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ORIGINAL ARTICLE

# Simultaneous optimization of fixture and joint positions for non-rigid sheet metal assembly

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**Abstract** This paper presents an optimization methodology for non-rigid sheet metal assembly variation by considering part variation, fixture variation, fixture layout, and joint positions, as well as the assembly spring back. The proposed algorithm integrates the finite element analysis (FEA) with a powerful global optimization algorithm, called the mode-pursuing sampling (MPS) method to simultaneously search for the optimal fixture and joint positions in order to minimize the assembly variation. An example application study is presented to demonstrate the optimization procedure and its effectiveness.

**Keywords** Sheet metal assembly · Fixture layout · Joint position · Finite element analysis · Mode-pursuing sampling method (MPS)

## 1 Introduction

Assuring assembly dimensional quality is one of the vital requirements in non-rigid sheet metal assemblies, which are widely applied in many industries such as aircraft, automobile, shipbuilding, and so on [1–4]. The fixture layout is recognized as one of the primary factors that influence the final assembly variation [5–7]. Many researchers have addressed this interesting issue. Rearick et al. [8] proposed an optimization algorithm to obtain the optimal number and location of clamps that minimize the deformation of compliant parts. Cai et al. [9] proposed the N-2-1 fixture principle for the compliant sheet metal assembly. They also presented an optimization algorithm to find the optimal location for $N$ fixtures that minimize the work-piece deflection under a given force. The work-piece deformation was calculated by using FEA. Dahlstrom and Camelio [10] proposed a general method to predict the effect of fixture design in compliant assembly. It focused on the impact of fixture layout, as well as locators and clamp positions on the dimensional quality of sheet metal assemblies. FEA and design of computer experiments were used to derive the response models. The response models were used to analyze the final assembly sensitivity to fixture, part and tooling variation for different assembly configurations. Camelio et al. [11] presented a new fixture design method for sheet metal assembly processes. Liao [12] proposed a genetic algorithm (GA)-based optimization method to automatically select the optimal number of locators and clamps as well as their positions for sheet metal assemblies. Lai et al. [13] presented a method that directly minimizes work-piece location errors due to its fixture elastic deformation. They developed a variation of genetic algorithm to solve the fixture layout problem.

It is also known that the joint technology is another challenging task for non-rigid sheet metal assembly. Liu and Hu [14] performed a study of joint performance in the sheet metal assembly. Since different joint configurations have different performance characteristics, they considered the level of dimensional variation in the assembly as one of the performance criteria. Three kinds of the most commonly used joints in sheet metal assembly—lap joints, butt joints and butt-lap joints—were parametrically modeled and evaluated based on the assembly variation levels. Lee

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and Hahn [15] conducted a comparison study on the existing joint technology, such as discrete fasteners, metal welds and adhesive bonds that are commonly used in the design and assembly of transportation systems. Bhalerao et al. [16] also discussed the FEM method and techniques for analyzing the functionality of a variety of joints. Zhang and Taylor [17] presented the optimization problem of a spot-welded structure that had the maximum stiffness or fatigue life by optimizing the spot weld positions. They did not, however, address the influence of joint positions on the assembly deformation.

From the above review, it is seen that all have only or separately studied the fixture layout or joint positions optimization. The presented work in this paper considers the fixture layout and the joint positions, and their influence on the assembly spring back, where both part variation and fixture variation are present. The fixture and joint positions are considered as the sources of assembly variation and treated as the design variables in this study since they simultaneously affect the final assembly dimensional quality.

In the current research, the conventional gradient-based programming, the genetic algorithms (GA), and the response surface method are used in the fixture layout design. In this paper, an alternative global optimization method, the mode-pursuing sampling (MPS) method developed by Wang et al. [18], is identified and employed to solve the simultaneous optimization problem of the fixture and joint positions. In fact, MPS is a sampling-based optimization algorithm, which uses Fu and Wang's sample method [19] to search the global optimal solution. In Fu and Wang's method [19], the sample points are generated to conform to a given probability density function (PDF), i.e., more points in the area has a high probability and fewer points in the area has a low probability, as defined by the PDF. MPS applies such a sampling process and creates a sampling guidance function, functioning similarly to the PDF, so that there are more points generated in areas having lower objective function values and fewer in other areas. At first, MPS constructs an approximation model from a few sample points; then, based on the approximation model, MPS generates a large number of points, sorts the points, and constructs a guidance function in the sorted point set. A sample is then drawn from the sorted point set according to this guidance function. As a result, more new sample points are generated around the current minimum and less in other regions in the design space. Such a procedure is iterated until the optimal criteria are met.

The remainder of this paper is organized as follows. First, mathematical formation of the fixture and joint position optimization problem is discussed in detail, based on the non-rigid sheet metal assembly process modeling. Following that, the characteristics of the MPS method and its optimization procedure will incorporate the simultaneous optimization issue of the fixture and joint positions. The implementation of this approach by integrating ANSYS [20, 21] and Matlab [22, 23] will be discussed and demonstrated through an application example.

## 2 Non-rigid sheet metal assembly process

Following the mechanistic simulation methodology developed by Liu and Hu [24], it is also assumed in this work that all process operations occur simultaneously; the component deformation is linear and elastic; material is isotropic; fixtures and tools are rigid; no thermal deformation occurs during the assembly process; and the stiffness matrix remains constant for deformed component shapes. The complex non-rigid assembly process in a typical assembly station can be modeled as a four-step procedure (see Fig. 1), i.e., placing components, clamping components, joining components, and releasing clamps/fixtures.

It is inevitable that the fabrication error $\{\delta_u\}$ of components will cause the matching gap (Fig. 1a) between components and subassemblies after the components are loaded and placed on work-holding fixtures using a locating scheme. This gap shall be forced to close by deforming components to the nominal position (Fig. 1b).

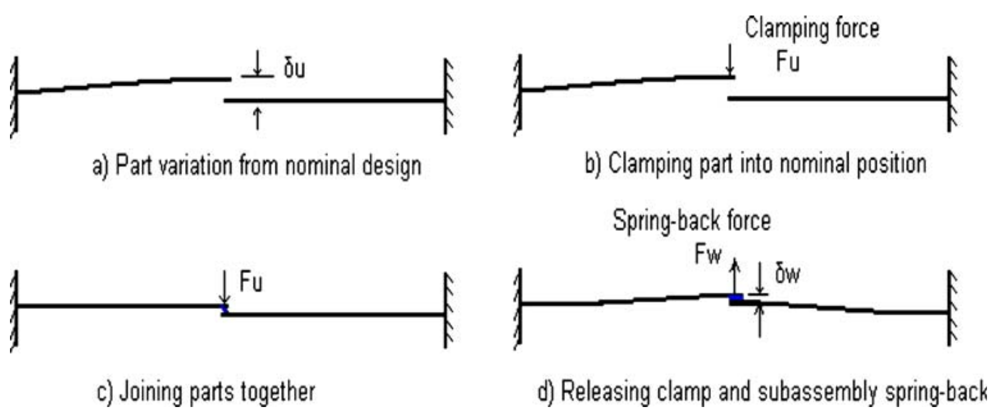


Fig. 1 The non-rigid assembly process

The required clamping forces $\{F_u\}$ can be obtained by Eq. 1

$$\{F_u\} = [K_u]\{\delta_u\} \quad (1)$$

where, $[K_u]$ is a stiffness matrix related to closing the component matching gap.

Meanwhile, the errors of fixtures $\{\delta_t\}$ that are in the direction of flexible deformation, i.e., the out-of-plane fixture variation would also contribute to the matching gap. The required force $\{F_t\}$ that is used to close the gap can be calculated with Eq. 2

$$\{F_t\} = [K_t]\{\delta_t\} \quad (2)$$

where, $[K_t]$ is a stiffness matrix related to closing the fixture gap.

Therefore, the assembly force $\{F_a\}$ would be the sum of $\{F_u\}$ and $\{F_t\}$.

$$\{F_a\} = \{F_u\} + \{F_t\} \quad (3)$$

When using a joining method such as welding, riveting, or gluing to join two components, deformation occurs at each joint point as the gap between components is closed. The assembly force $\{F_a\}$ is still being applied. After assembling the two components, some clamps and locators are removed. The joined components will spring back to release the stored strain energy during the assembly operation. It is reasonable to assume that the spring-back force $\{F_w\}$ is equal to the assembly force $\{F_a\}$ [24]. Therefore, by applying FEM, the assembly deformation $\{\delta_w\}$ can be calculated by adding the spring-back force and removing displacement boundaries both at the clamping points and releasing fixture locations to simulate the clamps/fixtures release.

From Eqs. 1–3, we can see that different joint positions will lead to different stiffness matrices $[K_u]$ and $[K_t]$. Therefore, different joint point selection schemes would have influence on the assembly deformation for given parts and fixture errors [25, 26]. Not only the joint positions, but the fixture positions also affect the assembly deformation if the assembly is still over-constrained after some fixtures are released [9–13]. The “3-2-1” fixture scheme (shown in Fig. 2a) is usually used in the assembly process. That means, during the assembly process, constraining 3 DOF (degree of freedom) in the first plane, 2 DOF in the second plane, and 1 DOF in the third plane. However, for non-rigid assemblies, Cai et al. [9] suggest that, due to the flexibility of sheet metal parts, it is better to use the “N-2-1” ($N$>3) fixture scheme (shown in Fig. 2b) that sets up $N$ (>3) DOF constraining in the first plane instead of the “3-2-1” scheme to assure the assembly quality because of the assembly deformation.

## 3 Modeling of the fixture and joint position optimization problem

In this study, the simultaneous optimization problem of fixture and joining positions for non-rigid sheet metal assembly can be described as the following: in the presence of part variation and fixture variation, as well as the constrains from assembly process and designed function requirements, find the best locations of fixtures and joining points so that the non-rigid sheet metal assembly can achieve the minimal assembly variation. The first task is hence to set up the objective function and corresponding constraints for this simultaneous optimization problem.

### 3.1 Objective function

For non-rigid sheet metal assembly, there is a set of points on components, called critical points (CPs), which are utilized to evaluate the assembly dimensional quality [11, 26–28]. The characteristics of the CPs usually significantly affect the target value of the controlled variation, performance of component function, and customer satisfaction. The optimization objective is to minimize the variations at

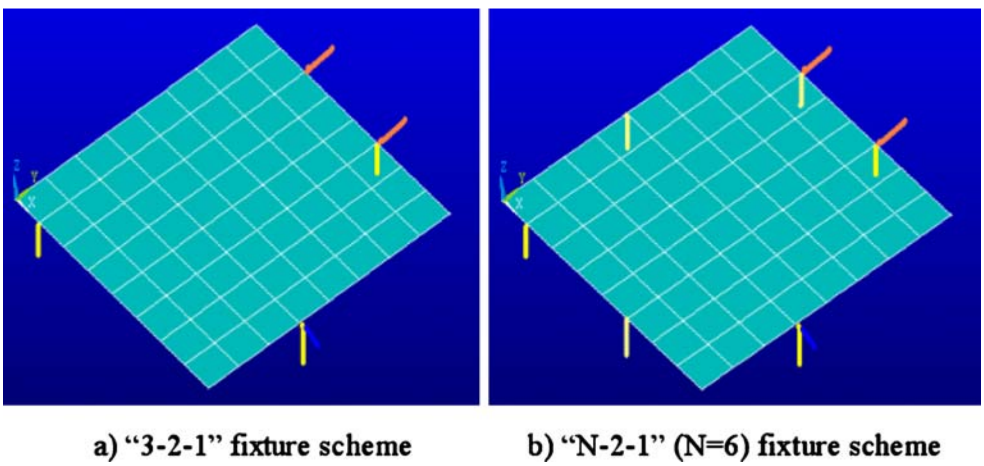


**Fig. 2** Two types of fixture schemes

these critical points to improve the assembly dimensional quality.

Suppose that there are $j$ fixtures and $k$ joints in an assembly. The position of fixtures and joints can be expressed as a vector $V=\{V_1, V_2, ..., V_j, V_{j+1}, ... V_{j+k}\}$, with $V_i = \left[V_x^i, V_y^i, V_z^i\right]$, $i = 1, 2, .., j, j+1, ..., j+k$.

According to the general computation formation of finite element analysis, we have

$$\mathrm{KU} = \mathrm{F_w} \tag{4}$$

where K is the total stiffness matrix of the sheet metal assembly, U is the vector of total node deformation, and $\mathrm{F_w}$ is the vector of spring-back force, which is equal to $\mathrm{F_a}$ in Eq. 3.

For a given fixture layout and joint positions $V$, the deformation at each node can be computed by applying the boundary conditions which correspond to both fixture and joint positions.

Assume there are $r$ number of CPs; the deformation of $r$ CPs in an assembly can be extracted as

$$\mathrm{U}_i = \left[\mathrm{U}_x^i\ \mathrm{U}_y^i\ \mathrm{U}_z^i\right], \quad i = 1, 2, \ldots, r \tag{5}$$

Confining the study to only the out-of-plane deformation ($z$ direction) and without losing its generality, the total absolute values of assembly deformation of the $r$ CPs can be obtained as

$$\Theta = \sum_{i=1}^{r} \mathrm{abs}\left(\mathrm{U}_z^i\right) \tag{6}$$

From Eq. 4, it is known that the assembly deformation is determined by the positions of fixtures and joints. Therefore, Eq. 6 can be rewritten as

$$\Theta = \sum_{i=1}^{r} \mathrm{abs}\left(\mathrm{U}_z^i\right) = f\left(V_1, V_2, \ldots, V_j, V_{j+1}, \ldots V_{j+k}\right) \tag{7}$$

Equation 7 is the objective function for the simultaneous optimization problem of non-rigid assembly fixture and joint positions.

## 3.2 Constraints

In the fixture design process, the positions of fixtures and joints are localized at some specific areas according to the design requirements [8, 9]. For example, in Fig. 3 the locators $P_1$ and $P_2$ will be inside a square, and the joint $S$ will be on a line segment.

Suppose the positions of $j$ fixtures and $k$ joints will be selected in areas $\Omega_1$ and $\Omega_2$, respectively, we have

$$V_i(x, y) \in \Omega_1 \quad i = 1, 2, \ldots, j \tag{8}$$

and

$$V_i(x, y) \in \Omega_2 \quad i = j+1, j+2, \ldots, j+k \tag{9}$$

Meanwhile, staying in consistence with the specifications of product function and assembly technique, any two of fixture/joint positions $V_i$, $V_n$ should not be very close, and should be kept a distance $L_{\mathrm{in}}$.

$$\|V_i - V_n\| \geq L_{\mathrm{in}} \quad i, n = 1, 2, \ldots, j, j+1, \ldots, j+k \tag{10}$$

## 3.3 Mathematical optimization model

From the above discussion, the mathematical formation for simultaneously optimizing the fixture layout and joint position can be summarized as follows

$$\begin{aligned} &\text{Minimize} \quad \Theta = \sum_{i=1}^{r} \mathrm{abs}\left(\mathrm{U}_z^i\right) = f\left(V_1, V_2, \ldots, V_j, V_{j+1}, \ldots V_{j+k}\right) \\ &\text{Subject to}: V_i(x, y) \in \Omega_1 \quad i = 1, 2, \ldots, j \\ &\qquad V_i(x, y) \in \Omega_2 \quad i = j+1, j+2, \ldots, j+k \\ &\qquad \|V_i - V_n\| \geq L_{\mathrm{in}} \ i, n = 1, 2, \ldots, j, j+1, \ldots, j+k \end{aligned}$$

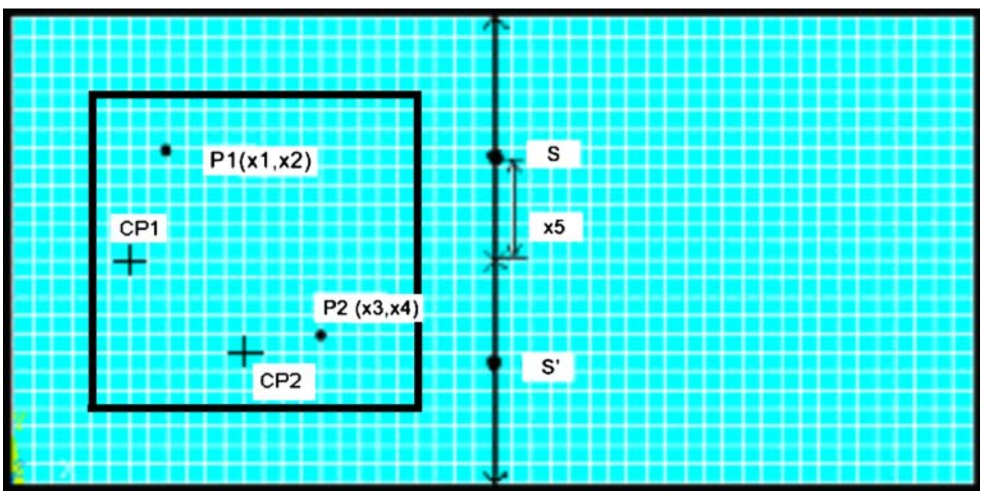


**Fig. 3** An assembly of two sheet metal parts

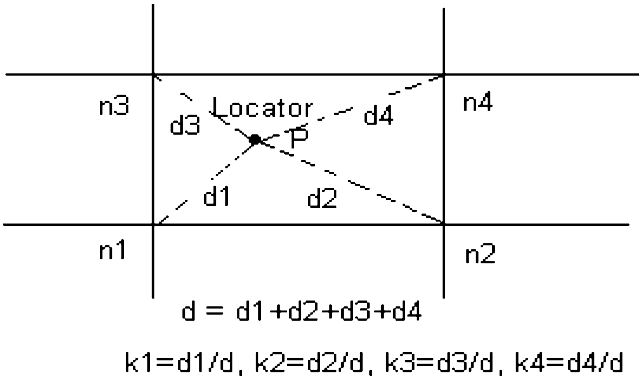


**Fig. 4** Parameters defined for MPC [9]

From the mathematical model, it is seen that this optimization problem has the following special characteristics:

1. The non-linear objective function is like a "black-box" function and its properties are unknown.
2. The conventional gradient-based method is not suitable for this kind of problem since the gradient calculated from FEA is usually noisy and not reliable [29].
3. Application of genetic algorithms (GA) would be too computation intensive since GA needs a large number of function evaluations, and in this case, a FEA is required for each objective function evaluation.

In this study, a new effective global optimization algorithm called the mode-pursuing sampling method (MPS) [18] will be applied to solve this problem. Details of MPS will be elaborated in Section 4.

During the optimization process, in order to ensure each fixture and joint position is applied on the finite element node, the multi-point constraint (MPC) method is employed to avoid re-meshing the FEA model [9–11].

The basic theory of the MPC method is illustrated in Fig. 4. Suppose one locator P is in a FEA element, which

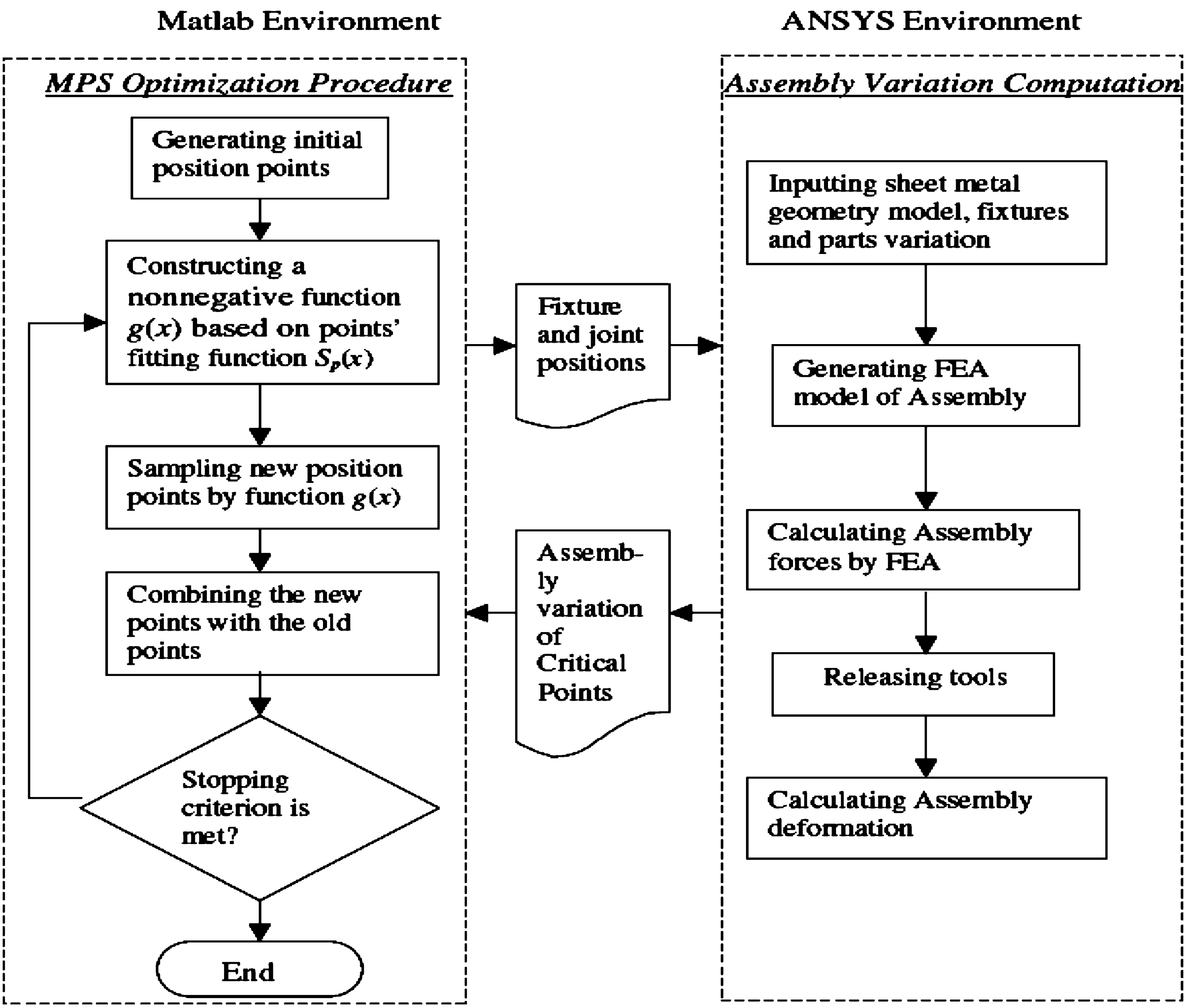


**Fig. 5** The workflow of optimization by integrating Matlab and ANSYS

has four nodes n1, n2, n3, and n4. The degree of freedom (DOF) in the Z direction for these four nodes is w1, w2, w3, and w4, respectively. Then, the boundary condition applied at locator P can be expressed as a linear function of w1, w2, w3, w4

$$\begin{aligned} \mathrm{wp} &= \mathrm{k1} * \mathrm{w1} + \mathrm{k2} * \mathrm{w2} + \mathrm{k3} * \mathrm{w3} + \mathrm{k4} * \mathrm{w4} \\ &= \mathrm{constant} \end{aligned} \tag{11}$$

where k1, k2, k3, k4 are coefficients, and $\mathrm{k1} + \mathrm{k2} + \mathrm{k3} + \mathrm{k4} = 1$. In this study, they are obtained by normalizing the computed distances between the locator P and nodes n1, n2, n3, and n4.

## 4 Mode-pursuing sampling (MPS) method

The mode-pursuing sampling (MPS) method, which was developed by Wang et al. [18], is a general global optimization algorithm on any expensive "black-box" objective function. MPS is an algorithm based on sampling technology and it searches the design space only with the objective function value and does not need any calculation of gradients. In terms of the total number of expensive function evaluations and the amount of computation, MPS is proven to be more efficient than genetic algorithms when design variables are a few.

Basically, MPS is tracked from the random-discretization based sampling method of Fu and Wang [19], which is a general-purpose algorithm to draw a random sample from any given multivariate probability distribution. To differentiate points evaluated by FEA from points calculated by an approximation model, we refer to the former "expensive points" and the latter "cheap points." Suppose a function $f(x)$ is to be minimized in a compact set $S(f)=[a, b]^n$, and $f(x)\geq 0$. The optimization procedure of MPS method can be described with the following steps:

1. Initialization

Firstly, based on function $f(x)$, $m$ uniformly distributed expensive points $x^{(i)}$, $i=1, m$ are generated on $S(f)=[a, b]^n$.

2. Construction of the sampling guidance function

An approximation function $S_p(x)$ can be obtained based on these $m$ points, such as a linear spline function $S_p(x)$

$$\begin{array}{l} S_p(x) = \sum_i \alpha_i \left\| x - x^{(i)} \right\| \\ \text{Subject to } S_p\left(x^{(i)}\right) = f\left(x^{(i)}\right),\ i = 1,\, m \end{array} \tag{12}$$

Then a nonnegative function $g(x)$ can be constructed by defining $g(x) = \mathrm{C}_0 - S_{\mathrm{p}}(x) \geq 0$ on $S(f) = [a, b]^n$, where $\mathrm{C}_0$ is a constant.

3. Sampling points by Fu and Wang's method [19]

By applying the sampling algorithms of Fu and Wang, another $m$ random sample points $y^{(i)}$, $i=1, m$ are drawn from $S(f)=[a, b]^n$ according to $g(x)$. These sample points tend to locate around the current maximum of $g(x)$, i.e., the minimum of $S_p(x)$. Then these $m$ points are evaluated by FEA and thus become expensive points.

4. Evaluation and stopping search

Combine the new $m$ points $y^{(i)}$ $(i=1, m)$ with the old points $x^{(i)}$ $(i=1, m)$, $x=[x\ y]$, and repeat steps 2–3 until a certain stopping criterion is met.

The above MPS optimization algorithm is proven to be very effective and applicable in global optimum through testing with well-known benchmark problems [18]. In this study, MPS will be employed to simultaneously optimize the fixture layout and joint positions for non-rigid sheet metal assemblies.

## 5 Implementation of simultaneous optimization for fixture and joint positions

Based on the mathematical optimization model and MPS algorithm discussed above, the overall simultaneous optimization for the fixture and joint positions of non-rigid sheet metal assemblies is implemented by integrating the finite element analysis software ANSYS into the Matlab environment. The workflow diagram is shown in Fig. 5.

The basic computation procedure of the sheet metal assembly deformation is constructed as a batch file by using ANSYS, while the mainstream of MPS optimization algorithm is coded by applying Matlab. Because the basic FEA model for the assembly deformation computation is parametric, the Matlab-based MPS optimizer is able to update the basic FEA model by changing its boundary conditions according to the given positions of fixtures and joints until the optimum is obtained.

The proposed optimization flowchart shown in Fig. 5 provides a seamless process to globally search the optimal positions of fixtures and joints for non-rigid sheet metal assemblies so that the assembly deformations at critical

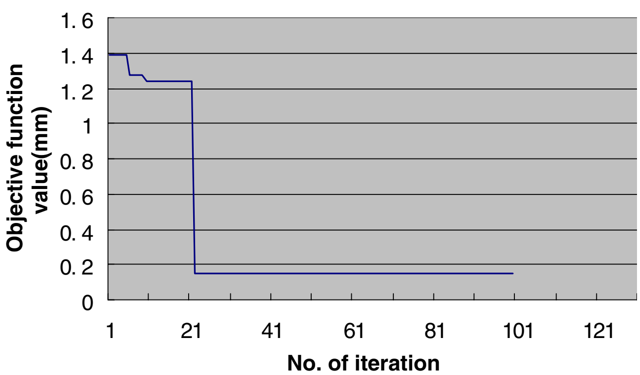


**Fig. 6** The iteration and convergence process of MPS

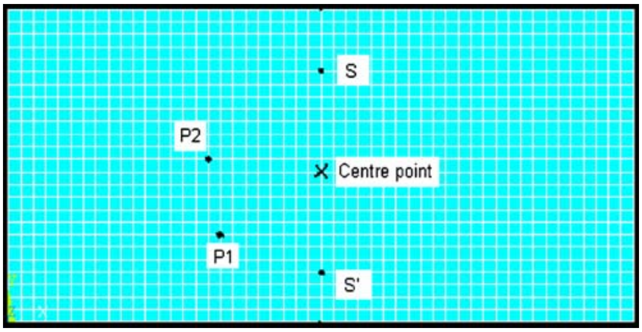


**Fig. 7** The optimal positions for fixtures P1, P2 and joints S, S'

points are minimal. The next section will show an example to which the proposed method is applied.

## 6 An application example

The assembly by lap joints of two identical flat sheet metal components shown in Fig. 3 is employed to illustrate the proposed method. Assuming that these two components are manufactured under the same conditions, their fabrication variations are expected to be the same. The size of each flat sheet metal parts is 100×100×1 mm, with Young's modulus $E = 2.62e + 4\,\mathrm{N/mm}^2$, and Poison's ratio $\nu$= 0.3. The finite element computation model of the assembly is created in ANSYS. The element type is SHELL63. The number of elements and the number of nodes are 1,250 and 1,352, respectively.

As shown in Fig. 3, the fixture scheme N-2-1 ($N$>3) [9] is applied and thus the assembly is over-constrained. Two locators at P1(x1, x2) and P2(x3, x4) that are assumed to have fixture errors, as well as one joint S(x5) (as well as its symmetrical point S' with respect to the part's center line) need to be optimized so that the assembly deformation is satisfactory. It is assumed that, according to the assembly requirements, locators P1 and P2 should be chosen in a rectangular area, and joints S and S' should be on a line segment. Locator P1 should not be too close to locator P2, and joint S should also be a distance from other joints. After assembling, locating fixtures P1 and P2 are released. The final assembly is still over-constrained.

The initial conditions applied are: the fixture variation at locators P1 and P2 is 1 mm; the part variation at joint points that are indicated by "**x**" in Fig. 3 is 1 mm. Thus, the multi-point constraint (MPC) is applied respectively on P1 and P2 to avoid re-meshing the FEA model as the following is specified.

$$k1 * w1 + k2 * w2 + k3 * w3 + k4 * w4 = 1\,\mathrm{mm} \qquad (13)$$

The assembly deformations at critical points CP1 and CP2 are shown in Fig. 3, i.e., the absolute values of $U_z^1$ and $U_z^2$ are determined by positions P1 (x1, x2), P2 (x3, x4), and S (x5). $U_z^1$ and $U_z^2$ are extracted to form the optimization objective value, and the design variables are x1, x2, x3, x4, and x5, which are gathered into a vector $X$= [x1, x2, x3, x4, x5].

Suppose that we set up a constraint on the locators P1 and P2: ||P1−P2||≥10 mm. The search range is

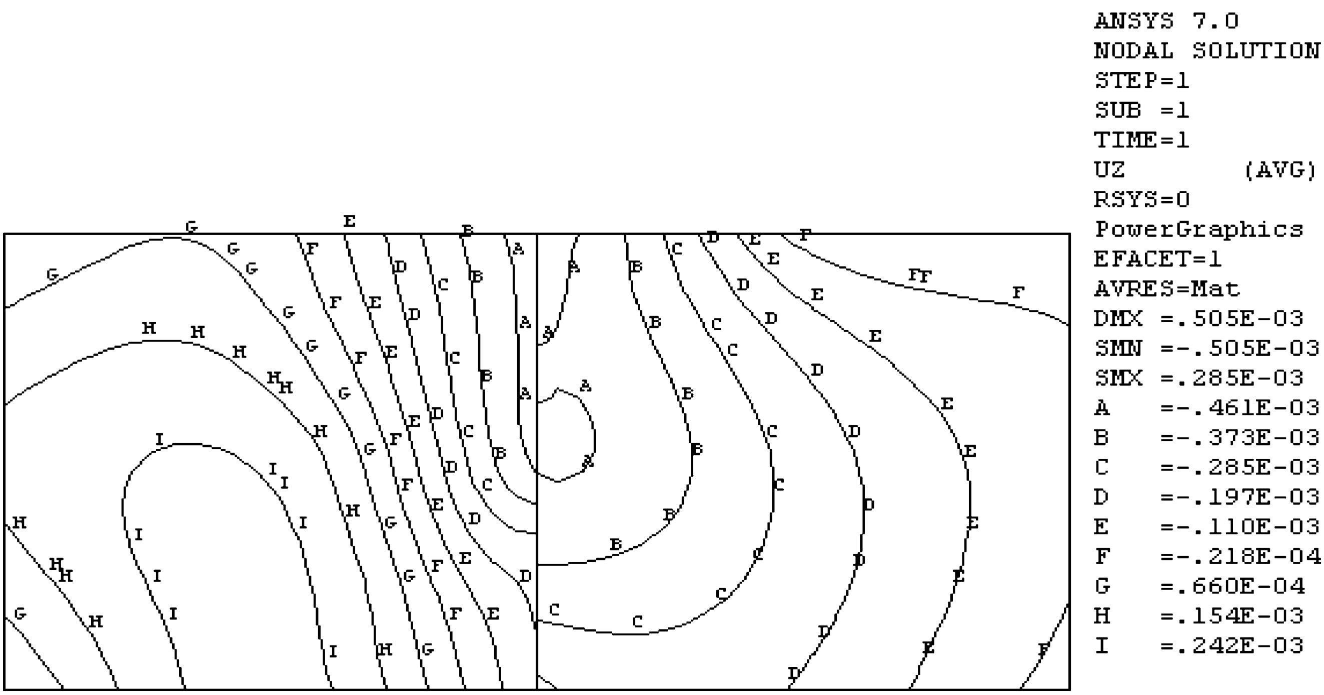


**Fig. 8** The assembly deformation distribution under optimal fixture and joint positions

x1, x2, x3, x4 ∈ [20, 80] and x5 ∈ [10, 40]. Therefore, the mathematical optimization model for this specific example can be written as the following,

$$\text{Minimize } \Theta(X) = \text{abs}\left(\text{U}_z^1\right) + \text{abs}\left(\text{U}_z^2\right) \tag{14}$$

$$\text{Subject to } (x1 - x3)^2 + (x2 - x4)^2 \geq 100 \tag{15}$$

$$20 \leq x1, x2, x3, x4 \leq 80 \tag{16}$$

$$10 \leq x5 \leq 40 \tag{17}$$

By applying the proposed algorithm that integrates the mode-pursuing sampling method (MPS) and the FEM-based assembly simulation approach, the optimal fixture and joint positions are obtained as follows

$$\text{P1} = (66.44, 26.38),\ \text{P2} = (63.63, 52.24),\ \text{S} = 16.0$$

The minimal objective function value is

$$\Theta_{\min} = 0.148\,\text{mm}$$

The convergence process is shown in Fig. 6. It took only 22 iterations and 100 function evaluations (of the finite element analysis) for the MPS algorithm to obtain the optimal results. The optimal positions for fixtures P1, P2 and joint S as well as its symmetrical S' are shown in Fig. 7, and the assembly deformation distribution under such optimal fixture and joint positions is shown in Fig. 8.

## 7 Conclusion

It is known that not only the fixture layout but also the joint positions have impact on the non-rigid sheet metal assembly deformation. In this paper, a simultaneous optimization method for fixture and joint positions is proposed by combining the global optimization algorithm MPS and the FEM-based assembly deformation analysis. The overall optimization procedure is implemented by integrating ANSYS into the Matlab environment. The contributions of the proposed methodology can be summarized as follows.

1. To the authors' best knowledge, it is the first time that the fixture and joint positions are optimized simultaneously for non-rigid assembly.
2. The global optimal solution is obtained by employing a new powerful global search procedure based on the mode-pursuing sampling method (MPS), which can significantly reduce the amount of computation while obtaining a global optimum, in contrast with other algorithms, such as gradient-based non-linear programming and genetic algorithms.
3. ANSYS and Matlab are integrated to implement the proposed FEM-based non-rigid sheet metal assembly modeling and optimization algorithms. An assembly of two identical flat sheet metal components by lap joints is employed to validate the proposed methods and technology.

Since the current MPS method cannot deal with integer variables, the future work is to develop such MPS algorithms that can solve the optimization problems with mixed discrete and continuous variables, which can be applied in more complex optimization problems for non-rigid sheet metal assembly.

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