Odds ratio, Odds ratio test for independence, chi-squared statistic.
Announcements:

Assignment 5 is live on webpage. Due Wed Aug 1 at 4:30pm. (9 days, 1 hour, 58.5 minutes...)

Final exam is Aug 9.

Review session Aug 7? 8? What works for you?

Deadline for suggesting questions for final: July 31 Monday at 11:59pm. (I need to submit it to printers soon after that)
Last time, we looked at odds, a modification of probability.

The odds of something happening is how many times more likely the event happen is to something not happening.

Example: If the probability of something 0.6,
Then the odds of the effect are $0.6 / 0.4 = 3/2$ or 1.5

That means the event is 1.5 times as likely as it not happening.
The formula for getting odds from probability $P$ is:

$$Odds = \frac{P}{1-P}$$

Other equivalent interpretations:

$$Odds = \frac{Pr(\text{Event})}{Pr(\text{Not Event})}$$

$$Odds = \frac{Pr(\text{Success})}{Pr(\text{Failure})}$$
Example 1: If something is impossible, it has probability **zero**.

An impossible event has odds **zero** as well.

\[
\text{Odds} = \frac{P}{1-P} = \frac{0}{1-0} = 0
\]

So an impossible event happening is zero times as likely as it not happening.
Example 2: If an event has 0.5 probability, it has odds 1.

\[
\text{Odds} = \frac{0.5}{1-0.5} = \frac{0.5}{0.5} = 1
\]

This means if an event has probability 0.5, it’s just as likely that the event will happen as it not happening.

(Flipping a coin as heads has probability 0.5. This event is just as likely as flipping and getting a tails.)
Example 3: If something is certain to happen, it has probability \(1\). This event has odds infinity. \((1 / 0)^*\)

\[
\text{Odds} = \frac{1}{1 - 1} = \frac{1}{0^+}
\]

A certain event is infinitely more likely to occur than it not happening.

These examples also provide the limits of odds. Odds are always between 0 and infinity, and never negative.
for interest: You can get the probability from odds by using

\[ P = \frac{\text{Odds}}{1 + \text{Odds}} \]

... it’s handy to check your work.

for interest *: 1/0 should actually be 1 / 0^+ to make it infinity instead of undefined.

0^+ means “zero, but always from the positive side”, which we use because 1 – P is always positive because P is never above 1.

This is covered in calculus, so it’s here only for completion.
We estimate Odds directly from a sample.

Example 1: Of those from a recent driver training course. 16 passed the licence exam on their first attempt and 7 failed.

The odds of passing someone passing if they took driver training are estimated to be:

\[
\text{# that passed} / \text{# that failed} = \frac{16}{7} = 2.29
\]

2.29 times as many passed as failed, so we estimate that passing is 2.29 times as likely.
Example 2: We also a sample of the exam results from those that didn’t take the driver training.

8 of them passed their first time, and

11 of them failed.

The odds of passing without driver training are:

\[
\frac{8}{11}, \text{ or } 0.73
\]

0.73 as many passed as failed, so passing is estimated to be

0.73 times as likely.
Sometimes it’s not odds, sometimes it’s just odd.
An odds ratio (OR) is a comparison between two odds. The odds ratio, OR, shows which odds are larger and by how much. Therefore, they also show which event is more likely, and by how much.

\[ OR = \frac{\text{Odds}_1}{\text{Odds}_2} \]
It allows us to see if an event is more likely under certain situations if we know the odds under each situation.

Example: The odds of passing the driving test with a course are 2.29. (16 / 7)
The odds of passing the test without a course are 0.73. (8 / 11)
The odds ratio of passing the test over taking the course are

\[ OR = \frac{16/7}{8/11} = \frac{2.29}{0.73} = 3.142 \]
This means that the odds of passing the test are \(3.142\) times as large if you take a driver training course first.

This could be flipped to say that the odds of passing the test are \(0.318\) times as large if don’t take driving training.
Quick formula for odds ratio

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>D</td>
</tr>
</tbody>
</table>

Odds Ratio = \( \frac{AD}{BC} \)

Multiply by these.

Divide by these.
Odds ratio \( = \frac{(11 \times 16)}{(7 \times 8)} \)

\[ = \frac{176}{56} = 3.142 \]

The odds of passing the test are 3.142 as large with a driver’s course.

This way we don’t have to compute the odds beforehand.
If OR = 1, then an event is equally likely under both situations. In other words, the event and the situation are independent.

If OR > 1, the event with the first odds are more likely.
If OR < 1, the event with the second odds are more likely.

The most common null hypothesis tested using a 2x2 cross tab is that the row category and the column category are independent.

So really, we’re testing if the OR is significantly far from 1.
The odds ratio is 3.142. It’s above 1, but is it significantly so?

The 95% confidence interval of the odds ratio is:
(0.881 to 11.215), which includes 1.

So OR = 1 is in the interval, we **fail to reject** the hypothesis that these pass the test and taking the class are independent.
OR from sample = 3.142, with a 95% CI of (0.881 to 11.215)

The confidence interval is very large in this case because the groups are small, especially the groups of size 7 and 8. Small groups leave a lot of uncertainty about the true odds ratio.

**Important note**: For the confidence level of the odds ratio, what matters is whether the confidence interval includes 1, not that it includes 0.

An odds ratio of zero would only happen if the first event were estimated to be impossible.
The formula for getting the confidence interval of the odds ratio involves taking the logarithm of the odds ratio.

The formula isn’t going to be used in this class, what you should know about it, and any other test involving cross tabs, is that every cell* should be large for the confidence interval to be small.

* A cell is a combination of a row and a column. The number of people that took the course (column) but failed the test (row) is the size of a cell.
SPSS: Computing odds ratio and its confidence interval.

First, recall how to build a crosstab.

**Analyze ➔ Descriptive Stats ➔ Crosstabs**
From there, put one variable of interest in the **row** category, and one in the **column** category. It doesn’t matter which goes in where.

That much gets you the crosstab. For the odds ratio, click on the **statistics** button in the upper right.
To get the odds ratio, check off *risk.*

It’s under “risk” because odds ratio is related to relative risk.

Click “Continue”, then “Okay”.

![Image showing the selection of risk in a statistics software interface.](image-url)
This will produce both the cross tab which we just saw, and the risk estimate table, which includes the Odds Ratio.
Unlike other confidence intervals that we’ve seen where the estimated value is in the middle of the confidence interval, the odds ratio appears closer to the lower side.

CI from odds ratio isn’t just a + or - like the other ones, the odds ratio is being X and / by a some amount to get these values.
Eyelash Crested Geckos. Good pets, but their bodies have the consistency of uncooked chicken strips.
Odds ratios are only useful for testing for independence in a 2x2 contingency square.

You can also make the odds ratio a one-sided test at half the alpha of the two-sided confidence interval.

If both ends of the 95% confidence interval in the driver testing problem were positive, we could have concluded at the 0.025 level that getting driving training didn’t change the odds of passing the test, it improved them.
The $\chi^2$, or chi-squared (pronounced ky squared) test for independence is useful for 2x3, 4x2, 5x5 cross tabs, or cross tabs of any number of row or columns.

However, the only thing it checks for is independence.

So you can’t tell if the odds in a particular situation are more than in another situation, only that there is a difference.

To understand the $\chi^2$ statistic, we first need to know about expected values.
Expected frequencies are the number of responses in each cell you would expect under the null assumption of independence.

To illustrate, say we found 40 people that like bearded dragons and 40 people that didn’t, and tested to see if they were clinically insane.

Our observed frequencies might look like:

<table>
<thead>
<tr>
<th>OBSERVED</th>
<th>Likes Dragons</th>
<th>Dislikes Dragons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sane</td>
<td>37</td>
<td>33</td>
</tr>
<tr>
<td>Insane</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>OBSERVED</td>
<td>Likes Dragons</td>
<td>Dislikes Dragons</td>
</tr>
<tr>
<td>------------</td>
<td>---------------</td>
<td>------------------</td>
</tr>
<tr>
<td>Sane</td>
<td>37</td>
<td>33</td>
</tr>
<tr>
<td>Insane</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>Total</td>
<td>40</td>
<td>40</td>
</tr>
</tbody>
</table>

From the totals, 70 of 80, or 7/8 of the people we tested were sane.

If liking dragons and being sane were independent, we would expect that 7/8 of the people in each group would be sane.

In other words, we’d expect the same proportion of people in each group to be sane or insane regard of dragon liking status.
Each like/dislike group has 40 people, and $7/8$ of 40 is 35, so the expected frequencies would be:

<table>
<thead>
<tr>
<th>EXPECTED</th>
<th>Likes Dragons</th>
<th>Dislikes Dragons</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sane</td>
<td>35</td>
<td>35</td>
<td>70</td>
</tr>
<tr>
<td>Insane</td>
<td>5</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>Total</td>
<td>40</td>
<td>40</td>
<td>80</td>
</tr>
</tbody>
</table>

Note that totals are the same as with the observed frequencies, but now the cells are evened out.

The OR would be: $(35 \times 5) / (35 \times 5) = 1$, showing independence.
The expected frequencies are calculated from the totals.

\[
\text{Expected Frequency} = \frac{(\text{Row Total}) \times (\text{Col. Total})}{(\text{Grand Total})}
\]

So the expected number of sane dragon likers would be...

\[
\text{Expected Frequency} = \frac{40 \times 70}{80} = 35
\]

...because there are 40 dragon likers, 70 sane people, and 80 people in total.
All the others are calculated likewise.

<table>
<thead>
<tr>
<th></th>
<th>Likes Dragons</th>
<th></th>
<th>Dislikes Dragons</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Sane</td>
<td>40 x 70</td>
<td></td>
<td>40 x 70</td>
<td></td>
</tr>
<tr>
<td></td>
<td>80</td>
<td>35</td>
<td>80</td>
<td>35</td>
</tr>
<tr>
<td>Insane</td>
<td>40 x 10</td>
<td></td>
<td>40 x 10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>80</td>
<td>5</td>
<td>80</td>
<td>5</td>
</tr>
<tr>
<td>Total</td>
<td>40</td>
<td>40</td>
<td>40</td>
<td>80</td>
</tr>
</tbody>
</table>

(See textbook page 319 for another example)

We can find how strong the evidence against the null hypothesis is by how large the differences between the observed frequencies and the frequencies expected by the null
<table>
<thead>
<tr>
<th>DIFFERENCES OBS - EXP</th>
<th>Likes Dragons</th>
<th>Dislikes Dragons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sane</td>
<td>37 - 35</td>
<td>33 - 35</td>
</tr>
<tr>
<td>Insane</td>
<td>5 - 7</td>
<td>5 - 3</td>
</tr>
</tbody>
</table>

Or, calculating...

<table>
<thead>
<tr>
<th>DIFFERENCES OBS - EXP</th>
<th>Likes Dragons</th>
<th>Dislikes Dragons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sane</td>
<td>2</td>
<td>-2</td>
</tr>
<tr>
<td>Insane</td>
<td>-2</td>
<td>2</td>
</tr>
</tbody>
</table>
If the differences in sanity levels was larger...

<table>
<thead>
<tr>
<th>OBSERVED</th>
<th>Likes Dragons</th>
<th>Dislikes Dragons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sane</td>
<td>37</td>
<td>23</td>
</tr>
<tr>
<td>Insane</td>
<td>3</td>
<td>17</td>
</tr>
</tbody>
</table>

The differences between the observed and expected values would become larger as well.

<table>
<thead>
<tr>
<th>DIFFERENCES OBS - EXP</th>
<th>Likes Dragons</th>
<th>Dislikes Dragons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sane</td>
<td>7</td>
<td>-7</td>
</tr>
<tr>
<td>Insane</td>
<td>-7</td>
<td>7</td>
</tr>
</tbody>
</table>
The $\chi^2$ statistic gets **larger** when the differences between the expected and the observed get larger.

\[
\chi^2 = \sum \frac{(\text{Obs.} - \text{Exp.})^2}{\text{Exp.}}
\]

\[\sum\] means “add up all the..” whatever is to the right of it.
With the small difference in sanity level, $\chi^2 = 1.83$

<table>
<thead>
<tr>
<th>DIFFERENCES</th>
<th>Likes Dragons</th>
<th>Dislikes Dragons</th>
</tr>
</thead>
<tbody>
<tr>
<td>OBS - EXP</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sane</td>
<td>2</td>
<td>-2</td>
</tr>
<tr>
<td>Insane</td>
<td>-2</td>
<td>2</td>
</tr>
</tbody>
</table>

$$\chi^2 = \frac{(2)^2}{35} + \frac{(-2)^2}{35} + \frac{(2)^2}{5} + \frac{(-2)^2}{5} = 1.83$$

With the large difference in sanity level, $\chi^2 = 22.40$
The $\chi^2$ statistic, like the t-statistic, is compared to a table of critical values in the back of your book, page 523 (or in the table that was sent out with assignment 5).

The degrees of freedom for a cross tab are

$$(\text{rows} - 1) \times (\text{columns} - 1)$$

So in a 2x2 cross tab as we have, we’d only have

$$(2 - 1) \times (2 - 1) = 1 \text{ degree of freedom.}$$
We then compare our $\chi^2$ value, with the critical values in the table and either reject or fail to reject the null hypothesis that sanity and liking dragons are independent.

Let alpha = 0.05 by default.

Critical $\chi^2$ at 0.05 level and 1 df = 3.841

Critical $\chi^2$ at 0.01 level and 1 df = 6.635

So for the small differences (37 and 33 sane people per group),

We would fail to reject because $\chi^2 = 1.83 < 3.841$
Next time: Getting expected values and chi-squared stats from SPSS. Lots of examples.