous state.

6. Progress in Prehistory

Section 5 showed that a positive neutral climate shock, if sufficiently large, leads to a causal cascade involving: (a) population growth; (b) greater dietary breadth; and (c) technological progress. The latter can feed back to population growth, generating further rounds of (a), (b), and (c) until the system settles into a new equilibrium. This dynamic appears to have been at work during the Upper Paleolithic and Mesolithic transitions.

The Upper Paleolithic transition. The warming period during 64 – 32 KYA led to population growth in southwest Asia by at least 44 KYA and probably earlier (Stiner et al., 2000). This growth is documented from the effects of human predation on the size distribution of prey species. The Upper Paleolithic, which is usually dated by the arrival of blade technology, began in southwest Asia around 44 KYA. Stiner et al. (1999: 193) observe that "in western Asia, . . . human populations increased substantially before the remarkable and rapid technologic innovations (radiations) that mark the Upper and Epi-Paleolithic periods". This sequence, in which climate stimulates population growth and the latter stimulates technical progress, is consistent with the predictions of our model.

Modern humans moved into Europe from southwest Asia around 45 – 40 KYA. Since blade technology already existed in southwest Asia at this time and had precursors in Africa, it can be assumed that this technology was brought to Europe rather than being invented there. Indeed, blade technology may have helped encourage the migration, but the milder climate played a major facilitating role. The availability of large prey on the European steppe (bison, horses, reindeer, mountain goats, woolly mammoth, woolly rhinoceros, oxen, deer) may have stimulated further refinements in this technology.

The diet was broadened in Italy to include birds around 35 KYA, a date that coincides with the arrival of the Upper Paleolithic there. Such prey were previously available but had been unexploited. Exploitation of birds began in modern-day Israel around 28-26 KYA (Stiner et al., 2000), simultaneously with the transition to the Upper Paleolithic in that region. This dietary expansion was evidently linked to the diffusion of blade technology and perhaps also to population growth.

The Mesolithic transition. The end of the Ice Age led to substantial population growth in southwest Asia and Europe (Gamble et al., 2005). An important feature of the associated Mesolithic transition was the inclusion of smaller prey (hares and rabbits) in the diet throughout West Asia, Europe, and North Africa. By 19 – 17 KYA hares and rabbits had been added to the diet in Italy, and the same occurred by 13 – 11 KYA in southwest Asia (Stiner et al., 2000). As with birds in the Upper Paleolithic transition, these prey species had previously been available but remained latent until the climate moderated and population grew. Stiner (2001: 6996) comments: "early indications of expanding diets in the eastern Mediterranean precede rather than follow the evolution of the kinds of tools (specialized projectile tips, nets, and other traps) needed to capture quick small animals efficiently." Again, this is consistent with our model.

By 14.5 KYA, the Natufian culture in southwest Asia showed rapid population growth, with site areas five times larger on average than in the preceding Geometric

Kebaran (see Bellwood, 2005: 51). These hunter-gatherers consumed a very wide range of prey shortly before the onset of agriculture, including gazelle, goat, sheep, cattle, bear, deer, badger, marten, beaver, hedgehog, boar, hare, rodents, fox, dog, cat, birds, fish, reptiles, amphibians, crab, and mollusk (Savard, Nesbitt, and Jones, 2006).

Dietary breadth with respect to plant foods during the Mesolithic is controversial. It is clear that around the LGM, the inhabitants of southwest Asia relied heavily on small-seeded grasses (Weiss et al., 2004). As the climate improved, large-seeded grasses grew more common (especially the wild precursors of wheat, barley, and rye). According to one school of thought, the people of the region increasingly specialized in these grasses, which smoothed the path to agriculture. The other school of thought argues that climate change and population growth led to a gradual broadening of the diet to include a wider range of wild plant species (the so-called 'broad spectrum revolution' or BSR). In this scenario, large-seeded grasses were a minor part of the overall Mesolithic diet, despite their later importance for agriculture.

In a re-analysis of data from sites in southwest Asia shortly before agriculture, Savard, Nesbitt, and Jones (2006) find that the plant component of the diet was highly diverse at each site, and also that there was substantial diversity in the composition of the diet across sites. At only one site (Mureybit) were the wild ancestors of domestic cereals a dominant component of the diet. The increased abundance of large-seeded grasses after the LGM did not drive out consumption of the small-seeded grasses that were important at the LGM. The results for the region as a whole "fit with elements of . . . the broad spectrum model" (Savard, Nesbitt, and Jones, 2006: 192). In a similar vein, Hillman et al. (2001) report that just before the Younger Dryas and the start of agriculture, more than

100 species of edible seeds and fruits were used in southwest Asia. This broad pattern of plant exploitation was accompanied by considerable technological paraphernalia such as querns, mortars, pestles, bowls, grinders, pounders, whetstones, choppers, and sickles.

The original BSR hypothesis from archaeology involved exogenous population pressure and a resulting deterioration in living standards, which drove people to use less desirable species. By contrast, in our model the BSR is a response to a positive climate shock, and the endogenous population growth following this shock is not to be regarded as 'pressure'. Rather, it is a response to improved living standards and expands dietary breadth as people find it profitable to exploit latent resources. This process stimulates technical innovation aimed at more efficient harvesting of the new resources.

We conclude that evidence for the Upper Paleolithic and Mesolithic transitions is consistent with the theory developed in section 5. In both cases, improved climate led to population growth, followed by increased dietary breadth and then technical progress that facilitated the exploitation of new food resources.

Our theory also provides insight into the geographic distribution of technological innovations. Important innovations before agriculture included boats; tailored clothing; better food storage methods; microblades; spear throwers; and the bow and arrow. Boats were useful not just for long-distance migration but also for efficient local transportation. Tailored clothing and cold-weather housing adaptations were vital for the colonization of northern Asia as well as migration to the Americas. Microblades emerged among mobile hunters in central Asia as a way of conserving on scarce raw materials (Fagan, 2006: 150-2). The utility of food storage, spear throwers, and the bow and arrow are evident.

In most of these cases, dates and locations are so uncertain that it is impossible to link innovations to particular climate events or population trends with any confidence. Even so, one pattern does emerge: most of the innovations listed above are empirically associated with temperate or arctic Eurasia rather than Africa or tropical Asia. The data could be biased by differences in archaeological research efforts across continents, but the concentration of technical advances in temperate or arctic climate zones is striking.

Our theory suggests an explanation. As we discussed at the end of section 5, the environment best suited to innovation in a foraging economy involves occasional neutral climate shocks, especially if knowledge about latent resources can be preserved through strong oral traditions when shocks are negative. Frequent shocks give new techniques less time to evolve. Large negative shocks or strongly biased shocks tend to shut down active resources and strain a society's ability to preserve existing knowledge, although progress could still occur. The environment least suited to innovation is one in which shocks are small or absent, because then it is impossible to escape a stagnation trap.

Until about 11.5 KYA, the temperate regions of the world were buffeted by very large climate shocks on time scales ranging from decades to millennia (Richerson, Boyd, and Bettinger, 2001). Although far from ideal, these conditions apparently generated a ratchet effect through which innovations gradually accumulated. This process was likely assisted by the diffusion of techniques across regions, which would have increased the effective number of experimenters and created a permanent reservoir of shared technical knowledge that could be drawn upon by individual societies (Diamond, 1997). Tropical regions were more sheltered from such Ice Age shocks, giving the people of Africa and southern Asia less reason to experiment with latent resources.

Nevertheless, technical progress was patchy and infrequent even in temperate and arctic Eurasia until the Holocene period of the last 11,500 years. This poses a puzzle: if climate shocks were a necessary stimulus, why did progress accelerate with the arrival of the more stable Holocene climate regime?

This acceleration was closely linked to the emergence of agriculture. We argue elsewhere that agriculture arose in southwest Asia due to a very large negative climate shock (Dow, Reed, and Olewiler, 2007). Following an interval of mild climate lasting 1500-2000 years during which regional population accumulated, an abrupt shift to colder and drier conditions led to large population spikes at a few sites, as people moved from arid areas to places with reliable water sources. The resulting local reductions in the marginal product of foraging labor made cultivation attractive.

Agriculture brought more of the reproductive cycle of plants and animals under human control. Rather than merely harvesting resources given by nature, humans could intervene at earlier stages in the production process, which led to learning by doing with respect to planting, irrigation, weeding, and the like. Over many generations, processes of artificial selection led to fully domesticated plants and animals, generating enormous productivity gains. Stone-age foraging techniques and low population densities persisted in areas where conditions were unsuitable for farming (the Arctic; the deserts of Australia and southern Africa; the rain forests of the Amazon basin, Africa, and southeast Asia). Elsewhere, agriculture brought an end to stagnation.

Furthermore, although the size and frequency of climate shocks has decreased dramatically in the last 11,500 years, there is a sense in which the Holocene itself has been a single huge positive shock. Around the time that agriculture arose and began to spread, world climate improved both in the mean (warmer and wetter conditions) and through reduced variance. This created a permissive environment in which population could grow, new resources could be explored, and new techniques could be perfected. The rest is history.

7. Concluding Remarks

In this paper we propose an explanation for why technological progress was largely absent before agriculture, a period that encompasses most of human existence. After agriculture such progress became the norm, initially resulting in rising population densities and more recently in rising per capita incomes. We conclude by developing some connections between our framework and theories of long run growth.

Unified growth theory, summarized by Oded Galor (2005), provides a useful focus. This theory views long run growth as having three stages. The first stage is 'Malthusian stagnation', in which the long run response to productivity growth involves population growth rather than growth in per capita income. Although rising population stimulates technological progress, parents tend to invest in the quantity rather than the quality of children. In the second stage, more rapid technical change leads parents to invest more in the human capital of their children. Per capita income begins to rise but only slowly. In the third stage, technical progress is very rapid, parents strongly prefer quality to quantity of children, and the resulting human capital investments lead to more technical progress. This virtuous circle ultimately yields modern economic growth with rapidly rising per capita income.

We would insert an initial foraging stage in which growth is dominated by the natural environment. This stage exhibits long intervals of stagnation punctuated by a few episodes in which climate change stimulates technical advance. It lasts until a specific sequence of climate shocks gives rise to agriculture and ushers in a second Malthusian stage, which we would call 'Malthusian growth' rather than 'Malthusian stagnation'.

Our analysis does not offer insight into the nature of the feedback mechanisms that promoted technological change prior to and during industrialization. In addition to the population-to-technology channel emphasized in unified growth theory (which is also a component of our model), the economic history literature offers alternative hypotheses emphasizing knowledge networks (Mokyr, 2002), mechanistic science (Lipsey, Carlaw, and Bekar, 2005), institutional innovation (North, 1981; Acemoglu, Johnson, and Robinson, 2005; Greif, 2006), and differential reproductive success (Galor and Moav, 2002; Clark, 2007).

Our analysis does, however, identify a missing element in the literature on long run growth. The amount of progress generated through endogenous growth mechanisms depends crucially on the degree to which production is isolated from environmental shocks. In foraging societies, technical advance was driven almost entirely by shocks from nature, to the extent that it occurred at all. Feedback from population or human capital investment to technical change had little quantitative impact in the absence of climate shifts that prompted experimentation with latent resources.

Agriculture offered more opportunity for technological advance through learning by doing. As nature's role decreased relative to these learning opportunities, Malthusian progress became possible. This was reinforced by the fact that large climate shocks no longer disrupted knowledge accumulation. Eventually, productivity gains in agriculture allowed the emergence of large manufacturing and service sectors that were even further removed from nature's influence. This unleashed the feedback effects to technology that underpin modern economic progress.

Such progress was hardly inevitable. Rather, one can imagine a range of progress trajectories conditional on technological opportunities and climate regimes. In a world of hunting and gathering with either very low or very high variance in climate, we expect to see very little progress, and progress will be episodic rather than continuous when it does occur. In a world where agriculture is combined with a mild and low-variance climate, we see Malthusian growth, and in a world where large manufacturing and service sectors are combined with the same climate conditions, we see modern economic growth.

Suppose, however, that world climate had continued to follow the normal pattern of glacial periods lasting 100,000 years interrupted by interglacial periods lasting 10,000 years. This would have implied the onset of glacial conditions around 5000 years ago, a time at which agriculture was spreading across southwest Asia, China, and Mesoamerica. The long run outcome would have been a major contraction in population as well as in regions of human occupation, and most likely a reversion to hunting and gathering -- in short, a world with little or no progress.

Appendix: Proofs of Propositions

Proof of Proposition 1.

Choose any resource r. Let $Z \equiv X(n_r^t/N^t)N^{t+1} \equiv Xn_r^t\rho^t$ be the number of copies of string k_r^t, where X is the number of observations per child, n_r^t/N^t is the fraction of these observations that pertain to resource r, and N^{t+1} is the number of children who survive to become adults in period t+1. Let z = 1 . . Z index the individual copies k_{rz} of k_r^t.

The number of digits in k_r^t that match k_r^* is $q_r^t \in \{0, 1 .. Q\}$ and the number of digits in k_{rz} that match k_r^* is $q_{rz} \in \{0, 1 .. Q\}$. The random variables q_{rz} are iid conditional on k_r^t. Fix the string k_r^t and define the probabilities

$$\pi_q(p) \equiv \Pr(q_{rz} = q \mid k_r^t; p) \qquad \text{for all } q = 0, 1 .. Q \text{ and } z = 1 .. Z.$$

Then define

$$\theta_q(p) \equiv \Pr(q_r^{t+1} = q \mid k_r^t; p) = \Pr(\max\{q_{r1} .. q_{rZ}\} = q \mid k_r^t; p)$$

$$= \Pr(q_{rz} \leq q \text{ for } z = 1 .. Z \mid k_r^t; p) - \Pr(q_{rz} \leq q\text{-}1 \text{ for } z = 1 .. Z \mid k_r^t; p)$$

$$= [\pi_0(p) + .. + \pi_q(p)]^Z - [\pi_0(p) + .. + \pi_{q-1}(p)]^Z$$

where $\pi_0(p) + .. + \pi_{q-1}(p) \equiv 0$ for q = 0.

We want to compute each $\theta_q(p)$ when $X \to \infty$ and $p \to 0$ such that $Xp \equiv \lambda > 0$ is constant. From the definition of Z we have $Z(p) = \lambda\rho^t n_r^t/p$. In what follows we drop the r subscript and the t superscripts in this expression. Next define

$$\theta_q^* \equiv \lim_{p\to 0} \theta_q(p)$$

$$= \lim_{p\to 0} [\pi_0(p) + .. + \pi_q(p)]^{Z(p)} - \lim_{p\to 0} [\pi_0(p) + .. + \pi_{q-1}(p)]^{Z(p)}$$

$$= \{\lim_{p\to 0} \exp[(1/p)\ln(\pi_0(p) + .. + \pi_q(p))]\}^{\lambda n\rho}$$

$$- \{\lim_{p\to 0} \exp[(1/p)\ln(\pi_0(p) + .. + \pi_{q-1}(p))]\}^{\lambda n\rho}$$

We have $\lim_{p\to 0} \pi_q(p) = 1$ for $q = q_r^t$ and $\lim_{p\to 0} \pi_q(p) = 0$ for $q \neq q_r^t$ because at least one mutation must occur whenever the number of correct digits differs from q_r^t. This implies that for each expression of the form $\lim_{p\to 0} \exp[(1/p)\ln(\pi_0(p) + .. + \pi_q(p))]$, there are two possible cases:

(a) If $q < q_r^t$ then $\lim_{p\to 0} \exp[(1/p)\ln(\pi_0(p) + .. + \pi_q(p))] = e^{-\infty} = 0$ and hence $\theta_q^* = 0$.

(b) If $q \geq q_r^t$ then $\lim_{p\to 0} (\pi_0(p) + .. + \pi_q(p)) = 1$ and hence $\lim_{p\to 0} \exp[(1/p)\ln(\pi_0(p) + .. \pi_q(p))] = \lim_{p\to 0} \exp[\pi_0'(p) + .. + \pi_q'(p)]$.

Now consider the derivatives $\pi_q'(p)$ in case (b) above. The probabilities $\pi_q(p)$ are polynomials in p, and all outcomes involving two or more mutations correspond to terms that are quadratic or higher. After taking derivatives, all such terms vanish in the limit. Thus we can confine attention to outcomes that involve either no mutations or just one mutation. This implies that only $q = q_r^t - 1$, $q = q_r^t$, and $q = q_r^t + 1$ are relevant.

(i) $q = q_r^t - 1$. This outcome can be obtained in q_r^t ways by having one mutation at a correct locus and no mutations elsewhere, which has the probability $q_r^t p(1-p)^{Q-1}$. All other ways to obtain the same result involve three or more mutations. This gives $\lim_{p\to 0} \pi_q'(p) = q_r^t$ so $\lim_{p\to 0} \exp[\pi_0'(p) + .. + \pi_q'(p)] = \exp(q_r^t)$.

(ii) $q = q_r^t$. This outcome can be obtained in one way with no mutations, which has probability $(1-p)^Q$. All other ways to obtain the same result involve two or more mutations. This gives $\lim_{p\to 0} \pi_q'(p) = -Q$ so $\lim_{p\to 0} \exp[\pi_0'(p) + .. + \pi_q'(p)] = \exp[-(Q-q_r^t)]$.

(iii) $q = q_r^t + 1$. This outcome can be obtained in $Q-q_r^t$ ways by having one mutation at an incorrect locus and no mutations elsewhere, with probability $(Q-q^t)p(1-p)^{Q-1}$.

All other ways to obtain the same result involve three or more mutations. This gives $\lim_{p\to 0} \pi_q'(p) = Q-q_r^t$ and thus $\lim_{p\to 0} \exp[\pi_0'(p) + . . + \pi_q'(p)] = e^0 = 1$. Recall from (a) above that $\theta_q^* = 0$ when $q < q_r^t$. To solve for θ_q^* when $q = q_r^t$ we first observe that the second limit in the last line for θ_q^* is zero due to (a). Substituting from (b) and (ii) in the first limit in the last line for θ_q^* gives $\theta_q^* = \exp[-\lambda n_r^t \rho^t (Q-q^t)]$.

To solve for θ_q^* when $q = q_r^t + 1$ we observe that (b) applies to both limits in the last line for θ_q^*. Substituting from (b) gives $\theta_q^* = 1- \exp[-\lambda n_r^t \rho^t (Q-q^t)]$.

Finally, we have $\theta_q^* = 0$ when $q \geq q_r^t + 2$ because (b) applies to both limits in the last line for θ_q^* and both of these limits equal unity. The latter result follows from (iii) and the fact that any outcome with $q \geq q_r^t + 2$ requires two or more mutations.

By construction all of the θ_q^* are conditional on k_r^t. However, the structure of the proof shows that the only relevant property of k_r^t is the number of correct digits q_r^t. Thus we can write the transition probabilities for q_r^{t+1} and q_r^r as in Proposition 1.

The limiting transition probabilities for strings, $\lim_{p\to 0} \Pr(k_r^{t+1} = k \mid k_r^t; p)$, follow from the preceding results. If k has fewer correct digits than q_r^t or more than $q_r^t + 1$, it has probability zero in the limit. The only way to have $q_r^{t+1} = q_r^t$ in the limit is by having $k_r^{t+1} = k_r^t$ so this has probability $\exp[-\lambda n_r^t \rho^t (Q-q^t)]$. There are $Q-q_r^t$ ways to obtain $q_r^{t+1} = q_r^t + 1$ by a single mutation that changes one incorrect digit to a correct one. Each of these strings k_r^{t+1} has probability $\{1- \exp[-\lambda n_r^t \rho^t (Q-q^t)]\}/(Q-q_r^t)$. This ends the proof.

Proof of Proposition 2.

Uniqueness follows from the strict concavity of the objective function and continuity follows from the theorem of the maximum.

(a) Let n' be optimal for (A, N') and let n'' be optimal for (A, N''). Suppose that $n_r'' \leq n_r'$. The first order conditions for (2) with parameters (A, N'') imply that all s with $n_s'' > 0$ have $A_s f_s'(n_s'') \geq A_r f_r'(n_r'')$. The fact that $n_r'' \leq n_r'$ gives $A_r f_r'(n_r'') \geq A_r f_r'(n_r')$. Finally, the first order conditions for (2) with parameters (A, N') and $n_r' > 0$ give $A_r f_r'(n_r') \geq A_s f_s'(n_s')$ for all $s = 1 \,..\, R$. This series of inequalities gives $A_s f_s'(n_s'') \geq A_s f_s'(n_s')$ for all s with $n_s'' > 0$ and thus implies $n_s' \geq n_s''$ for all s such that $n_s'' > 0$. Clearly $n_s' \geq n_s''$ also holds for all s such that $n_s'' = 0$. Summing over resources gives $N' \geq N''$, contradicting the assumption $N' < N''$. This shows that $n_r'' > n_r'$.

(b) Let n' be optimal for (A', N) and let n'' be optimal for (A'', N). Suppose $n_r'' \leq n_r'$. For all $v \neq r$ such that $n_v'' > 0$ we have $A_v'' f_v'(n_v'') \geq A_r'' f_r'(n_r'')$. Furthermore, $A_r'' f_r'(n_r'') > A_r' f_r'(n_r'') \geq A_r' f_r'(n_r') \geq A_v' f_v'(n_v')$ for all $v = 1 \,..\, R$. The first inequality follows from $A_r'' > A_r'$, the second from $n_r'' \leq n_r'$, and the last from $n_r' > 0$. The preceding series of inequalities and $A_v'' = A_v'$ for $v \neq r$ shows that $A_v' f_v'(n_v'') > A_v' f_v'(n_v')$ for all $v \neq r$ such that $n_v'' > 0$ and hence $n_v'' < n_v'$ for all $v \neq r$ such that $n_v'' > 0$. There must be at least one such $v \neq r$ since otherwise $n_r'' = N > n_r'$ due to $n_s' > 0$, but we have supposed $n_r'' \leq n_r'$. Clearly all $v \neq r$ with $n_v'' = 0$ have $n_v'' \leq n_v'$. Summing over all resources gives $N'' < N'$ because there is at least one $v \neq r$ with $n_v'' < n_v'$. This contradicts the fact that N is constant. Therefore $n_r'' > n_r'$. Next suppose $n_s'' \geq n_s'$. Consider $v \neq s$ and $v \neq r$. For all such v, $A_v'' f_v'(n_v'')$

$\leq A_s''f_r'(n_s'') \leq A_s''f_s'(n_s') = A_s'f_s'(n_s')$. Moreover, if $n_v' > 0$ we have $A_s'f_s'(n_s') = A_v'f_v'(n_v')$. This and $A_v'' = A_v'$ implies that for any $v \neq s$ and $v \neq r$ with $n_v' > 0$, it must be true that $n_v'' \geq n_v'$. Clearly the same inequality holds when $n_v' = 0$. Since $n_r'' > n_r'$ and we have supposed $n_s'' \geq n_s'$, summing over resources gives $N'' > N'$. This contradicts the fact that N is constant. Therefore $n_s'' < n_s'$.

(c) Fix $A > 0$. Choose any $N' \neq N''$ and $\mu \in (0, 1)$. Let n' be optimal for (A, N') and let n'' be optimal for (A, N''). Define $n^\mu = \mu n' + (1\text{-}\mu)n'' \geq 0$. This is a feasible allocation for the total population $N^\mu = \mu N' + (1\text{-}\mu)N''$. It follows that $H(A, N^\mu) \geq \sum A_r f_r(n_r^\mu) = \sum A_r f_r[\mu n_r' + (1\text{-}\mu)n_r''] > \sum A_r[\mu f_r(n_r') + (1\text{-}\mu)f_r(n_r'')] = \mu H(A, N') + (1\text{-}\mu)H(A, N'')$. The strict inequality occurs because due to the strict concavity of f_r we have $f_r[\mu n_r' + (1\text{-}\mu)n_r''] > \mu f_r(n_r') + (1\text{-}\mu)f_r(n_r'')$ whenever $n_r' \neq n_r''$, and the latter inequality must hold for at least one r because $N' \neq N''$. Due to $H(A, 0) = 0$, the strict concavity of H implies $H(A, \mu N) > \mu H(A, N)$ for all $N > 0$ and $\mu \in (0, 1)$. This yields $H(A, \mu N)/\mu N > H(A, N)/N$ for all $N > 0$ and $\mu \in (0, 1)$. Thus $y(A, N) \equiv H(A, N)/N$ is decreasing in N.

(d) Fix $A > 0$ and consider the (unique) optimal allocation $n(A, N)$. We first show that $\lim_{N\to\infty} n_s(A, N) = \infty$ must hold for some s. Suppose instead that for every r there is a finite upper bound $\underline{n}_r$ such that $n_r(A, N) \leq \underline{n}_r$ for all N. Then for any $N > \sum \underline{n}_r$ we have $\sum n_r(A, N) < N$, which contradicts optimality. Thus there is some s such that $\lim_{N\to\infty} n_s(A, N) = \infty$. From the assumption that $f_r'(n_r) \to 0$ as $n_r \to \infty$ for $r = 1 .. R$ we obtain $\lim_{N\to\infty} A_s f_s'[n_s(A, N)] = 0$. Next, define $m(A, N) = \max \{A_r f_r'[n_r(A, N)]\}$. There is some $\underline{N}$ such that $N > \underline{N}$ implies $n_s(A, N) > 0$. From the first order conditions for (2) this implies $m(A, N) = A_s f_s'[n_s(A, N)]$ for all $N >$

$\underline{N}$. Hence $\lim_{N\to\infty} m(A, N) = 0$, which implies $\lim_{N\to\infty} A_r f_r'[n_r(A, N)] = 0$ for all r = 1 . . R. Thus $\lim_{N\to\infty} n_r(A, N) = \infty$ for all r = 1 . . R. From part (c), H(A, N)/N is decreasing in N. Suppose that this ratio has a lower bound $\delta > 0$. This implies $\sum \{A_r f_r[n_r(A, N)] - \delta n_r(A, N)\} \geq 0$. However, we have $f_r(n_r)/n_r \to 0$ as $n_r \to \infty$ for r = 1 . . R. This is obvious if there is a finite upper bound on $f_r(n_r)$. If $f_r(n_r)$ is unbounded then using $f_r'(n_r) \to 0$ as $n_r \to \infty$ gives the same result. The facts that $\lim_{N\to\infty} n_r(A, N) = \infty$ and $f_r(n_r)/n_r \to 0$ as $n_r \to \infty$ for all r = 1 . . R together imply that there is some sufficiently large N such that $A_r f_r[n_r(A, N)] - \delta n_r(A, N) < 0$ for all r = 1 . . R. This contradicts the earlier inequality and gives the desired result $\lim_{N\to\infty} H(A, N)/N = 0$.

(e) Fix $A > 0$. By the envelope theorem H(A, N) is differentiable in N and $H_N(A, N)$ is the Lagrange multiplier for (2). Since $H(A, 0) = 0$, $\lim_{N\to 0} H(A, N)/N = \lim_{N\to 0} H_N(A, N)$. When $N > 0$, the first order conditions for (2) give $H_N(A, N) = \max \{A_r f_r'[n_r(A, N)]\}$. Because $n_r(A, N)$ is continuous in N with $n_r(A, 0) = 0$ for all r, we have $\lim_{N\to 0} H_N(A, N) = \max \{A_r f_r'(0)\}$.

Proof of Proposition 3.

Sufficiency. (K^S, N^S, n^S) is a VLRE if (i) $k_r^S = k_r^*$ for all r such that $n_r^S > 0$; (ii) $N^S = N[A(a, K^S)]$ is derived from (4); and (iii) $n^S = n[A(a, K^S), N^S]$ is derived from (2). The second and third conditions hold by construction. Condition (i) holds if $n_r^S = 0$ for all $r \notin S$. From the first order conditions for (2), this is true if $H_N(A^S, N^S) \geq A_r(k_r^{min})f_r'(0)$ for all $r \notin S$, which is true from (b) in Proposition 3. $N^S > 0$ holds if $A_r^S f_r'(0) > y^*$ for some $r \in \{1 .. R\}$, which is true from (a) in Proposition 3. This shows that (K^S, N^S, n^S) is a non-null VLRE of type S. Now suppose (K', N', n') is some other non-null VLRE of type S. This implies $H(A^S, N^S)/N^S = H(A', N')/N' = y^*$. By the construction of A^S, we have $A' \geq A^S$ where these vectors differ at most for $r \notin S$. By the definition of VLRE, $n_r' = 0$ for all $r \notin S$. Reducing the productivities of one or more resources that are not in use has no effect on $H(A, N)$ and therefore $H(A', N') = H(A^S, N')$. This implies $H(A', N')/N' = H(A^S, N')/N' = y^*$, which in turn gives $N' = N^S$. The uniqueness of the solution in (2) then gives $n' = n^S$.

Necessity. Suppose (K', N', n') is a non-null VLRE of type S but (a) in Proposition 3 does not hold. The definition of VLRE implies $n_r' = 0$ for all $r \notin S$. Setting $A' = A(a, K')$, we have $\max \{A_r'f_r'(0)\} = H_N(A', 0) > H(A', N')/N' > H_N(A', N') \geq A_r'f_r'(0)$ for all $r \notin S$. The equality follows from Proposition 2(e), the two strict inequalities follow from $N' > 0$ and Proposition 2(c), and the weak inequality follows from $n_r' = 0$ for $r \notin S$ and the first order conditions for (2). This series of results implies that $A_r'f_r'(0) = H_N(A', 0)$ for some $r \in S$ because otherwise there is a contradiction. But $H(A', N')/N' = y^*$ from (4) because (K', N', n') is a non-null VLRE. Moreover, $y^* \geq A_r'f_r'(0)$ because (a) in

Proposition 3 does not hold. Again this leads to a contradiction. Therefore if a non-null VLRE of type S exists, condition (a) in Proposition 3 must hold.

Now suppose (K', N', n') is a non-null VLRE of type S and (a) in Proposition 3 holds, but (b) in Proposition 3 does not. The definition of VLRE implies $n_r' = 0$ for all $r \notin S$. From the first order conditions for (2), this implies $H_N(A', N') \geq A_r(k_r')f_r'(0)$ for all $r \notin S$. Moreover, $A_r(k_r')f_r'(0) \geq A_r(k_r^{min})f_r'(0)$ for all $r \notin S$ by the definition of k_r^{min}, and $A_r(k_r^{min})f_r'(0) > H_N(A^S, N^S)$ holds for some $r \notin S$ because (b) in Proposition 3 is violated. This implies $H_N(A', N') > H_N(A^S, N^S)$. But this cannot be true because the only possible difference between A' and A^S is $A_r' > A_r^S$ for one or more $r \notin S$. Reducing productivity for a resource with $n_r(A', N') = 0$ has no effect on the optimal labor allocation or on the maximum value in (2). This implies $H(A', N')/N' = H(A^S, N')/N' = y^*$ and therefore $N' = N^S$. It follows that $H_N(A', N') = H_N(A^S, N^S)$. This contradiction shows that if a non-null VLRE of type S exists, condition (b) in Proposition 3 must hold.

Proof of Proposition 4.

(a) Consider any sample path $\{K^t, N^t\}$ for $t \geq 0$. There are finitely many possible repertoires, so at least one repertoire K' must be repeated infinitely many times. Because the sequence of productivity vectors $\{A^t\} = \{A(K^t)\}$ is non-decreasing by Proposition 1 and conservation of latent strings, it is impossible to return to an earlier repertoire after departing from it. Therefore only one repertoire can occur infinitely many times, and the occurrences of this K' must be consecutive. Let T be the first period in which K' occurs. Using $N^0 > 0$ and $\rho(y) > 0$ for $y > 0$, (3) implies $N^T > 0$. Due to $K^t = K'$ for all $t \geq T$, the condition MPA in section 4 gives $\{N^t\} \rightarrow N'$. The result $\{n^t\} \rightarrow n'$ follows from the continuity of solutions in (2).

(b) Suppose some terminal array (K', N', n') is not a VLRE. This implies that $n' = n[A(K'), N']$ has $n_r' > 0$ for some r with $k_r' \neq k_r^*$. Define M by $H_N(A', M) \equiv \max \{a_r g_r(k_r') f_r'(0)$ for r such that $k_r' \neq k_r^*\}$. Given the terminal productivities A', M is the largest population such that all improvable techniques are latent. A unique $M \in [0, \infty)$ exists because Proposition 2(e) gives $H_N(A', 0) = \max \{a_r g_r(k_r') f_r'(0)$ for $r = 1 \,..\, R\} \geq \max \{a_r g_r(k_r') f_r'(0)$ for r such that $k_r' \neq k_r^*\}$; Proposition 2(d) gives $H_N(A', \infty) = 0 < \min \{a_r g_r(k_r') f_r'(0)$ for r such that $k_r' \neq k_r^*\}$; and $H_N(A', N)$ is continuous and decreasing in N. Since (K', N', n') is not a VLRE, an improvable technique must be active in n' and therefore $M < N'$.

From (a), for each sample path having K' as the terminal repertoire there is some $T \geq 0$ such that $K^t = K'$ for all $t \geq T$. Consider any such sample path. There are two possibilities: (i) $N^T \in (0, M]$ or (ii) $N^T \in (M, \infty)$. In case (i), $K^t = K'$ for all $t \geq T$, $N^T \leq M < N'$, and MPA imply that after finitely many periods we must have $N^\tau \in (M, \infty)$. Setting $T = \tau$ if necessary, it therefore suffices to consider (ii). In this case, MPA ensures that if $N^T < N'$ then $N^t \geq N^T$ for all $t \geq T$, and if $N' \leq N^T$ then $N^t \geq N'$ for all $t \geq T$. Thus $M < \min \{N', N^T\} \leq N^t$ for all $t \geq T$.

The constancy of A' for $t \geq T$, the scale effect in Proposition 2(a), and the construction of M ensure that for some r with $k_r' \neq k_r^*$ there is a lower bound $\underline{n}_r$ such that $0 < \underline{n}_r \leq n_r^t$ for all $t \geq T$. Next, define $\rho^t \equiv N^{t+1}/N^t$ as in Proposition 1. MPA ensures that if $N^T \leq N'$ then $\rho^t \geq \underline{\rho} \equiv 1$ for all $t \geq T'$ and if $N^T > N'$ then $\rho^t \geq \underline{\rho} \equiv N'/N^T > 0$. Using the lower bounds $\underline{n}_r$ and $\underline{\rho}$ along with $k_r' \neq k_r^*$, Proposition 1 implies that for each $t \geq T$ the probability that k_r stays unchanged cannot exceed $\exp(-\lambda\underline{\rho}\underline{n}_r) < 1$. Over the unbounded interval $t \geq T$, the probability that k_r remains unchanged vanishes. Therefore $\Pr(K^t = K' \text{ for } t \geq T \mid K^T = K', N^T) = 0$.

Every sample path has a terminal repertoire K′ (which may or may not generate a VLRE) and the number of repertoires is finite, so we can partition the set of sample paths starting from (K^0, N^0) into finitely many sets indexed by K′. The probability of a particular terminal repertoire K′ is Prob(K′ is terminal) =

$$\sum_{T \in \{0, 1 \,..\}} \sum_{\underline{N} \in N(K',T)} \Pr(K^T = K', N^T = \underline{N} | K^0, N^0) \Pr(K^t = K' \text{ for } t \geq T | K^T = K', N^T = \underline{N})$$

where T is the first date on which K′ occurs and N(K′, T) is the set of population levels that are consistent with a first occurrence of K′ in period T. For any finite T there are finitely many possible mutation histories, and each of these histories determines a unique N^T, so N(K′, T) is a finite set. Moreover, $N^0 > 0$ implies $N^T > 0$ for any finite T from (3). We have shown that if the terminal array (K′, N′, n′) is <u>not</u> a VLRE then $\Pr(K^t = K' \text{ for } t \geq T | K^T = K', N^T = \underline{N}) = 0$ for all $\underline{N} > 0$ and all $T \geq 0$. Thus the probability that K′ is the terminal repertoire is zero if K′ does not generate a VLRE. Since there are finitely many terminal repertoires, the terminal array (K', N', n') is a VLRE with probability one.

(c) Suppose $0 < N^t < N[A(K^t)]$. From MPA, Proposition 1, the conservation of latent strings, and the fact that N(A) is non-decreasing we have $N^t < N^{t+1} < N[A(K^t)] \leq N[A(K^{t+1})]$. When $0 < N^0 < N[A(K^0)]$, we can repeat the argument to obtain $0 < N^t < N^{t+1}$ for all $t \geq 0$. When $0 < N^0 = N[A(K^0)]$, we have $N^t = N[A(K^0)]$ for $0 \leq t \leq T$ where T is the first period (if any) at which a mutation occurs for an active resource. This yields $N^0 = N^T < N[A(K^T)]$. For $t \geq T$, $\{N^t\}$ is increasing as before. When $N^0 > N[A(K^0)] \geq 0$, MPA ensures $N^0 > N^1 > N[A(K^0)]$. If there is a period $T \geq 1$ in which a mutation to an active resource yields $N^T \leq N[A(K^T)]$ then $\{N^t\}$ is non-decreasing for $t \geq T$ by the reasoning used above (and increasing if $N^T < N[A(K^T)]$ holds). Otherwise, we have $N^t > N[A(K^t)]$ for all $t \geq 0$. From MPA this implies $N^t > N^{t+1} > N[A(K^t)]$ for all $t \geq 0$ and $\{N^t\}$ is decreasing.

Proof of Proposition 5.

(a) Let the climate change permanently to a' at the start of period $t = 0$ before labor is allocated. The repertoire and population (K^0, N^0) at $t = 0$ are inherited from the previous VLRE. Due to the neutrality of the shock, the optimal labor allocation in (2) for $t = 0$ is unaffected (the solution $n(A, N)$ is homogeneous of degree zero in A). We use n^0 interchangeably for the labor allocation in the original VLRE and the allocation in period $t = 0$ after the climate change occurs. The productivity vector in period $t = 0$ is $\theta A^0 = \theta A(a^0, K^0) = A(\theta a^0, K^0)$ and the corresponding population in LRE is denoted by $N' = N(\theta A^0)$.

We will show that for any $T \geq 0$, $K^T = K^0$ and $N^T \in (N', N^0]$ implies $K^{T+1} = K^0$ and $N^{T+1} \in (N', N^0]$. This implies that the system converges to a VLRE with $K' = K^0$ and $N' = N(\theta A^0)$.

The labor allocation associated with (K^T, N^T) is $n^T = n(\theta A^0, N^T)$. Two necessary conditions for $k_r^{T+1} \neq k_r^T$ are (i) $k_r^T \neq k_r^*$ and (ii) $n_r^T > 0$. Any r satisfying (ii) has $n_r(\theta A^0, N^T) > 0$ and hence $n_r(\theta A^0, N^0) > 0$ from Proposition 2(a) and $N^0 \geq N^T$. But then $n_r(A^0, N^0) > 0$ by the homogeneity of $n(A, N)$ in A. This implies $k_r^0 = k_r^T = k_r^*$ because (K^0, N^0, n^0) is a VLRE for climate a^0. Thus (i) cannot hold. This shows that $k_r^{T+1} = k_r^T$ for all r and hence $K^{T+1} = K^0$. We also have $H(\theta A^0, N')/N' = y^* > H(\theta A^0, N^T)/N^T$ because $N^T > N'$. Therefore by MPA, $N^{T+1} \in (N', N^T) \subseteq (N', N^0]$. This establishes deterministic convergence to a unique VLRE (K', N', n') such that (i) $K' = K^0$ and (ii) $N' = N(\theta A^0) < N^0$.

Suppose $n_r^0 = n_r(A^0, N^0) = 0$. If $n_r' = n_r(\theta A^0, N') > 0$ then $n_r(A^0, N') > 0$ by homogeneity. Furthermore, $n_r(A^0, N^0) > 0$ from $N' < N^0$ and Proposition 2(a), which is false. Therefore $n_r' = 0$. This shows that the active resources in n′ are a subset of the active resources in n^0.

Starting from the (non-null) VLRE (K′, N′, n′) associated with climate a′, suppose in period t = 0 the climate returns permanently to $a^0 = a'/\theta$. We will show that for any $T \geq 0$, $K^T = K'$ and $N^T \in [N', N^0)$ implies $K^{T+1} = K'$ and $N^{T+1} \in [N', N^0)$. It follows that the system converges to (K^0, N^0).

Using $K^T = K' = K^0$, the labor allocation associated with (K^T, N^T) is $n^T = n(A^0, N^T)$. As before, two necessary conditions for $k_r^{T+1} \neq k_r^T$ are (i) $k_r^T \neq k_r^*$ and (ii) $n_r^T > 0$. Any r satisfying (ii) has $n_r(A^0, N^T) > 0$ and hence $n_r(A^0, N^0) > 0$ from $N^0 > N^T$ and Proposition 2(a). But every r with $n_r^0 = n_r(A^0, N^0) > 0$ has $k_r^0 = k_r^T = k_r^*$ because (K^0, N^0, n^0) is a VLRE for a^0 and $K^T = K' = K^0$. Thus (i) cannot hold. This shows that $k_r^{T+1} = k_r^T$ for all r and therefore $K^{T+1} = K'$. We also have $H(A^0, N^0)/N^0 = y^* < H(A^0, N^T)/N^T$ because (K^0, N^0, n^0) is a VLRE for a^0 and $0 < N^T < N^0$. Therefore by MPA, $N^{T+1} \in [N^T, N^0) \subseteq [N', N^0)$. This shows deterministic convergence to the original VLRE (K^0, N^0, n^0).

(b) Suppose the terminal array (K′, N′, n′) is a VLRE as in Proposition 4(b). From the fact that (K^0, N^0, n^0) is a VLRE for the climate a^0 and H(A, N) is linearly homogeneous in A, $y^* = H(A^0, N^0)/N^0 < H(\theta A^0, N^0)/N^0$. This shows that $N^0 < N(\theta A^0)$. Using Proposition 4(c), the latter inequality implies $\{N^t\}$ is increasing and therefore $N^0 \leq N^t < N'$ for all $t \geq 0$.

$\underline{\text{Necessity}}$. Suppose $n_r[\theta A^0, N(\theta A^0)] = 0$ for all r such that $k_r^0 \neq k_r^*$. We will show that $K^T = K^0$ and $N^T \in [N^0, N(\theta A^0))$ implies $K^{T+1} = K^0$ and $N^{T+1} \in [N^0, N(\theta A^0))$. Repeating the argument then yields $K' = K^0$ and $N' = N(\theta A^0)$. Two necessary conditions for $k_r^{T+1} \neq k_r^T$ are (i) $k_r^T \neq k_r^*$ and (ii) $n_r^T > 0$. From $N^T < N(\theta A^0)$ and Proposition 2(a), (ii) implies $n_r[\theta A^0, N(\theta A^0)] > 0$. Our initial supposition and $K^T = K^0$ imply that (i) is false. Thus $k_r^{T+1} = k_r^T$ for all r and $K^{T+1} = K^T$. We have $H[\theta A^0, N(\theta A^0)]/N(\theta A^0) = y^* < H(\theta A^0, N^T)/N^T$ from the definition of $N(\theta A^0)$ and the fact that $N^T < N(\theta A^0)$. Therefore by MPA, $N^{T+1} \in (N^T, N(\theta A^0)) \subseteq [N^0, N(\theta A^0))$ as claimed.

$\underline{\text{Sufficiency}}$. Suppose $n_r[\theta A^0, N(\theta A^0)] > 0$ for some r such that $k_r^0 \neq k_r^*$, but $K' = K^0$. The conditions for VLRE require $H(\theta A^0, N')/N' = y^*$ so $N' = N(\theta A^0)$. The conditions for VLRE also require $n_r' = n_r[\theta A^0, N(\theta A^0)] = 0$ for all r such that $k_r' \neq k_r^*$. According to our supposition this is false. Therefore $K' \neq K^0$.

We have already shown that if (*) does not hold then $N' = N(\theta A^0)$. Suppose that (*) does hold. We need to show that $N' > N(\theta A^0)$. Define $A' \equiv A(a', K')$. Because (K', N', n') is a VLRE for the climate a' we have $H(A', N')/N' = y^* = H[\theta A^0, N(\theta A^0)]/N(\theta A^0)$. We cannot have $N' < N(\theta A^0)$ since then $A' \geq \theta A^0$ gives $y^* = H[\theta A^0, N(\theta A^0)]/N(\theta A^0) < H(\theta A^0, N')/N' \leq H(A', N')/N' = y^*$, which is a contradiction. Therefore to show $N' > N(\theta A^0)$ it suffices to rule out $N' = N(\theta A^0)$. Assume $N' = N(\theta A^0)$ holds. It follows from $H(A', N')/N' = y^* = H[\theta A^0, N(\theta A^0)]/N(\theta A^0)$ that $H(A', N') = H[\theta A^0, N(\theta A^0)]$. Consider the case $n' \neq n[\theta A^0, N(\theta A^0)]$. From the uniqueness of solutions in (2) and the fact that both of the labor allocations involved are feasible, we have $H(A', N') = \sum \theta a_r^0 g_r(k_r')f_r(n_r') > \sum \theta a_r^0 g_r(k_r')f_r[n_r(\theta A^0, N')] \geq \sum \theta a_r^0 g_r(k_r^0)f_r[n_r(\theta A^0, N')] = H[\theta A^0, N(\theta A^0)]$. This contradicts $H(A', N') = H[\theta A^0, N(\theta A^0)]$. Next consider the case $N' = N(\theta A^0)$ and $n' = n[\theta A^0, N(\theta A^0)]$. Now $H(A', N') = H[\theta A^0, N(\theta A^0)]$ implies that $\sum \theta a_r^0 g_r(k_r')f_r(n_r') = \sum \theta a_r^0 g_r(k_r^0)f_r(n_r')$ or $\sum a_r^0 f_r(n_r')[g_r(k_r') - g_r(k_r^0)] = 0$ where $g_r(k_r') \geq g_r(k_r^0)$ for all r. Because (*) holds with $n' = n[\theta A^0, N(\theta A^0)]$, there exists some r for which $n_r' = n_r[\theta A^0, N(\theta A^0)] > 0$ and $k_r^0 \neq k_r^*$. Because (K', N', n') is a VLRE, $n_r' > 0$ implies $k_r' = k_r^*$. Hence there is at least one r with $f_r(n_r') > 0$ and $g_r(k_r') > g_r(k_r^0)$. This contradicts $\sum a_r^0 f_r(n_r')[g_r(k_r') - g_r(k_r^0)] = 0$. Therefore (*) implies $N' > N(\theta A^0)$.

Suppose a^0 is permanently restored and (*) does not hold. We want to show that starting from (K', N', n') the system converges to (K^0, N^0, n^0). We have already shown that if (*) does not hold then $K' = K^0$. The reversion to a^0 from θa^0 is a neutral negative shock. Proposition 5(a) shows that the system converges to a VLRE (K'', N'', n'') such that $K'' = K' = K^0$. The productivity vector for this VLRE is $A'' = A(a^0, K'') = A^0$ and the new VLRE must satisfy $H(A'', N'')/N'' = H(A^0, N'')/N'' = y^*$. This implies $N'' = N^0$. The uniqueness of the solution in (2) gives $n'' = n(A^0, N^0) = n^0$.

Suppose a^0 is permanently restored and (*) does hold. We want to show that starting from (K', N', n') the system converges to a VLRE (K'', N'', n'') with $K'' = K' \neq K^0$ and $N' > N'' \geq N^0$. We have already shown that (*) implies $K' \neq K^0$. Proposition 5(a) shows that the system converges to a VLRE with $K'' = K'$ and N'

$> N''$. Thus it suffices to show $N'' \geq N^0$, and to establish conditions under which this inequality is strict. First we show that $N'' \geq N^0$. Define $A'' = A(a^0, K'')$, where $A'' \geq A^0$ due to Proposition 1. If $N'' < N^0$ then $y^* = H(A'', N'')/N'' > H(A'', N^0)/N^0 \geq H(A^0, N^0)/N^0 = y^*$. This contradiction implies $N'' \geq N^0$.

Now suppose $N'' = N^0$ with $n'' \neq n^0$. Because n^0 is feasible in the labor allocation problem for (A'', N'') and the solutions in (2) are unique, $H(A'', N'') = \sum a_r^0 g_r(k_r'')f_r(n_r'') > \sum a_r^0 g_r(k_r'')f_r(n_r^0) \geq \sum a_r^0 g_r(k_r^0)f_r(n_r^0) = H(A^0, N^0)$. This gives $y^* = H(A'', N'')/N'' > H(A^0, N^0)/N^0 = y^*$. This contradiction shows that if $N'' = N^0$ then $n'' = n^0$. We thus have two possibilities: (i) $N'' = N^0$ and $n'' = n^0$ or (ii) $N'' > N^0$ and $n'' \neq n^0$. In either case, because (K^0, N^0, n^0) is a VLRE we have $k_r^0 = k_r^*$ for all r with $n_r^0 > 0$. Proposition 1 implies $k_r'' = k_r' = k_r^*$ for all r with $n_r^0 > 0$.

(i) If $H_N(A^0, N^0) \geq a_r^0 g_r(k_r'')f_r'(0)$ for all r such that $n_r^0 = 0$ then n^0 satisfies the first order conditions for problem (2) with parameters (A'', N^0) because $k_r'' = k_r^0$ for all r with $n_r^0 > 0$. The first order conditions for (2) are sufficient for a solution so $H(A'', N^0)/N^0 = H(A^0, N^0)/N^0 = y^*$. This shows that (K'', N^0, n^0) is a VLRE for the climate a^0. But from Proposition 5(a), the VLRE (K'', N'', n'') is unique. Therefore $N'' = N^0$ and $n'' = n[A(a^0, K''), N^0] = n^0$.

(ii) If $H_N(A^0, N^0) < a_r^0 g_r(k_r'')f_r'(0)$ for some r such that $n_r^0 = 0$ then n^0 does not satisfy the first order conditions for problem (2) with parameters (A'', N^0). Thus $n[A(a^0, K''), N^0] \neq n^0$. Using $N'' \geq N^0$ and the uniqueness of solutions in (2), this gives $H(A'', N'') \geq H(A'', N^0) > \sum a_r^0 g_r(k_r'')f_r(n_r^0) \geq \sum a_r^0 g_r(k_r^0)f_r(n_r^0) = H(A^0, N^0)$. Now suppose $N'' = N^0$. This implies $y^* = H(A'', N'')/N'' = H(A'', N^0)/N^0 > H(A^0,$

$N^0)/N^0 = y^*$, which is a contradiction. Hence $N'' > N^0$. If $n_r'' = 0$ for all r such that $n_r^0 = 0$, there must be at least one s with $0 < n_s^0 < n_s''$. Using $k_r'' = k_r^*$ for all r with $n_r^0 > 0$ and the envelope theorem, this gives $H_N(A'', N'') = a_s^0 g_s(k_s'')f_s'(n_s'') < a_s^0 g_s(k_s^*)f_s'(n_s^0) = H_N(A^0, N^0)$. But then $H_N(A'', N'') < H_N(A^0, N^0) < a_r^0 g_r(k_r'')f_r'(0)$ for some r such that $n_r^0 = 0$, which contradicts the optimality of $n_r'' = 0$ for all r such that $n_r^0 = 0$. Therefore $n_r'' > 0$ for at least one r such that $n_r^0 = 0$.

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