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Wavelets-based method for variation analysis of non-rigid assemblies

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Abstract

This paper proposes a new method based on wavelets analysis and finite element method (FEM) for the variation analysis of non-rigid assemblies. It is well known that the part fabrication variation, coupled with the part's deformation during the assembly process, is one of the main factors affecting the assembly quality. But little investigation has been done on how component variations with different scales contribute to the final dimensional variation of non-rigid assemblies. The proposed approach takes the part variation as a signal and applies wavelets transform to decompose it into different scale components. The deformation of non-rigid assemblies that corresponds to these different scale components is calculated by using FEM. Since the part variation is resulted from manufacturing, manufacturing engineers can apply this method to get valuable information to avoid major variation causes in manufacturing process and make a better process plan. The proposed method is illustrated through a case study on an assembly of two flat sheet metal parts.

Keywords: Variation analysis; Non-rigid assembly; Finite element method; Wavelet transform

1. Introduction

Product quality is one of the most concerned issues in product design and manufacturing. Reducing and controlling the assembly dimensional variation plays an important role in product quality improvement in today's competitive market [1,2]. According to the material flexibility/stiffness, mechanical components can be roughly classified into two categories: rigid parts and no-rigid parts. Non-rigid parts, like sheet metal parts, are widely applied in many industries such as aerospace, automobile and electronic industries. Since sheet metal parts tend to deform during the assembly process, the manufacturing variation of parts will be coupled with other factors, such as the tool variation, fixture layout, and assembly sequence, to impact the assembly dimension variation [3,4]. Methods for rigid assemblies are not directly applicable to non-rigid assemblies [5]. The dimensional analysis and control for non-rigid assemblies are apparently more difficult than that of rigid assemblies [2].

For variation analysis of non-rigid assemblies, the component variation is generally recognized as a major problem in elastic assembly processes [5,6]. The uncertainties in a manufacturing environment would result in different scale structures of the part variation. Even if two parts reach the same dimensional tolerance, they may have quite different scale components in the tolerance zone [7]. In order to reduce and control the final assembly variation, it is very important to analyze the variation structure of the parts and its influence on the final assembly quality.

There are in the literature a number of modeling and analysis approaches for non-rigid assemblies to simulate the assembly processes and to analyze the assembly variation in the past a few years [5,6,8,9]. However, no method addresses components with different scales in the part manufacturing variation, and no one can be used to analyze the contribution of component variations with different scales in the tolerance zone to the final dimensional variation of non-rigid assemblies. In this paper, a new method based on wavelet analysis and finite element method (FEM) to analyze the non-rigid assembly variation is developed and investigated. The part variation will be considered as a signal and decomposed into different scale components by wavelet transform. The deformation of non-rigid assemblies with respect to these different scale components is calculated by using FEM. Manufacturing

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engineers can apply this method to get valuable information on major variation causes in the manufacturing process. A case study on an assembly of two flat sheet metal parts is presented to illustrate the proposed method.

2. Literature review

The variation analysis for non-rigid assemblies is an emerging research area [3–6,8,9]. Liu and Hu [5] considered the compliant nature of sheet metal parts and proposed an influence coefficients method to analyze the effect of component variation and assembly spring-back on the assembly variation by applying linear mechanics and statistics. The influence coefficients method was a key technique to get the component stiffness matrix. Camelio et al. [10] successfully extended this approach to model the product variation in multi-station assembly systems. Hu [3] set up the 'stream of variation' theory for the automotive body assembly variation analysis. Ceglarek et al. [4] made a detailed review on the stream of variation theory in terms of the state space model characterizing variation propagation in the multistage assembly. Ceglarek and Shi [8] proposed a new variation analysis methodology for the sheet metal assembly based on physical/functional modeling of the fabricated error using a beam-based model. Hu et al. [9] developed a numerical simulation method for the assembly process incorporating compliant non-ideal components. The effects of various variation sources were analyzed. In addition, Heieh and Oh [6] represented a procedure for simulating the combined effects of deformation and dimensional variation in the elastic assembly. Cai et al. [11] discussed the fixture schemes and demonstrated that the N-2-1 fixture scheme was better than the 3-2-1 scheme for non-rigid assemblies. Recently, Liao and Wang [7] applied the fractal geometry to include variations of surface micro-geometry of components into the modeling of variation analysis of non-rigid assemblies. A fractal function, named Weierstrass–Mandelbrot (W–M) function, is used to extract and represent the characteristics of the component variation microstructure. The reconstructed variation profile by the W–M function is taken as an input of the finite element analysis to calculate the deformation of the final assembly. This method is efficient for variation analysis of the non-rigid assembly, when the part variation is fractal.

In this paper, a general method based on the wavelet analysis is proposed to predict the contribution of different scale components in the part tolerance zone to the final assembly variation. In the next section, the non-rigid assembly process modeling is introduced. The wavelet transform and its application to the decomposition of part variation will be presented in Section 4. The flowchart of variation analysis for non-rigid assemblies based on wavelet transform and finite element method (FEM) is discussed in Section 5, and a case study in Section 6 is provided to illustrate the proposed method.

3. Non-rigid assembly process modeling

The 'real' complex non-rigid assembly process in a typical assembly station can be modeled as a four- step procedure (see Fig. 1) through the mechanistic simulation methodology developed by Liu and Hu [5]. This methodology assumes: all process operations occur simultaneously; the component deformation is linear and elastic; material is isotropic; fixtures and tools are rigid; no thermal deformation occurs during the assembly process; and the stiffness matrix remains constant for deformed component shapes.

(i) Placing components (Fig. 1(a)). Components are loaded and placed on work-holding fixtures using a locating scheme (Fig. 1(a)). Since the fabrication error of components is a natural phenomenon in component manufacturing, the component variation $\{\delta_u\}$ offset from the design nominal will inevitably cause the initial matching gap. Here, the index u refers to un-joined

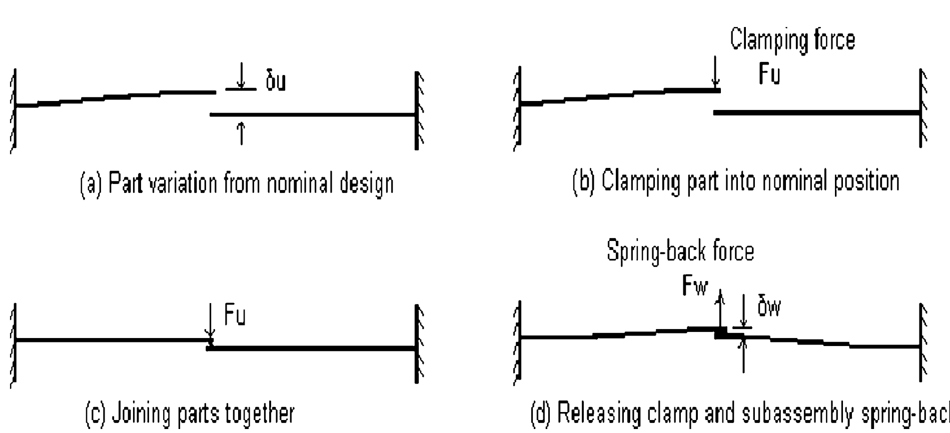

Fig. 1. The non-rigid assembly process [5].

components. Cai et al. [11] suggest that it is better to use the N-2-1 ($N>3$) fixture scheme than the 3-2-1 scheme for non-rigid assemblies to assure the assembly quality because of the assembly deformation. That means, constraining $N(>3)$ DOF (degree of freedom) in the first plane, 2 DOF in the second plane, and 1 DOF in the third plane.

(ii) Clamping components (Fig. 1(b)). The initial matching gap between components and subassemblies is forced to close by deforming components to the nominal position. Considering the component stiffness matrix $[K_u]$ that could be built through the finite element method, the relationship of the required clamping forces $\{F_u\}$ to the closed gap $\{\delta_u\}$ can be described by Eq. (1)

$$\{F_u\} = [K_u]\{\delta_u\} \tag{1}$$

(iii) Joining components (Fig. 1(c)). When using a joining method, such as welding, riveting, or gluing, to join two components, deformation occurs at each joint point as the gap between components is closed. The assembly force $\{F_u\}$ is still being applied.

(iv) Releasing clamps/fixtures and subassembly spring-back (Fig.1(d)). After assembling the two components, the clamps/fixtures are removed. The joined components will spring back to release the stored strain energy during the assembly operation. It is reasonable to assume that the spring-back force $\{F_w\}$ is equal to the clamping force $\{F_u\}$. Therefore, by applying FEM to get the component and assembly stiffness matrix, the value of spring-back variation $\{\delta_w\}$ can be calculated by removing displacement boundaries both at the clamping points and releasing fixture locations to simulate the clamps/fixtures release.

For a given specific assembly process and station, getting the stiffness matrix $[K_u]$ and $[K_w]$ by using commercial FEM software is the key issue to the assembly variation analysis procedure, because most software provides no direct means for users to access and operate the FEM stiffness matrix. The influence coefficients method, which is developed by Liu and Hu [5], could be used to indirectly construct the stiffness matrix $[K_u]$ and $[K_w]$ if the commercial FEM software embeds an application-oriented development language. The procedure to achieve the stiffness matrix of assembly and/or component can be described as follows: a unit force is applied at each source of variation with the same direction of the deviation; FEM is then used to calculate the response at some specific points; after the response computation for all sources of variations, a response matrix can be constructed; the stiffness matrix can be obtained by inverting the response matrix since it is symmetric. Details about the influence coefficients method are in the Ref. [5].

4. Wavelets-based analysis for component variation

It is well known that Fourier transform is a popular method for signal processing [12]. A signal can be represented as the sum of a series of sines and cosines by using Fourier transform. Since the sines and cosines that comprise the base of Fourier analysis are non-local functions that have only frequency resolution and no time resolution, the suitable signals for Fourier analysis should be stationary and their statistics do not change with time. If we calculate the frequency composition of non-stationary signal by Fourier theory, the result is the frequency composition averaged over the duration of the signal, which cannot adequately describe the characteristics of the signals in lower frequencies. Although we can use the short-time Fourier transform (STFT) method to deal with non-stationary signals, a high resolution in both time and frequency domain is hardly reached.

Wavelet transform is a fundamentally different approach from Fourier theory [13,14]. In this novel transformation, a signal is not decomposed into its harmonics, which are global functions that have support on $[-\infty, +\infty]$, but into a series of local basis functions called wavelets, which are of a waveform of effectively limited duration and having an average value of zero. At the finest scale, the wavelets may be very long. By wavelet transform, any particular local features of signals can be detected and identified from the scale and the position of the wavelets. The structure of non-stationary signals can be analyzed with local features represented by a close-packet wavelet of short length.

Given a time varying signal $f(t)$, wavelet transform (WT) consists of computing a coefficient that is the inner product of the signal and a family of wavelets. In the continuous wavelet transform (CWT), the wavelet base is constructed by dilating and translating a single function $\psi(t)\in L^2(R)$, which is named the mother function and has zero average

$$\int \psi(t)\,dt = 0 \tag{2}$$

$$\psi_{a,b} = \frac{1}{\sqrt{|a|}}\psi\left(\frac{t-b}{a}\right) \quad a,b\in R, a\neq 0 \tag{3}$$

where a and b are the dilation and translation parameters, respectively.

The continuous wavelet transform of $f(t)$ at scale a and position b is defined as follows:

$$w_f(a,b) = \int f(t)\psi^*_{a,b}(t)\,dt \tag{4}$$

where '*' denotes the complex conjugation.

With respect to $w_f(a,b)$, the signal $f(t)$ can be reconstructed by

$$f(t) = \frac{1}{c_\psi}\int_{-\infty}^{+\infty}\int_0^{+\infty} w_f(a,b)\frac{1}{\sqrt{|a|}}\psi\left(\frac{t-b}{a}\right)\,da\,db \tag{5}$$

where c_ψ is a constant depending on the base function.

Similar to the Fourier transform, $w_f(a,b)$ and $f(t)$ constitute a pair of wavelet transforms.

When $a=2^j$, $b=k2^j$, $j, k\in$ Z, the wavelet is

$$\psi_{j,k} = 2^{-j/2}\psi(2^{-j}t - k) \tag{6}$$

The discrete wavelet transform (DWT) is defined as follows:

$$c_{j,k} = \int f(t)\psi_{j,k}(t)\,\mathrm{d}t \tag{7}$$

Where $c_{j,k}$ is defined as the wavelet coefficient, it may be considered as a time-frequency map of the original signal $f(t)$.

Multi-resolution analysis is used in discrete scaling function:

$$\phi_{j,k} = 2^{-j/2}\phi(2^{-j}t - k) \tag{8}$$

Set

$$d_{j,k} = \int f(t)\phi^*_{j,k}(t)\,\mathrm{d}t \tag{9}$$

Where $d_{j,k}$ is called the scaling coefficient.

Wavelet coefficients $c_{j,k}$ ($j=1, 2, \ldots, J$) and the scaling coefficient $d_{j,k}$ can be represented as the follows

$$c_{j,k} = \sum_n f[n]h_j[n - 2^jk] \tag{10}$$

and

$$d_{j,k} = \sum_n f[n]g_j[n - 2^jk] \tag{11}$$

Where $f[n]$ is discrete-time signals; $h_j[n-2^jk]$ is the analysis discrete wavelets, and the discrete equivalents to $2^{-j/2}\psi(2^{-j}t-k)$; $g_j[n-2^jk]$ are called the scaling sequence [13].

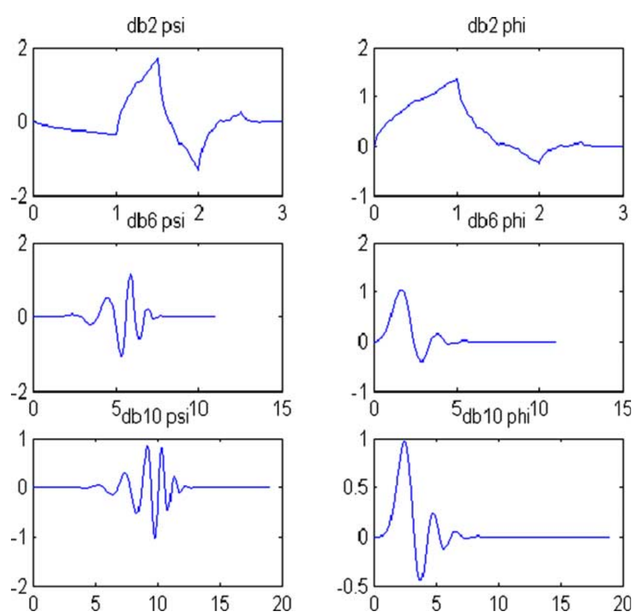

Fig. 2. The scaling function (left) and wavelet function (right) of db2, db6, and db10.

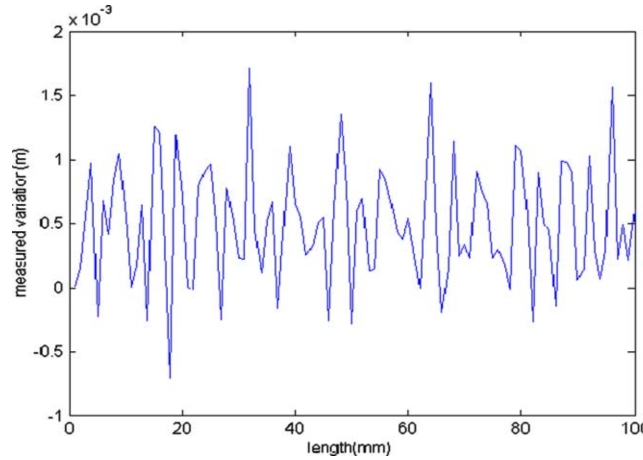

Fig. 3. The sampled part variation.

At each resolution $j>0$, the scaling coefficients and the wavelet coefficients can be written as follows:

$$c_{j+1,k} = \sum_n g[n - 2k]d_{j,n} \tag{12}$$

$$d_{j+1,k} = \sum_n h[n - 2k]d_{j,n} \tag{13}$$

The above two equations state that the scaling coefficients on the scale j can be decomposed into the wavelet coefficients and the scaling coefficients on the next higher scale $j+1$. On the other hand, the scaling coefficients on the scale j can be

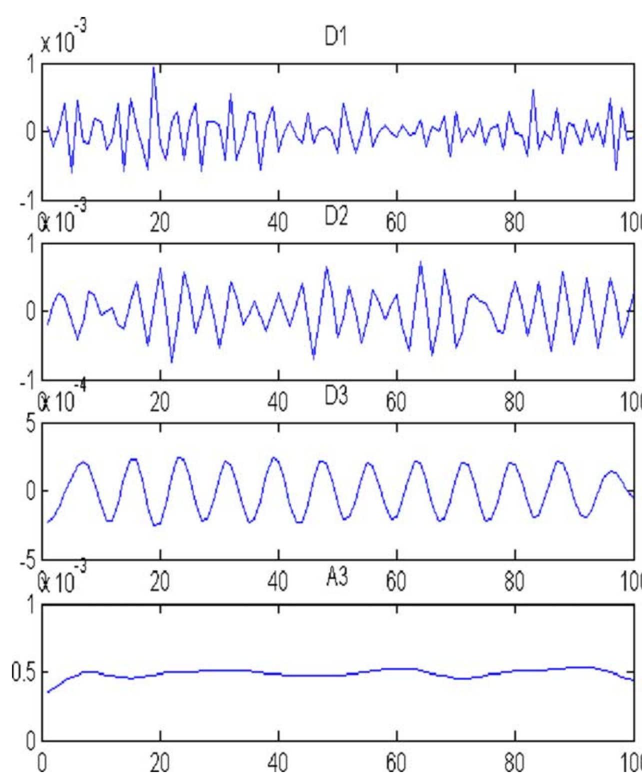

Fig. 4. The wavelet analysis of the part variation of different components by db10.

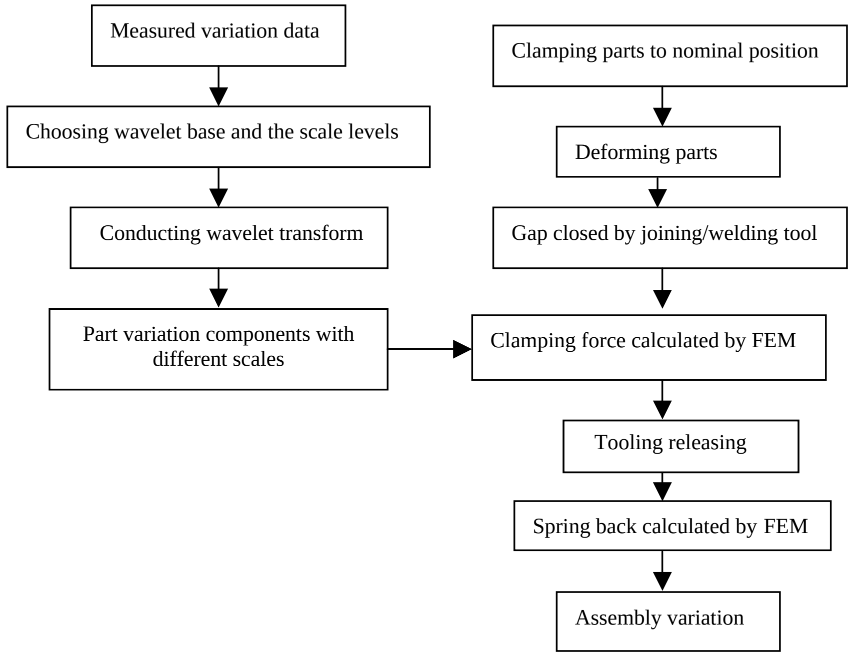

Fig. 5. Flowchart of the assembly variation simulation procedure.

also reconstructed by the wavelet coefficients and the scaling coefficients on the scale $j+1$.

$$d_{j,k} = \sum_n h[k-2n]d_{j+1,n} + \sum_n g[k-2n]c_{j+1,n} \tag{14}$$

The term g and h in Eqs. (12–14) can be considered as high-pass and low-pass filters derived from the analysis wavelets and the scaling function, respectively.

From Eqs. (12–14), it is found that the wavelet decomposition and reconstruction is calculated with discrete convolutions [13].

Based on the theory of wavelet transform mentioned above, a given signal can be analyzed by choosing a suitable wavelet base and the desired decomposition level. In the past decades, many wavelet bases with different characteristics have been developed. The Daubechies wavelet base is the most common orthogonal wavelet base and is widely applied in signal processing [14]. Fig. 2 shows the scaling function and wavelet function of Daubechies of order 2, 6 and 10. The higher order of Daubechies will result in better amplitude transmission.

The profile variation of manufactured parts can be considered as a signal [7]. This variation signal may be non-stationary due to the uncertainties in manufacturing system. Fig. 3 shows a measured sample profile variation of a flat sheet metal part. By applying wavelet function of Daubechies of order 10 (db10), the part variation showed in Fig. 3 can be decomposed into different components. Fig. 4 shows its decomposition up to level 3. It represents that 4 different components in part variation are identified, including level 3 approximation (A3), level 3 detail (D3), level 2 detail (D2), and level 1 detail (D1). Choosing the decomposition level depends on the specific problem at hand and the goal you want to reach. Once the different scale components in part variation are available, their contribution to the final non-rigid assembly variation can be investigated.

5. Assembly variation simulation procedure

Based on the four steps of the assembly process of components and subassemblies in a typical assembly station (shown in Fig. 1) and the method on the component variation analysis by using the wavelet transform, the non-rigid assembly variation simulation flowchart is summarized in Fig. 5. The entire analysis procedure shown in Fig. 5 consists mainly of two portions. One is the component variation analysis by using the wavelet transform; another is

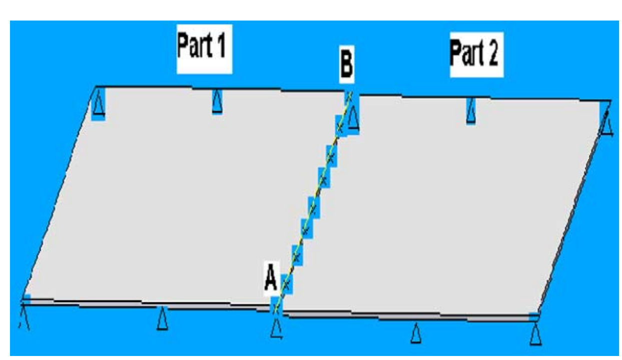

Fig. 6. Assembly of two flat sheet metal components.

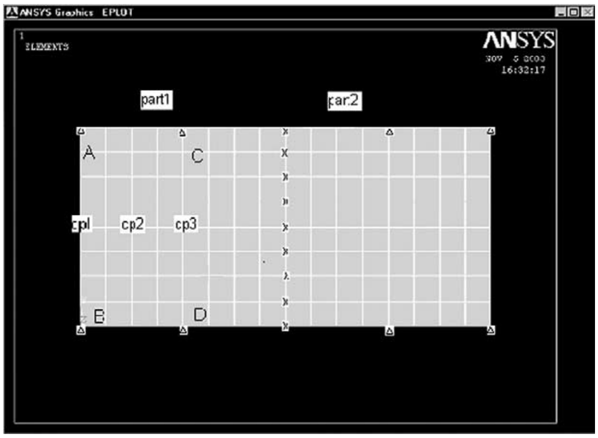

Fig. 7. The FEM model for analyzing the assembly of two flat sheet metal components.

the four-step assembly process simulation based on the finite element analysis method.

Generally, the FEM model can be created by 'map-mesh' with structural elements so that the jointed spots are definitely together. The component joining process is simulated through coupled nodes in the FEM model, while the tool releasing process is simulated by removing the displacement boundaries at the released clamp/fixture points. The whole assembly process is assumed to be non-frictional and linear.

In the variation analysis procedure of non-rigid assembly, some points on components are chosen to be critical points (CPs), which are used to measure if the assembly dimensional quality meets the design requirements [5–9]. However, it is not easy to decide on the locations of CP's. The determination of CP's is based on many factors, for

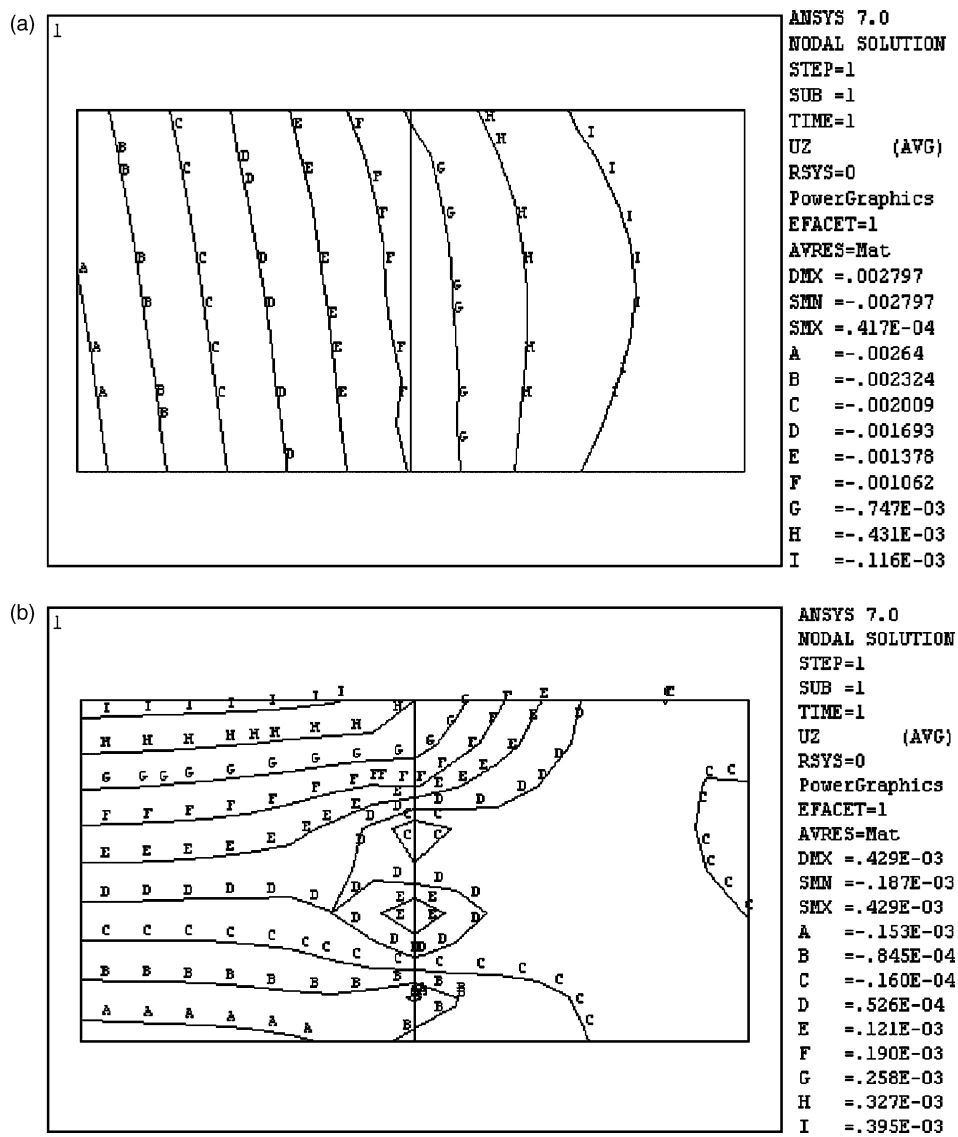

Fig. 8. Assembly variation distribution (unit is metre). (a) assembly variation corresponding to the level 3 approximation (A3) of part variation. (b) assembly variation corresponding to the detailed level 3 component (D3) of part variation. (c) assembly variation corresponding to the detailed level 2 component (D2) of part variation. (d) assembly variation corresponding to the detailed level 1 component (D1) of part variation.

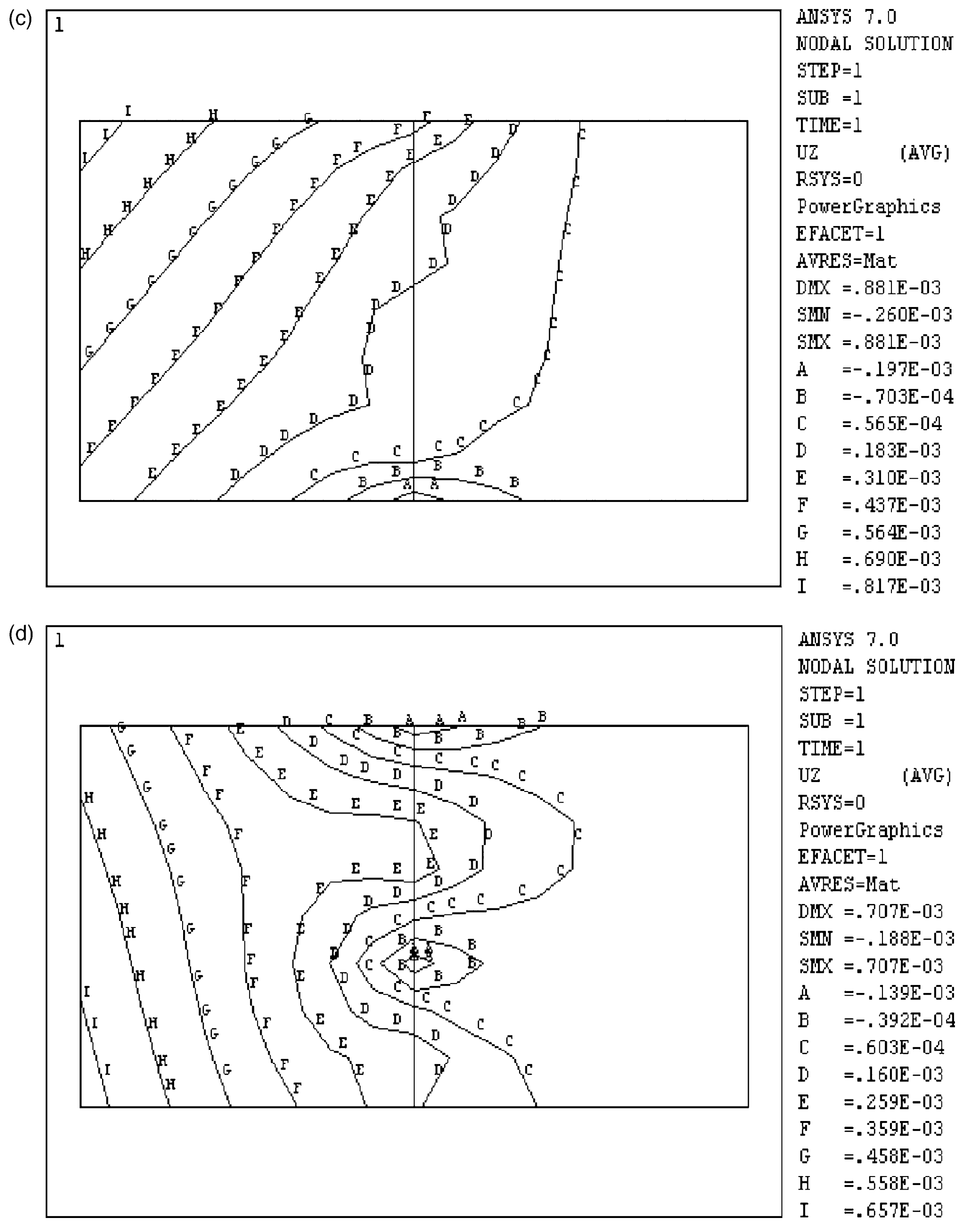

Fig. 8 (*continued*)

example, the component shape, the assembly process, the component or subassembly performance, and/or the assembly variation requirements [9].

The proposed assembly variation simulation procedure shown in Fig. 5 provides a method to analyze the different scale components of part variation in the tolerance zone and their contributions to the final assembly variation. It is implemented by using the software ANSYS [15,16] and Matlab [17,18]. ANSYS is used to generate the FEM model, compute component deformation and the clamping forces, simulate the joining and releasing process, and to calculate the spring back and the assembly variation. Matlab is applied to develop the program for the component variation analysis.

6. Case study: assembly of two flat sheet metal components

An assembly of two identical flat sheet metal components by lap joints is selected as an example to verify the proposed approach. Assuming that these two components are manufactured under the same conditions, their fabrication variations are expected to be the same. The size of the flat sheet metal parts is 100 mm $\times$ 100 mm $\times$ 1 mm, Young's modulus $E = 2.62e+9$ N/mm^2, and Poison's ratio $\nu = 0.3$.

The fixture scheme N-2-1 ($N > 3$) [11] is applied, shown in Fig. 6. The positions of symbol 'Δ' indicate the fixture locations. All pair joint spots (indicated by symbol 'x') are simultaneously assembled together.

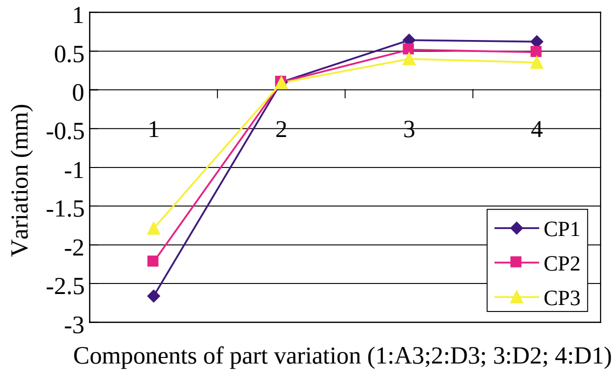

Fig. 9. Assembly variation of 3CPs.

The finite element computation model of the assembly of two flat sheets, shown in Fig. 7, is created in ANSYS by assuming that the small elastic deformation does not significantly change the component geometry size. The element type is SHELL63. The number of elements and the number of nodes are 128 and 162, respectively. There are 9 pairs of nodes to be connected together in this model, corresponding to the x symbols in Fig. 7.

Suppose we have the measured variation signal from the component profile and its decomposition by wavelet functions (see Figs. 3 and 4). The assembly variation that results from different scale components of part variation can be computed by the procedure shown in Fig. 5. In this example, corresponding to the 4 different components of part variation, the assembly variation distribution (Fig. 8) is obtained, respectively, when releasing all fixtures in Part 1. The assembly variations of 3 CPs (shown in Fig. 7) are also extracted from computation results, and are shown in Fig. 9. The computational procedure is coded by APDL (ANSYS Parametric Design Language) in ANSYS.

From Figs. 8 and 9 we can see that the different scale components of part variation have a different impact on the final assembly variation. The main component, i.e. the level 3 approximation (A3) of part variation in this case study, contributes much more than other detailed components (D3, D2, D1). However, the influence of detailed components to the final assembly variation cannot be ignored. For example, at CP3, the contribution of the components D1 is about 1/5 of that of A3 (see Fig. 9). Furthermore, it is also revealed in this case study that the detailed component D2 has a higher contribution than D3 and D1. Since the different components of part variation are resulted from a kind of uncertainty in manufacturing system, the uncertainty that corresponds to D2 can be identified by detecting the signal that has the same frequency characteristics as D2. Therefore, we are able to find the uncertainty cause and take action to control it.

In addition, the assembly variations of 3 CPs (shown in Fig. 7) with different scales are listed in Table 1 together with the fractal based analysis results from Ref. [7] and the results by directly using the measured data shown in Fig. 3, where assemblies are released of all fixtures in Part 1.

From Table 1, it is apparent that the assembly variations of the critical points by wavelets method studied in this paper are equal to the results by directly using the measured data, while the results by fractal method in Ref. [7] are not. It is because that the wavelets method can analyze all scales detailed part variation, while the fractal method only extracts the fractal component of part variation and eliminates the rest. Therefore, the proposed assembly variation analysis method based on wavelets transform is in general more accurate than the fractal method studied in Ref. [7] and is applicable to both non-fractal and fractal variations.

7. Summary and conclusion

This paper presents a methodology to analyze the contribution of variation components with different scales to the final dimensional variation for non-rigid assemblies. In this approach, the wavelet transform is used to identify the different scale components of part variation in the tolerance zone, while the finite element method (FEM) is utilized to calculate the deformation of non-rigid assemblies that corresponds to these different scale components. The integrated procedure of wavelet transform and FEM for non-rigid assembly variation analysis is set up and implemented by using ANSYS and Matlab. A case study on an assembly of two flat sheet metal parts is presented to illustrate the proposed approach. Basically, this methodology is more advantageous and more accurate than the approach based on the fractal geometry that was previously studied by Liao and Wang [7] in that the wavelets method can analyze all scales detailed part variation, while the fractal method only extracts the detailed part variation with fractal property. Furthermore, the wavelet-based method can deal with all kinds of non-stationary part variations, not only limited to the fractal variation.

Table 1
Comparison of assembly variation analysis results

Critical points	Assembly variation analysis results by fractal method in [7], (mm)	Assembly variation analysis results by wavelet method in this study (mm)	Assembly variation analysis results by inputting original measurement data (shown in Fig. 3), (mm)
CP1	−2.0	−1.3	−1.3
CP2	−1.75	−1.1	−1.1
CP3	−1.47	−0.95	−0.95

It is inevitable that the uncertainties in manufacturing environment would result in different scale structures of the part variation. The methodology set up in this paper provides an interesting opportunity to identify the components of large variation, to avoid such variation causes in the manufacturing process, and to design a better assembly process plan.

Acknowledgements

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